Optimal Interest Rate Rules, Asset Prices and Credit Frictions *

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Abstract

We study optimal Taylor-type interest rate rules in an economy with credit market imperfections. Our analysis builds on the agency cost framework of Carlstrom and Fuerst (1997), which we extend in two directions. First, we embed monopolistic competition and sticky prices. Second, we modify the stochastic structure of the model in order to generate a *countercyclical* premium on external finance. This is achieved by linking the mean distribution of investment opportunities faced by entrepreneurs to aggregate total factor productivity. We model monetary policy in terms of simple welfare-maximizing interest rate rules. We find that monetary policy should respond to increases in asset prices by *lowering* interest rates. However, when monetary policy responds strongly to inflation, the marginal welfare gain of responding to asset prices vanishes. Within the class of linear interest rate rules that we analyze, a strong anti-inflationary stance always attains the highest level of welfare.

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1 Introduction

In this paper we study optimal Taylor-type interest rate rules in an economy with nominal rigidities and credit market imperfections. Our interest is twofold. First, we aim at driving the attention of the recent literature on a typology of market distortions whose role has been largely neglected in the normative analysis of monetary policy. This is surprising, considering the increasing emphasis - starting with Bernanke and Gertler (1989) - placed on financial factors in the study of business cycles. Second, we aim at assessing - from a welfare-based perspective - the role that asset prices and/or other financial indicators should play in the optimal setting of monetary policy rules.

We model credit frictions following the *agency cost* framework of Carlstrom and Fuerst (1997) (CF henceforth). Financial imperfections originate from a problem of asymmetric information between an entrepreneur (the borrower) and a financial intermediary (the lender). The entrepreneur engages in a risky investment activity and seeks resources in excess of internal funds. Since the outcome of the investment activity is private information to the entrepreneur, the latter has an incentive to misreport the same outcome. This moral hazard problem induces the lender to monitor only the defaulting entrepreneurs, and transfer this implicit cost onto the average cost of credit. The appeal of this framework is that agency costs are endogenous over the business cycle and default emerges as an equilibrium phenomenon.[1]

We depart from CF in *two* main respects. First, we introduce monopolistic competition and sticky prices, a necessary step in order to study monetary policy. Second, and in order for the model to better comply with the empirical evidence on the cyclical behavior of the cost of credit, we modify the stochastic structure of the model to generate a *countercyclical* behavior of the finance premium, i.e., the cost of external finance in excess of the safe rate of return. This is accomplished by assuming that the mean distribution of investment outcomes across entrepreneurs is linked to the state of aggregate productivity in the economy. Importantly, this variation allows to overcome an often-cited limitation of the CF framework, which generates a *procyclical* cost of external finance, a feature at odds

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[1] In this respect, the agency cost framework differs from the credit cycle model of Kiyotaki and Moore (1997), in which the collateral constraint always binds but default never occurs.

with a large empirical evidence.[2] As a result, in our model, credit frictions generate both an effect of persistence (i.e., hump-shaped dynamics of investment and output, as in CF) and magnification in response to shocks. To clarify, while persistence is obtained relative to a nested model with frictionless credit markets, amplification is obtained relative to a model with agency costs but with time-invariant mean distribution of investment outcomes for entrepreneurs.

The issue of whether monetary policy should respond to asset prices has been recently the object of an intense debate, within both policy and academic circles.[3] However, the theoretical literature linking asset prices, monetary policy and financial frictions in business cycle models has been scant. Bernanke and Gertler (2001) and Gilchrist and Leahy (2002) argue that there is negligible stabilization gain from including asset prices as independent arguments in monetary policy rules. However, Cecchetti et al.(2002) argue that this gain is likely to depend on the underlying source of shocks.[4]

The common shortcoming of this literature is that it completely abstracts from strict *welfare* considerations. The metric adopted for the evaluation of the relative performance of policy rules is typically an output-inflation volatility frontier. This makes it hard to correctly rank alternative specifications for monetary policy, and to safely draw any conclusion about the desirability for monetary policy to react to asset price movements. It is this consideration that essentially motivates the present paper.

Our baseline economy will feature three types of distortions. Monopolistic competition in goods markets, sticky prices (both common to the recent stream of New Keynesian business cycle models) and endogenous agency costs. Agency costs, per se, have a twofold effect. In the long run, they produce an inefficiently low level of capital and investment, and hence output, since the economy suffers a deadweight loss associated to the monitoring activity of the lender. In the short-run, the presence of an endogenous premium on external finance distorts the dynamic allocation of capital and investment. In practice, agency costs act as an implicit - and time-varying - tax on investment.

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[2]See for instance Levin et al. (2004).

[3]See Gilchrist and Leahy (2002) for a survey.

[4]Iacoviello (2004) analyzes monetary policy in a model with credit cycles a la Kiyotaki and Moore (1997) and housing, and concludes that reacting to asset prices does not improve macroeconomic stability.

In this context, asset price movements are genuinely a symptom of financial distortions, for the relative price of capital goods is determined by the interaction of demand and supply in a lending market characterized by moral hazard. Hence reacting to asset prices may in principle bear a public finance motivation for monetary policy. The cyclicality of asset prices being strictly related to the presence of financial imperfections contrasts with a large business cycle literature in which asset prices merely reflect the shadow price of investment in the presence of physical adjustment costs on capital.

From a methodological viewpoint, our approach, as in Kollmann (2003a, 2003b) and Schmitt-Grohé and Uribe (2006), and unlike much of the so-called New Keynesian litera-ture, allows to study policy rules in a dynamic economy that evolves around a steady state which remains inefficient. Importantly, in our context, the steady state will be distorted also by the presence of monitoring costs in credit markets. As emphasized by Kim et al. (2003) and Schmitt-Grohé and Uribe (2003), this strategy requires that an accurate evaluation of welfare be based on a higher order approximation of all the conditions that characterize the competitive equilibrium of the economy.

Our main finding is that monetary policy should react to increases in the asset price by *lowering* nominal interest rates. The intuition lies in the fact that, as hinted above, the asset price (or relative price of capital goods) is akin to a tax which distorts the dynamic evolution of investment. In response to positive productivity shocks (which are the chief source of variability in our model) such tax evolves procyclically, thereby restraining investment.

However, responding (negatively) to the asset price in the interest rate rule improves welfare only for values of the inflation feedback coefficient which are low and in the range of those typically adopted in the specification of Taylor-type monetary policy rules (i.e., roughly between 1 and 2). In general, the marginal welfare gain of responding to the asset price flattens out when monetary policy responds more aggressively to inflation. In fact, a strong response to inflation always attains the highest level of welfare. On the other hand, though, the behavior of asset prices is only barely affected by the degree to which monetary policy stabilizes inflation. Thus, under a policy rule inducing strict stability of inflation, credit frictions generated by agency costs remain operative. This is

suggestive of a tradeoff faced by the monetary authority, which confronts two distortions (price stickiness and agency costs) with only one instrument, as well as of the fact that, in this context, the marginal gain of dampening the distortion induced by price stickiness quantitatively outweighs the benefit of neutralizing the distortion induced by agency costs and asset price fluctuations.

The remainder of the paper is as follows. Section 2 presents the model. Section 3 describes the calibration strategy. Section 4 discusses the implications of financial frictions for equilibrium dynamics. Section 5 analyzes the welfare effect of alternative interest rate rules. Section 6 concludes.

2 A Sticky Price Economy with Agency Costs

The economy is populated by two types of firms, *final* goods and *intermediate* goods producers, and by two types of consumers, named *households* and *entrepreneurs.* We describe their respective problems below.

