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by Thomas S. Lontzek and Wilfried Rickels

**No. 1475** | December 2008

Web: www.ifw-kiel.de

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We study the socially optimal anthropogenic intervention of the global carbon cycle using a non-renewable resource stock. We find that this kind of carbon capture and storage facilitates achieving strict stabilization targets for the atmospheric carbon content. It accelerates the slow natural flux within the carbon cycle, and because of its temporary abatement character it dampens the overshooting of the atmospheric reservoir. Furthermore, we analyze the optimal paths of the carbon tax. The carbon tax shows to be inverted u-shaped but depending on the initial sizes of the reservoirs and the speed of carbon fluxes between the reservoirs we also find the optimal tax to be increasing, decreasing or u-shaped. Finally, we suggest to link the level of the carbon tax to the declining ability of the deep ocean to absorb atmospheric carbon.

Keywords: exhaustible resource, CCS, ocean sinks, ocean sequestration, air capture, carbon tax, carbon cycle

JEL classification: Q32 Q54 C61

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# Carbon Capture and Storage & the Optimal Path of the Carbon Tax

Thomas S. Lontzek*, Wilfried Rickels†

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# 1 Introduction

Scarcity of resources, such as fossil fuels has been for a long time considered as the major obstacle to sustainability. Meanwhile policy makers seem to realize that the external cost of $CO_2$ emissions is the true limiting factor of using fossil fuels. Regulatory instruments, such as the EU emission trading system and carbon taxes aim at internalizing these external costs. However, in order to design a carbon tax policy makers need information about its optimal level and the shape of its long run path. Obtaining this information has been a major goal in the theoretical literature on optimal resource extraction. Obviously, the problem at hand is much too complex since one cannot simply include all relevant components of the global carbon cycle within a theoretical modeling framework.

One major component which has been neglected traditionally is the role of the deep ocean in absorbing anthropogenic carbon. The ocean itself is by far the largest reservoir of carbon and possesses a large uptake capacity. However, on long time scales the $CO_2$ uptake capacity becomes exhausted with rising $CO_2$ concentrations because each unit of carbon emitted will remain in the carbon cycle. Thus, in order to reduce the carbon concentration in the atmosphere significantly, we must sooner or later replace the conventional energy resources by ecologically friendly renewables. But, drastic reduction are not viable yet because the use of renewables on a large scale is both, economically and technologically not feasible.

Nevertheless, the rising demand for energy and the ecological urgency character of the problem call for alternatives that allow us to use the fossil fuels much more efficiently and at the same time, reducing its impact on the environment. These alternatives must come into action as soon as possible if we want to guarantee a smooth transition towards the on-scale usage of renewables in the long run.

Carbon capture and storage (CCS) is considered to be such an alternative. As for the *capture* part one usually assumes capturing carbon directly from the power plants. This however, requires building of new, CCS-ready power plants or retrofitting old ones. Air capture on the other hand, is a completely independent technology. It does not require any adjustment of energy generation processes. Regarding the *storage* of carbon, the most prominent kind of storage is underground storage, e.g. in oil and (depleted) gas fields, coal beds or saline aquifers. Applying these kinds of CCS technologies, one effectively removes carbon from the carbon cycle, provided leakage rates are low. Ocean sequestration via deep see injection is different. It has the unique characteristic that carbon removed from the atmosphere remains in the carbon cycle. Ocean sequestration alters the relative distribution of carbon between its reservoirs (e.g. ocean and atmosphere) because the natural transfer of carbon from the atmosphere to the ocean is accelerated.

The purpose of this paper is (i) to study the properties of the optimal carbon tax by explicitly including the ocean as an additional carbon reservoir and (ii) to investigate the role of air capture and ocean sequestration as an additional control in managing the global carbon cycle.

For the global carbon cycle we introduce two reservoirs: an upper reservoir containing the atmosphere and the upper ocean layer and a lower reservoir containing the deep ocean. We show that starting at the pre-industrial levels of both reservoirs, the optimal extraction rates are initially high and decreasing. As a consequence, the upper reservoir overshoots its long run equilibrium level while the carbon content in the lower reservoir is monotonically increasing.

Because of its delayed damage effect, ocean sequestration proves to be an effective instrument to dampen the overshooting of the upper reservoir. Comparing steady states, the usage of ocean sequestration will result in lower atmospheric carbon concentrations and, at the same time, higher total resource extraction. But it also reduces the natural uptake ability of the deep ocean. Furthermore, we show that the optimal path of the carbon tax can be decreasing, increasing, u-shaped or inverted u-shaped. Its shape depends on the initial values of the stock variables and the speed of the flux between the carbon reservoirs. Its level is heavily linked to the ability of the deep ocean to absorb additional atmospheric carbon. Since we observe a decreasing potential of the deep ocean to serve as a carbon sink, we conclude that the level of the carbon tax should be adjusted for this effect.

There are two externalities in our model. First, the negative external effect of high carbon concentration levels on welfare and second, the positive externality which results from removing carbon from the atmosphere via CCS. The latter in fact constitutes an abatement option which is only temporary since the carbon which is removed from the atmosphere and injected into the deep see will eventually be transferred back to the atmosphere as the carbon reservoirs keep on mixing. Studying the decentralized economy we find that the revenues of a carbon tax should be used to subsidize the CCS technology.

So far, the idea of additional carbon stocks has been theoretically applied in the literature on nonconstant pollution decay. In contrast to the standard resource extraction problem in which carbon simply vanishes from the carbon cycle, this strand of literature takes account of the fact that the uptake potential of anthropogenic carbon by other reservoirs is limited. Among the most prominent studies are Forster [1975], Tahvonen and Withagen [1996], and Toman and Withagen [2000]. In addition, the mostly numerical literature on Integrated Assessment Models (IAM), e.g. Nordhaus [1994] and Nordhaus and Boyer [2000] has been dealing with several stocks of carbon. However CCS has not been included into these models.

CCS has so far mainly been analyzed using complex integrated assessment models (e.g., Akimoto et al. [2004], McFarland et al. [2003], McFarland and Herzog [2006] or Edmonds et al. [2004]). Ocean sequestration via deep see injection has been suggested by [IPCC, 2005] and is analyzed by Herzog et al. [2003]. However, Herzog et al. [2003] do not put ocean sequestration into an optimization framework. Besides deep-see injection there are other ways of applying CCS, e.g. injection into geological formations, such as alkaline mineral strata or into natural off-shore storage facilities like oil and gas fields such as in the North Sea. [Lackner, 2003] provides an excellent survey of sequestration from an economic, ecological as well as technological point of view.

The air capture technology has been developed by Klaus S. Lackner from Columbia University and Global Research Technologies. It requires the installation of specific devices. These *artificial trees* (c.f. Lackner) have sorbents which can capture carbon dioxide from the air (see Zeman and Lackner [2004], Keith and Minh [2003] and Keith et al. [2005]. Air capture has been applied successfully on a very low scale so far. Its major advantage over other carbon capture technologies is that the an air capture device can be installed in any location, preferably ones very close to sequestration sites.

Ulph and Ulph [1994] show that the optimal carbon tax is increasing if the stock of $CO_2$ is below its steady state. We can show that including the deep ocean as an additional carbon reservoir the carbon tax could also be increasing in that case if e.g. carbon is absorbed very quickly by the deep ocean. To our best knowledge Farzin and Tahvonen [1996] is the only study in which four different paths of the carbon tax can occur (monotonically increasing, decreasing, U-shaped, or inversely U-shaped). The resulting path depends on the initial conditions of their two atmosphere stocks and the factors that determine the allocation of total emissions between these two stocks. If in addition, no resource constraint is included, steady states with positive pollution and decay rates can emerge. We are able to obtain this result even with a resource constraint because we assume that the total carbon content of the global carbon cycle may not fall at any time.

The next section describes our simplified version of the global carbon cycle. In section 3 we present the modeling framework and analyze the system dynamics, first in closed form and then using specific functional forms. In Section 4 we parameterize the model and discuss the model results focusing strongly on the optimal carbon tax. Section 5 concludes.

