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Noise and Bias in Eliciting Preferences

by

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Abstract:

In the context of eliciting preferences for decision making under risk, we ask the question: "which might be the 'best' method for eliciting such preferences?". It is well known that different methods differ in terms of the bias in the elicitation; it is rather less well-known that different methods differ in terms of their noisiness. The optimal trade-off depends upon the relative magnitudes of these two effects. We examine four different elicitation mechanisms (pairwise choice, willingness-to-pay, willingness-to-accept, and certainty equivalents) and estimate both effects. Our results suggest that economists might be better advised to use what appears to be a relatively inefficient elicitation technique (i.e. pairwise choice) in order to avoid the bias in better-known and more widely-used techniques.

Key words: pairwise choice, WTP, WTA, errors, noise, biases

JEL-classification: C91, C81

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1 INTRODUCTION

For all sorts of reasons economists frequently need to elicit people's preferences. There are different elicitation methods; sometimes economists use one method, sometimes another. This paper is concerned with trying to understand what might be the 'best' method. In this paper we concentrate attention on a particular context – the elicitation of individuals' preferences when taking decisions under risk – though our concerns and results clearly have implications in other contexts. To keep our analysis simple and concentrate on the key issues, we presume that all the individuals whose preferences we are eliciting obey expected utility theory, and that we are concerned with the estimation of their (von Neumann-Morgenstern) utility functions using different kinds of elicitation mechanisms.

There exist various methods for the assessment of von Neumann-Morgenstern utility functions, see e.g. Farquahar (1984) for a review. The specific mechanisms that we are considering are the principal ones used in the recent literature and are:

1. elicitation of preferences through pairwise choice preference questions;
2. elicitation of certainty equivalents through the statement of willingness-to-pay in a second-price auction;
3. elicitation of certainty equivalents through the statement of willingness-to-accept in a second-price offer auction;
4. elicitation of certainty equivalents using the Becker-DeGroot-Marschak mechanism.

According to Tversky et al. (1988) the latter three mechanisms can be categorized as matching procedures. In most practical applications preferences are elicited by matching procedures, e.g. willingness-to-pay and willingness-to-accept in contingent valuation studies or the time-trade-off method in health economics. Many empirical studies have shown that choices and matching procedures may lead to fundamentally different results. These phenomena are generally referred to as response mode effects. A well known response mode effect in decision making under risk is the preference reversal phenomenon first observed by Lichtenstein and Slovic (1974). This phenomenon occurs if a subject prefers a safe lottery to a risky one in direct choice but assigns a higher certainty equivalent to the risky lottery. Response mode effects do also occur when comparing the single matching procedures. Most prominent in this context seems to be the disparity between willingness-to-pay and willingness-to-accept discussed by Coursey et al. (1987) and Knetsch and Sinden (1984, 1987). This disparity is often explained by a status-quo bias (Samuelson and Zeckhauser, 1988) and leads to the question which of both measures should be used in contingent valuation studies. Our results may help to answer this question.

In general, response mode effects may be caused by errors or biases in the subjects´ responses. Consequently, answering our question – which elicitation method might be best – requires a comparison of the single methods in terms of their noisiness and in terms of involved biases. It is well known that subjects in experiments are noisy in their responses to pairwise choice questions (in that they give different answers when asked the same question on several occasions), cf. e.g. Camerer (1989), Starmer and Sugden (1989), Wu (1994), Harless and Camerer (1994), and Hey and Orme (1994). There is no reason to believe that subjects are not also noisy when it comes to stating their certainty equivalents although we are not aware of empirical studies investigating this noise explicitly.[4] Elicitation of certainty equivalents may also involve biases: Even if it has been explained carefully to subjects that their stated willingness-to-pay in a second-price auction ought to be equal to their certainty equivalent, it is apparent that there are subjects who deliberately and consistently under-bid, see Coppinger et al. (1980) and Cox et al. (1982). Similarly, in attempts to elicit certainty equivalents through willingness-to-accept in second-price auctions, it would appear that many subjects over-ask. The Becker-DeGroot-Marschak mechanism appears to be neutral in that there is no obvious bias in the procedure – but, nevertheless, it may be the case that subjects find the procedure too complicated and adopt some simple heuristic with an inbuilt bias.

[4] The study of Schmidt and Hey (2004) may be regarded as exception as it analyses the role of pricing errors for explaining preference reversals.

The purpose of this paper is to see if such biases exist and how noisy the various elicitation methods are.

Our analysis is important because it would appear on *a priori* grounds that one of other of the certainty equivalents methods is potentially more informative than the pairwise choice method, since the latter only tells us which choice is preferred – but not by how much. However, if there is more noise in the certainty equivalents methods this might outweigh their inherent superiority. Moreover, if there is bias in the certainty equivalents methods, it may be better to elicit utility functions through the unbiased preference method.

We have estimated the noise and bias in the various elicitation methods using experimental data. Section 2 describes the experimental design, while section 3 discusses the techniques used to estimate the noise and bias in the various methods. Section 4 presents our results where we first analyse the noise of the single methods, then their bias, and finally take both into account for a concluding evaluation. Section 5 summarises our results.

2 EXPERIMENTAL DESIGN

The experiment was conducted at the Centre of Experimental Economics at the University of York with 24 participants. Each participant had to attend five separate sessions on five different days. After a subject had completed all five sessions, one question of one session was randomly selected and played out for real. The average payment to the subjects was £34.17 with £80 being the highest and £0 being the lowest payment.

In each of the five sessions subjects were presented the same 30 lottery pairs, 28 risky ones and two ambiguous ones (which are not analysed in this paper). All risky lotteries were composed of the four consequences £0, £10, £30, and £40. The probabilities of these consequences are recorded in the Appendix Table for all 28 lottery pairs. In the experiment lotteries were presented as segmented circles on the computer screen – see in Figure 1.

Insert Figure 1 here

In the five sessions subjects had to perform altogether eight tasks six of which will be analysed in the present paper:

- three times report a preference in 28 pairwise choice questions (between two risky lotteries). We call this the PC task;
- report a maximal buying price (bid) for each of the 56 lotteries. We call this the BID task;
- report a minimal selling price (ask) for each of the 56 lotteries. We call this the ASK task;
- report a certainty equivalent (CE) for each of the 56 lotteries. We call this the BDM task.

For all tasks we used incentive-compatible elicitation mechanisms. Bids and asks were elicited with second-price sealed-bid auctions while for the certainty equivalents we employed the Becker-DeGroot-Marschak mechanism. We now describe the various tasks with a little more detail.

In the PC task, on the subject's computer screen both lotteries of the pair appeared as circles and subjects had to indicate whether they preferred the left lottery, or the right lottery, or neither. After pressing the corresponding key they had to confirm their choice by pressing the return key. If a pairwise choice question was selected as reward the subject could simply play out the preferred lottery. In case of stated indifference one of the lotteries was selected by the experimenter.

In the BID task the following question appeared under each lottery: "Submit your bid for this lottery in a second-price sealed-bid auction." That is, subjects were asked to assume they did not have the lottery and had to bid to get it. They had to type in their bid and confirm it by pressing the return key. If a question of the bid treatment was selected for the reward, the subject received a payment of £*y* where *y* is the highest amount in the corresponding lottery. Moreover, if the subject submitted the highest bid for this lottery (among all subjects in the group with whom he or she completed the bid treatment) then the subject would additionally pay the second highest bid and then play out the lottery (receiving whatever outcome resulted).

