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ABSTRACT

NEW TRADE MODELS, SAME OLD EMISSIONS?

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This paper investigates the elusive role of productivity heterogeneity in new trade models in the trade and environment nexus. We contrast the Eaton-Kortum and the Melitz models with firm heterogeneity to the Armington and Krugman models without heterogeneity. We show that if firms have a constant emission share in terms of sales — as they do in a wide range of trade and environment models — the three models' emission predictions exactly coincide. Conversely, if firms have a constant emission intensity per quantity — a prominent alternative in the literature — the emission equivalence between the three models breaks. We provide a generalization that nests both constant emission shares in sales and constant quantity emission intensities as special cases. We calibrate the models to global production and trade data and use German firm-level data to estimate the key elasticity of how emission intensity changes with productivity. Our multi-industry quantification demonstrates that the role of firm heterogeneity depends both on the model and the estimated parameters. Moving from the Armington model to the EK model increases the emissions effect on trade, while moving from the Krugman model to the Melitz model decreases the emission effects on trade.

Keywords: International trade; carbon emissions; firm heterogeneity

JEL classification: F11; F12; F18

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1 Introduction

Are exporting firms cleaner than domestic producers? How do different firms react to episodes of trade liberalization? Do firms' international sourcing decisions affect their emissions intensity? The focus on firm heterogeneity by "New New Trade Theory"' and the availability of firm-level data has strongly influenced the literature on the relationship between international trade and environmental outcomes. Here, the focus has shifted toward new micro-level questions. However, for the "pure" trade models, the implications of micro-level insights for macro-level outcomes deserve special attention: Do new models with firm heterogeneity deliver different predictions for *aggregate* emissions? In their seminal paper, Arkolakis, Costinot, and Rodríguez-Clare (2012) (henceforth ACR) show that a broad class of trade models with very different micro-foundations — including Armington (1969)-type models, Krugman (1980), Eaton and Kortum (2002), and Melitz (2003) (henceforth Armington, Krugman, EK, and Melitz) — yield the same expression for gains from trade. Hence, even though firm heterogeneity offers a new source of gains from trade — the reallocation of production towards more productive firms — this new channel does not translate into larger overall gains. The ACR insight begs the question of whether heterogeneous firm-level responses to trade liberalization could potentially change overall environmental outcomes or, in a similar vein, are neutralized in aggregate consideration.

The first key contribution of our study is an ACR-type equivalence result for single-industry models: if the emissions of firms are proportional to the value of their production and if this proportion is common across firms, the aggregate emission predictions of the Armington-, Krugman-, EK-, and Melitz-type models are identical. The equivalence condition applies to production emissions in the majority of new quantitative trade and environment models, including models based on Armington (e.g. Larch and Wanner, 2017, 2024; Klotz and Sharma, 2023), Krugman (Farrokhi and Lashkaripour, 2024), EK (e.g. Egger and Nigai, 2015; Duan et al., 2021; Caron and Fally, 2022; Mahlkow and Wanner, 2023), and Melitz (Shapiro and Walker, 2018; Sogalla, 2023).[1] The intuition behind this equivalence is straightforward: the EK- and Melitz-type models do take into account a

[1]All examples given here are multi-industry models. We will return to this distinction below but have already prefigured that a broad equivalence result will continue to hold.

crucial insight from the empirical trade and environment literature (see e.g. Cole, Elliott, Okubo, and Zhou (2013), Richter and Schiersch (2017), and Forslid, Okubo, and Ulltveit-Moe (2018), as well as Cherniwchan, Copeland, and Taylor (2017) and Copeland, Shapiro, and Taylor (2022) for recent surveys), which is that exporters have lower emission intensities. Trade liberalization induces a reallocation of production towards these cleaner firms, leading to an environmentally beneficial technique effect. However, as these models also capture the key feature that exporters are more productive (see, e.g. Bernard, Jensen, Redding, and Schott, 2007), they produce *more*, resulting in an emission-increasing scale effect. In this class of models, these two effects exactly offset each other when the relative emissions price is fixed.

An alternative specification to model emissions in trade models is to link them to produced quantities with fixed proportions. A prominent example is Shapiro (2016). Our second main contribution is showing that the Armington-, Krugman-, EK-, and Melitz-type models are *not* all equivalent in this case. Instead, models with heterogeneous producers may lead to worse emissions outcomes in the case of trade liberalization. The intuition can be linked to the previously mentioned scale and reallocation effects: Productivity differences are still considered, and the resulting scale effect drives up emissions, while emission intensity differences are absent in this case. Hence, there is no counteracting reallocation effect. Trade liberalization may lead to a shift in production to high-productivity firms and, therefore, to larger quantities produced and, in turn, higher emissions. In the specific case of movement from an economy in autarky to equilibrium with trade, we show that global emissions increase in the models with firm heterogeneity.

Third, we consider a generalized modeling strategy for emissions in trade and environment models along the lines of Kreickemeier and Richter (2014) (henceforth KR). In this case, emissions are linked to the quantities produced, but more productive producers are allowed to be cleaner. The productivity-emission intensity relationship is disciplined by an additional parameter: the productivity elasticity of emissions. We derive expressions for aggregate emissions in Armington, Krugman, EK, and Melitz models with KR-type emission intensities. Similar to the quantity-based approach, the different models do *not* yield the same emission outcomes. The EK and Melitz models imply a scale effect that drives emissions up and a counteracting technique effect due to reallocations between producers.

The magnitude of the latter depends on the KR elasticity parameter. If the emission intensity bonus of highly productive firms becomes sufficiently strong, firm heterogeneity may lead to *lower* aggregate emissions in response to trade liberalization. In this setting, we can show that when emissions are elastic [inelastic], moving from autarky to a trade equilibrium results in lower [higher] global emissions. Note that the KR approach nests the value- and quantity-based approaches as special cases. If the KR elasticity equals one, emission outcomes correspond to the case in which emissions are proportional to values, and the corresponding equivalence results hold. If the KR elasticity equals zero, the emission outcomes equal the ones from the case in which emissions are proportional to quantities.

Conditional on equal trade outcomes, our (non-)equivalence results for the three emission modeling strategies carry over to multi-industry models. In multi-industry environments, reallocations across sectors induce an additional composition effect on emissions: Some countries specialize in emission-intensive industries and face higher emissions, while others focus on clean industries and hence reduce their emissions. Central to pollution haven and carbon leakage concerns, this mechanism is crucially important and should be a part of full-fledged quantitative trade and environment models. However, our distinction between different types of trade models is decisive for scale and technique effects *within* sectors.

Due to economies of scale, trade outcomes for multi-sector Armington and EK models differ from those of the multi-sector Krugman and Melitz models. Hence, in the value emission case, The Armington and EK models again imply the same emission responses to trade, and these two models have different emissions responses compared to the Melitz and Krugman models only because the trade responses are different. In the flexible emission specification, emission effects again vary across models, with the emission effect of firm heterogeneity depending on the sectoral KR elasticities.

In our quantitative exercise, we calibrate the flexible emissions version of the multi-industry Armington, Krugman, EK, and Melitz models. The key parameter is the elasticity with which the emissions intensity reacts to a firm's productivity. We use a German firm-level data estimation procedure to identify this parameter. Otherwise, the Armington, Krugman, and EK models only rely on readily available trade and production data, and the Melitz model comes with only one additional data requirement, namely the country-industry level share of exporting firms.

We simulate a global reduction in trade costs to quantify the effect of trade on global emissions. In the Armington model (and equivalently, in an EK model with unit KR elasticities in all sectors), global emissions increase only mildly in response to sectoral production shifts across countries. Using the estimated elasticities in the EK setup instead, the global emissions increase in response to trade liberalization triples. The increase indicates that the emission-increasing scale effect induced by firm heterogeneity is stronger than the emission-decreasing technique effect. In other words, in the perfect competition models, once we incorporate firm heterogeneity in a way that can influence how trade affects emissions, our quantification suggests that firm heterogeneity turns out to be bad news. The opposite is true in the monopolistically competitive frameworks. The Krugman model (and hence, equivalently, the Melitz model with unit KR elasticities) yields a mild global emission increase similar to that of the Armington model. If, however, we use the estimated elasticities in the multi-sector Melitz model, global emissions *decrease* in response to the trade liberalization shock, implying that in this case, reallocation across firms induces a stronger technique than scale effect. These results indicate the importance of the model structure in terms of both competitive environment (and hence scale economies) and firm heterogeneity, as well as of the specific parametrization.

Our work is related to several strands of literature. First, it contributes to the literature on the interplay between international trade and the environment (recently surveyed e.g. by Copeland et al., 2022).

Second, it relates to the new quantitative trade literature (see Costinot and Rodriguez-Clare, 2014; Caliendo and Parro, 2022, for overviews) and, more specifically, environmental extensions of quantitative trade models (see e.g. Egger and Nigai, 2015; Larch and Wanner, 2017; Shapiro and Walker, 2018). We clarify how modeling choices in the trade component affect the environmental effects they predict.

Third, we contribute to a strand of literature that studies the role of firm heterogeneity in the trade and environment context, which has been very active in terms of both theory (e.g. Kreickemeier and Richter, 2014; Cherniwchan, Copeland, and Taylor, 2017; Forslid, Okubo, and Ulltveit-Moe, 2018; Egger, Kreickemeier, and Richter, 2021; Chang, Cheng, and Peng, 2022) and empirics (e.g. Cole, Elliott, Okubo, and Zhou, 2013; Holladay, 2016; Cherniwchan, 2017; Richter and Schiersch, 2017; Rodrigue, Sheng, and Tan, 2022). In this firm heterogeneity literature, we take a bird's-eye view of how incorporating (some) insights from the micro-level into a macro-level quantitative framework affects aggregate outcomes.

Finally, there is a long tradition in the trade and environment literature of decomposing emission changes into scale, composition, and technique effects (Grossman and Krueger, 1993; Copeland and Taylor, 1994; Levinson, 2009; Shapiro and Walker, 2018). We elucidate how different trade modeling strategies shape these effects and whether differences in decomposition translate into different aggregate outcomes.

The rest of the paper proceeds as follows. Section 2 shows a simple example of incorporating emissions into the Melitz model. Section 3 establishes our general (non-)equivalence results and applies them to three types of trade and emissions modeling choices. Section 4 describes the multi-industry extension and the corresponding analytical results. In Section 5, we introduce flexible versions of the models to the data and quantify how firm heterogeneity alters the emission effects of international trade. Section 6 concludes.

2 A primer on trade and emissions with firm heterogeneity

To gain an intuition for our equivalence result, we start with a single-industry Melitz model with two emission settings: endogenous abatement in the spirit of Copeland and Taylor (2003) and emissions linked to quantities in a fixed proportion.

Emissions proportional to value – In the model, firms have heterogeneous productivity φ. The physical output from country i for market n is produced using labor l_{ni}:

$$q_{ni}(\varphi) = (1 - \xi_i(\varphi))\varphi l_{ni}(\varphi),$$

where ξ_i is the share of labor devoted to abatement. The emissions in the production process are given by:

$$z_{ni}(\varphi) = (1 - \xi_i(\varphi))^{1/\alpha_i} l_{ni}(\varphi),$$

where α_i is the elasticity of the pollution emissions intensity with respect to pollution abatement intensity. With this abatement function, emissions can be equivalently expressed as a second factor of production (Copeland and Taylor, 2003):

$$q_{ni}(\varphi) = \varphi(z_{ni}(\varphi))^{\alpha_i}(l_{ni}(\varphi))^{1-\alpha_i}.$$

Given a price for emissions, t_i, and wages, w_i, the firm-level emissions embodied in the trade flow from i to n are a constant share of the revenues, $x_{ni}(\varphi) = p_{ni}(\varphi)\tilde{q}_{ni}(\varphi)$, :

$$z_{ni}(\varphi) = \alpha_i \frac{x_{ni}(\varphi)}{\tilde{\sigma} t_i},$$

where $\tilde{q}_{ni}$ is the quantity of goods consumed in country n and $\tilde{\sigma}$ is the markup ratio, which is common and constant. The aggregate emissions embodied in the trade flow from i to n are obtained by aggregating the emissions of each exporting firm.[2] Under standard assumptions on the relative magnitude of fixed costs, a unique productivity cutoff exists for every market, φ^*_{ni}, that determines the set of firms that export from i to n. Only firms with productivity greater than or equal to this cutoff export. The emissions embodied in the total exports from i to n are given by:

$$Z_{ni} = M_{ni} E[z_{ni}(\varphi) | \varphi \geq \varphi^*_{ni}] = M_{ni} \int_{\varphi_{ni*}} \frac{\alpha_i}{\tilde{\sigma} t_i} x_{ni}(\varphi) dG(\varphi \geq \varphi^*_{ni}) d\varphi = \frac{\alpha_i}{\tilde{\sigma} t_i} X_{ni} \tag{1}$$

where $X_{ni} \equiv M_{ni} \int_{\varphi_{ni*}} x_{ni}(\varphi) dG(\varphi \geq \varphi^*_{ni}) d\varphi$ is the aggregate export value from country i to country n. M_{ni} is the mass of firms producing in i and selling to n. It is evident from (1) that the change in the aggregate bilateral export value is a sufficient statistic for the change in embodied emissions.

Emissions proportional to quantities – We now provide an alternative assumption

[2] We assume fixed costs of exporting are not associated with emissions, or we can also allow the fixed cost to be paid in terms of goods.

regarding the generation of emissions. Specifically, we assume that emissions are proportional to the quantity produced:

$$z_{ni}(\varphi) = \mu_i q_{ni}(\varphi).$$

With an analogous calculation as in the previous case, the emissions embodied in exports from i to n are given by:

$$Z_{ni} = M_{ni} E[z_{ni}(\varphi)|\varphi \geq \varphi^*_{ni}] = M_{ni} \int_{\varphi_{ni*}} \beta_i q_{ni}(\varphi) dG(\varphi \geq \varphi^*_{ni}) d\varphi = \mu_i Q_{ni} \tag{2}$$

where $Q_{ni} \equiv M_{ni} \int_{\varphi_{ni*}} q_{ni}(\varphi) dG(\varphi \geq \varphi^*_{ni}) d\varphi$ is the aggregate quantity of goods produced to export from i to n. This quantity includes an iceberg component. In this case, the change in the aggregate export *quantity* is a sufficient statistic for the change in embodied emissions.

There are two main differences in the treatment of the two cases. The first concerns how firms' productivity is related to their emission intensity. When emissions are proportional to the value, more productive firms use less labor and emit less per output unit.[3] In contrast, when emissions are proportional to quantity, more productive firms may use less labor but emit the same amount per output unit. Trade liberalization changes the productivity composition of firms, which, in the value case, affects both the quantity and emissions per unit of output. However, in the case of quantity, it only changes the quantity produced. Regarding the decomposition of emission changes (Grossman and Krueger, 1993; Copeland and Taylor, 1994), trade in both cases induces an emission-increasing scale effect. It is the sole effect in the quantity case but counteracted by an emission-decreasing technique effect in the value case.[4]

The second difference lies in the applicability of the ACR results. In a scenario where emissions are proportional to the trade value, the model benefits from the ACR finding that changes in trade value coincide across a range of models. If emissions per value are common across different models (e.g., Armington, Krugman, EK, and Melitz) and the ACR result of equal changes in aggregate trade flow holds, the changes in aggregate

[3]Emission per quantity inversely proportional to productivity ($z_{ni}/q_{ni} \propto 1/\varphi_{ni}$).

[4]The exact decompositions for each case and each model are shown in Section 3.5.

emissions will be identical. However, in a scenario where emissions are proportional to quantity, even though the emission intensity is the same across models, the changes in trade quantity differ, breaking emission equivalence. In the next section, we formalize this intuition and present the analytical results for changes in aggregate emissions associated with trade shocks.

3 (Non-)Equivalence of emission effects

3.1 General model set-up

This section introduces a general model encompassing the Armington, Krugman, EK, and Melitz models with different emission mechanisms, including endogenous abatement as in Copeland and Taylor (2003). In this section, we consider a single-industry setting. We will show how this Section's results can be embedded into a multi-sector setting in Section 4.

The global economy comprises $i = 1, ..., N$ countries. Each country has a mass of consumers l_i, each supplying a unit of labor inelastically. There is a mass of varieties Ω, which is potentially endogenous.[5] A variety can be produced by multiple firms or a single firm.

Preferences – The preference of a representative consumer in country i are of the Dixit-Stiglitz form, maximizing utility from consuming a variety of goods. The associated price index is

$$P_i = \left(\int_{i \in \Omega_i} p_i(\omega)^{1-\sigma} d(\omega) \right)^{1/(\sigma-1)},$$

where Ω_i is the set of varieties available in country i and σ is the elasticity of substitution between varieties.

[5]Following ACR, Ω may include either a continuum or a discrete number of goods. This allows us to include the Armington model and models with heterogeneous producers.