## 2 The Simplified Global Carbon Cycle

71% of the earth surface is ocean. The ocean is also by far the earth's largest reservoir of heat and carbon. The net transfer of carbon between atmosphere and the ocean is caused by differences in the partial pressure of $CO_2$ across the air-sea interface. If the partial pressure in the atmosphere is larger than that in the ocean, we will observe a net transfer into the ocean, and vice versa.

In our model, we assume that both, the upper and the lower reservoir are homogenous systems, meaning that their carbon contents are distributed uniformly and that the uptake capacity of the surface ocean is homogenous. In reality, this is not true since alkalinity and temperature varies across the ocean surface, i.e. the oceans capacity to take up additional $CO_2$ varies with location. High capacity to absorb $CO_2$ is generally found at low latitudes in warm tropical and subtropical waters, whereas low uptake capacity is found in the cold waters at high latitudes.

Two processes are responsible for the transport of carbon from the surface ocean into the deep ocean: the physical circulation (i.e. the solubility pump), and the biological pump in which dead organic matter is remineralized at depth. We assume that the biological pump is constant, i.e. does not change with anthropogenic forcing, whereas we add some constrains to the effectiveness of the physical pump. The solubility pump is driven by, among other factors, the thermohaline circulation which transports high-latitude surface waters into the deep sea. The deep waters of the world ocean are ventilated on the time-scales of a few thousand years, which is the time it would take to equilibrate the lower reservoir with the upper reservoir in our model.

We focus on the fact that the $CO_2$ uptake capacity becomes exhausted with rising $CO_2$ concentrations [Lackner, 2003]. It is estimated that the oceans will eventually absorb about 80 percent of the carbon in the atmospheric reservoir and transfer it to the deep ocean. But ocean uptake is also not constant over time. The study by Canadell et al. [2007] indicates that the decrease in efficiency of ocean sinks has contributed substantially to the increasing growth rate of atmospheric $\mathrm{CO_2}$ for the period 2000-2006. It is exactly these natural forces which we want to implement in our simplified version of the global carbon cycle.

# 3 An Economic Model of the Global Carbon Cycle

In our formulation of the global carbon cycle we incorporate both, a natural exchange of carbon between the two reservoirs and an anthropogenic influence on the reservoirs' size via burning fossil fuels and sequestering. The stocks of the upper reservoir $S$ and the lower reservoir $W$ are driven by the following dynamics:

$$\dot{S} = q - a - \gamma(\sigma S - \omega W) \quad (1)$$

$$\dot{W} = a + \gamma(\sigma S - \omega W) \quad (2)$$

At each point in time $q \geq 0$ is the amount of carbon emissions that is added to the carbon system. It is obtained by extracting the same amount of a fossil resource. Furthermore, $a \geq 0$ denotes the amount of carbon which is captured from the air and injected into the deep ocean (i.e. removed from the upper reservoir). We interpret both, $q$ and $a$ as anthropogenic components of the global carbon cycle. On the contrary, the term $\gamma(\sigma S - \omega W)$ represents the natural component. It describes the natural force of the carbon system to equilibrate and to neutralize the difference in partial pressure of its components. Whereas the term $\sigma S - \omega W$ describes the nominal difference of the carbon content in the two reservoirs the parameter[1] $\gamma$ is an indicator for the pace of the net flux between the two reservoirs. The cut-off line between the two reservoirs is located at an ocean depth of around 100 meters. The reason for such a division is that whereas it takes several centuries for the deep ocean to mix with the atmosphere, the exchange between the atmosphere and the upper ocean layer (upper 100 meters) takes place at a much lower time scale. It takes about one year for the upper reservoir

[1]Alternatively, one could think of treating $\gamma$ as a variable, e.g. $\gamma(S)$ with $\gamma_S < 0$ and $\gamma_{SS} > 0$. This specification would explicitly take into account the weakening of the ocean's uptake capability due to increasing carbon concentrations in the atmosphere.

to mix. Therefore, we assume instant equilibration between the surface ocean and the atmosphere which is justified in this context.

In addition to the two carbon reservoirs $S$ and $W$ we introduce $R$, the stock of a non-renewable resource stock with carbon content. It is extracted at the rate $q$.

$$\dot{R} = -q \tag{3}$$

Figure (1) illustrates the functioning of our model economy. Notice that $a$, the CCS activity is completely independent of $q$, the extraction of the non-renewable resource stock.

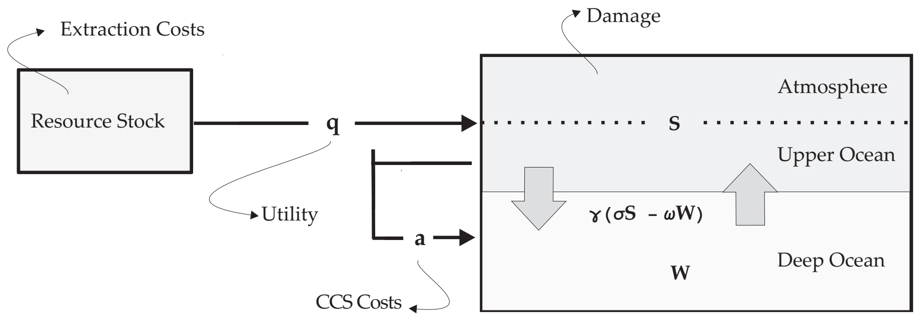


Figure 1: CCS & the Global Carbon Cycle

With this formulation negative emission can occur because carbon can be extracted from the air and sequestered into the deep see even if the resource stock is economically/physically depleted.

## 3.1 The Carbon System Diagram

Before we turn to the formulation of the dynamic problem we introduce the carbon system diagram which should simplify the understanding of the climate module within our model.

We observe that equations (1)-(3) imply a balanced carbon content system from which carbon cannot vanish. Each unit of carbon extracted from the stock of the non-renewable resource must flow either to the upper reservoir or (partly) to the lower reservoir. This feature stands in contrast to that part of the literature which assumes a constant decay rate. In the latter case carbon may simply vanish from the carbon cycle. This feature is at the core of our model and can be illustrated in in a diagram. Figure (2) shows the carbon balance diagram which illustrates the dynamics of the

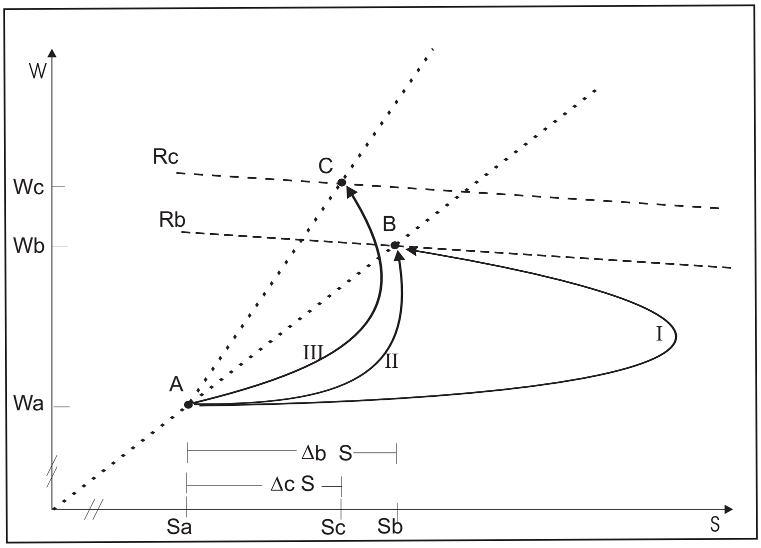


Figure 2: The Carbon System Diagram

simplified carbon cycle. $S$, the upper reservoir's carbon content is displayed along the horizontal axis, and $W$, the carbon content of the lower reservoir is displayed along the vertical axis. Let $A$ denote an initial steady state (e.g. the preindustrial state) of the system (1)-(3) which is characterized by $a = q = 0$, $R = R_0$ and $Wa = \frac{\sigma}{\omega}Sa$. Recall that, since the dotted line passing through points $A$ and $B$ ($\overline{AB}$) has the slope $\frac{\sigma}{\omega}$, we observe no carbon transfer via the natural component of the carbon cycle at $A$. In the following, we want to demonstrate how air capture and ocean sequestration affect the global carbon cycle. We investigate three possible paths of the carbon cycle, each being subject to a different constraint on the anthropogenic disturbance of the carbon system. These constraints are: (I) sequestration not possible. In this scenario the only control option is the extraction of the non-renewable resource stock. (II) sequestration possible with $(0 \leq a \leq q)$. The second scenario does not allow for negative emissions. It mimics the effects of CCS on the global carbon cycle when a carbon capture technology is installed directly at a power plant. Scenario (III) on the other hand assumes that sequestration is possible with $a \geq 0$.