The ASK task was identical to the BID task except that for each lottery a different question was asked: "Submit your offer for this lottery in a second-price offer auction". That is, subjects were asked to assume that they owned the lottery and had to make an offer to dispose of it. If a question from the ask treatment was selected for the reward, and if the subject had not submitted the lowest ask then the subject could play out the corresponding lottery. However, if he or she had submitted the lowest ask (among all subjects in the group with whom he or she completed the ask task), he or she received the second lowest ask instead of playing out the lottery.

In the BDM task the following question appeared under each lottery: "State the amount of money such that you do not care whether you will receive this amount or the lottery". If a question of the BDM task was chosen as reward we employed the standard BDM mechanism: a number z was randomly drawn between zero and y where y is the highest possible prize in the given lottery. If z was greater or equal to the answer, the subject received £z, otherwise she or he could play out the given lottery.

3 ESTIMATION METHOD

We use the different kinds of data to estimate the subjects' (Neumann-Morgenstern) utility functions. In this section we discuss the main conceptual issues of our estimation method; details are presented in a technical appendix.

The estimation of the parameters of the utility function from *pairwise choice* data follows the procedure adopted in Hey and Orme (1994). Let us denote the two lotteries in the pairwise choice by L and R, and the expected utility of them by EUL and EUR respectively. Then, if there is no noise or error in the subject's responses, he or she will report a preference for $L(R)$, if and only if $EUL > (<) EUR$. This is equivalent to saying that L (R) is reported as preferred if and only if $CEL > (<) CER$, where CEL (CER) denotes the certainty equivalent of L (R). However, as we know from the existing literature, subjects' responses are typically affected by noise. We assume that this noise affects the certainty equivalents. Let us denote the error in measuring the *difference* between the certainty equivalents by ε. With this error the subject will report a preference for *L(R)*, if and only if *CEL - CER* + ε > (<) 0, that is, if and only if ε > (<) *CER - CEL*. We can now write the probability that the subject reports a preference for *L (R)* as Prob{ε > (<) *CER - CEL*}.

To proceed to the estimation of the parameters using maximum likelihood methods, we need to specify the distribution of the measurement error. We assume this to be normally distributed with mean *0* and variance s^2. The magnitude of *s* measures the noisiness of the subject's responses: if $s = 0$ then the subject makes no mistakes – as *s* increases, the noise gets larger and larger. In the limit, when *s* is infinite, there is no information content in the subject's responses. There is a slight complication when the subject reports indifference (as was allowed in the experiment[5]). Following Hey and Orme (1994) we assume that those subjects expressing indifference do so when $-\tau < CEL - CER + \varepsilon < \tau$ where τ is some *threshold.* We estimate τ along with the other parameters.

For the certainty equivalent methods, we follow the same route. If the subject is asked to provide his or her certainty equivalent for some gamble *G*, we assume that the subject calculates the Expected Utility of the gamble, *EUG*, according to his or her utility function, and then calculates the certainty equivalent *V* - that is, certain amount of money that yields the same utility. We can now write $V = u^{-1}(EUG)$. Incorporating the error and modelling it as above, then we have that $V = u^{-1}(EUG) + \varepsilon$, and hence that the probability density of *V* being reported as the certainty equivalent of the gamble is given by $f[V - u^{-1}(EUG)]$, where *f(.)* is the probability density function of ε. If we now make the same assumption about the distribution of the measurement error ε - namely that it is $N(0,s^2)$ – we can proceed to the estimation of the parameters of the utility function. As will be seen, we allow for a different variance s^2 for each of the elicitation mechanisms.

[5] It is not clear why a subject should report indifference, and the modelling we have done is only one of several ways to proceed.

We assume that subjects have a Constant Relative Risk Aversion (CRRA) utility function[6] and we adopt the following specific form, which embodies the normalisation that $u(0) = 0$ and $u(40) = 1$:

$$u(x) = (x/40)^{r}$$

We need to estimate only the parameter r (the relative risk aversion coefficient) as it fully describes the utility function of the individual. As noted above, we assume that the standard deviation of the noise - that is, the magnitude of s - is different for the four different elicitation methods and we estimate them individually. We also test to see if there is any bias in the certainty equivalent elicitation methods - in the following way. We estimate a *true* valuation v from the *reported* valuation V according to the formula

$$v = a + bV$$

Here the parameters a and b determine the *bias* in the reporting of the certainty equivalents. If $a=0$ and $b=1$ there is no bias. With the certainty equivalent methods, particularly with the willingness-to-pay and the willingness-to-accept questions, there are well known biases: when asked how much they are willing to pay, it is well-known that subjects underbid; when they are asked how much they are willing to accept, they over-ask. This is partly because subjects do not appear to fully understand the question and perceive it as some kind of strategic game. In contrast, a pairwise choice question seems not to be open to such a misinterpretation, particularly in the context of the usual incentive mechanism; in other words, if the subject knows that his or her stated choice on any pairwise choice question is to be played out, what (conscious or unconscious) reason is there for not replying according to his or her true preferences? Nevertheless, we also estimated the bias for pairwise choice and it turned out to be not significantly different from zero for all individuals.[7] Consequently, in the following analysis we take PC as unbiased.

[6] We have investigated other specifications – most notably that of CARA. CRRA fits significantly better. Details are available on request.
[7] Details are available upon request.

4 RESULTS

We now report the results obtained from the experiment. We present results obtained from the certainty equivalent data (BDM, BID, and ASK) combined with the pairwise choice data (PC). We estimate individual preferences functions subject by subject as subjects are clearly different. That is, we estimate the parameter *r* for each subject using all of the choice and certainty equivalent data. We also estimate the standard deviation *s* for each of the elicitation methods and the parameters *a* and *b* which determine the bias in the reporting of the certainty equivalents[8].

Insert Table 1 here

In Table 1 we report the estimations obtained at the individual level.[9] We comment first, on noise, the on bias, and finally try to combine both measures for a concluding evaluation.

(i) Noise of the elicitation methods

Our initial hypothesis that PC involves less noise than the certainty equivalent methods is confirmed by our data. Considering the average standard deviation of the methods given in the second column of Table 2, we can conclude that the noise of PC is lowest while the noise of BID is highest.[10] Moreover, the BDM mechanism seems to be less prone to errors than ASK. Wilcoxon tests confirm that the noise of PC is significantly lower than that of BID at the 5%-level and significantly lower than the noise of BDM and ASK at the 10%-level. The differences between the single certainty equivalent methods are, however, insignificant.

Insert Table 2 here

(ii) Bias of the elicitation methods

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[8] It may be of interest to report the results of tests of whether the estimated functions differ from data source to data source. Using standard likelihood ratio tests we cannot reject the following null hypotheses: that the utility function estimated from each of BDM, BID and ASK is the same; that the utility function estimated from the choice data and from the certainty equivalents is the same; that the utility function estimated from the choice data and from the BDM, BID and ASK is the same.

[9] We omit two of the 24 subjects (subjects 21 and 22) who answered all questions as if they were perfect expected-value maximisers. For them r = 1, all the *s* and *a* values are 0, and all the *b* values are 1.

[10] For PC one clear outlier (subject 10) was deleted.