2.1 Final Good Producers

The aggregate final good is produced by perfectly competitive firms. It requires assembling a continuum of intermediate goods, indexed by $i \in (0, 1)$, via the production function:

$$Y_t = \left(\int_0^1 Y_t(i)^{\frac{\varepsilon-1}{\varepsilon}} di \right)^{\frac{\varepsilon}{\varepsilon-1}} \quad \varepsilon > 1 \tag{1}$$

Maximization of profits yields typical demand functions:

$$Y_t(i) = \left(\frac{P_t(i)}{P_t} \right)^{-\varepsilon} Y_t \tag{2}$$

for all i, where $P_t \equiv \left(\int_0^1 P_t(i)^{1-\varepsilon} di \right)^{\frac{1}{1-\varepsilon}}$ is the price index consistent with the final good producers earning zero profits.

2.2 Production of Intermediate Goods

There is a continuum of firms producing differentiated intermediate goods. The shares of these firms are owned by the households.[5] Each firm assembles labor and capital (supplied by both households and entrepreneurs) to operate a constant return to scale production function for the variety i of the intermediate good:

$$Y_t(i) = A_t F(N_t(i), K_t(i)) \tag{3}$$

where A_t is a total factor productivity shifter common to all firms, $N_t(i)$ and $K_t(i)$ are total inputs of labor and capital demanded by firm i. We defer to a later section the analysis of the price setting problem of these firms.

2.3 Households

There is continuum of households with measure $1 - \eta$. Households derive income from renting labor and capital to intermediate goods firms, and from receiving profits of the same monopolistic firms. They use their income to purchase final *consumption* goods and *capital* goods from the entrepreneurs. Capital goods are originally final consumption goods transformed via the entrepreneurial activity to be described later.

The representative household's sequence of budget constraints reads (in nominal terms):

$$P_t C_t^h + Q_t \left[K_{t+1}^h - (1-\delta) K_t^h \right] + D_{t+1} \leq (1 + R_t) D_t + W_t^h N_t^h + Z_t K_t^h + \Gamma_t + T_t \tag{4}$$

where K_t^h is capital held by the households, Q_t is the price of capital goods, Z_t is the nominal rental rate of capital, Γ_t is profits earned on the shares of the monopolistic competitive firms, W_t^h is the nominal wage, N_t^h is labor hours, R_t is the nominal interest rate on one-period risk-free bonds D_t, and T_t are net lump-sum transfers.[6]

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[5]An alternative ownership structure could be explored, in which the entrepreneurs directly own the shares of the intermediate goods firms that employ capital in production. In this case monopolistic profits would be part of capital income, as in Cook (2002).

[6]Notice that we are implicitly assuming that each household can have access also to a full set of state contingent securities. This implies that each household will face the same intertemporal budget constraint, and therefore the same level of expenditure. The specification of those assets would be redundant.

Let $z_t \equiv \frac{Z_t}{P_t}$ be the real rental rate of capital and $q_t \equiv \frac{Q_t}{P_t}$ be the price of capital goods in units of consumption goods. Households maximize the utility program:

$$E_0 \left\{ \sum_{t=0}^{\infty} \beta^t \, U(C_t^h, \, N_t^h) \right\}$$

subject to (4). Efficiency conditions require:

$$U_{c,t}^h = \beta(1 + R_t) E_t \left\{ U_{c,t+1}^h \frac{P_t}{P_{t+1}} \right\} \tag{5}$$

$$U_{c,t}^h \frac{W_t^h}{P_t} = -U_{n,t}^h \tag{6}$$

$$U_{c,t}^h = \beta E_t \left\{ U_{c,t+1}^h \left(\frac{q_{t+1}(1-\delta) + z_{t+1}}{q_t} \right) \right\} \tag{7}$$

where $U_{c,t}^h$ is the marginal utility of households' consumption and $U_{n,t}^h$ is the marginal disutility of households' labor.

Equations (5) and (6) are standard conditions on bonds investment and labor supply. Equation (7) is an intertemporal investment demand condition, with $\left(\frac{q_{t+1}(1-\delta)+z_{t+1}}{q_t} \right)$ being the real rate of return on capital owned by the households. Notice that fluctuations in q_t are similar to a time-varying tax on investment. Equivalently, one can interpret this model as one with endogenous adjustment costs (see CF).

2.4   Entrepreneurs

The second set of consumers in the economy (of measure η), are the *entrepreneurs*. Their activity consists in purchasing I_t units of the final consumption good and transform them into $\omega_t I_t$ units of *capital* goods via an instantaneous risky technology.[7] The random variable ω_t has positive support with cumulative distribution $\Phi(x, A_t) \equiv prob(\omega_t \leq x)$, with mean $\omega_{m,t}$, variance σ_ω^2, and density function $\varphi(\omega_t, A_t)$. Notice that the average productivity of each entrepreneur is time varying. We assume that each entrepreneur is

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[7] As will be appear more clear below, due to the linearity of the model, it is not essential to distinguish between individual and aggregate entrepreneurial variables. In fact, aggregation is straightforward in the model.

on average more productive when total factor productivity A_t increases. In particular we specify:

$$\omega_{m,t} = V(A_t) \tag{8}$$

where the function $V(\cdot)$ is increasing and convex. Hence, in our context, aggregate productivity shocks will have the additional effect of shifting the distribution of the risky invest outcome ω_t. As will appear more clear below, this feature will be key in driving the cyclical properties of the cost of external finance.

At the beginning of time t an entrepreneur with nominal net worth NW_t seeks for a loan (in excess of internal funds) $P_t I_t - NW_t$. The nominal lending rate on the loan is R_t^L. Default occurs when the return from the investment activity falls short of the amount that needs to be repaid. Hence the *default space* is implicitly defined as that range for ω_t such that:

$$\omega_t < \overline{\omega}_t \equiv \frac{(1 + R_t^L)(P_t I_t - NW_t)}{Q_t I_t} \tag{9}$$

where $\overline{\omega}_t$ is the cut-off value (determined endogenously in the general equilibrium) for the idiosyncratic productivity shock. When default occurs, the entrepreneur cannot repay the amount $(1 + R_t^L)(P_t I_t - NW_t)$, and can only repay what is available, namely $\omega_t I_t Q_t$. To avoid misreporting by the defaulting entrepreneur, the lender verifies this, but needs to expend a nominal amount $\mu I_t Q_t$ for monitoring, with $\mu \in [0,1]$.

At this stage we can compute the average *nominal income of the entrepreneur* at each point in time:

$$\int_{\overline{\omega}_t}^{\infty} \left[\omega_t Q_t I_t - (1 + R_t^L)(P_t I_t - NW_t) \right] \, d\Phi(\omega_t, A_t)$$

Using (9) we obtain:

$$
\begin{aligned}
& Q_t I_t \left\{ \int_{\overline{\omega}_t}^{\infty} (\omega_t - \overline{\omega}_t) \, d\Phi(\omega_t, A_t) \right\} \\
= \; & Q_t I_t \left\{ \int_{\overline{\omega}_t}^{\infty} \omega_t \, d\Phi(\omega_t, A_t) - \overline{\omega}_t (1 - \Phi(\overline{\omega}_t, A_t)) \right\} \\
\equiv \; & Q_t I_t f(\overline{\omega}_t, A_t)
\end{aligned}
\tag{10}
$$

where $f(\overline{\omega}_t, A_t)$ can be interpreted as the fraction of the expected investment outcome that accrues to the entrepreneur.