Production and emissions – If a firm in country i produces variety ω, it requires labor:

$$q_i(\omega) = f_{i,\omega}(l_i^p(\omega)),$$

where $l_i^p(\omega)$ is a labor input for production. We allow the production technology of variety ω to be country-specific. Emissions are associated with production and can be partially abated by labor input:

$$z_i(\omega) = g_{i,\omega}(q_i, l_i^z(\omega))$$

where $l_i^z(\omega)$ is the labor input for abatement of variety ω in country i. Again, we allow this abatement technology to be country-specific. We assume $g_{i,\omega}$ is increasing in y_i and nonincreasing in l_i^z.[6] Firms have to pay a carbon tax, t_i, per emission. The firm's cost minimization problem is:

$$\begin{aligned} C_{i,\omega}(w_i, t_i, \bar{q}) &= min_{l^p, l^z, z} w_i(l^p + l^z) + t_i z \\ \text{subject to } \bar{q} &= f_{i,\omega}(l_i^p) \\ z &= g_{i,\omega}(\bar{q}, l_i^z), \end{aligned}$$

where w_i and t_i are the wage and emission taxes in country i, respectively. The solution to this minimization problem is characterized by a cost function, $C_{i,\omega}(w_i, t_i, q)$, labor demand function, $l_{i,\omega}(w_i/t_i, q)$ and the emission output function, $z_{i,\omega}(w_i/t_i, q)$.[7]

Trade – Firms face an iceberg trade cost τ_{ni} and a fixed cost f_{ni} when exporting. Specifically, as variable trade costs, firms in country i have to produce τ_{ni} amount of goods to sell one unit to country n. For a fixed cost, firms have to pay f_{ni} amount of labor in the destination country.[8] We denote the shipped quantity and the value of export as $q_{ni}(\omega)$ and $x_{ni}(\omega)$, respectively, and denote associated emissions as $z_{ni}(\omega)$.

[6] We allow the case where firms cannot abate emissions.

[7] Because $l_{i,\omega}(w_i/t_i, q)$ and $z_{i,\omega}(w_i/t_i, q)$ are factor demand functions, they only depend on the factor price ratio.

[8] We assume there is no emission associated with fixed costs. We can allow fixed-cost payments involving abatement and emissions, like in the variable cost.

Market Structure – The market structure can be either (i) perfect competition or (ii) monopolistic competition. In the case of perfect competition, anyone can produce variety ω, and there are large numbers of potential firms and consumers. Firms and consumers take the prices, wages, and emission taxes as given. In this case, there are no fixed costs of exporting ($f_{ni} = 0$).

For monopolistic competition, we consider both free and fixed entry. In free entry, each firm can produce a unique variety ω by paying a fixed entry cost $F_i > 0$ in terms of labor. This fixed entry cost is not associated with emissions.[9] Firms enter until their expected profit from entry is zero. We denote the mass of firms in country i by M_i. In the case of fixed entry, each country has a fixed mass of firms, each producing a unique variety ω. The profits earned by the firms in i in the fixed entry case are given to consumers in the country. Firms take the aggregate price index and wage as given in both free and fixed entry, and maximize their profit.

Equilibrium – The carbon tax in country i is exogenously given, and tax revenue is repatriated to consumers in the country. In the equilibrium, the labor and goods markets clear.

Before discussing specific assumptions on how emissions are related to production, we briefly state how the three canonical trade models we consider throughout the paper fit into the general model structure outlined thus far.

Armington – The Armington model arises as a special case of the general model structure if we assume perfect competition and a fixed discrete number of varieties, namely one per country.

Krugman – In the Krugman case, firms engage in monopolistic competition; the number of firms in each country is endogenous, and each firm produces its unique variety. The production technology is the same for all firms within a country.

EK – The EK model is the second perfect competition special case with a fixed set of varieties. In this case, the set of varieties is a unit interval, and every variety can be

[9] We can allow entry costs and the export fixed cost to have emissions. In that case, we have to have equal emission intensities for fixed and variable costs.

produced in every country. Countries differ in their efficiency of producing these varieties and draw their productivities from a Frechet distribution with location parameter T_i and dispersion parameter θ.

Melitz – The Melitz model is similar to Krugman's, but firms differ in their productivity. Firms in every country draw their productivities from a Pareto distribution with scale parameter T_i and shape parameter θ.

3.2 Emissions linked to values

We explain the assumptions necessary for our propositions. The first assumption is that within a country, emissions embodied in sales are proportional to the output value, and this proportionality is constant across varieties (firms).

Restriction 1. *Denote the export value of a variety ω from country i to country n as $x_{ni}(\omega)$. The first assumption is that the emission embodied in the export of variety ω from country i to country n is proportional to the export value, and the proportion is common across varieties:*

$$z_{ni}(\omega) = \frac{\beta_i(w_i/t_i)}{t_i} x_{ni}(\omega), \tag{3}$$

where β_i can be fixed or depend on the country's wages relative to the emission cost.

A simple interpretation of this formula is that the proportion of emissions embodied in trade is common across firms and can be written as the emission cost share divided by the emission cost. First, we discuss the three assumptions that achieve this restriction when combined. Notice that these assumptions are slight generalizations of the example we discussed in the previous section:

Common cost share – In country i, for any variety ω, the emission (tax) cost share is common and constant for different quantities:

$$\frac{t_i z_{i,\omega}(w_i/t_i, q)}{C_{i,\omega}(w_i, t_i, q)} = \alpha_i(w_i/t_i),$$

where $\alpha_i(w_i/t_i)$ is a cost share function that only depends on the relative wages.[10]

Constant marginal cost – In any country and any variety, the marginal cost is constant:

$$C_{i,\omega}(w_i, t_i, q) = c_{i,\omega}(w_i, t_i)q,$$

where $c_{i,\omega}(w_i, t_i)$ is the marginal cost function, which differs across the country of production and depends on the factor prices.

Constant markup – The factory gate price of variety ω in country i is as follows:

$$p_{i,\omega}(w_i, t_i) = \tilde{\sigma} c_{i,\omega}(w_i, t_i),$$

where $\tilde{\sigma}$ is a markup ratio in the economy.

Constant markup and constant marginal cost jointly imply that the price of variety ω produced in country i and sold in country n is:

$$p_{ni,\omega}(w_i, t_i) = \tilde{\sigma}\tau_{ni} c_{i,\omega}(w_i, t_i)$$

These assumptions indicate that the emissions per monetary unit of export sales from i to n are constant across varieties:

$$\begin{aligned} \frac{z_{ni}(\omega)}{x_{ni}(\omega)} &= \frac{z_{ni}(\omega)}{\tilde{\sigma} c_{i,\omega}(w_i, t_i) q_{in}(\omega)} \\ &= \frac{\alpha_i(w_i/t_i)}{\tilde{\sigma} t_i}. \end{aligned}$$

These common assumptions in the literature result in Restriction 1 with $\beta_i(w_i/t_i) = \alpha_i(w_i/t_i)/\tilde{\sigma}$. Examples include the endogenous abatement modeling a la Copeland and Taylor (2003) in Shapiro and Walker (2018) and the Cobb-Douglas or CES energy input with proportional emissions in Larch and Wanner (2017) and Farrokhi and Lashkaripour (2024).

[10] Notice that the cost function is homogeneous of degree one with respect to the factor prices ($C_{i,\omega}(w_i, t_i, q) = t_i C_{i,\omega}(w_i/t_i, 1, q)$). Therefore, we have $\frac{t_i z_{i,\omega}(w_i, t_i, q)}{C_{i,\omega}(w_i, t_i, q)} = \frac{t_i z_{i,\omega}(w_i/t_i, q)}{t_i C_{i,\omega}(w_i/t_i, 1, q)} = \frac{z_{i,\omega}(w_i/t_i, q)}{C_{i,\omega}(w_i/t_i, 1, q)}$. This implies that the cost share is only a function of the relative factor price.

3.2.1 Aggregate implications

Denote Z_{ni} as the total emissions associated with the exports from country i to country n. This can be written as

$$Z_{ni} = \int_{\omega \in \Omega_i} z_{ni}(\omega) d\omega = \int_{\omega \in \Omega_i} \frac{\beta_i(w_i/t_i)}{t_i} x_{ni}(\omega) d\omega = \frac{\beta_i(w_i/t_i)}{t_i} X_{ni}$$

where $X_{ni} \equiv \int_{\omega \in \Omega} x_{ni}(\omega) d\omega$ is the total export from country i to country n. This equation suggests that the emissions associated with trade do not depend on the micro-composition of emissions.

Building on the ACR, we introduce the notion of trade shocks and explore their impact on aggregate emissions.

Definition 1. *A trade shock with constant carbon tax is a change from* $\{\boldsymbol{\tau}\}$ *to* $\{\boldsymbol{\tau'}\}$*, while fixing the relative price of emission* $(\boldsymbol{t}/\boldsymbol{w} = \boldsymbol{t'}/\boldsymbol{w'})$.

We choose a constant relative emission price to fix the climate policy and focus on trade liberalization.[11] We fix the emission tax relative to the wages because t_i is a nominal variable, and $\boldsymbol{t} = \boldsymbol{t'}$ is not a meaningful policy rule.[12] We denote the economic variable v after trade shock as v', and denote use $\hat{v} \equiv v'/v$ as the relative changes in the economic variable v following Dekle et al. (2007, 2008). Using the trade shocks, we show that the changes in emissions can be derived only by tracking the aggregate trade flows, wages, and emission taxes:

Proposition 1. *Suppose Restriction 1 is satisfied. The change in the aggregate emission for country i through a trade shock can be written as*

$$\hat{Z}_i = \frac{\hat{R}_i}{\hat{w}_i} \tag{4}$$

where $R_i \equiv \sum_{n=1}^{N} X_{ni}$ *is the total revenue in country i .*

[11] Similar assumptions are discussed in Copeland and Taylor (2003, pp. 112, 146)

[12] When $\boldsymbol{t} = \boldsymbol{t'}$, the emission outcome crucially depends on the normalization of wages. Note that this rules out any factor price-driven within-firm emission intensity changes. Any technique effect will, therefore, come from reallocation between firms with different productivities (see Egger, Kreickemeier, and Richter, 2021, for a discussion of these two different sources of the technique effect).

Proof. By Restriction 1, we showed that:

$$Z_{ni} = \frac{\beta_i (w_i/t_i)}{t_i} X_{ni},$$

and this implies

$$Z_i = \sum_{n=1}^{N} Z_{ni} = \frac{\beta_i (w_i/t_i)}{t_i} R_i.$$

Notice we fix the relative wages (w_i/t_i), and the emission tax moves proportional to the wages ($\hat{w}_i = \hat{t}_i$). Taking the ratio between the emissions of the baseline equilibrium and the equilibrium after a trade shock yields:

$$\hat{Z}_i = \frac{\hat{R}_i}{\hat{t}_i} = \frac{\hat{R}_i}{\hat{w}_i}.$$

□

3.2.2 Combining the restriction with ACR

The results of Theorem 1 are complemented with the restrictions imposed in Arkolakis, Costinot, and Rodríguez-Clare (2012) (henceforth ACR). These are macro restrictions that models may satisfy.[13].We slightly modify their restriction (to introduce an emission tax) and state the restrictions similar to R1, R2, and R3' in ACR.

ACR R1: Trade balance – For any country i, trade is balanced:

$$\sum_{i=1}^{N} X_{ni} = \sum_{i=1}^{N} X_{in}.$$

This assumption implies that the total revenue R_i is the same as total expenditure $Y_i \equiv \sum_{n=1}^{N} \int_{\omega \in \Omega} x_{in}(\omega) d\omega$.[14] ACR further imposes a constant aggregate profit share for each country:

ACR R2: Constant aggregate profit – Denote the aggregate profit of country i by

[13]In all the models we discuss, these restrictions are satisfied.

[14]We could include trade deficits as an exogenous transfer between countries.

$\Pi_i \equiv \int_{\omega \in \Omega} \pi_i(\omega) d\omega$, and the aggregate revenue of country i by R_i. For any country i, Π_i / R_i is constant.

For the third assumption, we introduce a slightly modified version of R3' in ACR to incorporate carbon taxes:

ACR R3': CES import demand – The import demand system is such that for any exporting country i and importing country n, the expenditure share of country n on goods from country i is:

$$\lambda_{ni} = \frac{\chi_{ni} \cdot M_i \cdot (\tilde{c}_i(w_i, t_i)\tau_{ni})^{\varepsilon}}{\sum_{k=1}^{N} \chi_{nk} \cdot M_k \cdot (\tilde{c}_k(w_k, t_k)\tau_{nk})^{\varepsilon}}.$$

where χ_{nk} is a parameter, M_i is a number of varieties, and $\tilde{c}_i(w_i, t_i)$ is a function that combines wages and the emission tax, which is homogeneous degree of one. The trade elasticity is denoted by ε.

We can show that as in the ACR, the changes in trade flows associated with trade shocks are the same across models. Combining this result with our proposition 1, we obtain the following proposition:

Proposition 2. *Consider a model that satisfies Restriction 1, ACR R1, ACR R2, and ACR R3'. The relative change in emissions associated with trade shocks with constant carbon tax is*

$$\hat{Z}_i = 1$$

where variable changes are determined by:

$$\hat{\lambda}_{ni} = \frac{(\hat{\tau}_{ni}\hat{w}_i)^{\epsilon}}{\sum_{k=1}^{N} (\hat{\tau}_{nk}\hat{w}_k)^{\varepsilon}}, \tag{5}$$

$$R_i^{'} = \hat{w}_i R_i, \tag{6}$$

$$\hat{w}_i = \frac{1}{R_i} \sum_j \hat{\lambda}_{ni} \lambda_{ni} R_n^{'}, \tag{7}$$

where R_i is the total revenue of country i.

The proof is provided in Appendix A. The result is striking; the proposition shows that global emissions remain constant following any trade shocks.[15] Our result highlights that in a single-industry trade model with emissions proportional to values, trade shocks do not change the aggregate emissions. We briefly discuss the rationale behind the absence of an emission effect in the Armington, EK, and Melitz models.

Armington – In the single-industry Armington model, trade only affects where goods are sold and leave production unaffected. As there are no heterogeneous producers, the same quantity is produced using the same technology (and hence emission intensity), irrespective of trade openness. In terms of the emission decomposition discussed in the primer, there is a zero scale effect and a zero technique effect (and — in the absence of a sectoral structure for now — a zero composition effect).

Krugman – As in the Armington case, trade only affects the destination products are sold to, not the overall quantity produced. In the Krugman case, this is less obvious because each firm features a production firm with an increasing return to scale. However, if the ACR Restrictions hold, the mass of active firms does not change with trade, and the scale economies hence are not affected, and the overall quantities and production techniques stay the same.

EK – In EK, trade allows countries to focus on goods they can produce efficiently. Hence, X trade increases the overall quantity produced. Simultaneously, specialization in goods for which a country has high productivity implies specialization in goods that the country can produce at a low emission intensity. These two effects — a positive scale effect and a negative emission effect — perfectly offset each other.

Melitz – In Section 2, we discussed that in the Melitz model, trade leads to a reallocation towards highly productive firms and, hence, to higher quantities but lower emission intensities. Proposition 2 implies that, as in the EK case, the scale and technique effects cancel each other out exactly. Notably, while EK and Melitz share this property, the magnitudes of the scale and technique effects do not coincide across the two models.

[15] While it is possible to generate changes in emissions by fixing $\boldsymbol{t}$ and changing $\boldsymbol{w}$, the aggregate effect on emissions crucially depends on the wage normalization scheme. In Appendix B, we show that any changes in global emissions can be realized by choosing the appropriate normalization.

3.3 Emissions linked to quantities

In an environmental context, physical units naturally play an important role, as we are interested in how much firms pollute rather than how much they spend on pollution. Hence, a clear alternative suggestion for introducing emissions into trade models is to link them directly to physical output.

Restriction 2. *Denote the export quantity of a variety ω from country i to country n as $q_{ni}(\omega)$. Emissions embodied in the export of variety ω from country i to country n are proportional to the export quantity, and the proportion is common across varieties:*

$$z_{ni}(\omega) = \mu_i q_{in}(\omega), \tag{8}$$

where μ_i is fixed.

3.3.1 Aggregate implications

The aggregate implication is similar to the common value assumption case:

$$Z_{ni} = \int_{\omega \in \Omega_{in}} z_{ni}(\omega) d\omega = \int_{\omega \in \Omega_{in}} \mu_i q_{ni}(\omega) d\omega = \mu_i Q_{ni}, \tag{9}$$

where Q_{ni} is the aggregate *quantity* produced to export from country i to country n.[16] For a trade shock, we propose the following:

Proposition 3. *Suppose Restriction 2 is satisfied. The change in the aggregate emission for country i in response to a trade shock can be written as:*

$$\hat{Z}_i = \sum_{k=1}^{N} \iota_{ni} \hat{Q}_{ni}, \tag{10}$$

where $\iota_{ni} \equiv \frac{Z_{ni}}{Z_i}$, and in this specific case, we also have $\iota_{ni} = \frac{Q_{ni}}{\sum_k Q_{ki}}$.

The change in export quantity in the Armington and Krugman models is derived as $\hat{Q}_{in} = \hat{X}_{in}/\hat{p}_{in}$. However, as discussed, it is not trivial to derive changes in the quantity in the EK and Melitz models.

[16] To be clear, the quantity includes the amount of goods used to pay the iceberg costs.

We adopt another restriction to Restriction 2 so that the model is compatible with ACR:

No emission tax – Producers do not pay a price for their emissions:

$$t_i = 0.$$

This assumption implies that labor is the only relevant cost in production.[17] We combine these assumptions with the ACR restrictions:

Proposition 4. *Consider a model that satisfies Restriction 2, ACR R1, ACR R2, and ACR R3'. The percentage change in emissions associated with any change in variable trade costs in country i can be expressed as*

$$\hat{Z}_i = \sum_{i=1}^{N} \iota_{ni} \cdot \hat{Q}_{ni}$$

Variable changes are determined by:

$$\begin{aligned}
\hat{\lambda}_{ni} &= \frac{(\hat{\tau}_{ni}\hat{w}_i)^{\epsilon}}{\sum_{k=1}^{N} (\hat{\tau}_{nk}\hat{w}_k)^{\varepsilon}}, \\
R_i^{'} &= \hat{w}_i Y_i, \\
\hat{w}_i &= \frac{1}{R_i} \sum_j \hat{\lambda}_{ni} \lambda_{ni} R_n^{'}.
\end{aligned}$$

The change in the quantity, $\hat{Q}_{ni}$, and hence the emission change varies across different models (e.g., Armington, Krugman, EK, and Melitz).