Columns 4 and 5 of Table 1 report the bias estimates for the BDM method; columns 7 and 8 report those for the BID method; and columns 10 and 11 report those for the ASK method. As noted earlier, lack of bias would have implied a value of *a* equal to 0 and a value of *b* equal to 1. Since this is clearly not always the case here, we can conclude that there is some degree of bias in the stated certainty equivalents.

Clearly the bias varies from subject to subject, as well as from elicitation method to elicitation method. Figure 2 gives an overall view, plotting the estimated true valuation as a function of the reported valuation for each subject. In these figures also the 45-degree line appears in which stated valuations are equal to the true valuations. It can be seen that for the BDM method the lines are clustered close to the 45-degree line, though there are some exceptions. In contrast, the BID lines are generally *above* the 45-degree line, indicating that subjects generally underbid in willingness-to-pay questions. The lines for the ASK method are somewhat dispersed though generally they are clustered around the 45-degree line, indicating no clear tendency to over-ask in willingness-to-accept questions.

Information on mean values of bias is given in columns 3 and 4 of Table 2 where the bias is calculated at £0 and £40. More precisely, the calculated bias is given by the difference between our estimated true valuation *v* and the reported valuation *V* such that *bias(£0) = a* and *bias(£40) = a + 40(b – 1)*. Table 2 shows that bias is always largest for BID. Comparing BDM and ASK, bias is higher for ASK at £0 and higher for BDM at £40. Wilcoxon tests confirm that the bias of BID is significantly higher than that of BDM at the 1%-level for both *£0* and *£40*. Also the bias of BID is significantly higher that that of ASK at the 1%-level (5%-level) for *£0 (£40)*. The difference between BDM and ASK is significant at the 5%-level for *£0* but insignificant for *£40*.

(iii) Bias and noise

The final evaluation of an elicitation method has to take into account both, bias and noise. In order to get an aggregated view of bias and noise, we calculate the mean square error (MSE) at *£0* and *£40* in the last two columns of Table 2. From this analysis we get a clear picture: PC performs best, BDM performs better then ASK, which, in turn, performs better then BID. Apart from the difference between ASK and BDM, all these differences are according to Wilcoxon tests significant at the 5%-level. Altogether, our results seem to confirm those of Schmidt and Hey (2004) who show that preference reversals are caused rather frequently by pricing errors whereas choice errors play a minor role.

To put our results in perspective, we return to our original question: suppose we want to elicit the true preferences of an individual, what might be the best method? One way of answering this is the following. Suppose we want to know whether our individual prefers a lottery *A* to a lottery *B* or vice versa. We could simply ask the individual which he or she prefers, or we could ask the individual to value the two lotteries and then see for which lottery the valuation is the highest. The problem – as is clear from the above – is that there is noise (and bias) in the subject's responses. So let us ask the more appropriate question: suppose *A* is genuinely preferred to *B* by the subject, what is the probability that (either through pairwise choice or by valuations) the individual actually expresses the correct preference? This clearly depends on how far apart *A* and *B* are in the subject's preferences. Accordingly, we consider three different cases in which the difference in the true evaluations are either £1, £2, or £3. Clearly as the difference increases the probability of expressing the true preference increases, but how it does so depends upon the noise and the bias.

Insert Table 3 here

For each subject we calculate these probabilities (using the estimates of Table 1) and present the results in Table 3. This table shows that *generally but not always* the PC method has the greatest probability of eliciting the true preferences. This may be because the PC method is more easily understandable by subjects in experiments than the other methods. Moreover, PC should not induce strategic behaviour which the other methods might do.

Figure 3 gives a graphical presentation of the results contained in Table 3. The figure shows the median (white line), 75% confidence interval (grey box), and 95% confidence interval for the probability that a subject expresses the correct preferences. It turns out that – consistent to previous results – the median probability is in each case highest for PC and lowest for BID. Wilcoxon tests confirm that the median probability for PC is for all three cases higher than that of BID at the 5% level and higher than those of BDM and ASK at the 10% level. The differences between the single certainty equivalent methods are, however, insignificant.

Insert Figure 3 here

6 CONCLUSIONS

In this study we have been concerned with the question: which is the 'best' method for eliciting preferences. We have analysed four standard elicitation methods, pairwise choice, willingness-to-pay, willingness-to-accept, and certainty equivalents obtained by the BDM mechanism. A particular feature of our analysis is the explicit distinction between noise and bias induced by the single methods. Our experimental data show that maximal buying prices induce the highest noise and, at the same time, the largest bias. Altogether, we find evidence that pairwise choice may be regarded as the best method in general, though for certain subjects one of the other methods may be preferable. But if one does not know anything about an individual subject, it may be best to use the pairwise choice method. Perhaps applied studies should reconsider the predominant use of matching procedures in order to elicit individual preferences.

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Table 1: Estimation using all data

			BDM			BID			ASK			PC	
	Relative risk aversion coefficient	Goodness of fit	Bias Intercept	Bias Slope	Standard deviation of the measurement Error	Bias Intercept	Bias Slope	Standard deviation of the measurement Error	Bias Intercept	Bias Slope	Standard deviation of the measurement error	Indifference	Standard deviation of the measurement Error
Subject	*r*	Log-likelihood	a-bdm	b-bdm	s-bdm	a-bid	b-bid	s-bid	a-ask	b-ask	s-ask	τ	s-pc
1	0.625	-19.189	-5.466	1.171	2.691	4.718	1.022	4.261	7.942	0.893	6.955	0.383	1.435
2	0.644	-18.395	-1.619	0.953	2.784	15.171	0.789	7.555	-0.115	0.958	2.511	1.234	1.196
3	1.005	-7.680	0.204	1.000	0.512	0.201	1.000	1.346	0.488	0.990	0.295	0.234	2.355
4	0.687	-17.483	-6.548	1.123	2.648	12.591	1.034	6.258	-3.213	1.015	2.596	0.028	1.144
5	0.531	-19.386	-8.045	1.191	5.675	8.073	1.004	5.770	-1.868	1.061	3.033	0.123	2.695
6	0.218	-19.206	-0.280	0.931	5.078	11.078	0.814	6.368	0.776	1.070	4.014	0.000	1.520
7	0.997	-12.34	-0.572	0.998	1.216	1.053	0.947	1.102	0.955	0.964	1.371	0.313	2.041
8	0.465	-21.131	9.764	0.412	8.632	6.868	0.568	5.541	4.931	0.711	5.262	0.000	4.051
9	1.445	-17.740	11.614	0.536	6.838	1.660	0.994	1.634	4.692	0.917	2.378	1.356	14.858
10	1.555	-19.132	0.357	0.963	3.175	13.419	1.138	6.495	2.819	0.812	3.466	0.000	95957.85
11	0.394	-18.656	-6.888	1.219	3.826	5.457	1.116	3.982	0.999	1.309	4.824	1.348	1.771
12	0.944	-11.672	1.508	0.977	1.12	1.306	0.985	0.815	1.279	0.928	1.445	0.251	1.636
13	4.366	-19.138	9.105	0.752	3.684	27.067	0.233	7.245	23.48	0.235	6.144	0.000	1.802
14	0.531	-14.946	-0.573	1.018	2.565	3.903	0.900	2.171	-1.788	1.001	2.360	0.090	1.481
15	0.630	-20.790	-2.613	1.114	5.942	4.426	0.986	5.676	-13.676	1.801	6.787	0.073	1.313
16	0.554	-22.314	-1.789	1.304	6.891	11.927	1.348	7.526	8.086	0.582	8.755	0.850	1.13
17	2.722	-20.339	9.259	0.751	3.171	21.559	0.156	7.503	21.372	0.123	6.165	0.000	10.595
18	0.607	-19.356	-4.311	1.091	3.535	6.617	0.963	4.873	2.329	0.901	5.888	0.265	1.232
19	0.711	-20.877	-7.902	1.009	6.181	9.026	0.641	4.381	5.143	0.685	6.052	0.226	1.322
22	1.013	-4.723	0.063	1.000	0.337	0.096	0.995	0.740	0.291	0.987	0.267	1.676	18.193
23	1.153	-13.904	1.874	0.857	2.609	0.567	0.966	0.897	0.817	0.949	2.057	0.257	2.257
24	0.712	-19.642	-4.263	1.057	4.046	8.456	1.079	4.878	0.971	0.779	6.335	0.000	1.292