Furthermore, the average *nominal income of the lender* (net of monitoring costs) can be computed as:

$$
\begin{aligned}
& Q_t I_t \left\{ \int_0^{\overline{\omega}_t} \omega_t \, d\Phi(\omega_t, A_t) - \mu \Phi(\overline{\omega}_t, A_t) \right\} + (1 - \Phi(\overline{\omega}_t, A_t))(1 + R_t^L)(P_t I_t - NW_t) \\
= \; & Q_t I_t \left\{ \int_0^{\overline{\omega}_t} \omega_t \, d\Phi(\omega_t, A_t) - \mu \Phi(\overline{\omega}_t, A_t) + \overline{\omega}_t(1 - \Phi(\overline{\omega}_t, A_t)) \right\} \\
\equiv \; & Q_t I_t g(\overline{\omega}_t, A_t)
\end{aligned}
\tag{11}
$$

where $g(\overline{\omega}_t, A_t)$ is the fraction of the expected investment outcome that accrues to the lender.

Notice that, according to the definitions above we have

$$
f(\overline{\omega}_t, A_t) + g(\overline{\omega}_t, A_t) = 1 - \mu \Phi(\overline{\omega}_t, A_t)
\tag{12}
$$

Hence a fraction of the investment income is lost in the monitoring activity. This fraction depends on the parameter μ and on the frequency of defaulting entrepreneurs $\Phi(\overline{\omega}_t, A_t)$.

2.5 Financial Contract

The financial contract assumes the form of an optimal risky debt contract à la Townsend (1979) and Gale and Hellwig (1985). The contract specifies a pair $(I_t, \overline{\omega}_t)$ that maximizes entrepreneur's expected nominal income

$$
\max \; Q_t I_t f(\overline{\omega}_t, A_t)
\tag{13}
$$

subject to the lender's participation constraint:

$$
Q_t I_t \; g(\overline{\omega}_t, A_t) \geq (P_t I_t - NW_t)
\tag{14}
$$

Efficiency requires that (14) holds with equality. Due to the lenders being perfectly competitive, P_t and Q_t are taken as given in the maximization problem. It is easy to show

that the solution to the problem above implies the following two first order conditions (expressed in real terms, i.e., units of final consumption goods) :

$$q_t f(\overline{\omega}_t, A_t) = \frac{f'(\overline{\omega}_t, A_t)}{g'(\overline{\omega}_t, A_t)} \left[q_t g(\overline{\omega}_t, A_t) - 1 \right] \tag{15}$$

$$I_t = \frac{nw_t}{1 - q_t g(\overline{\omega}_t, A_t)} \tag{16}$$

where $nw_t \equiv \frac{NW_t}{P_t}$. Notice that, for given q_t, (15) pins down the threshold value $\overline{\omega}_t$ uniquely. Given $\overline{\omega}_t$, (16) is then a linear relation between investment and net worth, where the factor of proportionality $\frac{1}{1 - q_t g(\overline{\omega}_t, A_t)}$ is independent of any idiosyncratic element. Hence (16) allows immediate aggregation.[8]

Equation (16) is a crucial condition in the model. It relates the *supply* of capital to the relative price of capital goods q_t and to the (aggregate) financial conditions proxied by the level of net worth nw_t. In the space (q_t, I_t) capital supply is positively sloped, and movements in net worth shift the investment supply schedule, in turn affecting the relative price of capital goods. This is the feature that genuinely signals the presence of monitoring imperfections in the lender-borrower relationship. Hence, as already hinted above, the relative price of capital is endogenous for reasons directly related to credit market imperfections, rather than to the eventual presence of adjustment costs on capital.

By combining (16) with (9) we can derive an expression for the real gross lending rate $\frac{(1+R_t^L)}{P_t}$ as a function of the cutoff $\overline{\omega}_t$. Since the loan contract is intra-period, the lending rate measures also the cost of credit in excess of the safe rate of return (in turn equal to 1), or *premium* on external finance:

$$\psi_t \equiv \frac{(1 + R_t^L)}{P_t} - 1 = \frac{\overline{\omega}_t}{g(\overline{\omega}_t, A_t)} - 1 \tag{17}$$

Notice that if the distribution of investment outcomes $\Phi(\cdot)$ has a constant mean (e.g., $\omega_{m,t} = 1$, as usually assumed in the agency cost framework of CF), it is easy to show that the finance premium ψ_t is a monotonic increasing function only of the threshold value

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[8]In turn, this is a consequence of both the capital production technology and the monitoring technology being constant return to scale.

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$\overline{\omega}_t$. However, in the case in which the mean $\omega_{m,t}$ varies with aggregate productivity, the lender's income share $g(\overline{\omega}_t, A_t)$ also depends on aggregate productivity. The behavior of the income share $g(\cdot)$ relative to the threshold value $\overline{\omega}_t$ is critical in driving the cyclical properties of the finance premium.

2.6   Entrepreneurs in the General Equilibrium

Entrepreneurs inelastically supply one unit of labor (hence their total labor supply is η) and are risk-neutral. They maximize the following utility function:

$$E_0 \left\{ \sum_{t=0}^{\infty} (\beta\gamma)^t \ C_t^e \right\}, \quad 0 < \gamma < 1 \tag{18}$$

Notice that the entrepreneurs discount utility at a higher rate, $\beta\gamma$, than the households (i.e., they are more impatient). This assumption insures that the entrepreneurs never hold enough wealth to overcome the financing constraints. Entrepreneurs earn income also from renting their capital stock K_t^e to intermediate goods firms. Hence their nominal net worth at the beginning of time t can be written:

$$NW_t = W_t^e + [Z_t + Q_t(1 - \delta)] K_t^e$$

Notice that risk-neutrality induces entrepreneurs to invest all their net worth in the investment technology.

The problem of a non-defaulting entrepreneur is to maximize (18) subject to the sequence of budget constraints:

$$Q_t K_{t+1}^e + P_t C_t^e \leq Q_t I_t f(\overline{\omega}_t, A_t) \tag{19}$$

The left hand side of (19) denotes uses of funds (expenditure in new capital goods and consumption goods), whereas the right hand side is nominal expected income, as from (10).

This intertemporal problem must satisfy the Euler equation:

$$1 = \beta \gamma E_t \left\{ \left(\frac{z_{t+1} + (1-\delta)q_{t+1}}{q_t} \right) \frac{q_{t+1} f(\overline{\omega}_{t+1})}{1 - q_{t+1} g(\overline{\omega}_{t+1})} \right\} \tag{20}$$

The left hand side of (20) is the marginal utility of entrepreneurs' consumption. The right hand side is the expected discounted rate of return for an entrepreneur who is not bankrupt in period t. The latter term has two components. The first is the safe rate of return on capital (i.e., the one gained by the households). The second component is the return on internal funds, which can be shown to strictly exceed unity, i.e., $\frac{q_t f(\overline{\omega}_t, A_t)}{1 - q_t g(\overline{\omega}_t, A_t)} > 1$ for all t (Carlstrom and Fuerst, 1997).

2.7   Pricing of Intermediate Goods

Each intermediate good firm i has monopolistic power in the production of its own variety and therefore has leverage in setting the price. In so doing it faces a quadratic cost equal to $\frac{\vartheta}{2} \left(\frac{P_t(i)}{P_{t-1}(i)} - \pi \right)^2$, where π is the *steady state* inflation rate and where the parameter ϑ measures the degree of nominal price rigidity.[9] The higher ϑ the more sluggish is the adjustment of nominal prices. In the particular case of $\vartheta = 0$, prices are flexible.