The key insight is that although trade values are common in the ACR class of models, trade quantities differ across models. Combining Restriction 1 with ACR restrictions gives rise to specific and common emission predictions. Meanwhile, combining Restriction 2 with ACR restrictions is insufficient to pin down the emission response to a trade shock. There is no general solution for $\hat{Q}_{ni}$, and a closed-form solution may not exist. Fortunately, we can show that there are closed-form solutions for the canonical models we consider (i.e., Armington, Krugman, EK, and Melitz).

[17] With positive emission taxes, the model cannot be reduced to the ACR class of trade models.

Armington – In the case of the Armington model, we have

$$\hat{Q}_{ni} = \frac{\hat{X}_{ni}}{\hat{w}_i}. \tag{11}$$

and $\iota_{ni} = \frac{X_{ni}}{Y_i}$. The equation implies $\hat{Z}_i = \hat{R}_i/\hat{w}_i = 0$, which coincides with the case of emissions associated with values. In the Armington model, emissions do not change with trade shocks. Without a change in technology, the production quantity remains unchanged. Trade shocks only change the allocation of goods used for iceberg transportation costs and consumption.

Krugman – The expressions for the quantity change $\hat{Q}_{ni}$ and the emission share ι_{ni} in the Krugman model coincide with the Armington case. Intuitively, as all producers use the same technology, the sales share coincides with the emission share, as there is no selection of exporters. Also, as trade liberalization does not affect the mass of active firms or their prodution technologies, aggregate emissions do not change.

EK – For the EK model, the quantity of production is not only a function of trade values but also of the import share $\lambda_{ni} = \frac{X_{ni}}{X_n}$. Specifically, changes in the production quantities associated with trade shocks are:

$$\hat{Q}^{EK}_{ni} = \frac{\hat{X}_{ni}}{\hat{w}_i} \hat{\lambda}_{ni}^{-1/\theta}, \tag{12}$$

where θ is a parameter of the Frechet distribution. The quantity share is given by:

$$\iota_{ni} = \frac{X_{ni} \lambda_{ni}^{-1/\theta}}{\sum_{k=1} X_{ki} \lambda_{ki}^{-1/\theta}}.$$

Appendix C provides the derivation of these results. When comparing to the Armington model, it's important to note that the EK model accounts for endogenous changes in productivity due to the selection of more productive varieties. In the EK framework, λ_{ni} represents the import share and the proportion of varieties that country i exports to country n. A key implication is that a lower λ_{ni} indicates a higher quantity for a given trade value X_{ni}, reflecting the export of only highly productive varieties charging lower prices. Furthermore, when calculating changes in quantity (and consequently emissions), it is essential to recognize that value shares do not precisely equate to quantity shares. This distinction is critical and is incorporated into the computation of ι_{ni}.

Melitz – For the Melitz model, the quantity of production is a function of the export value, domestic productivity cutoff φ^*_{ii}, and share of exporting firms to a particular destination, $S_{ni} = M_{ni}/M_i$:

$$\hat{Q}^{Melitz}_{ni} = \frac{\hat{X}_{ni}}{\hat{w}_i}\hat{S}^{-1/\theta}_{ni}\hat{\varphi}^*_{ii} \tag{13}$$

where θ is a shape parameter of the Paret distribution. and the quantity share is:

$$\iota_{ni} = \frac{S^{-1/\theta}_{ni} X_{ni}}{\sum_{k=1}^{N} S^{-1/\theta}_{ki} X_{ki}}$$

The derivation of $\hat{Q}^{Melitz}_{ni}, \hat{S}_{ni}$ and $\hat{\varphi}^*_{ii}$ are delegated to the appendix. Compared to the Armington model, changes in quantities in the Melitz model must again consider endogenous productivity changes due to selection. Changes in S_{ni} represent the productivity changes due to the selection in country i of being an exporter to country n. In addition to the changes in S_{ni}, we must consider the change in the domestic productivity distribution of producers, which is represented by the changes in φ^*_{ii}. Like the EK model, we need to consider that the quantity share differs from the value share.

Using these results, we derive the emission effects of trade opening:

Definition 2. *Trade opening is a change in trade costs from autarky (infinite iceberg trade costs) to finite trade costs.*

Proposition 5. *Consider a model that satisfies Restriction 2, ACR R1, ACR R2, and ACR R3'. Trade opening*

(i) leaves aggregate emissions in Armington and Krugman models unaffected

(ii) increases emissions in the EK and the Melitz models.

The proof is in appendix D.

For example, the quantity-based emission specification is used in Shapiro (2016) who embeds it into an Armington trade model. While the value-based approach is more common and has the advantage of allowing flexibility in substituting emissions/energy against other production inputs, there are some contexts in which a focus on quantities arises naturally. This includes transportation emissions (what matters is the physical quantity shipped from one country to another) and process-related emissions (different from emissions from fossil fuel combustion, they cannot be avoided by shifting inputs away from fossil fuels).

Next, we introduce an emissions specification that links emission intensity to productivity more flexibly and nests the value and quantity specifications as special cases.

3.4 A flexible emission specification

The two emission modeling approaches considered so far have very different implications. In the quantity-based setting, more productive firms are not at the same time cleaner than less productive firms. Hence, nothing counteracts the scale effect in the case of trade liberalization. In the value-based setting, more productive firms are cleaner firms, introducing a counteracting reallocation effect on emissions. However, for the value case, the magnitude of this effect turns out to be identical to the scale effect and, therefore, neutralizes the role of firm heterogeneity. In this section, we follow Kreickemeier and Richter (2014) and consider an emission-generating process linked to the produced quantities and link emission intensity to productivity with a technology parameter.

Restriction 3. *Assume that the production function is a simple one-factor function and emissions are generated according to the following expression:*

$$z_i(\omega) = \frac{\mu_i q_i(\omega)}{[\varphi(\omega)]^{\gamma_i}}. \tag{14}$$

The relationship between the physical emission intensity and firm productivity now depends on the value of γ_i. If $\gamma_i < 0$, more productive firms are dirtier than less productive firms. Conversely, $\gamma_i > 0$ relates to the empirically more relevant case in which highly productive firms produce less emission-intensively. Whether the associated reallocation effect suffices to offset the scale effect entirely depends on the precise value of γ_i rather than just on its sign. Unlike in Sections 3.2 and 3.3, as emission intensity across firms is constant neither in value nor quantity terms, we cannot write useful general expressions for aggregate national emissions or their relative change. Instead, we directly consider the implications of the third emission modeling approach in the three trade models.[18] The change in the aggregate emissions is again shown as a change in emissions weighted by the initial weight:

$$\hat{Z}_i = \sum_{n=1} \iota_{ni} \hat{Z}_{ni}.$$

Armington – The change in emissions is equal to the change in the quantity of production, which is given by

$$\hat{Z}_{ni}^{Armington} = \frac{\hat{X}_{ni}}{\hat{w}_i}, \tag{15}$$

and the emission share again equals the sales share ($\iota_{ni} = \frac{X_{ni}}{Y_i}$).

Krugman – As any differences in the KR case from the quantity case arises due to the heterogeneity of producers and firms are homogenous in the Krugman framework, it again simply coincides with the Arimington model ($\hat{Z}_{ni}^{Armington} = \hat{Z}_{ni}^{Krugman}$).

EK – For the EK model, the quantity of production is not only a function of trade values but also of the import share $\lambda_{ni} = \frac{X_{ni}}{X_n}$. The changes in emissions associated with trade shocks are:

$$\hat{Z}_{ni}^{EK} = \frac{\hat{X}_{ni}}{\hat{w}_i} \hat{\lambda}_{ni}^{-(1-\gamma_i)/\theta}, \tag{16}$$

and the emission share, ι_{ni}^{EK}, is

$$\iota_{ni}^{EK} = \frac{X_{ni} \lambda_{ni}^{-(1-\gamma_i)/\theta}}{\sum_{k=1} X_{ki} \lambda_{ki}^{-(1-\gamma_i)/\theta}}.$$

Melitz – For the Melitz model, the quantity of production is a function of the domestic

[18] We derive the exact expressions in appendix C

productivity cutoff φ_{ii} and a share of firms serving a particular destination, $S_{ni} = M_{ni}/M_i$:

$$\hat{Z}_{ni}^{Melitz} = \frac{\hat{X}_{ni}}{\hat{w}_i} \hat{S}_{ni}^{-(1-\gamma_i)/\theta} \left(\hat{\varphi}_{ii}^*\right)^{1-\gamma_i}, \tag{17}$$

and the emission share, ι_{ni}^{Melitz}, is

$$\iota_{ni}^{Melitz} = \frac{S_{ni}^{-(1-\gamma_i)/\theta} X_{ni}}{\sum_{k=1}^{N} S_{ki}^{-(1-\gamma_i)/\theta} X_{ki}}.$$

Using these results, we state the following proposition for the emissions effect of trade openings:

Proposition 6. *Consider a model that satisfies Restriction 3, ACR R1, ACR R2 and ACR R3'. Trade opening*

(i) leaves aggregate emissions in Armington and Krugman models, as well as in EK and Melitz models with $\gamma_i = 1$ unaffected

(ii) increases emissions in EK and Melitz models with $\gamma_i < 1$

(iii) lowers emissions in EK and Melitz models with $\gamma_i > 1$.

The proof is in appendix D.

3.5 Decomposing emission changes

As discussed above, the different effects in models with and without firm heterogeneity can intuitively linked to emission decompositions. We discuss the four cases by decomposing the changes in emissions into scale and technique effects. Log-linearizing the emissions embodied in trade yields:

$$d\ln Z_{ni} = \underbrace{d\ln Q_{ni}}_{\text{scale effect}} + \underbrace{d\ln\left(\frac{Z_{ni}}{Q_{ni}}\right)}_{\text{technique effect}}.$$

In the value case, we can show that:

$$\begin{aligned}
d\ln Z_{ni}^{\text{Arm/Krug}} &= \underbrace{d\ln X_{ni} - d\ln w_i}_{\text{scale effect}} + \underbrace{0}_{\text{technique effect}}, \\
d\ln Z_{ni}^{\text{EK}} &= \underbrace{-\frac{1}{\theta}d\ln\lambda_{ni} + d\ln X_{ni} - d\ln w_i}_{\text{scale effect}} + \underbrace{\frac{1}{\theta}d\ln\lambda_{ni}}_{\text{technique effect}}, \\
d\ln Z_{ni}^{\text{Melitz}} &= \underbrace{-\frac{1}{\theta}d\ln S_{ni} + d\ln\varphi_{ii}^{*} + d\ln X_{ni} - d\ln w_i}_{\text{scale effect}} + \underbrace{\frac{1}{\theta}d\ln S_{ni} - d\ln\varphi_{ii}^{*}}_{\text{technique effect}}.
\end{aligned}$$

Summing over all the destinations yields:

$$\begin{aligned}
dlnZ_i^{Arm/Krug} &= \sum_{n=1}^{N}\iota_{ni}\left(dlnX_{ni} - dlnw_i\right) + 0, \\
dlnZ_i^{EK} &= \sum_{n=1}^{N}\iota_{ni}\left(-\frac{1}{\theta}dln\lambda_{ni} + dlnX_{ni} - dlnw_i\right) + \sum_{n=1}^{N}\iota_{ni}\frac{1}{\theta}dln\lambda_{ni}, \\
dlnZ_i^{Melitz} &= \sum_{n=1}^{N}\iota_{ni}\left(-\frac{1}{\theta}dlnS_{ni} + dln\varphi_{ii}^{*} + dlnX_{ni} - dlnw_i\right) + \sum_{n=1}^{N}\iota_{ni}\left(\frac{1}{\theta}dlnS_{ni} - dln\varphi_{ii}^{*}\right),
\end{aligned}$$

where using $\sum_{i=1}^{N}\iota_{ni}\left(dlnX_{ni} - dlnw_i\right) = 0$[19], we obtain:

$$\begin{aligned}
d\ln Z_{i}^{\text{Arm/Krug}} &= \underbrace{0}_{\text{scale effect}} + \underbrace{0}_{\text{technique effect}}, \\
d\ln Z_{i}^{\text{EK}} &= \underbrace{\sum_{n=1}^{N}\iota_{ni}\left(-\frac{1}{\theta}d\ln\lambda_{ni}\right)}_{\text{scale effect}} + \underbrace{\sum_{n=1}^{N}\iota_{ni}\frac{1}{\theta}d\ln\lambda_{ni}}_{\text{technique effect}}, \\
d\ln Z_{i}^{\text{Melitz}} &= \underbrace{\sum_{n=1}^{N}\iota_{ni}\left(-\frac{1}{\theta}d\ln S_{i} + d\ln\varphi_{ii}^{*}\right)}_{\text{scale effect}} + \underbrace{\sum_{n=1}^{N}\iota_{ni}\left(\frac{1}{\theta}d\ln S_{ni} - d\ln\varphi_{ii}^{*}\right)}_{\text{technique effect}}.
\end{aligned}$$

The result indicates that in the Armington and Krugman models, the scale and technique effects are zero, whereas for the other two, the effects cancel out exactly. In the quantity case, the technique effect is absent, and only the scale effect is present.[20] For

[19]This is from $\sum_{i=1}^{N}\iota_{ni}\left(dlnX_{ni} - dlnw_i\right) = \sum_{i=1}^{N}\frac{X_{ni}}{R_i}dlnX_{ni} - dlnw_i = dlnR_i - dlnw_i = 0$

[20]In addition, ι_{ni} is different between the value case and the quantity case, since in the value case $\iota_{ni} = X_{ni}/R_n$ while in the quantity case $\iota_{ni} = Q_{ni}/Q_n$.

the flexible emission specification, the decomposition is expressed as follows:

$$
\begin{aligned}
dlnZ_i^{Arm/Krug} &= \sum_{n=1}^{N} \iota_{ni} \left(dlnX_{ni} - dlnw_i\right) + 0 \\
dlnZ_i^{EK} &= \sum_{n=1}^{N} \iota_{ni} \left(-\frac{1}{\theta} dln\lambda_{ni} + dlnX_{ni} - dlnw_i\right) - \gamma_i \sum_{n=1}^{N} \iota_{ni} \frac{1}{\theta} dln\lambda_{ni} \\
dlnZ_i^{Melitz} &= \sum_{n=1}^{N} \iota_{ni} \left(-\frac{1}{\theta} dlnS_{ni} + dln\varphi_{ii}^{*} + dlnX_{ni} - dlnw_i\right) - \gamma_i \sum_{n=1}^{N} \iota_{ni} \left(\frac{1}{\theta} dlnS_{ni} - dln\varphi_{ii}^{*}\right)
\end{aligned}
$$

where the sign and the magnitude of the technique effect now depend on the parameter γ_i.[21] When $\gamma_i = 0$, the model is equivalent to the quantity case, where no technique effect exists, and only the scale effect exists. When $\gamma_i = 1$, the model collapses to the value case, where the technique and scale effect cancel out.

4 Multi-industry extension

4.1 General model set-up

We extend the model in Section 3 to multiple sectors S. The production technology and market structure in each industry are the same as in the aggregate consideration above. Additionally, we follow Kucheryavyy et al. (2023), Farrokhi and Lashkaripour (2024), and Lashkaripour and Lugovskyy (2023) and assume that consumers have a three-tier nested CES utility. The upper-tier is Cobb-Douglas across sectors with spending share κ_{ns}. The second tier is CES across different origins within industries, with the elasticity of substitution η_s, and the last tier aggregates varieties within a country of origin with the elasticity of substitution σ_s.[22] This model implies the following expenditure share of consumers in n on products from i, sector s (see Appendix E for the derivation):

$$
\lambda_{ins} = \frac{\left(\beta_{is}(w_i/t_i)L_{is}\right)^{\delta_s} \left(c_{is}(w_i, t_i)\tau_{nis}\right)^{-\epsilon_s} \xi_{nis}}{\sum_{l \in N} \left(\beta_{ls}(w_l/t_l)L_{ls}\right)^{\delta_s} \left(c_{ls}(w_l, t_l)\tau_{nls}\right)^{-\epsilon_s} \xi_{nls}}, \tag{18}
$$

[21] Also, as mentioned in the previous section, ι_{ni} depends on γ_i.

[22] We introduce the nest compared to Section 3 to ensure a unique equilibrium in the monopolistic competition versions of the model (see Kucheryavyy et al., 2023, for a discussion).

where ϵ_s is the trade elasticity, ψ_s is the scale elasticity, and $\delta_s = \psi_s \epsilon_s$ is the product of the scale and trade elasticities and ξ_{ins} is a constant. We now briefly outline how the four canonical models deliver special cases of Equation (18).

Armington – In the multi-industry Armington model, the three-tier nested utility function reduces to two nests because every country produces a single variety per country. The trade elasticity simply relates to the elasticity of substitution across varieties within one sector from different countries ($\varepsilon_s = \sigma_s - 1$). $\delta_s = 0$ because there are no scale economies in this framework.

EK – In the multi-industry EK model, the three-tier nest reduces to two tiers. In this case, it is because firms from different countries produce varieties from the same unit interval per sector rather than distinct varieties. The trade elasticity coincides with the sectoral Frechet dispersion parameter ($\epsilon_s = \theta_s$). As in the other perfect competition framework, there are no scale economies, and hence $\delta_s = 0$. Note that for a given trade elasticity, trade adjustments will be identical in the multi-sector EK and Armington model because they only differ in the constant ξ_{ins}, which does not affect the equilibrium in changes.