Table 2: Average noise, bias and mean square error

	s	bias(£0)	bias(£40)	mse(£0)	mse(£40)
BDM	3.780	-0.324	-1.366	14.392	16.152
BID	4.410	7.965	3.744	82.895	33.461
ASK	4.044	3.032	-1.202	25.546	17.796
PC	3.587	-	-	12.864	12.864

Table 3: Probabilities of eliciting the true preference using the different methods

True difference = £1				*True difference = £2*				*True difference = £3*			
PC	BDM	BID	ASK	PC	BDM	BID	ASK	PC	BDM	BID	ASK
.76	.59	.56	.55	.92	.67	.63	.59	.98	.75	.69	.63
.80	.61	.55	.62	.95	.7	.59	.72	.99	.79	.64	.81
.66	.92	.7	.99	.80	1.00	.85	1.00	.90	1.00	.94	1.00
.81	.59	.54	.61	.96	.68	.59	.70	1.00	.76	.63	.79
.64	.54	.55	.59	.77	.58	.60	.67	.87	.62	.64	.75
.74	.56	.55	.57	.91	.62	.61	.63	.98	.67	.66	.69
.69	.72	.75	.70	.84	.88	.91	.86	.93	.96	.98	.95
.6	.58	.59	.57	.69	.65	.67	.65	.77	.72	.75	.71
.53	.58	.67	.63	.55	.65	.81	.74	.58	.72	.90	.83
.5	.59	.54	.60	.50	.68	.58	.69	.50	.76	.61	.77
.71	.56	.56	.54	.87	.62	.62	.59	.95	.68	.68	.63
.73	.74	.81	.70	.89	.90	.96	.85	.97	.97	1.00	.94
.71	.60	.66	.69	.87	.70	.80	.84	.95	.78	.90	.93
.75	.61	.64	.62	.91	.71	.77	.73	.98	.79	.86	.82
.78	.54	.55	.52	.94	.58	.60	.55	.99	.63	.65	.57
.81	.53	.53	.56	.96	.56	.56	.61	1.00	.59	.58	.66
.54	.62	.73	.82	.57	.72	.89	.97	.61	.81	.97	1.00
.79	.57	.56	.55	.95	.64	.62	.61	.99	.71	.67	.66
.78	.55	.60	.57	.93	.59	.69	.63	.99	.63	.77	.7
.52	.98	.83	1.00	.54	1.00	.97	1.00	.57	1.00	1.00	1.00
.67	.62	.79	.64	.81	.74	.95	.77	.91	.83	.99	.86
.78	.57	.55	.56	.94	.63	.61	.61	.99	.69	.66	.67