*Path I, sequestration not possible*

Since sequestration is not possible, the only control is the extraction of the non-renewable resource. Along the initial part of Path $I$ anthropogenic release of carbon into the upper reservoir is much higher then natural transfer to the lower reservoir. As a consequence, the upper carbon stocks overshoots. Beyond its maximum, $S$ decreases for two reasons. First, the extraction of the carbon stock is reduced constantly and second, the natural transfer is very high because we observe a large difference in partial pressure. The new steady state is at $B$ with $q = 0$ and equalization of the partial pressure, i.e. $Wb = \frac{\sigma}{\omega} Sb$.

**Definition 1.** *Consider the $\{S, W\}$ space of the system (1)-(3) and its steady state $(S^*, W^*)$. An carbon isocontent line passing through $(S^*, W^*)$ depicts all combinations of $S$ and $W$ with the same total carbon content.*

By Definition 1, *Rb* is the carbon isocontent line corresponding to the steady state at point $B$[2].

*Path II, sequestration possible and $0 \leq a \leq q$*

Now we assume the possibility of ocean sequestration but limit its volume to be lower than the volume of carbon obtained from processing the extracted resource. Notice, that we are now no longer considering air capture, but rather any other end of pipe carbon capture technology. Recall, that the unique characteristic of deep see injection of carbon is, that it accelerates the natural, but slow mixing of the two carbon reservoirs. As a consequence the overshooting of the upper reservoir is significantly reduced. Just like path $I$, path $II$ ends at $B$, implying that the total amount of carbon which has been extracted from the non-renewable resource is the same (same carbon isocontent line). Notice also, that since at $B$ extraction $q$ is zero, ocean sequestration does no longer take place, because we impose the constraint $0 \leq a \leq q$ [3]

[2] Notice that all paths starting at $A$ and ending at $B$ may never leave the area bounded by $Sa$ from the left, $Wa$ from below and *Rb* from above. This is due to the fact that carbon cannot vanish from the carbon cycle since we assume a balanced carbon content

[3] see Rickels and Lontzek [2008] for a extensive discussion of this case

*Path $III$, sequestration possible and $a \geq 0$*

For path $III$ we consider again air capture since we now allow carbon capture and sequestration to be positive even in the absence of extraction. This automatically implies that in a new steady state the natural component of the carbon cycle does not need to be in equilibrium. This is reflected in Figure 3 by point $C$ which is not on the line passing through AB with a slope $= \frac{\sigma}{\omega}$. Since $C$ is above AB the lower reservoir is supersaturated and hence, a net source of carbon release. As a consequence, the steady state level of carbon in the upper reservoir has been reduced ($Sc < Sb$) even though more of the non-renewable resource has been extracted. The latter fact is indicated by the higher carbon isocontent line $Rc$ corresponding to $C$.

This section's purpose was to emphasize some qualitative dynamic behavior of our simplified version of the global carbon cycle. Before we turn to the dynamic optimality analysis, we focus in the next section on equations' (1)-(3) major property of a balanced carbon system.

## 3.2 The carbon balance equation

Equations (1)-(3) reveal one fundamental characteristic about the carbon cycle used in this model.: The total carbon content of all reservoirs (the upper reservoir, the lower reservoir and the resource stock) must be constant at each point in time. This constant is determined by the initial contents of the carbon stocks.

$$R_t + W_t + S_t = constant = R_0 + W_0 + S_0 \qquad \forall t \tag{4}$$

Dropping the time index for convenience we can use the carbon balance equation to reduce the dimension of the dynamic system, as implied by (1)-(3). We solve (4) for

$W$ to obtain[4] :

$$W = R_0 + W_0 + S_0 - R - S \tag{5}$$

Inserting (5) in (1) we can reformulate the dynamic equation for the upper reservoir.

$$\dot{S} \;=\; q - a - \gamma(\sigma S - \omega(R_0 + W_0 + S_0 - R - S))$$

### 3.3 The optimal control problem

Usage of the fossil fuel generates utility $U(q)$ with

$$U_q(q) > 0, \quad U_{qq}(q) < 0, \quad U_q(0) = u_1.$$

The assumption $U_q(0) = u_1$ implies a choke price for the non-renewable resource. Extraction of the resource stock is costly and is stock dependent.

$$c(q, R) = qC(R) \tag{6}$$

with $C_R < 0, \quad C_{RR} \geq 0$. $A(a)$ are the costs of sequestration with $A_a > 0$ and $A_{aa} > 0$ [5]. $D(S)$ is the social damage that is caused by the stock of carbon in the atmosphere. We assume $D_S > 0$ and $D_{SS} > 0$. Taking into account equations (1)-(3) a social planer solves the following dynamic optimization problem:

$$\max_{q,a} \int_0^\infty e^{-\rho t} \left(U(q) - A(a) - qC(R) - D(S)\right) dt \tag{7}$$

[4]Notice, that due to the carbon balance equation the upper and lower reservoir have an implicit upper bound which is given respectively by $\overline{S} = \frac{\omega}{\sigma+\omega}(R_0 + S_0 + W_0)$ and $\overline{W} = \frac{\sigma}{\sigma+\omega}(R_0 + S_0 + W_0)$.

[5]For the rest of this paper we use term sequestration to describe air capture and ocean sequestration for convenience.

subject to

$$\dot{S} = q - a - \gamma(\sigma S - \omega(R_0 + W_0 + S_0 - R_t - S_t)) \tag{8}$$

$$\dot{R} = -q \tag{9}$$

and

$$S(0) = S_0 > 0, \quad R(0) = R_0 > 0, \quad W(0) = W_0 > 0$$

We formulate the current value Hamiltonian[6] as:

$$\begin{aligned} H &= U(q) - A(a) - qC(R) - D(S)) \\ &- \lambda_S(q - a - \gamma(S - \omega(R_0 + W_0 + S_0 - R_t - S_t))) \\ &+ \lambda_R \cdot (-q) \end{aligned} \tag{10}$$

The shadow value $\lambda_R$ can be interpreted as the resource rent, $\lambda_S$ as a carbon tax. Applying the maximum principle yields the following F.O.C.

$$U_q = C + \lambda_R + \lambda_S \tag{11}$$

$$A_a = \lambda_S \tag{12}$$

$$\dot{\lambda}_S - \rho\lambda_S = \lambda_S\gamma(1 + \omega) - D_S \tag{13}$$

$$\dot{\lambda}_R - \rho\lambda_R = qC_R - \gamma\omega\lambda_S \tag{14}$$

In order to calculate the optimal converging paths, we need the initial conditions and the transversality conditions:

$$\lim_{t\to\infty} \lambda_S \cdot e^{-\rho t} \geq 0, \quad \lim_{t\to\infty} R \cdot \lambda_R \cdot e^{-\rho t} \geq 0 \tag{15}$$

[6]Note, that we have changed the sign of $\lambda_S$ to facilitate their economic interpretation as taxes. We have also normalized $\sigma = 1$

The static efficiency condition (11) relates marginal utility from extracting and consuming $q$ units of the resource to the marginal costs of extraction, the resource rent and the carbon tax. From (12) we deduct that it is optimal to sequester the extracted carbon up to the amount where the marginal sequestration costs are equal to the shadow price of the upper carbon reservoir. Since we interpret $\lambda_S$ as a carbon tax, equation (12) states that the optimal sequestration path will follow the path of the carbon tax. Equation (14) is the dynamic efficiency condition for the carbon rent. It has the standard form, as implied by the literature assuming resource based extraction costs, except for the term $-\gamma\omega\lambda_S$ which results from the carbon balance equation. It reflects the fact that reducing the resource stock automatically implies that part of the amount extracted is transferred to the lower reservoir via the natural component of the carbon cycle. Thus, ceteris paribus the ocean's uptake capability is reduced and we obtain a higher carbon content in the atmosphere. We analyze the dynamic properties of the carbon tax as given by (13) in section 3.5.