Krugman – In the multi-sector Krugman model, substitutability has to differ between varieties from different firms, countries, and sectors to ensure a unique equilibrium. The trade elasticity is directly linked to the elasticity of substitution across different varieties in a given sector from the same country ($\epsilon_s = \eta_s - 1$). Further, in this monopolistically competitive framework, scale economies are present and linked to the elasticity of substitution between varieties from different countries, specifically $\nu_s = 1/(\sigma_s - 1)$ and hence $\delta_s = (\eta_s - 1)/(\sigma_s - 1)$.

Melitz – In the multi-industry Melitz model, substitutability again has to differ across the three layers of firms, countries, and sectors. The trade elasticity is linked to both the Pareto shape parameter θ_s and the elasticities of substitution between varieties from both different firms and countries, specifically $\epsilon_s = \theta_s/(1 + \theta_s \left(\frac{1}{\eta_s - 1} - \frac{1}{\sigma_s - 1}\right))$. As in the second monopolistically competitive framework, there are scale economies, which in this case are linked to the Pareto shape parameter, specifically $\nu_s = 1/\theta_s$ and hence

$\delta_s = 1/(1+\theta_s\left(\frac{1}{\eta_s-1} - \frac{1}{\sigma_s-1}\right))$. Note that trade responses are identical in the multi-sector Melitz and Krugman models for given values of trade and scale elasticities.

4.2 Emissions linked to values

Following the same lines as in 3.2, it is straightforward to show that emissions in sector s, country i are given by:

$$Z_{is} = \frac{\beta_{is}(w_i/t_i)}{t_i} R_{is}, \tag{19}$$

where revenues are equal to:

$$R_{is} = \sum_{n\in N} \lambda_{nis} E_{ns},$$

and the sector-level expenditure is defined as:

$$E_{ns} = \kappa_{ns}\left(w_n L_n + t_n Z_n\right).$$

Proposition 7. *Suppose we have a model where Equation (18) and Restriction 1 hold. Following a trade shock as in Definition 1,*

(i) Emission changes in country i are:

$$\hat{Z}_i = \sum_{s\in S} \frac{Z_{is}}{Z_i}\hat{L}_{is} = \sum_{s\in S} \frac{Z_{is}}{Z_i}\frac{1}{\hat{w}_i w_i L_{is}} \sum_{n\in N} \hat{\lambda}_{nis}\lambda_{nis}\hat{E}_{ns}E_{ns}, \tag{20}$$

where the change in the trade share is

$$\hat{\lambda}_{nis} = \frac{\left(\hat{L}_{is}\right)^{\delta_s}\left(\hat{w}_i\hat{\tau}_{ins}\right)^{-\epsilon_s}}{\sum_{l\in M}\lambda_{ljs}\left(\hat{L}_{ls}\right)^{\delta_s}\left(\hat{w}_l\hat{\tau}_{lns}\right)^{-\epsilon_s}}, \tag{21}$$

and the counterfactual expenditure is given by:

$$\hat{E}_{ns}E_{ns} = \kappa_{ns}\hat{w}_n\left(w_n L_n + \hat{Z}_n t_n Z_n\right),$$

where ϵ_s is the trade elasticity, ψ_s is the scale elasticity, and $\delta_s = \psi_s\epsilon_s$ is the product of the scale and trade elasticities.

Proof. See Appendix F. □

Corollary 1. *Suppose the model structure that satisfies equation (18) and that Restriction 1 holds. We further assume common trade elasticity across four models, and common scale elasticity for Melitz and Krugman (which is zero for Armington and EK). Following a trade shock as in Definition 1,*

(i) the canonical Armington and EK model lead to the same emission changes, $\hat{Z}_i$.

(ii) the Melitz and Krugman model lead to the same emission changes, $\hat{Z}_i$, which is different to the emission changes for EK and Armington due to scale economies.

(iii) $\hat{Z}_i = 1$ for all models if $\frac{Z_{is}}{Z_i} = \frac{L_{is}}{L_i} \quad \forall i \in N \; s \in S$.

Proof. See Appendix G. □

In the multi-industry case, a trade shock affects emissions even when the emission price is fixed to the wage rate. The reallocation of labor across sectors fully explains these emission changes. As is evident from equation 21, the trade response, which determines the magnitude and the direction of labor reallocation, differs across models. As scale economies are absent in the canonical Armington and EK models, the change in expenditure share depends only on the changes in iceberg trade costs and the wage rate. The monopolistic competition models of Krugman and Melitz generate external economies of scale, implying that the expenditure share also depends on sectoral employment. These differences in trade responses have been shown elsewhere (see e.g. Costinot and Rodriguez-Clare, 2014). Moreover, proposition 7 shows that emission responses are solely driven by the shift in the industry composition. A special case arises when the emission per employed labor is common across industries within the country, $Z_{is}/L_{is} = Z_i/L_i$. In this case, the composition effect is absent because the reallocation of labor does not alter the relative importance of emission-intensive and cleaner sectors. Hence, trade shocks do not affect emissions as in the single industry economy, regardless of the model considered.

Note that part (i) of Corollary 1 is of practical relevance because for a range of models used in the literature (including e.g., Egger and Nigai, 2015; Larch and Wanner, 2017, 2024; Duan et al., 2021; Caron and Fally, 2022; Mahlkow and Wanner, 2023), it implies that they could switch back and forth between an Armington representation without firm heterogeneity and an EK model with heterogeneous producers without any changes to their aggregate outcomes. The same is not quite true for the Melitz models by Shapiro and Walker (2018) and Sogalla (2023): Switching them to Armington models would change aggregate emission outcomes, however, *only because* the trade responses are different in the Melitz case, not because of the within-industry reallocation between more or less emission-intensive producers.

4.3 Flexible emission specification

The basic model structure is similar to Section 4.2. Moreover, Restriction 3 is assumed to hold for each sector, with sector-varying emissions intensities and elasticity μ_{is} and γ_{is}. The emission changes due to a trade shock according to Definition 1 is given by:

$$\hat{Z}_i = \sum_{s \in S} \frac{Z_{is}}{Z_i} \sum_{n \;\; N} \frac{Z_{ins}}{Z_{is}} \hat{Z}_{ins}. \tag{22}$$

The changes in emissions are given by the sector level equivalent of (15) to (17) as well as the corresponding embodied emission shares ι_{nis}. [23].

The remaining equilibrium conditions in the changes are given by equation 21 and the following equations:

$$\hat{E}_{ns} E_{ns} = \beta_{ns} \hat{w}_n w_n L_n, \tag{23}$$

$$\hat{w}_i = \frac{1}{w_i L_i} \sum_{s \in S} \sum_{n \in N} \hat{\lambda}_{ins} \lambda_{ins} \hat{E}_{ns} E_{ns}, \tag{24}$$

$$\hat{L}_{is} = \frac{1}{\hat{w}_i w_i L_{is}} \sum_{n \in N} \hat{\lambda}_{ins} \lambda_{ins} \hat{E}_{ns} E_{ns}. \tag{25}$$

When $\gamma = 0$, the formulation collapses to the quantity case with multiple industries. Unlike in the single industry case, the flexible model with $\gamma = 1$ differs from the model that links emissions to values. This is because collected emission taxes, which respond to the trade shock in multiple industry cases, are absent in the flexible model. If one assumes that environmental tax revenues are lost in rent-seeking as, e.g., Shapiro and Walker (2018), the two models are again isomorphic.

As the multi-sector model features, composition effects, and the role of firm hetero-

[23]The change in embodied emissions in Melitz is a function of the change in the exporting share, which is presented in Appendix H

geneity may differ across industries, general statements on the emission effects of trade shocks and trade opening and the role of firm heterogeneity become harder. Nevertheless, we still reach sharp conclusions for the *relative* emission effects between EK and Armington for a set of interesting special cases concerning the range of values that γ_{is} takes, summarized in the following proposition:Comparing Krugman and Melitz models is not straightforward, since trade liberalization may lower the productivity of certain industries in a multi-industry setting.

Proposition 8. *Consider a model that satisfies Equation 18 and Restriction 3. Trade opening*

(i) leads to equal aggregate emissions in Armington and EK models if $\gamma_{is} = 1 \; \forall i, s$.

(ii) increases emissions in EK relative to Armington models if $\gamma_{is} < 1 \; \forall i, s$.

(iii) lowers emissions in EK relative to Armington models if $\gamma_{is} > 1 \; \forall i, s$.

The proof is in appendix I. Note that generally, γ_{is} will be above one in some sectors and below one in others. Whether firm heterogeneity leads to relatively higher or lower emissions associated with trade liberalization then has to be answered by quantitative simulations.

5 Quantification

5.1 Data and parameters

Our main data source is the World Input-Output Database (WIOD) from Timmer et al. (2015), which contains trade flows and expenditure shares. CO_2 emissions consistent with WIOD classifications are provided by Amores et al. (2019). To quantify emission changes in the Melitz model, we require data on the shares of exporting firms. We gather these data from various sources. The number of active firms is given by the OECD Structural Statistics of Industry and Services database. For the number of exporting firms by partner country, we rely on the OECD Trade by Enterprise Characteristics Database, Eurostat, and the World Bank Exporter Dynamics Database. We limit the number of exporting/active firms to those operating in manufacturing. Unfortunately, we do not have data on the share of exporting firms for all trading pairs. We impute the remaining export shares to retain as many countries as possible (see Appendix J for details).

We rely on estimates from the literature for the standard model parameters. We obtain the demand substitution parameters, σ_s and η_s, from Lashkaripour and Lugovskyy (2023). We use the Pareto shape parameter estimate θ_s from Shapiro and Walker (2018). The translation of these parameters into the trade and scale elasticities, ϵ_s and δ_s, is model-specific. We calculate ϵ_s and δ_s in line with the multi-sector Melitz model structure. We then fix the resulting trade elasticities across all four canonical models. Similarly, we fix δ_s across the two monopolistically competitive models but put it equal to zero for the two perfectly competitive models as they do not feature scale economies. While this procedure implicitly assumes different values for underlying parameters (such as σ_s and θ_s) across models, it ensures maximal comparability across models in the quantitative results. We want all differences to stem from *model* differences rather than from differently calibrated elasticities.

The key parameter for how firm heterogeneity affects the effects of trade on emissions is the productivity elasticity of emissions. We estimate γ_s using administrative German firm-level data. We combine different modules of the official German manufacturing census *AFiD-Panel (Amtliche Firmendaten für Deutschland)*. The main module is the *AFiD Panel Industrial Firms*. It covers the universe of German manufacturing and mining firms with 20 or more employees and provides data on sales and employed labor. For a representative sample, we further observe material expenditure, different costs of the firm, and investments. We complement this module with the module *Energy Usage*, which contains the energy inputs in physical quantity. We combine these data with fuel-specific emission factors from Juhrich (2022), which enables us to calculate firm-level CO_2 emissions from fuel combustion. This method of calculating carbon emissions has been applied in several other studies such as Richter and Schiersch (2017) or Rottner and Von Graevenitz (2022). Finally, we combine these two modules with a customs data set to obtain firm-level exports.

5.2 Estimation of γ

Hereafter, we assume γ_{is} is common across countries but varies across sectors. We estimate γ_s by exploiting the relationship between domestic sales and the emission intensity. Here, we provide a derivation through the perspective of the Melitz model, while the estimation is also consistent with the EK model. Expressing emissions in terms of sales leads to:

$$z_{is}(\omega) = (\varphi(\omega))^{-\gamma_s} \mu_{is} \frac{\sum_{n \in N} \tau_{nis} x_{nis}(\omega)}{p_{nis}(\omega)} = (\varphi(\omega))^{1-\gamma_s} \mu_{is} \left(\frac{\sigma_s}{\sigma_s - 1} w_i \right)^{-1} \sum_{n \in N} x_{nis}(\omega),$$

and the emission intensity is then given by:

$$e_{is}(\omega) = \frac{z_{is}(\omega)}{x_{is}(\omega)} = (\varphi(\omega))^{1-\gamma_s} \mu_{is} \left(\frac{\sigma_s}{\sigma_s - 1} w_i \right)^{-1}.$$

Firm productivity is linked to the sales in the domestic market as follows:

$$\varphi^{\sigma_s - 1}(\omega) = x_{iis}(\omega) \left(\frac{\sigma_s}{\sigma_s - 1} w_i \right)^{\sigma_s - 1} B_{iis}^{-1},$$

where B_{iis} is the real market size of sector s in country i. This leads to the following expression of the emission intensity in terms of domestic sales:

$$e_{is}(\omega) = (x_{iis}(\omega))^{\frac{1-\gamma_s}{\sigma_s - 1}} \left(\frac{\sigma_s}{\sigma_s - 1} w_i \right)^{-\gamma_s} B_{iis}^{\frac{\gamma_s - 1}{\sigma_s - 1}}. \tag{26}$$

Hence, we can estimate γ_s by regressing emission intensity on the domestic sales:

$$\ln e_{\omega,is} = \beta_0 + \beta_1^{OLS} \ln x_{\omega,iis} + \epsilon_{\omega,is}, \tag{27}$$

where $\epsilon_{\omega,is}$ is an idiosyncratic component capturing measurement errors. With an additional estimate of σ_s, we can obtain an estimate for γ_s from the estimated coefficient:

$$\hat{\gamma}_s = 1 - \hat{\beta}_1^{OLS}(\sigma_s - 1).$$

We include direct emissions and those embodied in the electricity usage in our emission intensity measure. We estimate (27) by the whole sample (Mining and Manufacturing) and each WIOD sector. Table 1 shows the estimation results and the parameters sourced from the literature. Note that we calculate $\hat{\gamma}_s$ based on the Melitz-consistent σ_s values from Lashkaripour and Lugovskyy (2023) and then fix γ_s across all models considered — again ensuring that quantitative differences across models stem from model differences, rather than differences in the calibration of key elasticities.

The aggregate γ, estimated using manufacturing and mining firms, is 1.2. This value is close but above unity, which implies that the technique effect is stronger than the scale effect in the models with firm heterogeneity. In individual industries, while γ_s is always positive (more productive firms are not dirtier), the value of γ_s widely differs across sectors. In industries like the manufacture of basic metals (C24) and the manufacture of paper and paper products (C17), the value of γ_s is close to zero, which indicates that these industries exhibit emissions almost proportional to quantity. Conversely, industries like the manufacture of motor vehicles, trailers, and semi-trailers (C29) and repair and installation of machinery and equipment (C33) have γ_s above 2, indicating a strong positive relationship between productivity and emission intensity.[24] The relative strength of the scale and the technique effect differs by industry, and the aggregate implications are a-priori unclear. To further investigate the aggregate implications, we plot the relationship between emission intensity and γ_s in figure 1. In general, higher emission intensity is associated with lower γ_s, and these industries occupy a significant fraction of the global emissions. This overall tendency suggests that trade liberalization may increase aggregate emissions.

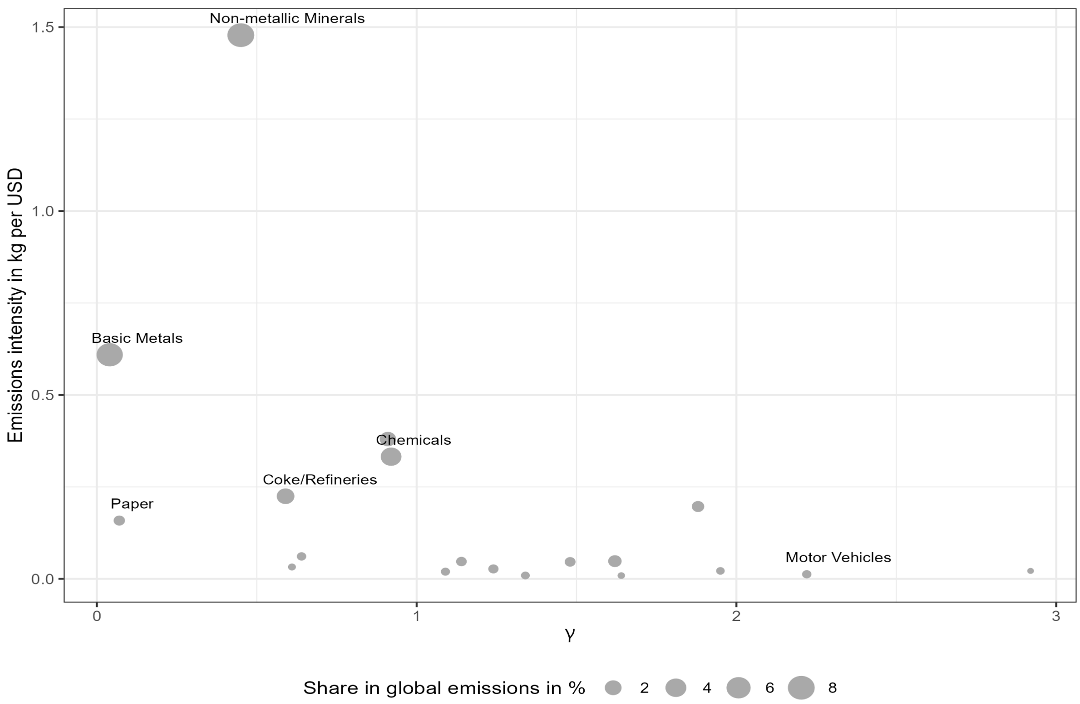
[24] Note that, as mentioned in Section 5.1, this is calculated using only combustion emissions.