Each monopolistic firm chooses the sequence $\left\{ K_t(i), \ N_t^h(i), \ P_t(i) \right\}$, taking nominal wage rates W_t^h and W_t^e as given, in order to maximize expected discounted profits:

$$E_0 \left\{ \sum_{t=0}^{\infty} \Lambda_{0,t} \left[P_t(i)Y_t(i) - \left(W_t^h N_t^h(i) + W_t^e \eta + Z_t K_t(i) \right) - \frac{\vartheta}{2} \left(\frac{P_t(i)}{P_{t-1}(i)} - \pi \right)^2 P_t(i) \right] \right\} \tag{21}$$

subject to the constraint $A_t F_t(.) \geq Y_t(i)$, where $\Lambda_{0,t}$ is the household's stochastic discount factor.

Let's denote by $\{mc_t\}_{t=0}^{\infty}$ the sequence of lagrange multipliers on the above demand constraint, and by $\widetilde{p}_t \equiv \frac{P_t(i)}{P_t}$ the relative price of variety i. The first order conditions of

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[9]Recall that, in our framework, $\pi > 0$. An alternative formulation may feature adjustment costs penalizing the deviation of the rate of change of prices from the *past* inflation rate π_{t-1}. However, Schmitt-Grohé and Uribe (2004b) show that the latter formulation (similar to the one employed in Christiano et al. (2003)) biases the optimal policy towards generating an inflation volatility significantly different from zero.

the above problem read:

$$\frac{W_t^h}{P_t(i)} = mc_t \ A_t F_{n,t}^h \tag{22}$$

$$\frac{W_t^e}{P_t(i)} = mc_t \ A_t F_{n,t}^e \tag{23}$$

$$\frac{Z_t}{P_t(i)} = mc_t \ A_t F_{k,t} \tag{24}$$

$$
\begin{aligned}
0 &= Y_t \widetilde{p}_t^{-\varepsilon} \left((1 - \varepsilon) + \varepsilon mc_t \right) - \vartheta \left(\pi_t \frac{\widetilde{p}_t}{\widetilde{p}_{t-1}} - \pi \right) \frac{\pi_t}{\widetilde{p}_{t-1}} \\
&\quad + \vartheta \left(\pi_{t+1} \frac{\widetilde{p}_{t+1}}{\widetilde{p}_t} - \pi \right) \pi_{t+1} \frac{\widetilde{p}_{t+1}}{\widetilde{p}_t^2}
\end{aligned}
\tag{25}
$$

where $F_{n,t}^h$ is the marginal product of households' labor, $F_{n,t}^e$ is the marginal product of entrepreneurs' labor, and $\pi_t \equiv \frac{P_t}{P_{t-1}}$ is the gross aggregate inflation rate. Notice that all firms employ an identical capital/labor ratio in equilibrium. The lagrange multiplier mc_t plays the role of the *real* marginal cost of production. In a symmetric equilibrium it must hold that $\widetilde{p}_t = 1$. This allows to rewrite (25) in the following form:

$$
\begin{aligned}
U_{c,t} \left(\pi_t - \pi \right) \pi_t &= \beta E_t \left\{ U_{c,t+1} \left(\pi_{t+1} - \pi \right) \pi_{t+1} \right\} \\
&\quad + U_{c,t} A_t F_t(\cdot) \frac{\varepsilon}{\vartheta} \left(mc_t - \frac{\varepsilon - 1}{\varepsilon} \right)
\end{aligned}
\tag{26}
$$

The above equation has the form of a (non-linear) forward-looking New-Keynesian Phillips curve, in which deviations of the real marginal cost from its desired steady state value are the driving force of inflation.[10]

2.7.1 Market Clearing

Equilibrium in the final good market requires that the production of the final good be allocated to total private consumption by households and entrepreneurs, investment, public spending, and to resource costs that originate from the adjustment of prices:

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[10] Galí and Gertler (1999), Woodford (2003).

$$Y_t = (1 - \eta)C_t + \eta\left(C_t^e + I_t\right) + G_t + \frac{\vartheta}{2}\left(\pi_t - \pi\right)^2 \tag{27}$$

In the above equation, G_t is government consumption of the final good which evolves exogenously and is assumed to be financed by means of lump sum taxes.

Using (12), total capital accumulation can be written

$$K_{t+1} = (1 - \delta)K_t + \eta I_t\left[1 - \mu\Phi\left(\overline{\omega}_t, A_t\right)\right] \tag{28}$$

where $K_t \equiv (1 - \eta)K_t^h + \eta K_t^e$. Notice that the presence of monitoring costs endogenously distorts aggregate capital accumulation.

Finally, equilibrium in the labor market requires

$$N_t = (1 - \eta)N_t^h + \eta \tag{29}$$

In the Appendix we describe the complete form of the recursive competitive equilibrium.

2.8   Monetary Policy

We assume that monetary policy is conducted by means of an interest rate reaction function, constrained to be linear in the logs of the relevant arguments:

$$
\begin{aligned}
\ln\left(\frac{1 + R_t}{1 + R}\right) = {}& (1 - \phi_r)\left(\phi_\pi \ln\left(\frac{\pi_t}{\pi}\right) + \phi_q \ln\left(q_t\right) + \phi_\pi \ln\left(\frac{Y_t}{Y}\right)\right) \\
& + \phi_r \ln\left(\frac{1 + R_{t-1}}{1 + R}\right)
\end{aligned} \tag{30}
$$

with $\{\phi_\pi\} \in (1, \infty]$, $\{\phi_q\} \in (-\infty, \infty]$, $\{\phi_y\} \in (0, \infty]$ and $\{\phi_r\} \in [0, 1]$, and where π, Y, R correspond to deterministic steady state values. A few observations on the specification of (30) are in order. First, as in Schmitt-Grohé and Uribe (2003), we model monetary policy in terms of an implementable rule, whereby the central bank sets the short-run nominal interest rate in response to *observable* variables only. This assumption explains why - as an argument of the rule - we are including output in deviations from steady state rather than

a measure of the output gap. The latter specification would in fact require, in accordance with some practice in the literature, a computation of either the efficient level or the natural level of output (i.e., the level of output under flexible prices and/or frictionless credit markets). In both cases, such computation would presume that the central bank be able to observe the stochastic joint distribution of the shocks. Second, this general specification allows for a reaction of the monetary policy instrument to deviations of the relative price of capital goods q_t from 1. Incidentally, the latter corresponds also to the hypothetical *efficient* value of q_t, i.e., its value in the absence of credit frictions (regardless of whether nominal rigidities are built into the model). Hence, in (30), monetary policy is also implicitly reacting to an asset price *gap*. This choice addresses the issue raised by Gilchrist and Saito (2006) according to which the implications of responding to asset prices may vary depending on whether the monetary authority is reacting to an asset price *level* as opposed to an asset price gap. In our case, responding to an asset price gap does not require any knowledge of the behavior of the efficient/natural level of the asset price. Any movement in q_t, in fact, can be interpreted as deviations from its efficient level.

Our approach consists in finding the policy specification $\left\{ \phi_\pi, \phi_y, \phi_q, \phi_r \right\}$ that maximizes private agents' welfare (see more below). While we will allow ϕ_π to vary only on a range of values exceeding 1 (to insure equilibrium determinacy), ϕ_q will potentially span both negative and positive values. We will return on this point below.