Figure 1: Presentation of the lotteries in the experiment

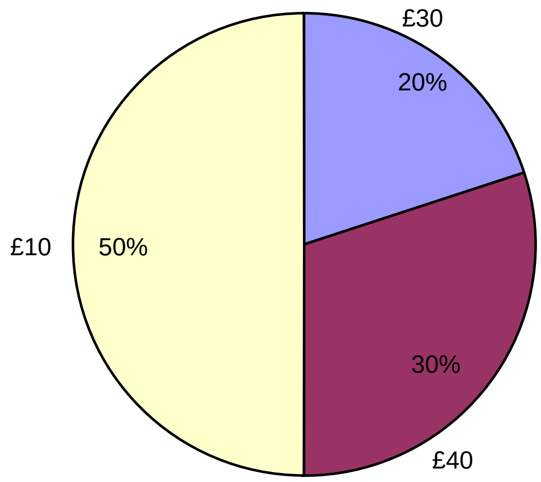


Figure 2: The relationship between true and stated valuations

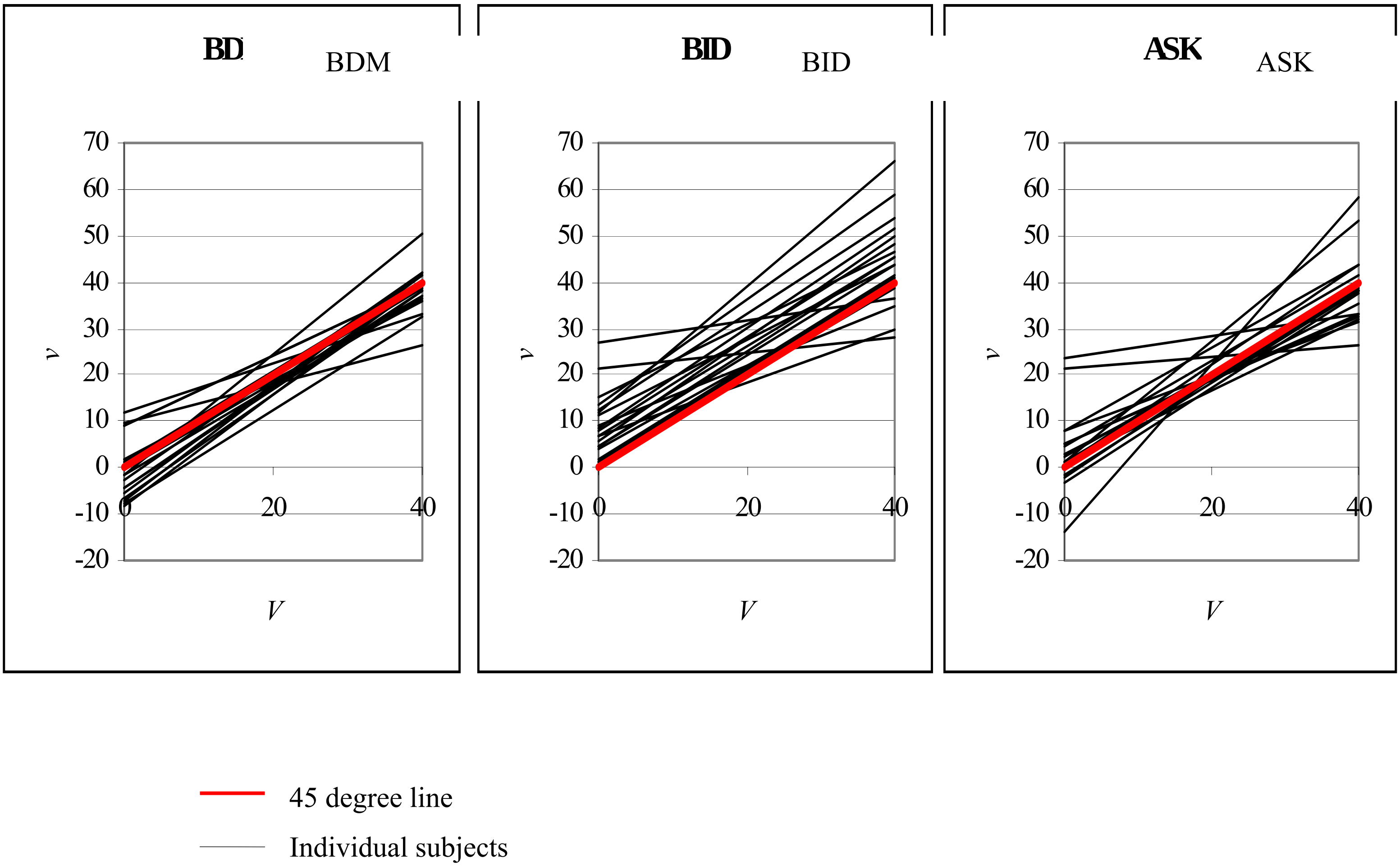
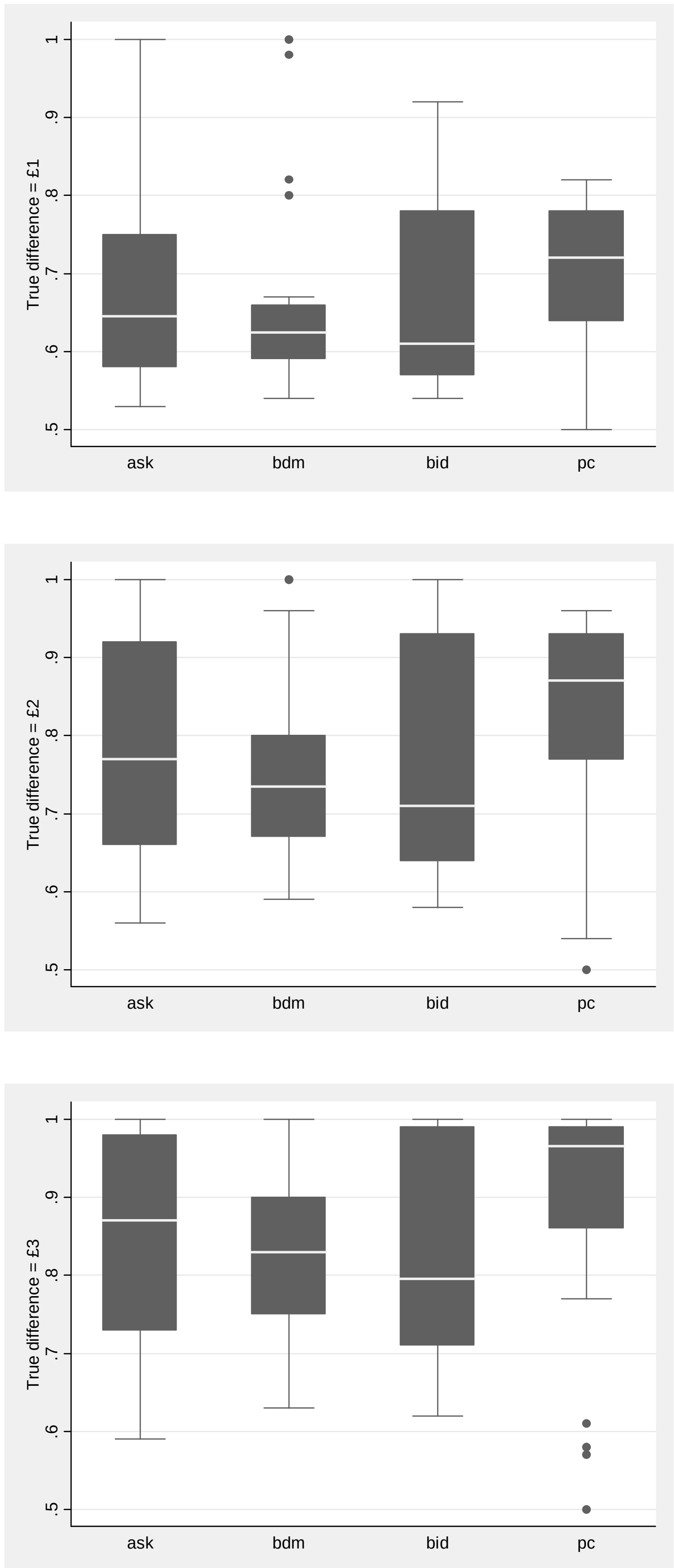


Figure 3: Box plot presenting the probabilities of observing the correct preferences



Appendix Table 1: The lotteries in the experiment

No.	£0	£10	£30	£40	*No.*	£0	£10	£30	£40	*No.*	£0	£10	£30	£40
1	.000	.000	1.000	.000	*20*	.000	.200	.700	.100	*39*	.000	.500	.000	.500
2	.750	.000	.250	.000	*21*	.000	.000	.500	.500	*40*	.500	.250	.000	.250
3	.300	.600	.100	.000	*22*	.500	.000	.500	.000	*41*	.200	.000	.400	.400
4	.000	.600	.100	.300	*23*	.250	.500	.250	.000	*42*	.100	.000	.200	.700
5	.000	1.000	.000	.000	*24*	.000	.500	.000	.500	*43*	.800	.000	.000	.200
6	.000	.500	.500	.000	*25*	.500	.250	.000	.250	*44*	.400	.000	.500	.100
7	.500	.500	.000	.000	*26*	.000	.250	.500	.250	*45*	.400	.000	.000	.600
8	.000	.000	.700	.300	*27*	.000	.000	.750	.250	*46*	.700	.000	.000	.300
9	.800	.000	.140	.060	*28*	.250	.250	.500	.000	*47*	.200	.000	.000	.800
10	.200	.000	.740	.060	*29*	.200	.000	.000	.800	*48*	.200	.000	.400	.400
11	.000	.200	.800	.000	*30*	.800	.000	.000	.200	*49*	.100	.000	.000	.900
12	.500	.100	.400	.000	*31*	.320	.600	.000	.080	*50*	.600	.000	.000	.400
13	.000	.200	.600	.200	*32*	.020	.600	.000	.380	*51*	.300	.500	.000	.200
14	.000	.100	.300	.600	*33*	.700	.000	.000	.300	*52*	.200	.200	.000	.600
15	.200	.800	.000	.000	*34*	.350	.000	.500	.150	*53*	.600	.100	.000	.300
16	.100	.400	.500	.000	*35*	.850	.000	.000	.150	*54*	.000	.350	.000	.650
17	.000	.400	.600	.000	*36*	.150	.000	.000	.850	*55*	.000	.100	.250	.650
18	.500	.200	.300	.000	*37*	.830	.000	.000	.170	*56*	.250	.350	.000	.400
19	.000	.200	.300	.500	*38*	.230	.000	.600	.170					

TECHNICAL APPENDIX

This Appendix describes the mathematics lying behind the estimation and the GAUSS programs used in the estimation. We assume throughout that the subjects make monetary evaluations of the various gambles with a normally distributed error.

In the experiment there were 4 possible outcomes £0, £10, £30 and £40. We denote the utilities of these by u_1, u_2, u_3 and u_4.