## 3.4 Dynamic properties and the MHDS

In a next step we study the dynamic properties of the model at hand in closed form. For that purpose we establish the modified hamiltonian dynamic system (MHDS). From equation (11) we can formulate $q$ as $q(R, \lambda_S, \lambda_R)$, with $q_R > 0$, $q_{\lambda_S} < 0$ and $q_{\lambda_R} < 0$. Similarly equation (12) defines $a = a(\lambda_S)$ with $a_{\lambda_S} > 0$. Using these specifications together with (8), (9), (13) and (14) we obtain the MHDS:

$$\dot{S} = q(R, \lambda_S, \lambda_R) - \gamma(\sigma S - \omega(R_0 + S_0 + W_0 - R - S)) - a(\lambda_S) \tag{16}$$

$$\dot{R} = -q(R, \lambda_S, \lambda_R) \tag{17}$$

$$\dot{\lambda_S} = \lambda_S(\gamma(1 + \omega) + \rho) - D_S \tag{18}$$

$$\dot{\lambda_R} = \rho\lambda_R - \gamma\omega\lambda_S + q(R, \lambda_S, \lambda_R)C_R \tag{19}$$

In a steady state we require the co-state and state variables to be constant. Applying $\dot{S} = \dot{R} = \dot{\lambda_S} = \dot{\lambda_R} = 0$ to (16)-(20) we obtain.

$$a(\lambda_S) = -\gamma(\sigma S - \omega(R_0 + S_0 + W_0 - R - S)) \quad (20)$$

$$q(R, \lambda_S, \lambda_R) = 0 \quad (21)$$

$$\lambda_S = \frac{D_S}{\gamma(1+\omega)+\rho} \quad (22)$$

$$\lambda_R = \frac{\gamma\omega\lambda_S}{\rho} \quad (23)$$

According to equation (21) extraction is zero in the steady state and hence, no additional carbon flows into the carbon cycle. The reason is that the marginal social benefit of extracting an additional unit of $R$ is less then its costs of extraction and the long run damage resulting from a higher carbon content. Since $q = 0$ the LHS of equation (20) can be interpreted as the net anthropogenic transfer of carbon from the upper reservoir to the lower reservoir. According to (20) this net anthropogenic transfer must be equal to the net natural transfer of carbon from the lower reservoir to the upper reservoir. Thus, the steady state corresponds qualitatively to point $C$ in figure 2. Equation (22) states that in the steady state the carbon tax must be equal to the marginal damage weighted by the discount rate and the parameters describing the lagged adjustment effect of the natural component of the carbon cycle. Since $S > 0$ in a steady state, $D_S > 0$ and the carbon tax must be positive. Finally equation (23) implies a steady state resource rent being linearly proportional to the carbon tax. Hence, the resource rent must be strictly positive as well. In a next step we derive saddle point properties of the MHDS.

**Proposition 1.** *The steady state of the MHDS system (18)-(21) is saddle point stable*

*Proof.* The Jacobian of the MHDS evaluated at the steady state is given by:

$$J = \begin{bmatrix} -\gamma(1+\omega) & -\gamma\omega + q_R & -a_{\lambda_S} + q_{\lambda_S} & q_{\lambda_R} \\ 0 & -q_R & -q_{\lambda_S} & -q_{\lambda_R} \\ -D_{SS} & 0 & \gamma+\rho+\gamma\omega & 0 \\ 0 & C_R q_R & -\gamma\omega C_R q_{\lambda_S} & \rho + C_R q_{\lambda_R} \end{bmatrix} \tag{24}$$

For this system of four linear first-order differential equations the four characteristic roots can be obtained by using [Dockner, 1985], Theorem 1, p.10:

$$p_{1,2,3,4} = \frac{\rho}{2} \pm [(\frac{\rho}{2})^2 - \frac{1}{2}\Omega \pm \frac{1}{2}(\Omega^2 - 4 * \Delta)^{0.5}]^{0.5}, \tag{25}$$

where $\Delta$ is the determinant of the Jacobian of (24) being:

$$\Delta = -\gamma\omega D_{SS}(\gamma\omega q_{\lambda_R} + \rho q_{\lambda_S}) + \rho\left(\gamma(1+\omega)(\gamma+\rho+\gamma\omega) + a_{\lambda_S} D_{SS}\right) q_R > 0$$

and

$$\Omega = -\gamma(1+\omega)(\gamma+\rho+\gamma\omega) + D_{SS}(q_{\lambda_S} - a_{\lambda_S}) - \rho q_R < 0 \tag{26}$$

Given that $\Delta > 0$ and $\Omega < 0$ the system has saddle point properties. In addition, by showing that $\Omega^2 - 4\Delta > 0$ we show that the roots are real.

$$\begin{aligned} \Omega^2 - 4\Delta \;&=\; \underbrace{-4\left(-\gamma(1+\omega)(\gamma+\rho+\gamma\omega) + D_{SS}(q_{\lambda_S} - a_{\lambda_S}) - \rho q_R\right)}_{>0} \\ &+\; \underbrace{\left(\gamma\omega D_{SS}(\gamma\omega q_{\lambda_R} + \rho q_{\lambda_S}) - \rho(\gamma(1+\omega)(\omega+\rho+\gamma\omega) + a_{\lambda_S} D_{SS}) q_R\right)^2}_{>0} \end{aligned} \tag{27}$$

□

### 3.5 Carbon Tax

From the dynamic efficiency condition (18) we can extract some information about the optimal paths of the carbon tax. We restate (18) as:

$$\dot{\lambda_S} = \lambda_S \cdot \theta - D_S \tag{28}$$

with $\theta = \rho + \gamma(1 + \omega)$. Consequently $\lambda_S$ increases with its own level weighted by $\theta$ and decreases with higher levels of the marginal damage. Notice that we can express the isocline of $\lambda_S$ by

$$\lambda_S(S)_{|\dot{\lambda_S}=0} = \frac{D_S(S)}{\theta} = z(S) \tag{29}$$

where $z_S > 0$. Using equation (29) we want to illustrate the possible shapes of the optimal carbon tax paths. These are: increasing, decreasing, u-shaped and inversely u-shaped. Figure 3 illustrates the implications of equation (28).

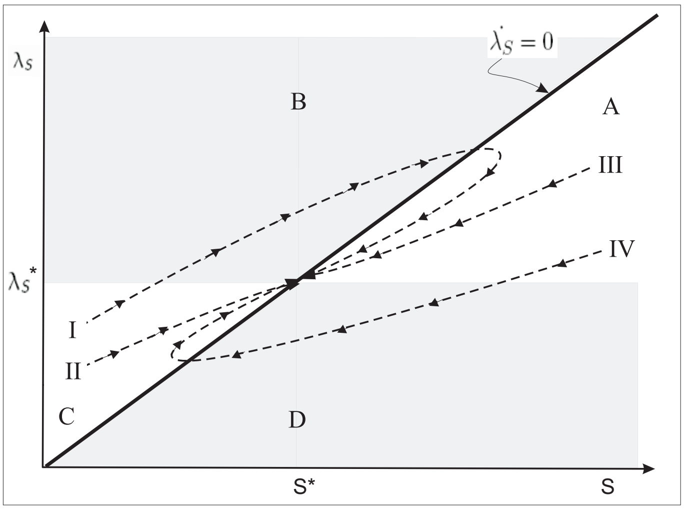


Figure 3: non-monotonic carbon tax paths

**Proposition 2.** *Consider Figure 3. Consider the isocline $\lambda_S = z(S)$ with $z_S > 0$ and the steady state $(\lambda_S^*, S^*)$. Define the set $B$ as $\{B\}$=$\{\lambda > \lambda_S^*\} \cup \{\lambda > z(S)\}$. All paths crossing $\{B\}$ at any $t \in [0, \infty]$ imply an inverse U-shaped carbon tax over the entire interval* $[0, \infty]$.