3   Calibration and Solution Strategy

The time unit is a quarter. We employ a period utility function for households $U(C_t^h, N_t^h) = \log(C_t^h) + \nu \log(1 - N_t^h)$, with parameter ν chosen in such a way to generate a steady state level of households' employment $N^h = 0.3$. We assume that the annualized steady state inflation rate is 4%, close to the historical average for the US. We set the discount factor $\beta = 0.99$, so that the annual real interest rate is equal to 4%. We assume that the production function for intermediate goods has the Cobb-Douglas form $F(\cdot) = K_t^\alpha \left(N_t^h \right)^{1-\alpha-\chi} \eta^\chi$, with $\alpha = 0.3$ and χ close to zero (see CF). The quarterly aggregate capital depreciation rate δ is 0.025, the elasticity of substitution between varieties is 8.

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In line with the evidence reported in CF, we set μ equal to 0.25. Notice that, in the steady state, the presence of non-zero monitoring costs ($\mu > 0$) implies that relative price of capital exceeds one ($q > 1$).

We calibrate the steady state to imply an annual (average) external finance premium $\psi = 1.02$ (two hundred basis points), and to generate an average bankruptcy rate of three percent ($\Phi(\overline{\omega}, 1) = 0.03$). We assume that ω_t has a uniform distribution, with positive support and mean $\omega_{m,t} = V(A_t) = A_t^{1+\sigma_\omega}$, with $\sigma_\omega = 2$.

Total factor productivity is assumed to evolve as:

$$A_t = A_{t-1}^{\rho_a} \exp(\varepsilon_t^a)$$

where the steady state value A is normalized to unity (which in turn implies $\omega_m = 1$) and where ε_t^a is an iid shock with standard deviation σ_a. In line with the real business cycle literature (King and Rebelo, 1999), we set $\rho_a = 0.95$ and $\sigma_a = 0.0056$. Log-government consumption is assumed to evolve according to the following process:

$$\ln\left(\frac{G_t}{G}\right) = \rho_g \ln\left(\frac{G_{t-1}}{G}\right) + \varepsilon_t^g$$

where G is the steady-state share of government consumption (set in such a way that $\frac{G}{Y} = 0.25$) and ε_t^g is an iid shock with standard deviation σ_g. We follow the empirical evidence for the United States in Perotti (2004) and set $\sigma_g = 0.008$ and $\rho_g = 0.9$.

In order to parameterize the degree of price stickiness ϑ, we observe that, by log-linearizing equation (26) we can obtain an elasticity of inflation to real marginal cost (normalized by the steady state level of output)[11] that takes the form $\frac{\varepsilon-1}{\vartheta}$. This allows a direct comparison with empirical studies on the New Keynesian Phillips curve such as Galí and Gertler (1999) and Sbordone (2002) using a Calvo-Yun approach. In those studies, the slope coefficient of the log-linear Phillips curve can be expressed as $\frac{(1-\widehat{\vartheta})(1-\beta\widehat{\vartheta})}{\widehat{\vartheta}}$, where $\widehat{\vartheta}$ is the probability of not resetting the price in any given period in the Calvo-Yun model. For any given values of ε, which entails a choice on the steady state level of the markup,

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[11] To produce a slope coefficient directly comparable to the empirical literature on the New Keynesian Phillips curve this elasticity needs to be normalized by the level of output when the price adjustment cost factor is not explicitly proportional to output, as assumed here.

we can thus build a mapping between the frequency of price adjustment in the Calvo-Yun model $\frac{1}{1-\hat{\vartheta}}$ and the degree of price stickiness ϑ in the Rotemberg setup. Traditionally, the sticky price literature has been considering a frequency of four quarters as a realistic value. Recently, Bils and Klenow (2004) argue that the observed frequency of price adjustment in the U.S. is higher, and in the order of two quarters. As a benchmark, we parameterize $\frac{1}{1-\hat{\vartheta}} = 4$, which implies $\hat{\vartheta} = 0.75$. Given $\varepsilon = 8$, the resulting stickiness parameter satisfies $\vartheta = \frac{Y \, \hat{\vartheta} \, (\varepsilon-1)}{(1-\hat{\vartheta})(1-\beta\hat{\vartheta})} \approx 20$, where Y is steady state output.

We solve the model by computing a second order approximation of the policy functions around the non-stochastic steady state (with positive average inflation, market power and monitoring costs). In the Appendix, we provide more details on the solution strategy.

4 Equilibrium Dynamics

In this section we study the dynamic behavior of the model. We first illustrate the role that financial frictions play in shaping the equilibrium response to shocks in contrast to a model in which the same frictions are absent. Next, we compare the standard model with agency costs (in practice, the CF model augmented with sticky prices) with our model in which the distribution of investment opportunities is assumed to be endogenous to aggregate productivity.

4.1 The Role of Financial Frictions

Figure 1 depicts impulse responses of selected variables to a positive total factor productivity shock in two cases: (i) a *baseline sticky price model* with capital and frictionless credit markets; (ii) the *agency cost* model of CF (i.e., with constant mean distribution of investment outcomes) augmented with sticky prices. The latter model nests the former for values of parameter μ close to zero. All numbers are in percent deviations from steady state values. This illustrative exercise is conducted under the temporary assumption that monetary policy is specified via a simplified Taylor rule $\ln\left(\frac{1+R_t}{1+R}\right) = 1.5 \ln\left(\frac{\pi_t}{\pi}\right)$.

The initial effect of the rise in productivity is to shift investment *demand* outward, and therefore to produce, for any given level of net worth, a rise in the price of capital.

The critical element to notice is that in the agency cost model the rise in investment is paralleled by a slow response in net worth. In fact, in the short-run, net worth is mostly composed of entrepreneurial capital, and its dynamic is sluggish. The result is an initial rise in borrowing needs (i.e., I_t rises relatively more than nw_t) and a consequent rise in the marginal cost of investment, an effect that tends to dampen the impact of the shock. In turn, this generates a rise in the default threshold $\overline{\omega}_t$. Since in the baseline agency cost model the finance premium ψ_t is a strictly increasing function of $\overline{\omega}_t$, the result is a *rise* in the external finance premium. However, over time, net worth accumulates, and its response shifts the investment *supply* schedule outward and to such an extent that the asset price starts to revert downward. It is this delayed response of net worth that induces a subsequent (supply-driven) boost to investment, thereby generating the observed hump-shaped dynamics in both investment and output. Noticeably, this feature of endogenous serial correlation is absent in a standard sticky price model (with frictionless credit markets).

Interestingly, the reactivity of net worth is inversely proportional to the degree of financial frictions summarized by μ. Hence here, as in the baseline CF framework, financial frictions are synonymous with persistence. Even more subtly, and as emphasized in Carlstrom and Fuerst (1998), magnification and persistence seem to be related by a trade-off.

4.2 Distribution of Investment Outcomes and Countercyclical Finance Premium

An often-cited limitation of the CF framework is that it generates a procyclical behavior of the external finance premium, which in the model is directly proportional to the behavior of the asset price. Levin et al. (2004) report micro-based evidence showing that, during the boom of the late 1990s in the US, credit spreads on corporate debt were historically low. Gomes et al. (2003) show that, in the data, alternative measures of the premium on external funds display a negative correlation with total factor productivity.[12]

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[12]This evidence is robust to alternative measures of the finance premium: (i) spread between Baa and Aaa corporate bonds; (ii) spread between prime bank loan rate and three-month commercial paper;

Figure 2 compares responses to a rise in total factor productivity in two versions of the agency cost model: (i) with *constant* mean distribution of investment outcomes: $\omega_{m,t} = \omega_m = 1$ for all t (solid line); (ii) with *endogenous* mean distribution of investment outcomes: $\omega_{m,t} = V(A_t)$ (dashed line).