Table 1: Parameter Estimates

Sector	Estimate	Std. Err.	γ
C10-C12	-0.09	(0.008)	1.62
C13-C15	-0.19	(0.027)	1.48
C16	0.08	(0.033)	0.64
C17	0.21	(0.026)	0.07
C18	0.12	(0.02)	0.61
C19	0.13	(0.122)	0.59
C20	0.09	(0.024)	0.92
C21	-0.15	(0.042)	1.64
C22	0.02	(0.014)	0.91
C23	0.08	(0.023)	0.45
C24	0.16	(0.024)	0.04
C25	-0.03	(0.01)	1.14
C26	-0.07	(0.019)	1.34
C27	-0.05	(0.017)	1.09
C28	-0.13	(0.01)	1.24
C29	-0.15	(0.02)	2.22
C30	-0.12	(0.038)	1.95
C31-C32	-0.11	(0.016)	1.88
C33	-0.29	(0.028)	2.92
Total	-0.05	(0.004)	1.20

Note: Estimation of γ according to (27). The first colum presents the estimation result. The second column presents the corresponding standard error. The implied γ is calculated based on the value of σ from Lashkaripour and Lugovskyy (2023).

Figure 1: Emission intensity and γ_s



Note: The figure shows the estimated γ and the emission intensity for all manufacturing sectors and mining. The size of the points reflects the sector's share in global emissions.

5.3 Results

5.3.1 Single Industry

To quantitatively illustrate the role of firm heterogeneity in shaping the emissions effect of international trade, we simulate a uniform 40% decrease in iceberg trade costs. We need a multi-industry model to fully understand the effects of trade on emissions. We nevertheless start with an aggregate consideration as it can accentuate the role of firm heterogeneity within the industry.

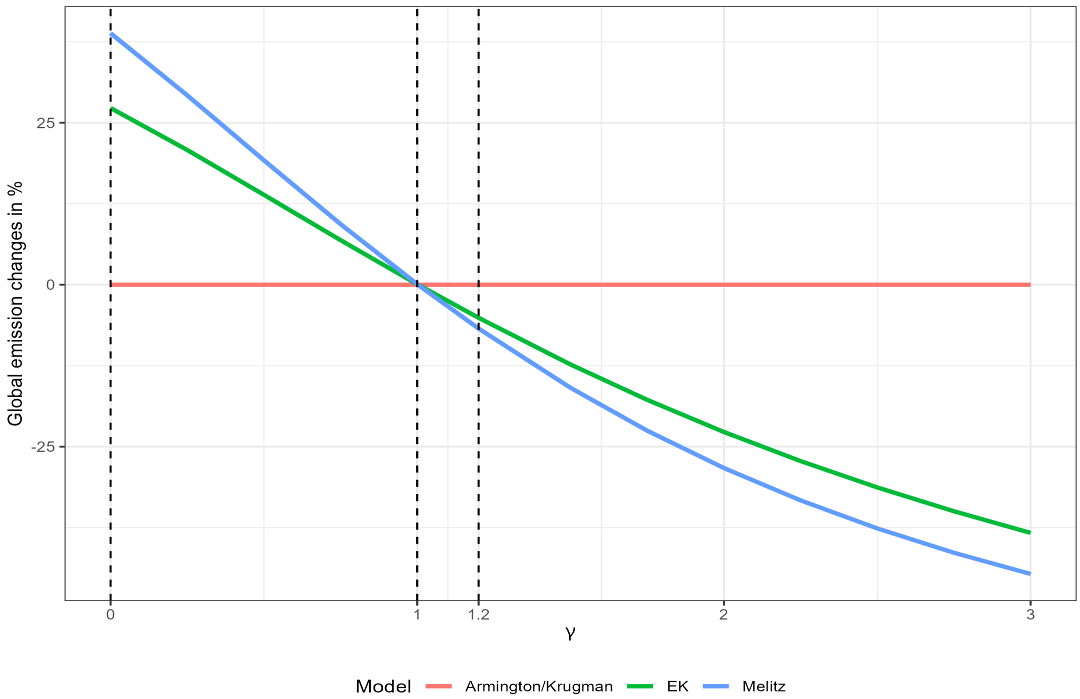
Figure 2 shows the global emission response for the different models and varying values for γ. Recall that γ captures the relationship between emission intensity and productivity: The larger γ, the cleaner and more productive firms are. The figure displays the effects for $\gamma \in [0, 3]$. The left end ($\gamma = 0$) corresponds to the quantity-based emission case in which the physical emission intensity is independent of the productivity. The two dashed vertical lines indicate the special case when emissions are proportional to value, i.e., $\gamma = 1$, and when γ lies at our estimated aggregate value of 1.2. The red line shows the emission effect in the aggregate Armington (and, by equivalence, Krugman) model. The blue and green curves refer to the Melitz and EK models.

In line with the intuition from Propositions 2 and 6, emissions do not change in response to the trade liberalization shock in the Armington and Krugman models for any value of γ, as well as in the EK and Melitz models if $\gamma = 1$. In the quantity case ($\gamma = 0$), consistent with the intuition from Proposition 5, emissions increase in the EK and Melitz models. The increase in global emissions is considerable: More than a quarter in EK and more than a third in Melitz. Global emissions increase in the two models with firm heterogeneity for all $\gamma < 1$. However, as γ increases, the emission increase becomes weaker because the scale effect (the only active effect in the quantity case) gets increasingly offset by the technique effect.

For all $\gamma > 1$, global emissions decrease in the EK and Melitz models in response to trade liberalization. While the production scale increases, reallocating production towards more productive firms significantly reduces the average emissions intensity. This technique effect is stronger than the scale effect, which reduces the aggregate emissions. For the estimated $\gamma = 1.2$, global emissions decrease by approximately 7% in Melitz and 5% in EK. However, it is premature to conclude that firm heterogeneity positively affects the emission effects of trade or that trade generally lowers global carbon emissions. First, the previous section shows that the aggregate estimated γ hides considerable heterogeneity across sectors. Second, a full quantification of the trade effects needs to consider sectoral reallocations — which can alter global emissions irrespective of sectoral differences in γ.

The Melitz model's emission response is slightly stronger than EK for all values of γ. However, the difference between this group of two models with producer heterogeneity and the two homogenous producer models with a zero-emission response is much more pronounced than the difference within the heterogenous producer model group.

Figure 2: Changes in emissions for different γ



Note: The figure shows the changes in global emissions due to a 40% uniform trade liberalization for different values of γ.

Table 3 in Appendix L additionally shows the *national* emission changes in EK and Melitz for the two cases in which emissions are assumed to be proportional to the physical quantity produced ($\gamma = 0$) or γ taking the estimated value of 1.2. As expected, emissions increase for all countries in the quantity case and decrease for $\gamma = 1.2$. However, there is significant heterogeneity across countries. In the quantity case, the emissions increase ranges from 17.79% in the US to 81.97% in Cyprus in the EK model and from 10.51% in Luxemburg to 51.21% in Cyprus in the Melitz model. For the case of $\gamma = 1.2$, the emission reductions range from 3.06% in the US to 15.05% in Lithuania in the EK model and from 4.32% in the US to 13.21% in Ireland in the Melitz model. Importantly, the composition effect of international trade, which involves *shifting* the industry composition, is inherently excluded from these results.

5.3.2 Multiple Industries

As discussed in Section 4, the emission response to trade liberalization can differ across models in multi-industry settings for two distinct and potentially interacting reasons: Firm heterogeneity as in the single-industry models and differential trade responses due to scale economies. We compare the multi-industry Armington, Krugman, EK, and Melitz models, and for the models with firm heterogeneity, consider four cases with $\gamma_s = 0$, $\gamma_s = 1$, $\gamma_s = 1.2$, and estimated γ_s. As with the aggregate quantification, we simulate a uniform cut in international trade costs by 40%.

The results are illustrated in Figure 3. Just as in the aggregate consideration, the choice of γ_s does not affect the results in the Armington and Krugman models because there is no firm heterogeneity. Therefore, only one result bar is shown for each of these models. However, global emissions are affected by trade liberalization, which is different from the single-industry case. The global emissions increase mildly by less than one percent (Armington) and 1.6% (Krugman).

Increased emissions in these models are driven by a global composition effect, i.e., trade liberalization induces countries to specialize in more emission-intensive industries. However, these findings suggest that these compositional shifts play a relatively minor role in global emissions for this specific counterfactual. Even though both the Armington and the Krugman models feature no heterogenous productivity, the results are not identical across these two models. This difference is because trade outcomes and the associated emission effects from sectoral reallocation are not identical, as the Krugman model features scale economies.

In line with Proposition 7, if $\gamma_s = 1$, the emission response is identical within the two groups of perfect competition models without scale economies (Armington and EK) and monopolistic competition models with scale economies (green bars in Figure 3). Within every industry, productivity heterogeneity does induce additional scale and technique effects. Still, as in the single industry case, these cancel out if the KR elasticities are unity.

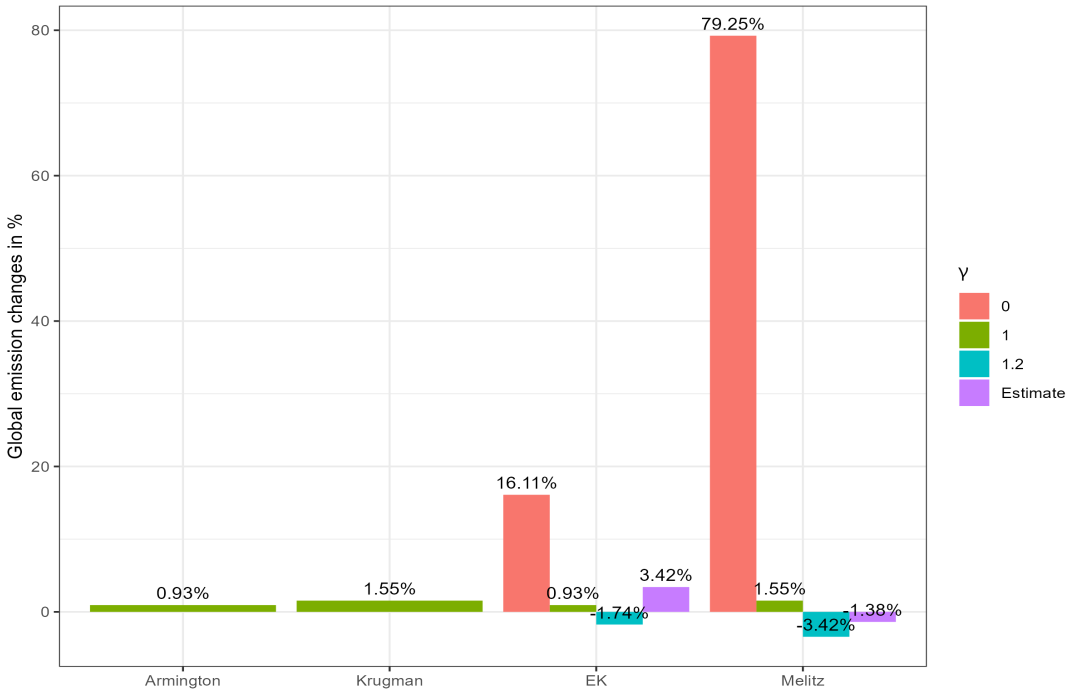
Further focussing on the EK model, when we use the estimated γ_s for each industry, a 40% reduction in trade costs leads to a 3.4% increase in global emissions (purple bar). Accounting for productivity heterogeneity, hence, in this case, more than triples the global emission increase associated with the trade liberalization shock. Note that this contrasts with the result of a single industry, where the trade liberalization resulted in a decrease in global emissions. This stresses the importance of taking the heterogeneity in the relationship between productivity and emission intensity across sectors. We can further illustrate this by using the aggregate γ estimate of 1.2 for all sectors in the multi-sector EK model (turquoise bar). In this case, within-sector reallocations across firms induce a net emission reduction and emissions decrease in response to the trade liberalization shock by 1.7%. As a final multi-sector EK consideration, assuming emissions are directly proportional to output ($\gamma_s = 0$), EK models would forecast a substantial increase in global emissions by over 16%.

Turning to the multi-industry Melitz case with emissions proportional to quantities (i.e. $\gamma_s = 0$), emissions increase dramatically by almost 80%. On the other hand, taking the optimistic $\gamma_s = 1.2$ case leads to an environmentally beneficial reallocation across firms that reduces global emissions by 3.4%. These two effects are qualitatively in line with the EK case, but the effects are considerably stronger. In the case in which we use the sectoral estimates of γ_s, even the qualitative prediction is different in the Melitz model: Global emissions *decrease* by 1.4% in response to the trade liberalization shock. These contrasting results illustrate the effect of scale economies and firm heterogeneity interact in a non-trivial way. Adding only scale economies (i.e. moving from Armington to Krugman) slightly increases the emission effect. Only adding firm heterogeneity (i.e. moving from Armington to EK) considerably increases the emission effect. However, adding both scale economies and firm heterogeneity (i.e., moving from Armington to Melitz) in a setting with flexible and sectorally varying KR elasticities alters the emission prediction altogether, turning the increase into a decrease.

The stark differences in global emissions prediction across models underscore the quantitative significance of firm heterogeneity and the importance of accurately modeling the emissions-productivity relations. Both the exact model specification and the parametrization make a sizable difference in the quantitative results.

The country-level results are shown in Table 4 in Appendix L. When we look at the individual countries, there is a huge heterogeneity in emission changes. When we use the estimated γ_s, while some countries like Denmark and Poland witness a stark increase in emissions, countries like Luxembourg and Malta witness a stark decline in emissions. Overall, we observe different models (i.e., Armington, Krugman, EK, and Melitz) with different γ_s values, resulting in quantitatively significant global and country-level differences.

Figure 3: Changes in global emissions by model and γ_s



Note: The figure shows the changes in global emissions due to a 40% uniform trade liberalization for Armington and EK. Values are shown for the quantity case $\gamma_s = 0$, the value case $\gamma_s = 1$, and the estimated values of γ_s from Table 1.

6 Conclusions

In their seminal paper, ACR raise the question to how much the rise of new trade models has altered the answer to the field's central question of how large the gains from trade are. Their answer is: "So far, not much." A decade later, we investigate how strongly new trade models have affected the key question in the trade and environment subfield: What are the effects of trade on emissions?

We initially arrived at a conclusion similar to ACR's. While new trade models add interesting mechanisms via which trade affects environmental outcomes, they tend to be incorporated into quantitative models, leaving aggregate emission effects unaltered. If emissions are linked to production value, the emission-saving effect of reallocating to cleaner producers is perfectly offset by an emission-increasing impact of higher overall production.

However, upon closer inspection, the answer is more nuanced. Even in cases where the ACR's trade equivalence holds, emissions equivalence breaks whenever emissions are linked to quantities rather than values. We further generalize the relationship between quantity and emissions to be productivity-dependent. Depending on how clean the productive firms are, accounting for firm heterogeneity can shed a better or worse light on the environmental consequences of international trade.

The basic intuition holds for multiple industries but requires additional caution because the ACR's trade equivalence no longer generally holds in multi-industry settings. In the model of multiple industries with emissions linked to value, the Armington and EK models still yield identical aggregate emission effects because the ACR-type trade equivalence continues to hold. A multi-industry Melitz model yields different aggregate emission effects, however, only because the trade effects and associated global changes in sectoral composition differ due to the presence of scale economies. It, therefore, coincides with a multi-sector Krugman model. If the emission intensity across industries is uniform, trade liberalization does not change global emissions in either model. We also provide a generalized multi-industry case in which emissions are flexibly linked to quantities, and firm heterogeneity matters for aggregate emission outcomes. Our quantitative exercises illustrate the importance of the elasticity with which emission intensity depends on productivity. If the link is relatively weak, firm heterogeneity worsens the emission effects of trade. If it is strong, firm heterogeneity can make trade environmentally beneficial. We estimate the key elasticity using German firm-level data and quantify our multi-industry model. Our results illustrate that the emission effect of firm heterogeneity and scale economies interact non-trivially. Firm heterogeneity fosters the emission increase induced by trade liberalization in a perfect competition environment while lowering global emissions in a monopolistically competitive world. These contrasting results illustrate the quantitative importance of getting both the model structure and the parametrization right.

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Appendix

A Proof for the exact hat algebra

We start with the change in the trade shares $\hat{\lambda}_{ni}$. For all i. We take the ratio of trade values:

$$\begin{aligned}\hat{\lambda}_{ni} &= \frac{\chi_{ij} \cdot N_i \cdot (\tilde{c}_i(w_i, t_i)\tau_{ni})^{\varepsilon}}{\sum_{k=1}^{N} \chi_{kj} \cdot N_k \cdot (\tilde{c}(w_k, t_k)\tau_{ki})^{\varepsilon}} \cdot \frac{\sum_{k=1}^{N} \chi_{kj} \cdot N'_k \cdot (\tilde{c}(w'_k, t'_k)\tau'_{ki})^{\varepsilon}}{\chi_{ij} \cdot N'_i \cdot (\tilde{c}_i(w'_i, t'_i)\tau_{ni'})^{\varepsilon}} \\ &= \frac{\hat{N}_i \left(\hat{\tau}_{ni}\hat{c}_i\right)^{\varepsilon}}{\sum_{k=1}^{N} \lambda_{nk}\hat{N}_k \left(\hat{\tau}_{nk}\hat{c}_k\right)^{\varepsilon}}\end{aligned}$$

where $\hat{c}_i = \frac{c_i(w'_i, t'_i)}{c_i(w_i, t_i)}$. Notice that c_i is homogeneous degree of one, and $\hat{t}_i = \hat{w}_i$. Therefore, we can simply calculate the change in costs as follows:

$$\hat{c}_i = \frac{c_i(w'_i, t'_i)}{c_i(w_i, t_i)} = \hat{w}_i \frac{c_i(w'_i/\hat{w}_i, t'_i/\hat{w}_i)}{c_i(w_i, t_i)} = \hat{w}_i \frac{c_i(w_i, t_i)}{c_i(w_i, t_i)} = \hat{w}_i.$$

Furthermore, as discussed in ACR, the first two restrictions imply $\hat{M}_i = 1$. Using these relationships, we have:

$$\hat{\lambda}_{ni} = \frac{\left(\hat{\tau}_{ni}\hat{w}_i\right)^{\varepsilon}}{\sum_{k=1}^{N} \lambda_{nk} \left(\hat{\tau}_{nk}\hat{w}_k\right)^{\varepsilon}}.$$

Now we discuss the goods market clearing. The goods market clearing assumption implies:

$$\hat{R}_i R_i = \sum_{n=1} \hat{\lambda}_{ni}\lambda_{ni}R'_k.$$

The change in the revenue (output) can be written as

$$\begin{aligned}\hat{R}_i &= \hat{w}_i \frac{w_i l_i}{w_i l_i + t_i Z_i + \Pi_i} + \hat{t}_i \hat{Z}_i \frac{t_i Z_i}{w_i l_i + t_i Z_i + \Pi_i} + \hat{\Pi}_i \frac{\Pi_i}{w_i l_i + t_i Z_i + \Pi_i} \\ &= \hat{w}_i \frac{w_i l_i}{w_i l_i + t_i Z_i + \Pi_i} + \hat{R}_i \frac{t_i Z_i}{w_i l_i + t_i Z_i + \Pi_i} + \hat{R}_i \frac{\Pi_i}{w_i l_i + t_i Z_i + \Pi_i}\end{aligned}$$

where the second equality uses the fact that $\hat{Z}_i = \hat{R}_i/\hat{t}_i$ and $\hat{\Pi}_i = \hat{R}_i$. This derivation implies $\hat{R}_i = \hat{w}_i$.