*Proof.* We argue that the carbon tax can be initially increasing and decreasing afterwards. Consider an optimal path originating at $(\lambda_{S_0}, S_0) \in \{B\}$. Equation (28) implies $\dot{\lambda_S} > 0$ and we observe an increasing carbon tax. Because in $\{B\}$ $\lambda_S > \lambda_S^*$, there exists one point in time $T_B \in [0, \infty]$ for which $\dot{\lambda_S} < 0$. According to (29), $T_B$ occurs at $\lambda = z(S)$. $\forall t \in [T_B, \infty]$ $\lambda_S$ is decreasing. □

Notice, that at the maximum of an inversely u-shaped carbon tax, the upper reservoir must be increasing. This can be seen by differentiating (28) w.r.t. time.

$$\ddot{\lambda_S} = \theta \dot{\lambda_S} - D_{SS}\dot{S} \tag{30}$$

Since $\dot{\lambda_S} = 0$ at a maximum and $D_{SS} > 0$ it follows that $\ddot{\lambda_S} < 0$ if $\dot{S} > 0$. In a next step we show the possibility of a U-shaped carbon tax.

**Proposition 3.** *Consider Figure 3. Consider the isocline $\lambda_S = z(S)$ with $z_S > 0$ and the steady state $(\lambda_S^*, S^*)$. Define the set D as $\{D\}$=$\{\lambda < \lambda_S^*\} \cup \{\lambda < z(S)\}$. All paths crossing $\{D\}$ at any $t \in [0, \infty]$ imply an U-shaped carbon tax over the entire interval* $[0, \infty]$.

*Proof.* This proof is similar to Proposition 2. We argue that the carbon tax can be initially increasing and decreasing afterwards. Consider an optimal path originating at $(\lambda_{S_0}, S_0) \in \{D\}$. Equation (28) implies $\dot{\lambda_S} < 0$ and we observe a decreasing carbon tax. Because in $\{D\}$ $\lambda_S < \lambda_S^*$, there exists one point in time $T_D \in [0, \infty]$ for which $\dot{\lambda_S} > 0$. According to (29), $T_D$ occurs at $\lambda = z(S)$. $\forall t \in [T_D, \infty]$ $\lambda_S$ is increasing. □

Proposition 3 implies that at a minimum of the carbon tax, the upper reservoir must be decreasing, since now $\ddot{\lambda_S} > 0$ if $\dot{S} < 0$.

While paths originating in area $B$ and $D$ must be non-monotonic, the converse is not necessarily true. In Figure 2 we have not explicitly considered the dynamics of the upper reservoir. The natural transfer rate $\gamma$ plays an important role. Consider e.g. a situation in which the carbon stock in the upper reservoir at $t = 0$ is extremely high. Whether $\lambda_S$ is high or low (i.e. paths originating in $\{A\}$ or $\{D\}$ at $t = 0$ depends among other model parameters, on the natural transfer rate. If the transfer rate is very hight as well, this implies that the flux from the upper reservoir to the lower reservoir is very high (lower $\lambda_S$ initially) and the possibility of undershooting the steady state carbon content in $S$ is more likely. As a consequence we obtain a u-shaped carbon tax (Path $III$). On the other hand, a low transfer rate implies a weaker flux from the upper reservoir to the lower reservoir (higher $\lambda_S$ initially). Hence, a monotonically falling carbon content in $S$ is more likely. As a consequence we obtain a monotonically decreasing carbon tax (Path $IV$).

By similar reasoning we cannot rule out paths originating in $C$ to be non-monotonic. For low initial levels of $S$ and a high transfer rate there need not be an overshooting of the upper reservoir, implying a monotonically increasing carbon tax (e.g. Path $II$). However, a low natural transfer rate of carbon combined with high emission levels can result in an overshooting of the upper reservoir which in turn relates to a inversely u-shaped carbon tax (Path $I$). The dynamic properties of the system variables, in particular the carbon tax are analyzed in section 4 in more detail.

# 4 System Dynamics in explicit form

We impose the following functional forms with $a_1, u_1, u_2, v_1, c_1, c_2, s_1 > 0$

$$U(q) = u_1 q - u_2 q^2 \tag{31}$$

$$A(a) = a_1 a^2 \tag{32}$$

$$C(R) = c_1 - c_2 R \tag{33}$$

$$D(S) = v_1 (s_1 S - s_2)^2 \tag{34}$$

Since $S$ is the stock of carbon in the upper box containing the atmosphere and the upper ocean layer, we introduce the parameter $s_1$, the percentage of the carbon stock in the upper box which is situated in the atmosphere. Using the FOC's (11) and (12) we can solve for $q$ and $a$:

$$q = \frac{-c_1 + c_2 R + u_1 - \lambda_R - \lambda_S}{2u_2} \tag{35}$$

$$a = \frac{\lambda_S}{2a_1} \tag{36}$$

Using the specific functional forms and the two previous equations we can rewrite the MHDS in canonical form:

$$\begin{pmatrix} \dot{S} \\ \dot{R} \\ \dot{\lambda_S} \\ \dot{\lambda_R} \end{pmatrix} = \begin{pmatrix} -\gamma(1+\omega) & -\gamma\omega + \frac{c_2}{2u_2} & \frac{-a_1-u_2}{2u_2 a_2} & \frac{-1}{2u_2} \\ 0 & \frac{-c_2}{2u_2} & \frac{1}{2u_2} & \frac{1}{2u_2} \\ -s_1^2 v_1 & 0 & \gamma\omega + \gamma + \rho & 0 \\ 0 & \frac{-c_2}{2u_2} & -\gamma\omega + \frac{c_2}{2u_2} & \rho + \frac{c_2}{2u_2} \end{pmatrix} \cdot \begin{pmatrix} S \\ R \\ \lambda_S \\ \lambda_R \end{pmatrix} + \begin{pmatrix} k_1 \\ k_2 \\ k_3 \\ k_4 \end{pmatrix}$$

with

$$\begin{aligned} k_1 &= \omega\gamma(R_0 + S_0 + W_0) - \frac{c_1 - u_1}{2u_2} \\ k_2 &= \frac{c_1 - u_1}{2u_2} \\ k_3 &= 2s_1 v_1 s_2 \\ k_4 &= \frac{c_2(c_1 - u_1)}{2u_2} \end{aligned}$$

We can solve for the steady state values of $\lambda_S$, $\lambda_R$, $S$ and $R$ which are obtained by setting $\dot{\lambda_S} = \dot{\lambda_S} = \dot{\lambda_W} = \dot{\lambda_R} = \dot{S} = \dot{W} = \dot{R} = 0$ The steady state values for the canonical system when $t \to \infty$ are:

$$\begin{aligned}
\tilde{S} &= \frac{1}{k_5} \cdot \big[ - a_2(c_1 - c_2 k_6)\gamma\rho\omega(\gamma + \rho + \gamma\omega) && (37)\\
&+ s_2 s_1 v_1(c_2\rho + 2a_2\gamma\omega(\rho + \gamma\omega)) + a_2\gamma\rho\omega(\gamma + \rho + \gamma\omega)u_1\big]\\
\tilde{R} &= \frac{1}{k_5} \cdot \big[2a_2 s_1 v_1 \gamma(\rho + \gamma\omega)(s_1 k_6 \omega - s_2(1 + \omega)) && (38)\\
&+ c1\rho(s^2 v + a_2\gamma(1 + \omega)(\gamma + \rho + \gamma\omega))\\
&- \rho(s^2 v + a_2\gamma(1 + \omega)(\gamma + \rho + \gamma\omega))u_1\big]\\
\tilde{\lambda_S} &= \frac{1}{k_5} \cdot \big[2a_2 s_1 v_1 \gamma\rho(s_1(-c1 + c_2 k_6)\omega - s_2 c_2(1 + \omega) + s\omega u_1)\big] && (39)\\
\tilde{\lambda_R} &= \frac{1}{k_5} \cdot \big[2a_2 s_1 v_1 \gamma^2\omega(s_1(-c1 + c_2 k_6)\omega - s_2 c_2(1 + \omega) + s\omega u_1)\big] && (40)
\end{aligned}$$

with

$$\begin{aligned}
k_5 &= 2a_2 s_1^2 v\gamma\omega(\rho + \gamma\omega) + c2\rho(s_1^2 v_1 + a_2\gamma(1 + \omega)(\gamma + \rho + \gamma\omega))\\
k_6 &= R_0 + S_0 + W_0
\end{aligned}$$