The rise in aggregate productivity once again produces an outward shift of investment demand, and therefore a rise in investment and in the price of capital. The crucial point is that in the model with endogenous $\omega_{m,t}$ a rise in aggregate productivity makes also each entrepreneur on average more productive. This effect is akin to an instantaneous shift of the investment *supply* schedule. Hence a rise in aggregate productivity produces now a simultaneous shift in both the investment demand and in the investment supply schedules. In contrast, in the baseline model with constant ω_m (as in CF), the shift in investment supply is delayed due to the slow accumulation of net worth.

Notice that, in the model with endogenous $\omega_{m,t}$, the price of capital is still procyclical in response to productivity shocks, a feature that appears empirically realistic. This happens because the contemporaneous shift in the investment supply schedule is initially not sufficient to produce an equilibrium fall in the price of capital. Hence, on impact, we still observe a positive *excess demand* of investment. However, because of the instantaneous supply effect, the price of capital rises less than in the baseline model with constant $\omega_{m,t}$, and even starts to fall below baseline after two quarters. At the same time, the share of income going to the lender $g(\overline{\omega}_t, A_t)$ (which depends positively on aggregate productivity) rises proportionally more than the threshold value $\overline{\omega}_t$, thereby producing a *fall* in the external finance premium (see equation (17)).

The fall in the finance premium and the more muted rise in the price of capital both induce investment and output to rise more on impact in the model with endogenous $\omega_{m,t}$ (relative to the model with constant $\omega_{m,t}$), generating a magnification effect. At the same time, though, the sluggish response of net worth still produces an effect of persistence in investment and output (although diminished relative to the case with constant $\omega_{m,t}$). Hence, our modified model is able to simultaneously generate both an effect of persistence

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(iii) spread between more and less financially constrained firms (as in Lamont et al.(2001)); (iv) default likelihood indicators, as in Vassalou and Xing (2002).

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and an effect of magnification. To clarify, the effect of persistence is to be understood relative to a baseline sticky price model with frictionless credit markets, whereas the effect of magnification is to be understood relative to a model with credit frictions but with constant $\omega_{m,t}$.

5 Welfare Evaluation

The critical feature of our analysis consists in the assessment of alternative interest rate rules based on social welfare evaluation. Some observations on the computation of welfare in this context are in order. First, one cannot safely rely on standard first order approximation methods to compare the relative welfare associated to each monetary policy arrangement. In fact, in an economy like ours, in which distortions exert an effect both in the short-run and in the steady state, stochastic volatility affects both first and second moments of those variables that are critical for welfare. Since in a first order approximation of the model's solution the expected value of a variable coincides with its non-stochastic steady state, the effect of volatility on the mean mean value of any endogenous variable is by construction neglected. Hence policy arrangements can be correctly ranked only by resorting to a higher order approximation of the policy functions.[13]

This last observation also suggests that our welfare metric needs to be correctly chosen. In particular, one needs to focus on the *conditional* expected discounted utility of the representative agent. This is necessary exactly to account for the transitional effects from the deterministic steady state (which defines the initial condition of the model) to the different stochastic steady states implied by each alternative policy rule, and conditional on the same distribution of the shocks.

Finally, it is important to recall that our framework features heterogeneity of consumers. However, entrepreneurs are risk-neutral agents. This implies that their mean level of consumption is unaffected by the underlying sources of stochastic volatility. Hence, alternative interest rate rules not only will imply the same (deterministic) steady state

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[13]See Kim and Kim (2003) for an analysis of the inaccuracy of welfare calculations based on log-linear approximations in dynamic open economies. See Kim et al. (2003) and Schmitt-Grohe and Uribe (2004a) for a more general discussion.

level of all variables, but they will also imply the same stochastic mean consumption for entrepreneurs. This implies that a correct measure of conditional welfare needs simply to be amended by adding the mean level of entrepreneurial consumption in the deterministic steady state. However, since this will be invariant to the alternative specifications of the monetary policy rule (30), we will measure conditional welfare simply by including households' utility:

$$\mathcal{W}_{0,t} \equiv E_0 \left\{ \sum_{t=0}^{\infty} \beta^t U(C_t^h, N_t^h) \right\} \tag{31}$$

or in recursive form:

$$\mathcal{W}_{0,t} = U(C_t^h, N_t^h) + \beta E_t \left\{ \mathcal{W}_{0,t+1} \right\} \tag{32}$$

We refer to the Appendix for more details on the correct computation of the conditional expected value $\mathcal{W}_{0,t}$.

5.1   Comparing Simple Rules

We begin by comparing the welfare performance of alternative (ad hoc) specifications of the monetary policy rule. The rules are the following: (i) *Strict Inflation Stabilization (IT)*; (ii) *Taylor rule (TR)* ; (iii) *Taylor rule with output* (iv) *Taylor rule with negative response to the asset price*; (v) *Taylor rule with positive response to the asset price;* (vi) *Taylor rule with output and negative response to the asset price.* Table 1 reports the specific parameter configuration that characterizes each rule.

We compare the rules both in terms of conditional welfare $\mathcal{W}_{0,t}$ and in terms of a compensating measure given by the fraction Ω of households' consumption that would be needed to equate conditional welfare $\mathcal{W}_{0,t}$ under a generic interest rate policy to the level of welfare $\mathcal{W}_{0,t}^*$ implied by the optimal rule. Hence Ω should satisfy the following equation:

$$E_0 \left\{ \sum_{t=0}^{\infty} \beta^t U((1+\Omega)C_t, N_t) \right\} = \mathcal{W}_{0,t}^*$$

Under our specification of utility one can solve for Ω and obtain:

$$\Omega = \exp\left\{\left(\mathcal{W}_{0,t}^* - \mathcal{W}_{0,t}\right)(1-\beta)\right\} - 1 \tag{33}$$

The results are displayed in Table 2. All rules are evaluated with and without interest rate smoothing ($\phi_r = 0$ and $\phi_r = 0.9$ respectively). A number of implications are worth emphasizing. First, strict stabilization of inflation always attains the highest level of welfare. However, the welfare loss of a simple Taylor rule with a mild inflation coefficient is very small. Second, responding to output in a Taylor rule is strongly welfare detrimental. Third, responding to asset prices improves welfare (relative to a Taylor rule) only if this response is negative. Fourth, interest rate smoothing improves the performance of all rules.

5.2   Responding to Inflation vs. Asset Prices

Next we search for a generalization of the results outlined above. Figures *3* and *4* depict the effects on the conditional welfare surface of varying both the inflation coefficient ϕ_π and the asset price coefficient ϕ_q in the monetary policy rule (30), respectively without ($\phi_r = 0$) and with ($\phi_r = 0.9$) interest rate smoothing. All results shown here correspond to cases in which the coefficient ϕ_y is set equal to zero. As hinted above, rules featuring a positive response to (deviations of) output (from steady state) are invariably welfare inferior to rules in which the same response is zero. This confirms similar results obtained in Schmitt-Grohé and Uribe (2003). In addition, in the light of the recent policy debate, we are particularly interested in evaluating the relative welfare performance of asset price versus goods price stabilization.