B Fixing absolute emission taxes

We focus on the global emissions, which is a summation of emissions across countries:

$$Z = \sum_{i=1}^{N} Z_i.$$

The changes associated with trade shocks are:

$$\hat{Z} = \sum_{i=1}^{N} \rho_i \hat{Z}_i = \sum_{i=1}^{N} \rho_i \hat{\beta}_i(\hat{w}_i, w_i/t_i)\hat{R}_i$$

where ρ_i is the share of emissions from country i (Z_i/Z) and $\hat{\beta}_i(\hat{w}_i, w_i/t_i) \equiv \beta_i(w_i'/t_i)/\beta_i(w_i/t_i)$ is a change in the emission intensity. We will need to calculate $\hat{R}_i$ given that $\hat{t}_i$ is fixed now:

$$\begin{aligned}
\hat{R}_i &= \frac{\hat{w}_i w_i L_i}{w_i L_i + t_i Z_i + \Pi_i} + \frac{\hat{Z}_i t_i Z_i}{w_i L_i + t_i Z_i + \Pi_i} + \frac{\hat{\Pi}_i \Pi_i}{w_i L_i + t_i Z_i + \Pi_i} \\
&= \frac{\hat{w}_i w_i L_i}{w_i L_i + t_i Z_i + \Pi_i} + \frac{\hat{\beta}_i(\hat{w}_i, w_i/t_i)\ \hat{R}_i \beta_i(w_i/t_i) R_i}{w_i L_i + t_i Z_i + \Pi_i} + \frac{\hat{R}_i \Pi_i}{w_i L_i + t_i Z_i + \Pi_i}
\end{aligned}$$

which implies:

$$\hat{R}_i = \frac{\hat{w}_i w_i L_i}{w_i L_i - \hat{\beta}_i(\hat{w}_i, w_i/t_i)\beta_i(w_i/t_i))R_i}$$

and hence:

$$\hat{Z} = \sum_{i=1}^{N} s_i \hat{\beta}_i(\hat{w}_i, w_i/t_i) \frac{\hat{w}_i w_i L_i}{w_i L_i - \hat{\beta}_i(\hat{w}_i, w_i/t_i)\beta_i(w_i/t_i))R_i}.$$

We also need to reconsider the trade share equations since now the changes in wages and taxes do not coincide:

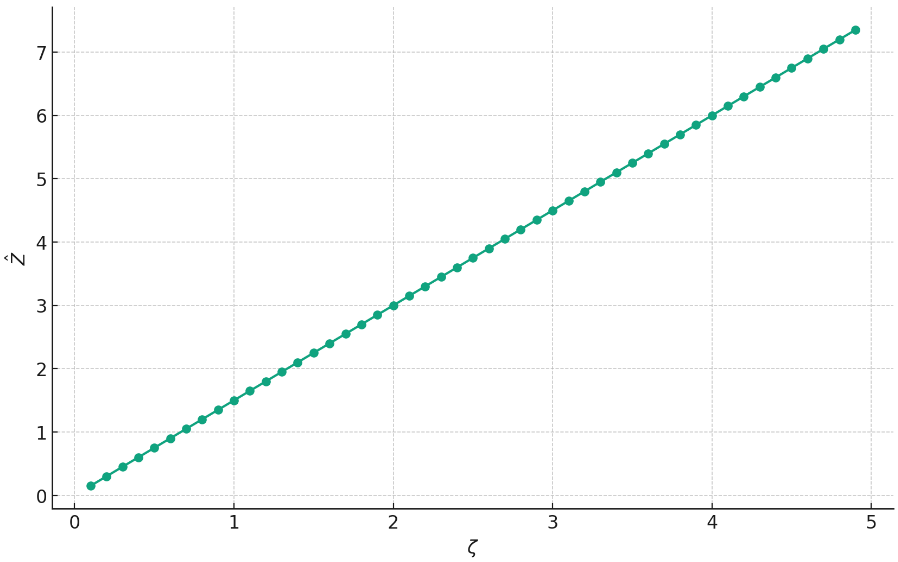
$$\hat{\lambda}_{ni} = \frac{(\hat{\tau}_{ni}\hat{c}_i)^{\varepsilon}}{\sum_{k=1}^{N} \lambda_{nk} (\hat{\tau}_{nk}\hat{c}_k)^{\varepsilon}}.$$

where $\hat{c}_i = \frac{c_i(w_i', t_i)}{c_i(w_i, t_i)}$. Using these equations, we construct an example where the global emissions depend on the normalization scheme. We choose the normalization scheme where we fix the wage change of country one as ζ. If $\zeta = 1$, the normalization scheme fixes the wages of country 1 to be the same before and after the counterfactual experiment. For the example, we use a symmetric two-country model with β fixed (this corresponds to the Cobb-Douglas emission function, which is common in the literature). Figure **??** shows the changes in global emissions for a 40% reduction in trade costs for various values of ζ. The figure indicates that the normalization scheme matters when we fix the absolute emission tax level. Especially the result suggests that increasing the price of the numeraire of goods results in higher global emissions.

C Deriving emission for the quantity case and the flexible specification case

Since the quantity case is a special case of the flexible specification with $\gamma_i = 0$, we omit the proof and only exhibit the proof of the flexible specifications.

Figure 4: Comparing emissions for different normalization schemes



C.1 EK

We start from the fact that the price of goods exported from country i to country n also follows a Frechet distribution. Specifically, the price distribution of goods exported from country i to country n ,$G_{ni}(p)$, only depends on the destination country:

$$\begin{aligned} &Pr\left(p_{in}(\omega) = p | p_{in}(\omega) \leq min_{k=1,\dots,N} p_{ki}(\omega)\right) \\ &= G_{ni}(p) = exp(-\Phi_n p^\theta), \end{aligned}$$

where $\Phi_n = \Gamma\left(\frac{\theta+1-\sigma}{\theta}\right)^{\theta/(1-\sigma)} P_n^{-\theta} = \int_0^\infty p^{1-\sigma} dG_{ni}(p)$ is a function of the price index in country n. The productivity of a product with price p_{ni} is $\frac{w_i \tau_{ni}}{p_{ni}}$, and hence the emissions of a particular producer with price p_{ni} is

$$z_{ni}(p_{ni}(\omega)) = \mathbf{1}(p_{ni}(\omega) \leq min_k p(\omega)_{nk}) \mu_i \left(\frac{p_{ni}}{w_i \tau_{ni}}\right)^{\gamma_i} \frac{p_{ni}(\omega)^{-\sigma}}{P_n^{1-\sigma}} \tau_{ni} X_i.$$

Hence, the aggregate emissions are

$$\begin{aligned}
Z_{ni} &= \int_0^1 z_{ni}(\omega)d\omega \\
&= \int_0^1 \mathbf{1}(p_{ni}(\omega) \leq min_k p_{nk}(\omega))\mu_i \left(\frac{p_{ni}}{w_i\tau_{ni}}\right)^{\gamma_i} \frac{\mu_i p_{ni}(\omega)^{-\sigma}}{P_n^{1-\sigma}} X_n d\omega \\
&= \lambda_{ni} \left(w_i\tau_{ni}\right)^{-\gamma_i} \int_0^\infty \frac{\mu_i p^{\gamma_i-\sigma}}{P_n^{1-\sigma}} \tau_{ni} X_n dG_{ni}(p) \\
&= X_{ni}\mu_i\tau_{ni} \left(w_i\tau_{ni}\right)^{-\gamma_i} \frac{1}{\Gamma\left(\frac{\theta+1-\sigma}{\theta}\right)} \Phi_n^{(\sigma-1)/\theta} \int_0^\infty p^{\gamma_i-\sigma} dG_{ni}(p) \\
&= X_{ni}\mu_i\tau_{ni} \left(w_i\tau_{ni}\right)^{-\gamma_i} \frac{1}{\Gamma\left(\frac{\theta+1-\sigma}{\theta}\right)} \Phi_n^{(\sigma-1)/\theta} \Phi_n^{(\gamma_i-\sigma)/\theta} \Gamma\left(\frac{\theta+\gamma_i-\sigma}{\theta}\right) \\
&= X_{ni}\mu_i\tau_{ni} \left(w_i\tau_{ni}\right)^{-\gamma_i} \frac{\Gamma\left(\frac{\theta+\gamma_i-\sigma}{\theta}\right)}{\Gamma\left(\frac{\theta+1-\sigma}{\theta}\right)} \Phi_n^{(\gamma_i-1)/\theta} \\
&= X_{ni}\mu_i \left(w_i^{-\theta}\tau_{ni}^{-\theta}T_i\right)^{(\gamma_i-1)/\theta} w_i^{-1} T_i^{(1-\gamma_i)/\theta} \frac{\Gamma\left(\frac{\theta+\gamma_i-\sigma}{\theta}\right)}{\Gamma\left(\frac{\theta+1-\sigma}{\theta}\right)} \Phi_n^{(\gamma_i-1)/\theta} \\
&= \frac{X_{ni}}{w_i}\mu_i \left(\frac{T_i}{\lambda_{ni}}\right)^{(1-\gamma_i)/\theta} \frac{\Gamma\left(\frac{\theta+\gamma_i-\sigma}{\theta}\right)}{\Gamma\left(\frac{\theta+1-\sigma}{\theta}\right)}.
\end{aligned}$$

This leads to the following exact hat algebra:

$$\hat{Z}_{ni}^{EK} = \frac{\hat{X}_{ni}}{\hat{w}_i} \hat{\lambda}_{ni}^{(\gamma_i-1)/\theta},$$

and the corresponding emission share ι_{ni} is given by

$$\iota_{ni}^{EK} = \frac{\lambda_{ni}^{(\gamma_i-1)/\theta} X_{ni}}{\sum_{k=1} \lambda_{ki}^{(\gamma_i-1)/\theta} X_{ki}}.$$

C.2 Melitz model

We start by expressing the physical quantity in terms of revenues, i.e.:

$$q_{ni}(\omega) = \frac{x_{ni}\tau_{ni}}{p_{ni,\omega}(w_i, t_i)}$$

Then emissions embodied in the trade flow of variety ω from i to n can be expressed in terms of

$$z_{ni}(\omega) = \frac{\mu_i \tau_{ni} x_{ni}(\omega)}{\tau_{ni}\tilde{\sigma} w_i (\varphi(\omega))^{\gamma_i}}.$$

In Melitz, the only firm-specific component of the unit costs is the productivity, $\varphi(\omega)$, i.e.

$$c_{i,\omega} = \frac{w_i}{\varphi(\omega)}.$$

Thus, emissions embodied in trade flows from i to n:

$$Z_{ni} = \frac{\mu_i}{\tilde{\sigma} w_i} M_{ni} \int_{\omega \in \Omega_{ni}} (\varphi(\omega))^{1-\gamma_i} x_{ni}(\omega) dG(\varphi(\omega)|\omega \in \Omega_{ni}).$$

With exports of firm ω from i to n of

$$x_{ni}(\omega) = (\varphi(\omega))^{\sigma-1} \left(\tilde{\sigma} w_i \tau_{ni}\right)^{1-\sigma} w_n L_n P_n^{\sigma-1},$$

To enter the foreign market, firms have to pay a fixed cost of $w_n f_{ni}$, which leads to a unique productivity cut-off of

$$(\varphi_{ni}^*)^{\sigma-1} = \frac{f_{ni}}{L_n P_n^{\sigma-1}} \left(\tilde{\sigma} w_i \tau_{ni}\right)^{\sigma-1}.$$

Because $\varphi(\omega)$ is the only parameter that varies at the firm level, we henceforth only index firms by their productivity. The embodied emissions are given by

$$Z_{ni} = \frac{\mu_i}{\tilde{\sigma} w_i} w_n L_n P_n^{\sigma-1} M_{ni} \int_{\varphi_{ni}^*}^{\infty} \varphi^{\sigma-\gamma_i} dG(\varphi|\varphi \geq \varphi_{ni}^*).$$

With the assumption of Pareto distributed productivity, i.e. $G_i(\varphi) = \left(\frac{T_i}{\varphi}\right)^{\theta}$ we get:

$$Z_{ni} = \frac{\mu_i}{\tilde{\sigma} w_i} w_n L_n P_n^{\sigma-1} \frac{\theta}{\theta + \gamma_i - \sigma} M_{ni} (\varphi_{ni}^*)^{\sigma-\gamma_i}.$$

Expressing $(\varphi_{ni}^*)^{\sigma-\gamma_i}$ in terms of revenues and the share of exporting firms:

$$S_{ni} = \left(\frac{\varphi_{ni}^*}{\varphi_{ii}^*}\right)^{-\theta},$$

yields:

$$Z_{ni} = \frac{\mu_i}{\tilde{\sigma} w_i} S_{ni}^{-(1-\gamma_i)/\theta} (\varphi_{ii}^*)^{1-\gamma_i} M_{ni} \frac{\theta}{\theta + \gamma_i - \sigma} x_{ni}(\varphi_{ni}^*).$$

Noting that

$$M_{ni} \frac{\theta}{\theta + \gamma_i - \sigma} x_{ni}(\varphi_{ni}^*) = M_{ni} E[x_{ni}|\varphi \geq \varphi_{ni}^*] = X_{ni},$$

yields:

$$Z_{ni} = \frac{\mu_i}{\tilde{\sigma} w_i} \frac{\theta}{\theta + \gamma_i - \sigma} S_{ni}^{-(1-\gamma_i)/\theta} (\varphi_{ii}^*)^{1-\gamma_i} X_{ni}.$$

This leads to the exact hat algebra expression of:

$$\hat{Z}_{ni}^{Melitz} = \frac{\hat{X}_{ni}}{\hat{w}_i} \hat{S}_{ni}^{-(1-\gamma_i)/\theta} (\widehat{\varphi_{ii}^*})^{1-\gamma_i},$$

which yields (17) and for $\gamma = 0$ (13). The corresponding weights to calculate aggregate emissions are given by

$$\iota_{ni}^{Melitz} = \frac{S_{ni}^{-(1-\gamma_i)/\theta} X_{ni}}{\sum_{k=1}^{N} S_{ki}^{-(1-\gamma_i)/\theta} X_{ki}}.$$

D Proof for proposition 5 and 6

We discuss the Armington, EK, and the Melitz model sequentially. We only prove the case for $\gamma_i < 0$ since we can prove the case of $\gamma_i > 0$ and $\gamma_i = 1$ in a similar manner. The proof of the quantity case is omitted since it is a special case with $\gamma_i = 0$.

D.1 Armington model

Because there is only a single variety in each country, we omit the notion of ω. The total emission of country i is proportional to the quantity produced:

$$Z_i = \mu_i \varphi^{-\gamma_i} Q_i,$$

and the produced quantity is:

$$Q_i = \varphi_i L_i.$$

Notice that the trade shocks that move the economy from autarky to trade economy do not alter the labor endowment l_i and the productivity φ_i. Therefore, the changes in overall emission of country i before and after the autarky are:

$$Z_i^{TRADE} = \mu_i \varphi_i^{1-\gamma_i} L_i = Z_i^{AUT}.$$

D.2 EK model

For the EK model, we utilize the expression for emissions embodied in trade:

$$Z_{ni} = \frac{X_{in}}{w_i} \left(\frac{T_i}{\lambda_{ni}} \right)^{(1-\gamma_i)/\theta} \frac{\Gamma\left(\frac{\theta+\gamma_i-\sigma}{\theta}\right)}{\Gamma\left(\frac{\theta+1-\sigma}{\theta}\right)}.$$

and the aggregate emission is

$$\begin{aligned} Z_i &= \frac{Y_i T_i^{1/\theta}}{w_i} \sum_{n=1}^{N} \frac{X_{ni}}{Y_i} \lambda_{in}^{(\gamma_i-1)/\theta} \frac{\Gamma\left(\frac{\theta+\gamma_i-\sigma}{\theta}\right)}{\Gamma\left(\frac{\theta+1-\sigma}{\theta}\right)} \\ &= l_i T_i^{1/\theta} \sum_{n=1} s_{in} \lambda_{in}^{(\gamma_i-1)/\theta} \frac{\Gamma\left(\frac{\theta+\gamma_i-\sigma}{\theta}\right)}{\Gamma\left(\frac{\theta+1-\sigma}{\theta}\right)}, \end{aligned}$$

where $s_{ni} = \frac{X_{ni}}{Y_i}$. Since λ_{ii} is unity in autarky, we have

$$Z_i^{AUT} = l_i T_i^{1/\theta} \frac{\Gamma\left(\frac{\theta+\gamma_i-\sigma}{\theta}\right)}{\Gamma\left(\frac{\theta+1-\sigma}{\theta}\right)}.$$

because $s_{ii} = 1$ and $\lambda_{ii} = 1$ in autarky. Notice that with finite trade costs, $0 < \lambda_{in} < 1$. If $\gamma_i < 1$, we have hence $\lambda_{in}^{(\gamma_i - 1)/\theta} > 1$. Combining these questions and $\sum_{n=1}^{N} s_{in} = 1$, we have:

$$
\begin{aligned}
Z_i^{AUT} = l_i T_i^{1/\theta} &= l_i T_i^{1/\theta} \sum_{n=1}^{N} s_{in} \frac{\Gamma\left(\frac{\theta+\gamma_i-\sigma}{\theta}\right)}{\Gamma\left(\frac{\theta+1-\sigma}{\theta}\right)} \\
&< l_i T_i^{1/\theta} \sum_{n=1}^{N} s_{in} \lambda_{in}^{(\gamma_i-1)/\theta} \frac{\Gamma\left(\frac{\theta+\gamma_i-\sigma}{\theta}\right)}{\Gamma\left(\frac{\theta+1-\sigma}{\theta}\right)} = Z_i^{TRADE}.
\end{aligned}
$$

We can prove the case for $\gamma_i = 1$ and $\gamma_i > 1$ in a similar manner.