## 4.1 Optimal Paths

In order to analyze the optimal paths of the model variables in detail we parameterize the model. Concerning the parameter space, there are some nature-given parameters which we have obtained from current estimates. The remaining (economic) parameters were chosen such as to calibrate the model. We assume that the pre-industrial ocean

| Nature-given Parameter | Value | Economic Parameter | Value |
|---|---|---|---|
| $\gamma$ | .005 | $\rho$ | .01 |
| $\sigma$ | 1 | $a_2$ | 2 |
| $\omega$ | .1 | $u_1$ | 50 |
| $W_0$ | 20,000 | $u_2$ | .5 |
| $R_0$ | 10,000 | $c_1$ | 50 |
| $S_0$ | 2,000 | $c_2$ | .004 |
| $s_2$ | 600 | $s_1$ | .3 |
| | | $v_1$ | .001 |

Table 1: Parameter Values: Base run

was in steady-state, i.e. there was a balance between carbon sources and sinks in the ocean. Since the onset of the industrial revolution (mid 19th century), this balance has been upset by release of $CO_2$ into the atmosphere, of which concentration of $CO_2$ has increased dramatically. About $36,100$ Pg-C[7] are currently stored in the deep ocean, compared to 910 Pg-C in the surface ocean and 820 Pg-C ( about 385 ppm) in the atmosphere[8]. For the preindustrial levels of the reservoirs we have chosen: $20,000$ Pg-C for the deep see reservoir $W_0$ [9], 600 Pg-C for the atmosphere and $1,200$ Pg-C for the upper ocean layer. The values for $\gamma$, the dynamic adjustment parameter and $\omega$, the factor of proportionality of the two reservoirs have been chosen such as to represent observed fluxes [10].

According to the German Federal Office for Geoscience and Natural Resources (BGR) current non-renewable *reserves* are estimated to be at an order of $1,350$ Pg-C equivalent [BGR, 2006]. Non-renewable carbon *resources* on the other hand are estimated between 5,000 Pg-C [Lackner, 2003] and $12,000$ Pg-C [BGR, 2006]. We have chosen $R_0$, i.e. the pre-industrial stock of $R_0$ to be at $10,000$ Pg-C. Since we include resource based extraction costs in our model, we can parameterize our model such that the resource stock will not be used entirely in finite time.

In order to obtain an expression for the optimal paths, we consider the Jacobian

[7] We measure these stocks in mass-units of carbon (i.e. Pg-C), as it appears in different chemical forms within the reservoirs.

[8] We have calculated these numbers by assuming that the average deep dissolved inorganic carbon (DIC) concentration is about $2.290 \cdot 10^6 \frac{mol}{km^3}$ and the volume of the deep ocean reservoir is about $1.3138 \cdot 10^9 km^3$. For the upper ocean (down to about 100m we assume an average DIC concentration of $2.100 \cdot 10^6 \frac{mol}{km^3}$ and a volume of $36.1 \cdot 10^6 km^3$

[9] The pre-industrial level of $W$, the deep-sea reservoir is difficult to calculate, with estimates ranging from $20,000$ - $40,000$ Pg-C

[10] Assuming $S_0 = 2,000$ and approximately today's $S_{2008}$ of $2,200$, using $\gamma = .005$, $\omega = .1$ total inactivity, i.e. no extraction, no sequestration would result in a half-life of $t = 126$ to the new system equilibrium of $S = 2,018.18$ and $W = 20,181.81$

of the MHDS applying the functional forms (31)-(34).

$$J = \begin{bmatrix} \gamma+\rho+\gamma\omega & 0 & -2s_1^2 v & 0 \\ -\gamma\omega+\frac{c_2}{2u_2} & \rho+\frac{c_2}{2u_2} & 0 & -\frac{c_2^2}{2u_2} \\ -\frac{a_2+u_2}{2a_2u_2} & -\frac{1}{2u_2} & -\gamma(1+\omega) & -\gamma\omega+\frac{c_2}{2u_2} \\ \frac{1}{2u_2} & \frac{1}{2u_2} & 0 & -\frac{c_2}{2u_2} \end{bmatrix} \tag{41}$$

Using the parameter values above we can then calculate the eigenvalues associated with this Jacobian. They are: $r_1 = 0.012$, $r_2 = -0.002$, $r_3 = 0.024$, $r_4 = -0.014$. Thus, the system has two negative eigenvalues, as we have shown in Proposition 1. Therefore, the steady state is saddlepoint stable. The steady state values are: $\tilde{S} = 2503.66$, $\tilde{R} = 1535.34$, $\tilde{\lambda_S} = 5.85$, $\tilde{\lambda_R} = .29$ and additionally, $\tilde{a} = 1.46$, $\tilde{q} = 0$. Furthermore, from the carbon balance equation we obtain $\tilde{W} = 27961.01$. Given the information above, we can formulate the optimal paths for $S(t)$, $R(t)$, $\lambda_S(t)$ and $\lambda_R(t)$ (where we denote optimal paths by an asterisk).

$$X_t^* = \tilde{X} + e^{r_1 t} \cdot \Theta_1 \cdot \Upsilon_{r_1,X} + e^{r_2 t} \cdot \Theta_2 \cdot \Upsilon_{r_2,X} \qquad \text{for } X = S, R, \lambda_S, \lambda_R \tag{42}$$

where $r_1$ and $r_2$ are the negative eigenvalues of the Jacobian above, $\Upsilon_{r_i,X}$ is the eigenvector of $X$ related to the eigenvalue $i$ and $\Theta_1$ and $\Theta_2$ are constants which are obtained by solving

$$X_0^* = X_0 \qquad \text{for } X = S, R \tag{43}$$

The optimal paths of $W$, $q$ and $a$ are obtained using equations (5), (35) and (36) respectively.

## 4.2 The base-run

Figure 4 depicts the results of the base run simulation. The abscissae denote time $t$. Recall that $S_0 = 2,000$ $R_0 = 10,000$ and $W_0 = 20,000$. Extraction is high in the begin-