We assume a CRRA utility function:

$$u(x) = (x/40)^{r} \qquad (1)$$

where we have normalised the function so that $u_1 = 0$ and $u_4 = 1$. A risk-neutral person has $r=1$. The inverse of the utility function is

$$x = u^{-1}(u) = 40u^{1/r}$$

Estimation using the Certainty Equivalent data.

Denote by c_j the certainty equivalent reported by the subject on question number j $(j = 1,..,J)$. Let us drop the subscript j to save notational clutter. Suppose the probabilities on the question are p_1, p_2, p_3 and p_4. Then the Expected utility of the gamble (for given parameters) is

$$EUG = p_1u_1 + p_2u_2 + p_3u_3 + p_4u_4 = 0.25^r p_2 + 0.75^r p_3 + p_4 \qquad (2)$$

Hence the true certainty equivalent, t, of the gamble G is given by

$$t = 40\,(0.25^r p_2 + 0.75^r p_3 + p_4)^{1/r} \qquad (3)$$

The difference between the stated certainty equivalent and the true one

$$c - t = c - 40\,(0.25^r p_2 + 0.75^r p_3 + p_4)^{1/r}$$

We assume that this difference $c - t$ is error, normally distributed with standard deviation s. The normal *pdf* of this is:

$$f(x) = \frac{1}{\sqrt{2\pi^2}} \exp\left[-\frac{x^2}{2s^2}\right] \qquad (4)$$

The log of the *pdf* is therefore

$$\ln f(x) = -x^2 / 2s^2 - 0.5\ln(2\pi s^2) \qquad (5)$$

It follows that the log of the probability density of the difference is:

$$\ln f(c-t) = -(c-t)^2 / 2s^2 - \ln(s) - 0.5\ln(2\pi) \qquad (6)$$

This is the contribution to the log-likelihood from the certainty equivalent data.

Estimation using the Preference data.

Again we assume that subjects make normally distributed errors when evaluating lotteries. When comparing two lotteries they compare the estimated certainty equivalents. Suppose we have two gambles *L* and *R*. The Expected Utilities are *EUL* and *EUR.* Their monetary evaluations are $ML = u^{-1}(EUL)$ and are $MR = u^{-1}(EUR)$ The treatment is different according as to whether the subject reports indifference or not.

1) The subject ***never*** *reports indifference*. In this case, we have that *L* is reported as preferred to *R* if $ML - MR + \varepsilon \geq 0$ and that *R* is reported as preferred if $ML - MR + \varepsilon < 0$. Hence the probability that *L* is reported as preferred is $Prob(\varepsilon \geq MR\text{-}ML)$ and the probability that *R* is reported as preferred is $Prob(\varepsilon < MR\text{-}ML)$. Hence the probability of *L (R)* is:

$$\text{Prob}(\varepsilon \geq MR - ML) \quad (\text{Prob}(\varepsilon < MR - ML)) \qquad (9)$$

Now we need to find expressions for the probabilities. If we denote the normal *cdf* by $\Psi(x/s)$ (this is the integral of (4)) we can then write that the probability of *L (R)* is

$$1 - \Psi((MR - ML)/s) \qquad (\Psi((MR - ML)/s)) \qquad (10)$$

Hence the *log-likelihood* is

$$\ln(1 - \Psi(MR - ML)) \qquad (\ln(\Psi(MR - ML))) \qquad (11)$$

2) The subject ***sometimes*** *reports indifference.* This is almost the same but we need some story about when the subject reports indifference. We say that *L* is reported as preferred when if *ML* –

MR + *ε* ≥ *τ* that *R* is reported as preferred if *ML* – *MR* + *ε* < -*τ* and that indifference is reported when –τ ≤ *ML* - *MR* + *ε* < *τ* Hence the probability that *L* is reported as preferred is *Prob(ε≥ MR-ML + τ)* the probability that *R* is reported as preferred is *Prob(ε< MR-ML - τ)* and the probability that indifference is reported is *Prob(MR-ML – τ ≤ ε < MR-ML + τ)*

Estimating bias in the certainty equivalents.

We simply assume that there is a *true* valuation *V* and a *reported* valuation *v* which are related by

$$V = a + bv$$

Here the parameters *a* and *b* determine the *bias* in the reporting of the certainty equivalents. If *a=0* and *b=1* there is no bias. In the text tables we report the estimated values of *a* and *b* for each of the certainty equivalent methods. We assume no bias in the pairwise choice elicitation method.