The main result that emerges in both pictures is that, when the inflation coefficient ϕ_π is sufficiently low and in the order of magnitude typically assigned in Taylor-type rules (i.e., between 1 and 2), there exists a positive effect on welfare of responding to asset prices. For sufficiently low values of ϕ_π, in fact, the welfare surface is concave in the asset price coefficient ϕ_q, with a maximum reached for values slightly below zero. A more detailed exposition of this result is in *Figure 5*, where we plot a section of our welfare surface. This corresponds to the effect of varying the asset price coefficient ϕ_q for

a given value $\phi_\pi = 1.5$ (and still maintaining $\phi_y = 0$), which is the value for the inflation coefficient usually chosen in the specification of a standard Taylor rule. Thus our results suggest that monetary policy should respond *negatively* (i.e., reduce the nominal interest rate) to increases in the asset price.

The intuition for why, at sufficiently low levels of ϕ_π, monetary policy should respond negatively to asset price rises is as follows. Consider a scenario with productivity shocks, which are the main source of variability in our context. Recall that, in the model, agency costs induce endogenous movements in the price of capital goods relative to consumption goods. Hence, when productivity increases, a higher relative price of capital goods is similar to a procyclical tax. As a result, investment dynamic is distorted below the level it would attain in an environment with frictionless credit markets. In this sense, agency costs restrict investment in a way similar to adjustment costs on capital (see also CF on this point). Thus the monetary authority wishes to *lower* interest rates (nominal and, in turn, real) in order to limit the tax distortion that constraints the expansion in investment (with the opposite prescription holding true in the case of negative productivity shocks).

However it is clear that the benefits of responding to asset price movements tend to vanish when monetary policy turns strongly anti-inflationary. As we see in Figure 3 and 4, at higher levels of ϕ_π, the welfare surface becomes completely flat in ϕ_q, and the scope for responding to asset prices tends to disappear. In general, the optimal rule prescribes a strong *anti-inflationary* stance.

5.2.1 Does Stabilizing Inflation Eliminate the Financial Distortions?

The above result bears the question of whether a policy of strict inflation stabilization, which emerges as welfare maximizing in our context, also manages to dampen or even neutralize credit market distortions, which manifest themselves in fluctuations of the asset price and/or the finance premium. In Figure 6 and 7 we depict impulse responses of inflation, output, asset price and finance premium to a productivity shock and to a government spending shock respectively. We compare the outcome under two rules: (i) a simple Taylor-type rule, featuring a mild response to inflation $\phi_\pi = 1.5$ (solid line); (ii) a strict inflation stabilization rule, which aims at fully stabilizing inflation ($\pi_t = 0$ for all t,

dashed line). The crucial point to notice is that, regardless of the underlying shock, the behavior of both the asset price and the finance premium is only barely affected by the degree to which monetary policy stabilizes inflation. Thus, under the welfare-optimal rule which prescribes stable inflation, credit frictions generated by agency costs are clearly not neutralized.

For one, this result is not surprising. In fact, the monetary authority faces two (cyclical) distortions (i.e., price stickiness and agency costs) and one policy instrument, and hence cannot simultaneously neutralize both distortions.[14] On the other hand, it suggests that, in this context, the marginal benefit of neutralizing the price stickiness distortion largely outweighs the marginal benefit of neutralizing the credit market distortion on a quantitative ground.

6 Conclusions

We have analyzed optimal interest rate rules in an economy featuring sticky prices and endogenous agency costs. In our context, and due to the presence of a countercyclical premium on external finance, credit frictions generate both an effect of persistence (relative to a model with frictionless credit markets) and an effect of magnification (relative to a version of our model featuring constant mean investment opportunities for entrepreneurs). We have focused our attention on the desirability of including asset prices, along with inflation, as a separate argument in the policy rule. For sufficiently low values of the inflation feedback coefficient in the policy rule, and in the range of those typically assigned in standard Taylor rules, responding to a rise in the asset price improves welfare. In particular, the prescription for the monetary authority is to lower interest rates when the asset price rises. This result depends on the fact that, in our context, an increase in the asset price is isomorphic to an increase in the price of capital relative to consumption goods, and hence acts as an implicit tax on investment. However, when monetary policy focuses on strongly stabilizing inflation, reacting to the asset price plays no significant

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[14]See also Dupor (2003) on this point, although in a context in which financial frictions are not explicitly modelled.

and independent welfare role in the specification of the interest rate rule. In general, rules with a strong anti-inflationary stance always attain the highest level of welfare.

A The Equilibrium

For any given policy instrument $\{R_t\}$ and stochastic processes $\{A_t, G_t\}$, a recursive (imperfectly) competitive equilibrium is a sequence of allocations for C_t^h, π_t, mc_t, N_t^h, q_t, I_t, z_t, C_t^e, $\overline{\omega}_t$, nw_t, K_{t+1}, K_{t+1}^e which solves the following system of equations:

$$U_{c,t}^h = \beta(1 + R_t)E_t \left\{ U_{c,t+1}^h \frac{P_t}{P_{t+1}} \right\} \tag{34}$$

$$U_{c,t}^h = \beta E_t \left\{ U_{c,t+1}^h \left(\frac{q_{t+1}(1 - \delta) + z_{t+1}}{q_t} \right) \right\} \tag{35}$$

$$1 = \beta\gamma E_t \left\{ \left(\frac{z_{t+1} + (1 - \delta)q_{t+1}}{q_t} \right) \frac{q_{t+1}\ f(\overline{\omega}_{t+1}, A_{t+1})}{1 - q_{t+1}g(\overline{\omega}_{t+1}, A_{t+1})} \right\} \tag{36}$$

$$\eta nw_t = \eta mc_t\ A_t F_{n,t}^e + [z_t + q_t(1 - \delta)\ ] K_t^e$$

$$K_{t+1} = (1 - \delta)K_t + \eta I_t\left[1 - \mu\Phi\left(\overline{\omega}_t, A_t\right)\right] \tag{37}$$

$$\eta I_t = \frac{nw_t}{1 - q_t g(\overline{\omega}_t, A_t)} \tag{38}$$

$$K_{t+1}^e = \eta nw_t \left(\frac{f(\overline{\omega}_t, A_t)}{1 - q_t\ g(\overline{\omega}_t, A_t)} \right) - \eta\frac{C_t^e}{q_t} \tag{39}$$

$$A_t F(\cdot) = (1 - \eta)C_t + \eta C_t^e + \eta I_t + \frac{\vartheta}{2}\left(\frac{\pi_t}{\pi} - 1\right)^2 + G_t \tag{40}$$

$$mc_t\ A_t F_{n,t}^h = \frac{-U_{n,t}^h}{U_{c,t}^h} \tag{41}$$

$$mc_t\ A_t F_{k,t} = z_t \tag{42}$$

$$q_t f(\overline{\omega}_t, A_t) = \frac{f'(\overline{\omega}_t, A_t)}{g'(\overline{\omega}_t, A_t)}\left[q_t g(\overline{\omega}_t, A_t) - 1\right] \tag{43}$$

$$(\pi_t - \pi) \, \pi_t = \beta E_t \left\{ \frac{U^h_{c,t+1}}{U^h_{c,t}} \left(\pi_{t+1} - \pi \right) \pi_{t+1} \right\} + A_t F(\cdot) \frac{\varepsilon}{\vartheta} \left(mc_t - \frac{\varepsilon - 1}{\varepsilon} \right) \tag{44}$$