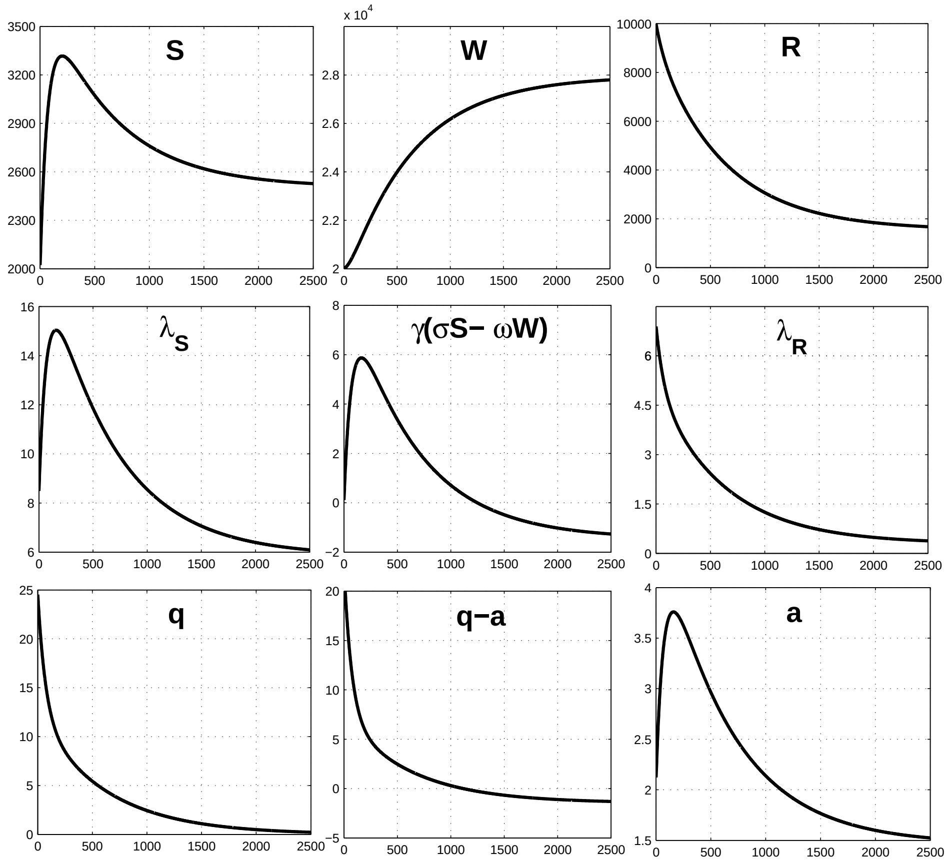


Figure 4: The base run scenario

ning and decreases monotonically to zero. The resource stock $R$ is only economically depleted and not fully used. Extraction and emission of the non-renewable resource has an instant damage effect. Sequestration of carbon into the deep ocean has a delayed damage effect. Thus, it is optimal to extract the resource rapidly and sequester a lot in the beginning. As a consequence of higher extraction rates, the carbon concentration in the upper reservoir and in turn, the carbon tax rises. The upper reservoir overshoots its steady state level. This is because the natural transfer of carbon to the deep ocean is not fast enough to absorb the carbon added to the upper reservoir. The lower reservoir is a net sink of carbon and its content increases monotonically. The resource scarcity rent is monotonically declining. In the new steady state the natural transfer of carbon to the lower reservoir is negative and the net anthropogenic transfer of carbon to the lower reservoir is positive. This implies for the new steady state that lower reservoir is supersaturated and we obtain an emitting ocean.

## 4.3 Sequestration vs. no sequestration

Using the base-run scenario we have analyzed the effect of not allowing for ocean sequestration. The upper plot in figure 5 depicts the carbon content diagram. The solid line represents the base run simulation while the dashed line depicts the situation without the option to sequester. Notice that without sequestration, the new steady state is characterized by equal partial pressure in both reservoirs since now sequestration and extraction are both zero. Without sequestration the overshooting of the upper

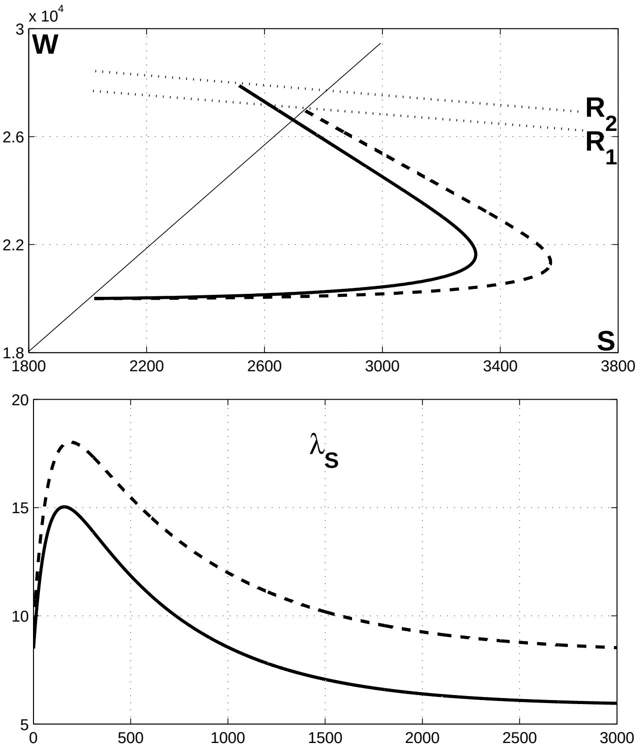


Figure 5: Sequestration vs. no sequestration

reservoir is much stronger and the steady state carbon content in the atmosphere is much larger when compared to the base run simulation. Most surprisingly, less of the resource stock has been extracted as can bee seen by the lower iso carbon content line $R1$. This is because sequestering carbon at a constant rate in the steady state allows for a larger equilibrium share of carbon in the lower reservoir. The lower graph in figure 5 shows the carbon tax. Without sequestration, the carbon tax path is shifted upwards, thus at any point in time the carbon tax is higher. The reason for this is the larger overshooting of the upper carbon reservoir which results in a higher damage.

## 4.4 Discounting

In a next step we investigate how the optimal solution changes if we decrease the discount rate. The solid lines in figure 6 depict the base run case again where the discount rate is 1%. There is an ongoing debate about the proper discount rate in presence of global warming. The Stern report stern postulate suggests a discount rate close to zero. The dotted line in figure 6 represents the optimal solution where we have lowered the discount rate to 0.1%. Since a high discount rate means that we

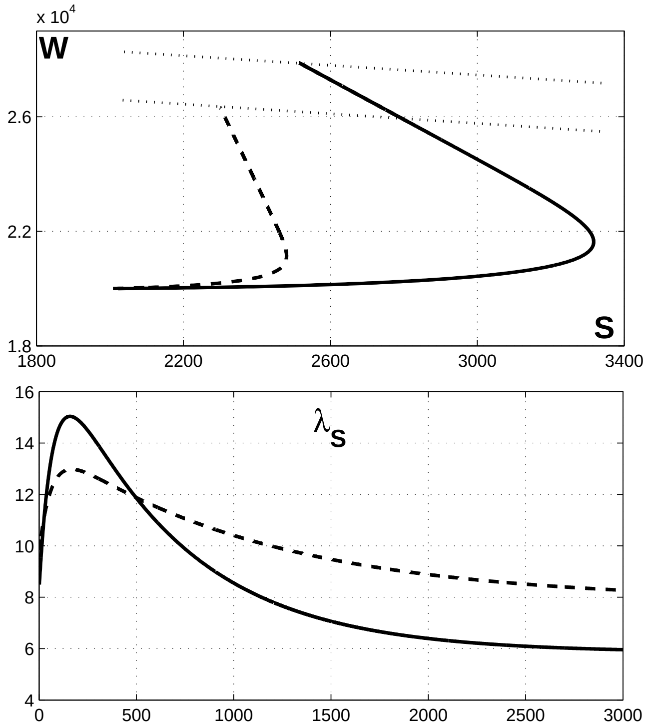


Figure 6: 1% vs. 0.1% discounting

value the future less, ocean sequestration will be used more intensively because of its lagged damage effect. On the other hand, placing more weight on the future damages leads to fewer total extraction as implied by the lower carbon isocontent line in Figure 6. In that case, future and current damages are perceived more equally. Thus, ocean sequestration as a tool in determining when the damages will occur becomes less powerful.

## 4.5 Carbon Taxes

In our base run scenario with the most reasonable parameter space the carbon tax is hump-shaped. This is because the path of $S$ is hump-shaped. In general, the carbon tax will have a similar shape to the upper reservoir. It is therefore possible to obtain different shapes of the carbon tax. Figure 7 displays four possible paths of the carbon tax within our modeling framework. These plots differ from each other w.r.t. the initial carbon stock size and the natural transfer speed of carbon between the two reservoirs. The base run simulation scenario (low $\gamma$, low $S_0$) is depicted in the upper