GAUSS PROGRAM – NOT FOR PUBLICATION

```
/*
This is CRRA.EST.

It uses GAUSS to fit preference functionals to combined
preference AND certainty equivalent data using the constant
relative risk aversion form for the utility function.
Thus, u(x) = (x/40)^r so u(0) = 0 and u(40) = 1.

This version combines the three preference data sets (from occasions A, B and C) with three
sources of certainty equivalent data: BDM, BID and ASK data.
The three occasions for the PC data are in the files p.a p.b and p.c.
For the CE methods we have that BDM is c.1, BID is c.2 and ASK is c.3.

This version uses the normal pdf for the error terms.

It is appropriate for subjects who sometimes declare indifference on the
preference questions. I use

indiff=-1 for subjects 20 and 21 who are always exactly risk-neutral (subjects 20 and 21).
indiff=0 for those subjects who never express indifference (excluding subjects 20 and 21).
indiff=1 for those subjects who sometimes express indifference.

I am going to estimate using various combinations of the data sets:

ds=1 just the BDM data
ds=2 just the BID data
ds=3 just the ASK data
ds=4 all the CE data together
ds=5 just the PC data
ds=6 all the CE data plus the PC data

THIS PROGRAM CORRECTS FOR THE BIAS
u(a+C*b)=Eu(G)+e.
a and b are the intercept and the slope of the function relating the expressed CE to the true value.
abdm, bbdm, abid, bbid, aask, bask are the intercept and the slope of the bias in the 3 data sets.

I assume no bias in the pairwise choice decisions.
*/

library maxlik;
maxset;

output file = d:/active/people/schmidt/two/crra/crra.out;
output off;


k = 24;                    /* k is the number of subjects */
n = 56;                    /* n is the number of certainty equivalent questions*/
m = 28;                    /* m is the number of pairwise choice questions*/

on=ones(n,1);
om=ones(m,1);
let _max_MaxIters = 1000;
let vars = 1 2 3 4 5 6 7 8;
eps=0.000000001;          /* drop this later     */

sp = 5.0;
output on;
print "starting s =  " sp;
print "";
```

```
output off;

load c1[k,n] = d:/active/people/schmidt/two/crra/c.1;     /* this is the matrix of BDM observations */
load c2[k,n] = d:/active/people/schmidt/two/crra/c.2;     /* this is the matrix of BID observations */
load c3[k,n] = d:/active/people/schmidt/two/crra/c.3;     /* this is the matrix of ASK observations */
load pa[k,m] = d:/active/people/schmidt/two/crra/p.a;     /* this is matrix of observations of preferences in occasion A*/
load pb[k,m] = d:/active/people/schmidt/two/crra/p.b;     /* this is matrix of observations of preferences in occasion B*/
load pc[k,m] = d:/active/people/schmidt/two/crra/p.c;     /* this is matrix of observations of preferences in occasion C*/


load p[n,4] = d:/active/people/schmidt/two/crra/p.inp;               /* this is matrix of probabilities in the lotteries in the
CE questions */
load q[m,8] = d:/active/people/schmidt/two/crra/pcprobs.inp;
                                                                      /* this is matrix of probabilities in the PC questions */

ql=q[.,1]~q[.,2]~q[.,3]~q[.,4];                                       /* this is the matrix of probabilities of the left gamble */
qr=q[.,5]~q[.,6]~q[.,7]~q[.,8];                                       /* this is the matrix of probabilities of the right gamble */

/* the next few lines work out the maximum payoff in each of the CE lotteries */
mx = ones(n,1);
i = 1;
do while i <= n;
if p[i,4] > 0; mx[i] = 40;endif;
if mx[i] <40;
if p[i,3] > 0; mx[i] = 30; endif;
endif;
if mx[i] < 30;
if p[i,2] > 0; mx[i] = 10; endif;
endif;
if mx[i] < 10;
if p[i,1] > 0; mx[i] = 0; endif;
endif;
i = i + 1;
endo;
/*end of working out max payoff */


ds=6;
do while ds<=6;
screen off; output on;format /rd 7,3;
print "";
print "";
if ds==1; print "Estimation of CRRA Model with BDM data" ; endif;
if ds==2; print "Estimation of CRRA Model with BID data" ; endif;
if ds==3; print "Estimation of CRRA Model with ASK data" ; endif;
if ds==4; print "Estimation of CRRA Model with (all) CE data" ; endif;
if ds==5; print "Estimation of CRRA Model with PC data" ; endif;
if ds==6; print "Estimation of CRRA Model with CE and PC data" ; endif;

if ds==1;print "   subj  exit c  log-lik   abdm    bbdm    sbdm      r";endif;
if ds==2;print "   subj  exit c  log-lik   abid    bbid    sbid      r";endif;
if ds==3;print "   subj  exit c  log-lik   aask    bask    sask      r";endif;
if ds==4;print "   subj  exit c  log-lik   abdm    bbdm    sbdm    abid    bbid    sbid    aask    bask    sask      r ";endif;
if ds==5;print "   subj  exit c  log-lik   spc       r      tau";endif;
if ds==6;print "   subj  exit c  log-lik   abdm    bbdm    sbdm    abid    bbid    sbid    aask    bask    sask    spc       r
 tau";endif;
print "";
output off; screen on;

j = 1;
do while j<=24;
```

```
if j==20 or j==21;indiff=-1;endif;
if j==1 or j==2 or j==3 or j==4 or j==5 or j==7 or j==9 or j==11 or j==12 or j==14 or j==15 or j==16 or j==18 or
j==19 or j==22 or j==23;indiff=1;endif;
if j==6 or j==8 or j==10 or j==13 or j==17 or j==24;indiff=0;endif;


/*reading in the decisions of the subjects*/
/* first the pairwise choice on the 3 occasions */
wa = (pa[j,.])';
wb = (pb[j,.])';
wc = (pc[j,.])';
/* now the certainty equivalents of the three types*/
w1 = (c1[j,.])';
w2 = (c2[j,.])';
w3 = (c3[j,.])';



if indiff==0;

if ds==1;_max_active=1|1|1|0|0|0|0|0|0|0|1|0;endif;
if ds==2;_max_active=0|0|0|1|1|1|0|0|0|0|1|0;endif;
if ds==3;_max_active=0|0|0|0|0|0|1|1|1|0|1|0;endif;
if ds==4;_max_active=1|1|1|1|1|1|1|1|1|0|1|0;endif;
if ds==5;_max_active=0|0|0|0|0|0|0|0|0|1|1|0;endif;
if ds==6;_max_active=1|1|1|1|1|1|1|1|1|1|1|0;endif;
start= 0.0|1.0|sp|0.0|1.0|sp|0.0|1.0|sp|sp|1.0|0.0;
/*starting values always 0 for a, 1 for b, sp for s, 0.0 for r and 0.0 for tau */
{x,f,g,c,retcode} = maxlik(q,vars,&ll,start);
output on;screen off;
if ds==1;print j~retcode~f~x[1]~x[2]~scf(x[3])~abs(x[11]);endif;
if ds==2;print j~retcode~f~x[4]~x[5]~scf(x[6])~abs(x[11]);endif;
if ds==3;print j~retcode~f~x[7]~x[8]~scf(x[9])~abs(x[11]);endif;
if ds==4;print j~retcode~f~x[1]~x[2]~scf(x[3])~x[4]~x[5]~scf(x[6])~x[7]~x[8]~scf(x[9])~abs(x[11]);endif;
if ds==5;print j~retcode~f~scf(x[10])~abs(x[11]);endif;
if ds==6;print j~retcode~f~x[1]~x[2]~scf(x[3])~x[4]~x[5]~scf(x[6])~x[7]~x[8]~scf(x[9])~scf(x[10])~abs(x[11]);endif;
screen on;output off;
endif;


if indiff==1;
if ds==1;_max_active=1|1|1|0|0|0|0|0|0|0|1|1;endif;
if ds==2;_max_active=0|0|0|1|1|1|0|0|0|0|1|1;endif;