D.3 Melitz model

We start by rewriting emissions embodied in trade flows as

$$Z_i = \frac{\mu_i L_i}{\tilde{\sigma}} \frac{\theta}{\theta + \gamma_i - \sigma} \left(\varphi_{ii}^*\right)^{1-\gamma_i} \sum_{n \in N} S_{ni}^{(1-\gamma_i)/\theta} \frac{X_{ni}}{w_i L_i}. \tag{28}$$

By the zero profit condition of exporting the domestic productivity cut-off is

$$\varphi_{ii}^* = \frac{f_{ii}}{P_i} w_i \tilde{\sigma} \tau_{ii}.$$

Hence $\varphi_{ii}^* \propto \frac{w_i}{P_i}$. As shown by Arkolakis et al. (2012) trade opening increases the real wage and thus must rise the domestic productivity cut-off. Hence $\varphi_{ii}^{*,AUT} < \varphi_{ii}^{*,TRADE}$. Further, under Autarky $\frac{X_{ii}}{w_i L_i} = S_{ii} = 1$ and under conventional parameter restrictions, $S_{ni} < 1$. As a consequence,if $\gamma_i < 1$, $S_{ni}^{(\gamma_i-1)/\theta} > 1$. In combination with $\sum_{n \in N} \frac{X_{ni}}{w_i L_i} = 1$ we get:

$$
\begin{aligned}
Z_i^{AUT} = \frac{\mu_i L_i}{\tilde{\sigma}} \frac{\theta}{\theta + \gamma_i - \sigma} \left(\varphi_{ii}^{*,AUT}\right)^{1-\gamma_i} &= \frac{\mu_i L_i}{\tilde{\sigma}} \frac{\theta}{\theta + \gamma_i - \sigma} \left(\varphi_{ii}^{*,AUT}\right)^{1-\gamma_i} \sum_{n \in N} \frac{X_{ni}}{w_i L_i} \\
&< \frac{\mu_i L_i}{\tilde{\sigma}} \frac{\theta}{\theta + \gamma_i - \sigma} \left(\varphi_{ii}^{*,TRADE}\right)^{1-\gamma_i} \sum_{n \in N} S_{ni}^{(1-\gamma_i)/\theta} \frac{X_{ni}}{w_i L_i}.
\end{aligned}
$$

The proof works similar for $\gamma_i = 0$ and $\gamma_i = 1$.

E Derivation of (18)

The multi-sector environment in Section 4 assumes the following nested demand structure. The upper nest is Cobb-Douglas with spending share ζ_{is} on sector s in country i. Consumers have CES preferences within each sector across goods from different countries with elasticity η_s and varieties from a specific origin with elasticity σ_s. Hence, consumers in i spend on variety ω from n in s:

$$p_{nis}(\omega) q_{nis}(\omega) = p_{nis}^{1-\sigma_s} (P_{nis})^{\sigma_s - 1} \left(\frac{P_{nis}}{P_{is}}\right)^{1-\eta_s} E_{is}, \tag{29}$$

with the price indices:

$$P_{is} = \left(\sum_{n \in N} P_{nis}^{1-\eta_s} \right)^{1/(1-\eta_s)}$$

$$P_{nis} = \left(\int_{\omega \in \Omega_{nis}} p_{nis}(\omega)^{1-\sigma_s} d\omega \right)^{1/(1-\sigma_s)}$$

Hence, the spending share of consumers in i on sector s country n is given by:

$$\lambda_{nis} = \frac{\int_{\omega \in \Omega_{nis}} p_{nis}^{1-\sigma_s}(\omega) d\omega (P_{nis})^{\sigma_s - \eta_s}}{\sum_{l \in N} \int_{\omega \in \Omega_{lis}} p_{lis}^{1-\sigma_s}(\omega) d\omega (P_{lis})^{\sigma_s - \eta_s}} \tag{30}$$

With the definition of the price index, this implies:

$$\lambda_{nis} = \frac{P_{nis}^{1-\eta_s}}{\sum_{l \in N} P_{lis}^{1-\eta_s}} \tag{31}$$

E.1 Armington

In Armington, the price index is

$$P_{nis} = \frac{\tau_{nis} c(w_n, t_n)}{\varphi_{ns}}$$

As a consequence, the expenditure share is given by

$$\lambda_{nis} = \frac{\left(\frac{\tau_{nis} c_n(w_n, t_n)}{\varphi_{ns}} \right)^{1-\eta_s}}{\sum_{l \in N} \left(\frac{\tau_{lis} c_l(w_l, t_l)}{\varphi_{ls}} \right)^{1-\eta_s}} \tag{32}$$

which implies equation (18) by setting the trade elasticity to $\epsilon_s = \eta_s - 1$ and $\xi_{ins} = \varphi_{ns}^{\eta_s - 1}$.

E.2 EK

In the EK model, there are only two nests ($\eta_s = \sigma_s$) since there is no distinction between the foreign and the domestic varieties. The expenditure share is given by

$$\lambda_{nis} = \frac{\left(\tau_{nis} \tilde{c}_i(w_i, t_i) \right)^{-\theta_s} T_{is}}{\sum_{l \in N} \left(\tau_{nls} \tilde{c}_l(w_l, t_l) \right)^{-\theta_s} T_{ls}} \tag{33}$$

which implies equation (18) by setting the trade elasticity to $\epsilon_s = \theta_s$ and $\xi_{ins} = T_{ls}$.

E.3 Krugman

In Krugman, firms are homogeneous and engage in monopolistic competition. This market structure implies the following constant markup price for all varieties $\omega \in \Omega_{nis}$

$$p_{nis}(\omega) = \frac{\sigma_s}{\sigma_s - 1} \frac{\tau_{nis} c_n(w_n, t_n)}{\varphi_{ns}} \tag{34}$$

To enter the market, firms have to pay a fixed cost of $w_n f_{ns}^e$. Free entry implies that firms enter as long as they earn positive profits, i.e.:

$$M_{ns} w_{ns} f_{ns}^e = \sum_{i \in N} M_{ns} \frac{p_{nis} q_{nis}}{\sigma_s}$$

Because labor is the only factor of production, aggregate revenues equal labor payments. Hence, the number of firms is given by:

$$M_{ns} = \frac{\gamma_n (w_n/t_n) L_{ns}}{f_{ns}^e \sigma_s} \tag{35}$$

Thus, the price index becomes:

$$P_{nis} = \left(\frac{\gamma_n (w_n/t_n) L_{ns}}{f_{ns}^e \sigma_s} \right)^{\frac{1}{1-\sigma_s}} \frac{\sigma_s}{\sigma_s - 1} \tau_{nis}) w_n \tag{36}$$

and the expenditure share can be expressed as:

$$\lambda_{nis} = \frac{\left(\frac{\gamma_n (w_n/t_n) L_{is}}{f_{is}^e} \right)^{\frac{1-\eta_s}{1-\sigma_s}} \left(\frac{\tau_{nis} c_n(w_n, t_n)}{\varphi_{ns}} \right)^{1-\eta_s}}{\sum_{l \in N} \left(\frac{L_{ls}}{f_{ls}^e} \right)^{\frac{1-\eta_s}{1-\sigma_s}} \left(\frac{\tau_{lis} c_l(w_n, t_n)}{\varphi_{ls}} \right)^{1-\eta_s}} \tag{37}$$

equation (18) can be obtained by setting $\epsilon_s = \eta_s - 1$, $\delta_s = \frac{1-\eta_s}{1-\sigma_s}$ and $\xi_{ins} = \left(\frac{\gamma_n (w_n/t_n)}{f_{is}^e} \right)^{\frac{1-\eta_s}{1-\sigma_s}} (\varphi_{ns})^{\eta_s - 1}$.

E.4 Melitz Model

In Melitz, firms are heterogeneous in their productivity $\varphi(\omega)$, which leads to a firm-spefic price of:

$$p_{nis}(\omega) = \frac{\sigma_s}{\sigma_s - 1} \frac{\tau_{nis} c_n(w_n, t_n)}{\varphi(\omega)} \tag{38}$$

Productivity is Pareto distributed with cumulative density

$$G_{ns}(\varphi(\omega)) = 1 - \left(\frac{T_{ns}}{\varphi(\omega)} \right)^{-\theta_s}$$

Firms face two different types of fixed costs. First, they pay $w_n f_{ns}^e$ to enter the economy. Second, for each market i, which they serve, they need to pay $w_n f_{nis}$. This second fixed cost type implies that only firms with productivity higher than φ_{nis}^* serve market i from

n, s. Thus, the equilibrium price index is determined by:

$$P_{nis}^{1-\sigma_s} = M_{nis}(1-G(\varphi_{nis}^*)) \left(\frac{\sigma_s}{\sigma_s-1}\tau_{nis}c_n(w_n,t_n)\right)^{1-\sigma_s} \int_{\varphi_{nis}^*} \varphi(\omega)^{\sigma_s-1} dG(\varphi(\omega)|\varphi(\omega) \geq \varphi_{nis}^*)$$

With the Pareto distribution, the price index can be simplified to:

$$P_{nis}^{1-\sigma_s} = M_{nis} \left(\frac{\sigma_s}{\sigma_s-1}\tau_{nis}c_n(w_n,t_n)\right)^{1-\sigma_s} \theta_s T_{is}^{\theta_s}(\varphi_{nis}^*)^{\sigma_s-1-\theta_s} \tag{39}$$

Free entry into markets implies the following cut-off productivity:

$$\varphi_{nis}^* = \frac{\sigma_s}{\sigma_s-1}\tau_{nis}c_n(w_n,t_n)\left(\frac{\sigma_s w_i f_{nis}}{B_{nis}}\right)^{1/(1-\sigma_s)} \tag{40}$$

where

$$B_{nis} = (P_{nis})^{\sigma_s-1}\left(\frac{P_{nis}}{P_{is}}\right)^{1-\eta_s} E_{is}$$

Plugging into the expression for the price index and defining $\zeta_s = \frac{\theta_s+1-\sigma_s}{\sigma_s-1}$ leads to:

$$P_{nis}^{1-\sigma_s} = M_{nis}\left(\frac{\sigma_s}{\sigma_s-1}\tau_{nis}c_n(w_n,t_n)\right)^{1-\sigma_s} \theta_s T_{is}^{\theta_s}\left(\frac{\sigma_s}{\sigma_s-1}\tau_{nis}c_n(w_n,t_n)\right)^{\sigma_s-1-\theta_s}\left(\frac{\sigma_s w_i f_{nis}}{B_{nis}}\right)^{\zeta_s}$$

Defining $\zeta_s = \frac{\theta_s+1-\sigma_s}{\sigma_s-1}$, this can be expressed as:

$$P_{nis}^{1-\sigma_s+\zeta_s(\sigma_s-\eta_s)} = M_{nis}\left(\frac{\sigma_s}{\sigma_s-1}\tau_{nis}c_n(w_n,t_n)\right)^{-\theta_s} \theta_s T_{is}^{\theta_s}\left(\frac{\sigma_s w_i f_{nis}}{E_{is}P_{is}^{\eta_s-1}}\right)^{\zeta_s}$$

Now, note that

$$1-\sigma_s+\zeta_s(\sigma_s-\eta_s) = 1-\eta_s+\theta_s\left(-1+\frac{1-\eta_s}{1-\sigma_s}\right) = (1-\eta_s)\left(1+\theta_s\left(\frac{1}{\eta_s-1}-\frac{1}{\sigma_s-1}\right)\right),$$

then the price index can be rewritten as:

$$P_{nis} = M_{nis}^{\frac{\delta_s}{1-\eta_s}}\left(\frac{\sigma_s}{\sigma_s-1}\tau_{nis}c_n(w_n,t_n)\right)^{\frac{-\theta_s\delta_s}{1-\eta_s}}\left(\theta_s T_{is}^{\theta_s}\right)^{\frac{\delta_s}{1-\eta_s}}\left(\frac{\sigma_s w_i f_{nis}}{E_{is}P_{is}^{\eta_s-1}}\right)^{\frac{\zeta_s\delta_s}{1-\eta_s}} \tag{41}$$

where $\delta_s = \frac{1}{(1+\theta_s\left(\frac{1}{\eta_s-1}-\frac{1}{\sigma_s-1}\right)}$. The number of firms can be calculated from the free entry condition:

$$M_{ns}w_n f_{ns}^e = \sum_{i\in N} M_{ns}\frac{p_{ins}q_{ins}}{\sigma_s} \tag{42}$$

Firms enter the economy as long as they make positive profits in expectation, i.e:

$$w_n f_{ns}^e = \sum_{i\in N}(1-G(\varphi_{nis}^*))\left(\frac{1}{\eta_s}E[p_{ins}(\omega)q_{nis}(\omega)|\varphi \geq \varphi_{ins}^*] - w_i f_{ins}\right) \tag{43}$$

With the Pareto distribution, the average revenues are given by

$$E[p_{nis}(\omega)q_{nis}(\omega)|\varphi \geq \varphi^*_{nis}] = \frac{\theta_s}{\theta_s + 1 - \sigma_s}\left(\frac{\sigma}{\sigma_s - 1}c_n(w_n, t_n)\right)^{1-\sigma_s}(\varphi^*_{nis})^{\sigma_s - 1}B_{nis} \quad (44)$$

and fixed costs of entry must equal the revenues of the marginal firm, i.e.:

$$w_n f_{nis} = \left(\frac{\sigma}{\sigma_s - 1}c_i(w_i, t_i)\right)^{1-\sigma_s}(\varphi^*_{nis})^{\sigma_s - 1}B_{nis} \quad (45)$$

Hence, the free entry condition implies:

$$w_n f^e_{ns} = \frac{\sigma_s - 1}{\theta_s + 1 - \sigma_s}\left(\frac{\sigma}{\sigma_s - 1}\sum_{i \in N} c_i(w_i, t_i)\right)^{1-\sigma_s}(\varphi^*_{nis})^{\sigma_s - 1}B_{nis} \quad (46)$$

or expressed in terms of aggregate revenues X_{ns}:

$$M_{ns} w_n f^e_{ns} = \frac{\sigma_s - 1}{\theta_s + 1 - \sigma_s}X_{ns} \quad (47)$$

With the labor market clearing condition, we get the following:

$$M_{ns} f^e_{ns} = \gamma_n(w_n/t_n)\frac{\sigma_s - 1}{\theta_s + 1 - \sigma_s}L_{ns} \quad (48)$$

Hence, the expenditure share is given by

$$\lambda_{ins} = \frac{\left(\frac{\gamma_n(w_n/t_n)L_{ns}}{f^e_{ns}}\right)^{\delta_s}(\tau_{ins}c_n(w_n, t_n))^{-\theta_s\delta_s}\left(T^{\theta_s}_{ns}\right)^{\delta_s}(f_{ins})^{\zeta_s\delta_s}\xi_{ins}}{\sum_{l \in N}\left(\frac{\gamma_l(w_l/t_l)L_{ls}}{f^e_{ls}}\right)^{\delta_s}(\tau_{lns}c_l(w_l, t_l))^{-\theta_s\delta_s}\left(T^{\theta_s}_{ls}\right)^{\delta_s}(f_{ils})^{\zeta_s\delta_s}\xi_{ils}} \quad (49)$$

Defining

$$\xi_{ins} = \left(\frac{\gamma_n(w_n/t_n)}{f^e_{ns}})^{\delta_s}T^{\theta_s}_{ns}\right)f^{\zeta_s\delta_s}_{ins}$$

and $\epsilon = \theta_s\delta_s$ yields (18)

F Proof of proposition 7

Part (i) The change in country level emissions is $\hat{Z}_i = \sum_{s \in S}\frac{Z_{is}}{Z_i}\hat{Z}_{is}$. From (**??**), the counterfactual change in emissions in sector s country i is given by

$$\hat{Z}_{is} = \frac{\widehat{\beta(w_i/t_i)}}{\hat{t}_i}\hat{R}_{is} \quad (50)$$

Fixing $\hat{t}_i = \hat{w}_i$ and noting that from the labor market clearing condition $\hat{R}_{is} = \hat{w}_i\hat{L}_{is}$ yields:

$$\hat{Z}_{is} = \hat{L}_{is} \quad (51)$$

and thus the first expression of (20). Because aggregate revenues are given by $R_{is} = \sum_{n\in N}\lambda_{ins}E_{ms}$ the change i the labor allocation can be expressed as:

$$\hat{L}_{is} = \frac{1}{\hat{w}_i}\frac{\sum_{n\in N}\hat{\lambda}_{ins}\lambda_{ins}\hat{E}_{ns}E_{ns}}{w_i L_{is}} \tag{52}$$

which directly yields the second equality of (20). The change in expenditure in the expenditure is obtained by applying the exact hat-algebra to (18)

G Proof of Corollary 1

Part (i) From the derivation of (18), the change in the trade share for an Armington and EK model is obtained by setting $\delta_s = 0$ in (21). Hence, the trade share response is exactly the same across models. From proposition 7, the difference in emissions across models is fully captured via differential responses in the trade share. Thus, the emission response is the same for Armington and EK.