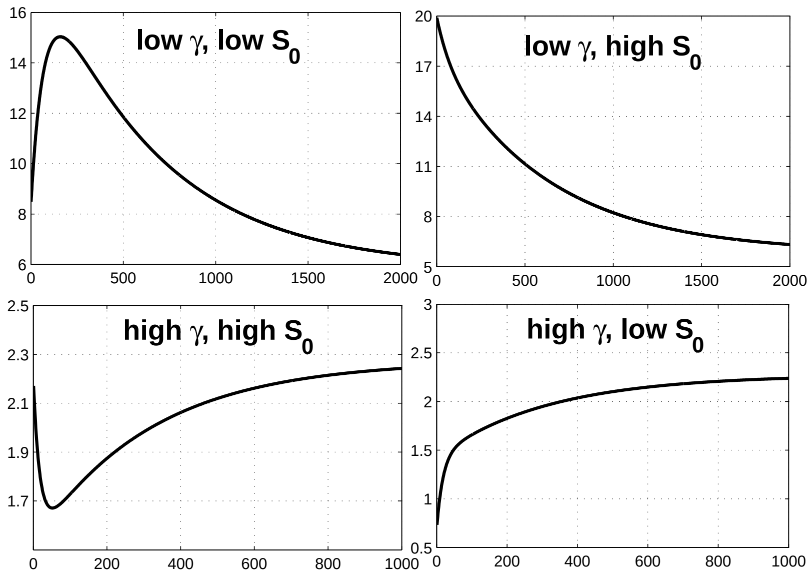


Figure 7: Possible paths of the carbon tax

left plot. It shows the same inverted u-shaped pattern as seen earlier. The upper right plot on the contrary shows the optimal carbon tax resulting from increasing the initial stock of carbon in the upper reservoir (low $\gamma$, high $S_0$). As a result, the carbon tax is monotonically decreasing to the same steady state as in the upper left scenario. The decrease occurs for two reasons. (1) A high initial $S_0$ induces much higher rates of sequestration (equation 12) and (2), since the carbon balance equation must hold at any point in time, increasing $S_0$ automatically implies a lower $W_0$. As a consequence, there is a very high difference in the partial pressure between the two reservoirs and the natural transfer is very strong.

The first thing to notice about the 2 lower plots is the time scale. In both cases we have significantly increased $\gamma$, the natural transfer speed. Consequently, carbon moves much faster between the two reservoirs and a steady state is reached sooner. In the lower right plot in particular (low $\gamma$, high $S_0$), the optimal carbon tax is monotonically increasing. The only difference to the base run scenario is a higher transfer speed of carbon. As a consequence the overshooting of the upper reservoir does no longer occur since now carbon is absorbed by the deep ocean much faster. This absorption effect is at its highest in the lower left plot. Here, the scenario (low $\gamma$, low $S_0$) implies that because (1) the transfer speed is very high and (2) the difference in the partial pressure of the two reservoirs is very high as well, we obtain an undershooting of the upper carbon reservoir and consequently, a u-shaped carbon tax.

As a general remark, notice that the level of the carbon tax is much higher in the upper plots. The natural transfer speed is the major determinant of the carbon tax level. Because it determines for "how long" the emitted carbon will remain in the atmosphere and hence, contribute to the damage resulting from higher carbon concentration levels in the atmosphere. Figure 8 is analogous to figure 2. We depict the four possible carbon tax paths in the $S - \lambda_S$ space. Notice that the supplots differ w.r.t the initial level of $S$ while within the supplots only $\gamma$ has been changed. In the upper plot we observe the base run case where we start with a carbon tax above its steady state level, but the carbon tax must increase first before it can monotonically fall towards its steady state.

In the lower plot we observe that the carbon tax may be monotonically increasing even if resource extraction goes to zero.[11] This is contrary to what most of the literature suggests. The reason is that the emission of carbon is not "forgotten", as it would be with a constant decay rate. The way we model the carbon cycle accounts

[11] This can be due to either economic or physical depletion

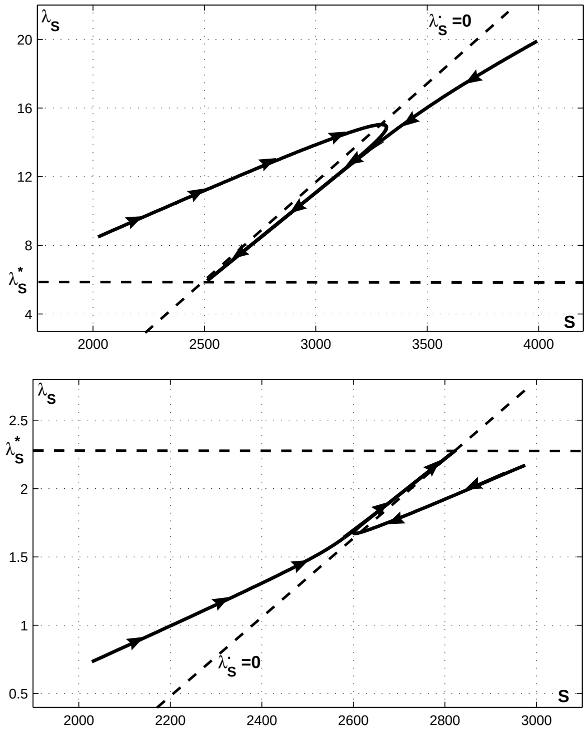


Figure 8: Carbon tax paths

for the fact that each unit of carbon is persistent and has a instant or lagged damage effect, with the latter being due to sequestration.

## 4.6 The decentralized case

There are two externalities in our model, (i) the negative external effect of high carbon concentration levels on welfare and (ii), the positive externality which results from removing carbon from the atmosphere via CCS. In this section we consider very briefly the decentralized economy and study how the externalities should be internalized. Consider a representative firm that extracts the resource stock and possesses the technology to capture carbon and sequester it into the deep ocean. Because the firm's profits are solely determined by selling the extracted resource, we introduce a subsidy on sequestration which we denote by $\theta$. At the same time we impose a tax $\tau$ on the price of carbon. The firm's profit maximization problem reads:

$$\max_{q,a} \int_0^\infty e^{-\rho t} \left(pq - \tau q + \theta a - A(a) - qC(R)\right) dt \tag{44}$$

subject to

$$\dot{R} \;=\; -q \tag{45}$$

$$R(0) = R_0 > 0 \tag{46}$$

From the first order condition of the current value Hamiltonian we obtain conditions for the optimal controls $a$ and $q$.

$$0 \;=\; p - \tau - C(R) \tag{47}$$

$$0 \;=\; \theta - A_a \tag{48}$$

Assuming $U_q = p$ we can compare (11)-(12) to (47)-(48) we obtain the optimal decentralized policy $\tau = \lambda_S = \theta$. This implies that the revenues from the carbon tax should be distributed as a subsidy on sequestering carbon. The reason for this is that ocean sequestration has a temporary abatement potential because of its lagged damage effect.

# 5 Conclusion

In order to assess the problem of the appropriate path and level of the carbon tax one has to take into account the lagged and persistent effect of emitting carbon on the ocean's capacity to absorb carbon from the atmosphere. However, the role of the oceans has not received much attention in theoretical models of optimal resource extraction. Especially, when analyzing global warming, not only the stock of carbon in the atmosphere is important, but also the functioning of the deep ocean as a carbon sink.

We take account of the latter effect by assuming two carbon reservoirs: The upper reservoir and the lower reservoir. The upper reservoir consists of the stock of carbon in the atmosphere and the upper ocean layer. The lower reservoir comprises the carbon stock in the deep ocean. The natural flux of carbon is driven by the relative size of the carbon reservoirs. The relatively "carbon-abundant" reservoir will therefore be a natural source of carbon outflow. This is the natural component of the global carbon cycle. We have added an anthropogenic component to this by introducing an exhaustible resource with carbon content.

The economic extraction of the fossil resource releases carbon that is pumped into the global carbon cycle. Without CCS, the whole amount of carbon released will be captured by the atmosphere and only slowly transferred into the deep ocean. In order to accelerate the slow natural mix of the deep ocean with the atmosphere and upper ocean layer we focus on the possibility of carbon capture and storage via deep see injection.

From our simulation results we conclude that CCS is an important tool for stabilizing the global carbon system because it accelerates the free, but slow natural flux within the carbon cycle. CCS may help achieving stricter stabilization targets in the coming decades without relying to much on the expensive and subsidy intensive renewables. Policy makers are well advised to consider investments into modern and efficient coal fired power plants while at the same time to support R&D of CCS technologies which have a huge potential to ensure a smooth transition towards the usage of renewables in the long run.

For the optimal carbon tax our findings suggest that it is inverted u-shaped and its level should be adjusted with the uptake capacity of other carbon sinks, such as the oceans.

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