if ds==3;_max_active=0|0|0|0|0|0|1|1|1|0|1|1;endif;
if ds==4;_max_active=1|1|1|1|1|1|1|1|1|0|1|1;endif;
if ds==5;_max_active=0|0|0|0|0|0|0|0|0|1|1|1;endif;
if ds==6;_max_active=1|1|1|1|1|1|1|1|1|1|1|1;endif;
start= 0.0|1.0|sp|0.0|1.0|sp|0.0|1.0|sp|sp|1.0|0.0;
/*starting values always 0 for a, 1 for b, sp for s, 1.0 for r and 0.0 for tau */
{x,f,g,c,retcode} = maxlik(q,vars,&ll,start);
output on;screen off;
if ds==1;print j~retcode~f~x[1]~x[2]~scf(x[3])~abs(x[11]);endif;
if ds==2;print j~retcode~f~x[4]~x[5]~scf(x[6])~abs(x[11]);endif;
if ds==3;print j~retcode~f~x[7]~x[8]~scf(x[9])~abs(x[11]);endif;
if ds==4;print j~retcode~f~x[1]~x[2]~scf(x[3])~x[4]~x[5]~scf(x[6])~x[7]~x[8]~scf(x[9])~abs(x[11]);endif;
if ds==5;print j~retcode~f~scf(x[10])~abs(x[11])~1/(1+exp(-x[12]));endif;
if ds==6;print
j~retcode~f~x[1]~x[2]~scf(x[3])~x[4]~x[5]~scf(x[6])~x[7]~x[8]~scf(x[9])~scf(x[10])~abs(x[11])~tcf(x[12]);endif;
screen on;output off;
endif;
```

```
j = j+1;
endo;
ds=ds+1;
endo;


proc ll(x,y);
/* THIS IS THE CRRA LIKELIHOOD FUNCTION*/

local ll,lla,lla1,lla2,lla3,llb,llb1,llb2,llb3,llc,llc1,llc2,llc3,f1,f2,f3,ll1,ll2,ll3;
local abdm,bbdm,sbdm,abid,bbid,sbid,aask,bask,sask,spc,tau,r;
local nw1,nw2,nw3;
local u1,u2,u3,u4,uv,eul,eur,eug,cel,cer,ced,ced1,ced2,ced3;
local x1a,x2a,x3a,x4a, u1a,u2a,u3a,u4a,ua,euga,cega;

x1a=ones(n,1);
x2a=ones(n,1);
x3a=ones(n,1);
x4a=ones(n,1);
euga=ones(n,1);
ced2=ones(n,1);
u1a=ones(n,1);
u2a=ones(n,1);
u3a=ones(n,1);
u4a=ones(n,1);
cega=ones(n,1);

abdm=x[1];
bbdm=x[2];
sbdm=scf(x[3]);
abid=x[4];
bbid=x[5];
sbid=scf(x[6]);
aask=x[7];
bask=x[8];
sask=scf(x[9]);

spc=scf(x[10]);

r=abs(x[11]);
if indiff==1;tau=tcf(x[12]);endif;
if indiff==0;tau=0.0;endif;


/* we start with the PC estimation */
u1=0;u2=0.25^r;u3=0.75^r;u4=1;
uv=u1|u2|u3|u4;
eul=ql*uv;eur=qr*uv;
cel=40*(eul^(1/r));cer=40*(eur^(1/r));        /*these are the CE of left and the CE of right      */
ced=cer-cel;                                  /*this is the difference between the two CEs        */


/*print r;print cer~cel~wa~wb~wc;pause(5);*/

/*if ds==5;print ced~(ced-tau);pause(5);endif;*/

if indiff==0;
f1 = cdfnc(ced/spc);                   /*this is the prob that z is greater than ced      */
f3 = cdfn(ced/spc);                    /*this is the prob that z is smaller than ced      */
lla1= 0.5*(3*om-wa).*(2*om-wa).*ln(f1+eps*om);
lla3=0.5*(wa-2*om).*(wa-om).*ln(f3+eps*om);
```

```
lla=lla1+lla3;                              /* this is the log-likelihood for PC occasion A      */
llb1= 0.5*(3*om-wb).*(2*om-wb).*ln(f1+eps*om);
llb3=0.5*(wb-2*om).*(wb-om).*ln(f3+eps*om);
llb=llb1+llb3;                              /* this is the log-likelihood for PC occasion B      */
llc1= 0.5*(3*om-wc).*(2*om-wc).*ln(f1+eps*om);
llc3=0.5*(wc-2*om).*(wc-om).*ln(f3+eps*om);
llc=llc1+llc3;                              /* this is the log-likelihood for PC occasion C      */
endif;


if indiff==1;
f1 = cdfnc((ced+tau)/spc);                  /*this is the prob that z is greater than ced+tau      */
f3 = cdfn((ced-tau)/spc);                   /*this is the prob that z is smaller than ced-tau      */
f2=om-f1-f3;                                /*this is the prob that z is between ced-tau and ced+tau */
lla1= 0.5*(3*om-wa).*(2*om-wa).*ln(f1+eps*om);
lla2=(3*om-wa).*(wa-om).*ln(f2+eps*om);
lla3=0.5*(wa-2*om).*(wa-om).*ln(f3+eps*om);
lla=lla1+lla2+lla3;                         /* this is the log-likelihood for PC occasion A      */
llb1= 0.5*(3*om-wb).*(2*om-wb).*ln(f1+eps*om);
llb2=(3*om-wb).*(wb-om).*ln(f2+eps*om);
llb3=0.5*(wb-2*om).*(wb-om).*ln(f3+eps*om);
llb=llb1+llb2+llb3;                         /* this is the log-likelihood for PC occasion B      */
llc1= 0.5*(3*om-wc).*(2*om-wc).*ln(f1+eps*om);
llc2=(3*om-wc).*(wc-om).*ln(f2+eps*om);
llc3=0.5*(wc-2*om).*(wc-om).*ln(f3+eps*om);
llc=llc1+llc2+llc3;                         /* this is the log-likelihood for PC occasion C      */
endif;


nw1=abdm*on+bbdm*w1;
nw2=abid*on+bbid*w2;
nw3=aask*on+bask*w3;

eug=p*uv;                                   /* this is the EU of the Gambles                  */
ced1=nw1-40*(eug^(1/r));                    /* this is the difference between the stated CE and
                                            that implied by the utility function: BDM data    */
x1a=(mx-nw2)/40;
x2a=(mx-nw2+10)/40;
x3a=(mx-nw2+30)/40;
x4a=(mx-nw2+40)/40;

i=1;
do while i<=n;
if x1a[i]>0;u1a[i]=(x1a[i])^r;endif;
if x1a[i]==0;u1a[i]=0;endif;
if x1a[i]<0;u1a[i]=-(-x1a[i])^r;endif;
if x2a[i]>0;u2a[i]=(x2a[i])^r;endif;
if x2a[i]==0;u2a[i]=0;endif;
if x2a[i]<0;u2a[i]=-(-x2a[i])^r;endif;
if x3a[i]>0;u3a[i]=(x3a[i])^r;endif;
if x3a[i]==0;u3a[i]=0;endif;
if x3a[i]<0;u3a[i]=-(-x3a[i])^r;endif;
if x4a[i]>0;u4a[i]=(x4a[i])^r;endif;
if x4a[i]==0;u4a[i]=0;endif;
if x4a[i]<0;u4a[i]=-(-x4a[i])^r;endif;
i=i+1;
endo;

ua=u1a~u2a~u3a~u4a;


i=1;
```

```
do while i<=n;
euga[i]=p[i,.]*ua[i,.]';
i=i+1;
endo;


i=1;
do while i<=n;                                   /* this is the EU of the augmented gamble   */
if euga[i]>0;cega[i]=40*((euga[i])^(1/r));endif;
if euga[i]==0;cega[i]=0;endif;
if euga[i]<0;cega[i]=-40*((-euga[i])^(1/r));endif;                   /* this is the difference between xmax and .... */
i=i+1;
endo;

ced2=mx-cega;


/*
if r<0.00000001;
print mx~w2~nw2~r*on~x1a~x2a~x3a~x4a~ua~euga~cega~ced2;pause(5);
endif;
*/

ced3=nw3-40*(eug^(1/r));                         /* this is the difference between the stated CE and
                                         that implied by the utility function: ASK data    */

/* below is some old code for the BID data*/
/* ced2=nw2-40*(eug^(1/r)); */
                      /* this is the difference between the stated CE and
                                         that implied by the utility function: BID data    */


/* coming up are some pdfs */
ll1=-ced1.*ced1/(2*sbdm^2)-ln(sbdm)-0.5*ln(6.283185308);          /*this is the log density of ced1*/
ll2=-ced2.*ced2/(2*sbid^2)-ln(sbid)-0.5*ln(6.283185308);          /*this is the log density of ced2*/
ll3=-ced3.*ced3/(2*sask^2)-ln(sask)-0.5*ln(6.283185308);          /*this is the log density of ced3*/


if ds==1;ll=on'*(ll1);endif;
if ds==2;ll=on'*(ll2);endif;
if ds==3;ll=on'*(ll3);endif;
if ds==4;ll=on'*(ll1+ll2+ll3);endif;
if ds==5;ll=om'*(lla+llb+llc);endif;
if ds==6;ll=om'*(lla+llb+llc)+on'*(ll1+ll2+ll3);endif;


retp(ll);
endp;

proc scf(s);
retp(abs(s));
/*retp(100/(1+exp(-s)));*/
endp;

proc tcf(t);
/*retp(abs(t));*/
retp(10/(1+exp(-t)));
endp;

closeall;
end.
```