Part (ii) The derivation of equation (18) shows that the change in expenditure shares in Krugman is obtained by setting $\delta_s = \frac{\eta_s - 1}{\sigma_s - 1}$, which is larger than zero because $\eta_s > 1$ and $\sigma_s > 1$. Hence, there is an additional term affecting expenditure changes. From (18), these scale economies are also operative in Melitz. Moreover, ζ_s is generally different from zero.

Part (iii) If $\frac{Z_{is}}{Z_i} = \frac{L_{is}}{L_i}$, the change in emission is given by:

$$\hat{Z}_i = \sum_{s\in S}\frac{L_{is}}{L_i}\hat{L}_{is} \tag{53}$$

Because the labor endowment is fixed, for any trade shock $\hat{L}_i = 1\forall i$. Hence,

$$\sum_{s\in S}\frac{L_{is}}{L_i}\hat{L}_{is} = \hat{L}_i = 1$$

H Counterfactual exporter share

The change in the exporter share is given by:

$$\hat{S}_{nis} = \left(\frac{\widehat{\varphi^*_{nis}}}{\widehat{\varphi^*_{iis}}}\right)^{-\theta_s}$$

The change in the productivity cut-off is equal to:

$$\widehat{\varphi^*_{nis}} = \hat{\tau}_{nis}\hat{w}_i\left(\frac{\hat{w}_n}{\hat{B}_{nis}}\right)^{1/(\sigma_s-1)}, \tag{54}$$

with the counterfactual value of aggregate market level for goods in sector s shipped from i to n given by:

$$\hat{B}_{nis} = (\hat{P}_{nis})^{\sigma_s - \eta_s}(\hat{P}_{ns})^{\eta_s - 1}\hat{E}_{ns}, \tag{55}$$

with the change in price indices of:

$$\hat{P}_{nis} = (\hat{L}_{is})^{\frac{\delta_s}{(1-\eta_s)}} (\hat{\tau}_{ins}\hat{w}_i)^{\frac{-\epsilon_s}{(1-\eta_s)}} \left(\hat{P}_{ns}\right)^{-\zeta_s\delta_s}, \tag{56}$$

$$\hat{P}_{ns} = \left[\sum_{i\in M} \lambda_{nis} L_{is}^{\delta_s} (\hat{\tau}_{nis}\hat{w}_{is})^{-\epsilon_s}\right]^{\frac{-1}{\epsilon_s}}. \tag{57}$$

I Proof of proposition 8

We start from the same observed autarky equilibrium with Z_{is}^{AUT} and R_{is}^{AUT}. We only prove the statement (ii), since proving (i) and (iii) is a straightforward extension.

I.1 EK

We first prove that if $\gamma_i < 1$, trade opening implies higher emissions in the EK than in the Armington model. Denote the industry's emissions per output in autarky as $\tilde{\mu}_i = Z_{is}^{AUT}/R_{is}^{AUT}$ where Z_{is}^{AUT} and R_{is}^{AUT} are emissions and revenue in autarky for industry s in country i, respectively. The emissions under trade opening in the Armington model are:

$$Z_{is}^{ARM} = \tilde{\mu}_i R_{is}$$

where R_{is} is a revenue of industry s in country i after the trade opening. Notice that R_{is} is common across the two models. In the EK model, the emissions after trade opening are:

$$Z_{is}^{EK} = \sum_{i=1}^{N} X_{nis}\tilde{\mu}_i\lambda_{nis}^{(\gamma_i-1)/\theta} > Z_{is}^{ARM} R_{is} = Z_{is}^{ARM}.$$

Aggregating this over industries will result in the proposition 8 (ii).

J Imputation of the exporting share

We impute the share of exporting firms based on a model-driven gravity equation. Recall that the share of exporters from n to i in sector s is given by

$$S_{nis} = \frac{M_{nis}}{M_{iis}} \tag{58}$$

The number of exporters can be expressed as the ratio of total to average exports, i.e. :

$$M_{nis} = \frac{X_{nis}}{\bar{x}_{nis}} \tag{59}$$

and thus

$$S_{nis} = \frac{X_{nis}}{X_{iis}} \frac{\bar{x}_{iis}}{\bar{x}_{nis}} \tag{60}$$

In the Melitz-Chaney model, the ratio of the average sales is equal to the ratio of fixed costs:

$$\frac{\bar{x}_{iis}}{\bar{x}_{nis}} = \frac{f_{nis}}{f_{iis}} \tag{61}$$

We proxy the relative fixed costs by the distance between n and i, $dist_{ni}$. Hence, we get the following estimation equation:

$$log(S_{nis}) = log(\lambda^x_{nis}) + log(dist_{ni}) + \xi_n \tag{62}$$

where $\lambda^x_{ni} = \frac{X_{nis}}{X_{iis}}$, and ξ_n are destination fixed effects. We drop all internal trade flows for the estimation and estimate the equation for each WIOD sector and the entire economy.

K Parameter calibration

Calibration of parameters for the multiple industries is done as follows. We take σ_s and η_s from Lashkaripour and Lugovskyy (2023) and θ_s from Shapiro and Walker (2018). Using these parameters, we construct trade elasticity and scale elasticity using the formula from the Melitz model ($\epsilon_s = \frac{\theta_s}{(1+\theta_s(\frac{1}{\eta_s-1}-\frac{1}{\sigma_s-1}))}$, $\delta_s = \frac{\epsilon_s}{\theta_s}$). We use the trade elasticity derived from the Melitz model for other models. For the scale elasticity, we use the derived scale elasticity for the Melitz model while setting the parameter to zero for the Armington and the EK. The parameter γ_s is calculated from the estimate in column 9 and the same σ_s we use to calculate elasticities. Apart from γ_s the counterfactual emissions in EK and Melitz depend on the value of θ_s, which is different for these two models. For Melitz we use the value presented in table 2, whereas for EK, θ_s equals the trade elasticity, i.e. we set $\theta^{EK}_s = \epsilon_s$.

We acknowledge that this is not internally consistent. If we take the parameters seriously, we should have different trade elasticity across the model. If we fix the trade elasticity, we should use the implied σ_s and θ_s to calculate γ^s_i and the counterfactual emissions, which will differ across models. We keep our calibration this way to highlight the key differences from the firm heterogeneity while keeping the elasticity intact.

Table 2: Parameters

Sector	δ	ϵ	θ	η	σ
A01	0.00	1.85		7.23	7.99
A02	0.00	0.52		7.23	7.99
A03	0.00	11.98		7.23	7.99
B	0.00	13.97		7.23	7.99
C10-C12	0.86	3.35	3.89	3.30	3.54
C13-C15	0.74	3.55	4.8	4.36	5.46
C16	0.85	5.29	6.2	4.90	5.37
C17	0.77	4.00	5.21	3.65	4.12
C18	0.77	4.00	5.21	3.65	4.12
C19	0.22	2.21	9.91	1.64	1.82
C20	0.93	3.27	3.5	4.97	5.31
C21	0.93	3.27	3.5	4.97	5.31
C22	0.80	3.70	4.62	6.16	8.14
C23	0.92	3.71	4.05	6.28	6.99
C24	0.45	4.47	10.01	4.00	5.78
C25	0.63	3.01	4.8	4.00	5.78
C26	0.42	2.24	5.32	2.24	2.81
C27	0.45	2.14	4.77	2.24	2.81
C28	0.96	4.04	4.19	8.75	9.33
C29	0.44	2.46	5.6	3.81	8.75
C30	0.53	2.06	3.87	3.81	8.75
C31-C32	0.96	3.61	3.75	7.17	7.58
C33	0.96	3.61	3.75	7.17	7.58
D-E	0.00	4.00		4.00	5.00
F	0.00	4.00		4.00	5.00
G	0.00	4.00		4.00	5.00
H	0.00	4.00		4.00	5.00
I-U	0.00	4.00		4.00	5.00
Total		4.00			5.00

Note:
Parameters used in the analysis. σ_s and η_s are sourced from citeLashkaripour2023 and $theta_s$ from citeShapiro2018. The remaining parameters are calculated based on these estimates and consistent with the Melitz model, i.e $\epsilon_s = \frac{\theta_s}{(1+\theta_s\left(\frac{1}{\eta_s-1}-\frac{1}{\sigma_s-1}\right)}$, $\delta_s = \frac{\epsilon_s}{\theta_s}$. For sectors outside manufacturing and without an estimate of θ_s we take the estimated trade elasticity from Fontagné et al. (2018).

L Country-level results

Table 3: Changes in emissions by country

	$\gamma = 0$		$\gamma = 1.2$	
Country	EK	Melitz	EK	Melitz
AUS	47.72	44.66	-7.91	-7.55
AUT	50.89	34.91	-10.78	-8.12
BEL	38.80	28.59	-10.64	-8.17
BGR	69.38	45.28	-13.92	-9.18
BRA	42.28	48.04	-6.17	-6.46
CAN	43.13	30.31	-8.95	-6.54
CHE	48.38	38.76	-12.33	-10.59
CHN	18.21	35.49	-3.71	-5.61
CYP	81.97	51.21	-14.68	-9.24
CZE	46.00	42.45	-11.57	-11.52
DEU	30.80	37.58	-8.35	-9.99
DNK	52.67	32.73	-12.93	-9.16
ESP	45.77	42.76	-8.30	-8.14
EST	60.06	32.80	-13.97	-7.86
FIN	65.60	43.51	-11.92	-8.81
FRA	39.59	43.60	-7.24	-7.99
GBR	40.34	38.86	-7.28	-7.25
GRC	67.19	44.41	-10.43	-7.24
HRV	73.38	42.97	-14.69	-9.20
HUN	40.45	35.47	-12.23	-11.46
IDN	56.02	44.57	-9.42	-7.96
IND	42.20	39.03	-6.89	-6.50
IRL	20.40	16.89	-14.34	-13.21
ITA	41.63	37.07	-7.77	-7.28
JPN	34.95	39.08	-6.00	-6.44
KOR	43.62	41.62	-10.29	-9.66
LTU	50.86	30.97	-15.08	-9.52
LUX	20.47	10.51	-12.98	-9.34
LVA	78.09	40.62	-14.14	-7.99
MEX	46.06	36.00	-9.29	-7.53
MLT	42.75	31.58	-13.73	-9.36
NOR	54.20	38.75	-12.69	-10.28
POL	48.96	49.08	-9.81	-10.35
PRT	66.25	45.70	-11.84	-8.69
ROU	64.59	47.43	-11.74	-9.22
ROW	20.78	48.20	-3.33	-8.10
RUS	43.52	42.55	-8.45	-8.40
SVK	48.96	31.94	-12.88	-8.33
SVN	59.00	29.27	-14.77	-7.63
SWE	51.37	40.97	-10.79	-9.49
TUR	52.53	48.72	-9.93	-9.33
TWN	42.67	32.45	-13.19	-10.46
USA	17.79	27.10	-3.06	-4.32

Note:
Changes in emissions by country in the single industry case. Results shown for the Eaton-Kortum (EK) and Melitz model and the quantity case $\gamma = 0$ and the estimated $\gamma = 1.2$. All values in percentage changes compared to the baseline.

Table 4: Changes in emissions by country

	Armignton	Krugman	EK				Melitz			
Country	$\gamma = 1$	$\gamma = 1$	$\gamma = 1$	$\gamma = 0$	$\hat{\gamma}$	$\gamma = 1.2$	$\gamma = 1$	$\gamma = 0$	$\hat{\gamma}$	$\gamma = 1.2$
AUS	8.73	15.61	8.73	23.11	9.48	6.17	15.61	53.85	1.98	13.72
AUT	4.42	-1.11	4.42	37.38	13.87	-2.66	-1.11	7.92	-21.61	-1.16
BEL	15.63	7.90	15.63	43.49	17.44	8.47	7.90	22.77	0.53	-1.18
BGR	12.97	26.16	12.97	77.07	15.00	0.70	26.16	51.18	23.68	-1.57
BRA	5.65	8.78	5.65	26.76	14.34	1.66	8.78	37.40	-3.82	6.06
CAN	6.56	2.02	6.56	30.75	7.95	2.15	2.02	25.65	-3.32	-5.68
CHE	-5.54	-7.07	-5.54	39.16	-2.48	-13.59	-7.07	38.74	-11.36	-18.65
CHN	-0.18	1.14	-0.18	8.80	2.11	-1.61	1.14	101.85	-2.06	2.57
CYP	12.29	558.73	12.29	57.01	22.56	3.89	558.73	374.17	344.24	348.95
CZE	5.87	11.79	5.87	54.42	7.64	-2.39	11.79	56.45	3.38	-4.14
DEU	3.53	2.61	3.53	35.98	5.92	-2.32	2.61	89.15	-0.42	-6.23
DNK	31.93	19.90	31.93	60.39	32.65	24.67	19.90	33.47	20.66	-7.81
ESP	6.07	66.16	6.07	33.08	11.75	0.56	66.16	98.54	55.61	63.02
EST	24.63	25.48	24.63	87.42	26.95	10.81	25.48	57.82	21.52	-3.09
FIN	4.16	15.91	4.16	56.70	7.46	-4.21	15.91	63.57	-13.08	9.22
FRA	1.21	-23.88	1.21	28.62	5.49	-4.25	-23.88	17.97	-27.30	-31.00
GBR	-1.98	5.23	-1.98	22.56	-2.28	-6.11	5.23	60.80	-8.99	-0.51
GRC	-3.67	1.64	-3.67	45.60	0.54	-10.34	1.64	90.32	-3.94	-7.17
HRV	10.08	-0.41	10.08	60.46	15.43	-1.02	-0.41	36.59	-6.15	-12.03
HUN	5.24	12.55	5.24	57.09	6.91	-5.46	12.55	51.59	7.92	-7.74
IDN	-0.40	-7.84	-0.40	17.73	2.07	-3.44	-7.84	22.01	-10.02	-14.39
IND	-4.79	1.45	-4.79	9.71	0.36	-7.28	1.45	15.95	-1.02	-5.51
IRL	-13.35	-22.45	-13.35	8.17	-10.64	-18.33	-22.45	12.19	-18.65	-34.71
ITA	-3.39	0.54	-3.39	25.60	0.87	-8.31	0.54	72.25	-4.31	-0.86
JPN	-0.01	-3.24	-0.01	15.36	5.26	-2.82	-3.24	40.46	-8.84	-4.05
KOR	-0.45	-4.36	-0.45	29.73	8.04	-5.44	-4.36	94.85	-10.87	-7.06
LTU	19.32	22.47	19.32	47.73	21.83	9.67	22.47	8.40	16.40	1.02
LUX	-28.40	-38.37	-28.40	4.43	-25.56	-36.58	-38.37	-24.78	-40.06	-53.15
LVA	5.51	19.37	5.51	52.58	8.28	-4.60	19.37	42.93	16.04	4.73
MEX	0.15	-12.64	0.15	21.51	4.84	-3.38	-12.64	34.70	-14.29	-15.00
MLT	-31.31	-31.95	-31.31	0.99	-31.47	-38.22	-31.95	-18.62	-32.02	-61.21
NOR	32.70	32.74	32.70	59.77	33.64	26.97	32.74	16.10	19.50	22.05
POL	-3.03	-7.22	-3.03	34.95	-0.23	-8.88	-7.22	66.62	-9.57	-19.17
PRT	11.05	18.63	11.05	52.47	16.48	2.47	18.63	61.83	11.12	10.30
ROU	-2.21	-13.54	-2.21	45.19	4.04	-9.73	-13.54	36.71	-15.15	-27.55
ROW	-0.49	-2.94	-0.49	10.69	0.08	-2.68	-2.94	88.26	-2.74	-11.22
RUS	1.62	-0.62	1.62	15.26	5.95	-0.68	-0.62	53.37	-4.82	-6.04
SVK	-3.21	-3.47	-3.21	30.36	-0.05	-9.08	-3.47	28.43	-23.14	-0.57
SVN	23.69	14.68	23.69	78.50	26.39	10.05	14.68	29.33	9.29	-2.66
SWE	2.93	1.12	2.93	41.25	7.44	-4.25	1.12	38.95	-11.18	-8.76
TUR	10.83	35.45	10.83	41.35	16.15	6.00	35.45	119.14	30.12	31.32
TWN	-1.62	-18.35	-1.62	27.50	5.85	-7.38	-18.35	49.47	-21.15	-25.95
USA	5.45	8.75	5.45	23.83	6.54	2.28	8.75	92.01	7.62	-3.40

Note:
Changes in emissions by country in the multi industry case for Armington and EK. Results shown for the quantity case $\gamma = 0$, the value case, $\gamma = 1$, the estimated one, γ^e, and if $\gamma = 1.2$ for all sectors. All values in percentage changes compared to the baseline.