B Solution of the Model and Welfare

The set of conditions characterizing the optimal plan can be described as follows:

$$E_t \left\{ \, \mathcal{H}(\mathcal{Y}_{t+1}, \mathcal{Y}_t, \, X_{t+1}, X_t) \right\} = 0 \tag{45}$$

where E_t denotes the mathematical expectations operator, conditional on information available at time t, $\mathcal{Y}_t$ is the vector of endogenous non-predetermined variables, and $X_t \equiv [x_{1,t}, x_{2,t}]$ is the state vector. Here $x_{1,t}$ denotes the vector of endogenous predetermined variables, while $x_{2,t}$ is the vector of exogenous variables which follows a stochastic process:

$$x_{2,t+1} = F \, x_{2,t} + \overline{b\xi} \, \varepsilon_{t+1}, \qquad \varepsilon_t \sim i.i.d.N(0, \Sigma) \tag{46}$$

where the scalars $\overline{\xi}$ and $\overline{b}$ are known parameters, and Σ is the variance-covariance matrix of the innovations. The solution of the model is of the form (Schmitt-Grohé and Uribe (2004a)):

$$\mathcal{Y}_t = \gamma(X_t, \overline{\xi}) \tag{47}$$

$$X_{t+1} = h(X_t, \overline{\xi}) + \overline{b\xi} \, \varepsilon_{t+1} \tag{48}$$

Equation (47) and (48) describe the policy function and the transition function respectively. Our solution methods consists in computing a second order expansion of the functions $\gamma(X_t, \overline{\xi})$ and $h(X_t, \overline{\xi})$ around the deterministic steady state. Schmitt-Grohé and Uribe (2004a) show two results. First, in a first-approximation of the model, the expected value of any variable belonging to either $\mathcal{Y}_t$ or X_t coincides with its value in the non-stochastic steady state. Second, in a second-order approximation of the model, the expected value of any variable differs from its deterministic steady-state value only by a constant term. Let $\frac{1}{2}\Delta$ be the vector of such constant "correction" terms corresponding

to the set of endogenous variables. Hence our measure for the conditional expected value $\mathcal{W}_{0,t}$ can be written:

$$\mathcal{W}_{0,t} = \overline{\mathcal{W}}_0 + \frac{1}{2}\boldsymbol{\Delta}\left[\mathcal{W}_0\right] \tag{49}$$

where $\overline{\mathcal{W}}_0 = \frac{U(C^h, N^h)}{1-\beta}$ is the value of $\mathcal{W}_{0,t}$ in the deterministic steady state and $\boldsymbol{\Delta}\left[\mathcal{W}_0\right]$ is the (constant) correction element of the vector $\boldsymbol{\Delta}$ corresponding to the endogenous variable $\mathcal{W}_{0,t}$. To analytically compute (and numerically evaluate) the first and second order derivatives of the relevant policy functions we employ the Matlab codes compiled by Schmitt-Grohé and Uribe, available at the website http://www.econ.duke.edu/~Grohé.

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<p style="text-align:center">Table 1. Simple Rules</p>

Rule	Description		
IT	$\pi_t = 0$ all t		
TR	$\phi_\pi = 1.5$	$\phi_y = 0$	$\phi_q = 0$
TR +output	$\phi_\pi = 1.5$	$\phi_y = 0.5$	$\phi_q = 0$
TR + $\phi_q < 0$	$\phi_\pi = 1.5$	$\phi_y = 0$	$\phi_q = -0.2$
TR + $\phi_q > 0$	$\phi_\pi = 1.5$	$\phi_y = 0$	$\phi_q = 0.2$
TR + output + $\phi_q < 0$	$\phi_\pi = 1.5$	$\phi_y = 0.5$	$\phi_q = -0.2$

<p style="text-align:center">Table 2. Welfare Results</p>

	No Smoothing ($\phi_r = 0$)		Smoothing ($\phi_r = 0.9$)	
Rule	**Welfare**	**Ω**	**Welfare**	**Ω**
IT	-76.02	0	-76.02	0
TR	-76.03	0.003	-76.02	6.4513e-004
TR + output	-77.43	1.413	-76.27	0.2399
TR + $\phi_q < 0$	-76.03	0.0013	-76.028	5.5352e-004
TR + $\phi_q > 0$	-76.033	0.0045	-76.029	0.0010
TR + output + $\phi_q < 0$	-77.43	1.416	-76.27	0.2399

Notes: Ω is the % fraction of consumption required to equate welfare under any given policy rule to the one under the optimal policy (see equation (33)). Welfare is calculated as conditional to the initial deterministic steady-state.
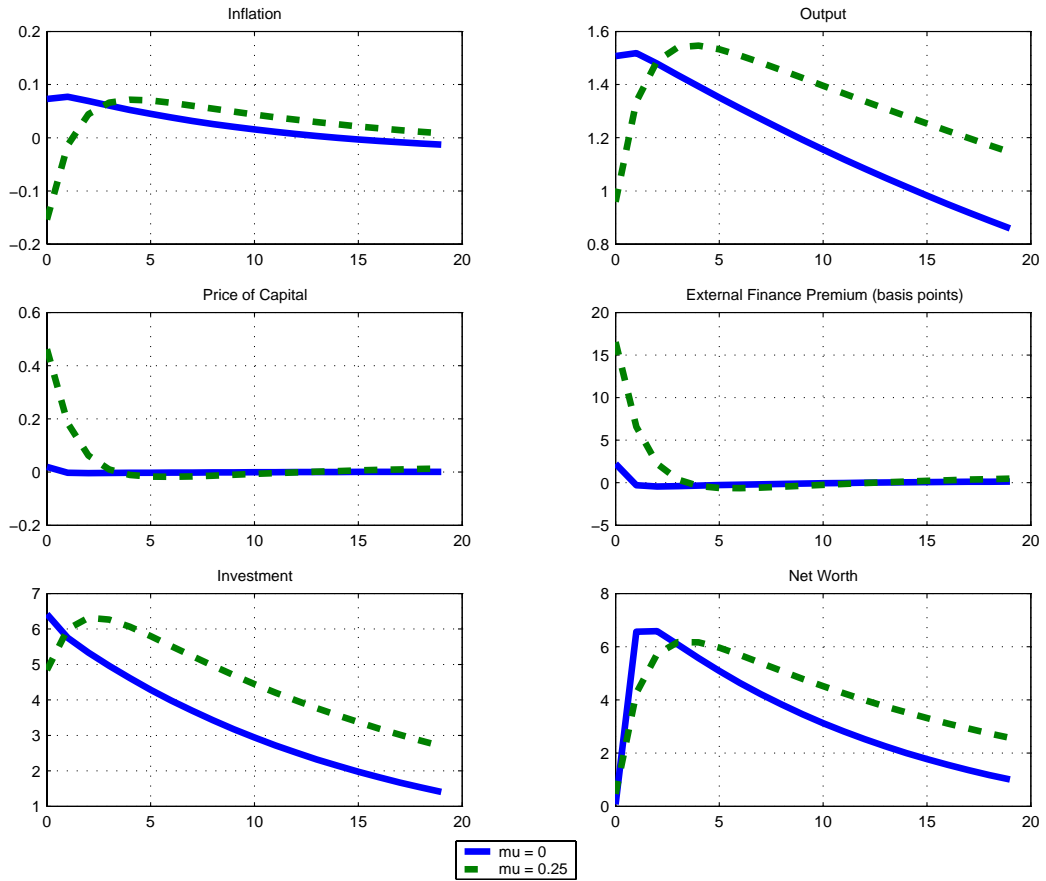
Figure 1: Impulse Responses to a Total Factor Productivity Shock: No Credit Frictions (solid line, $\mu = 0$) vs. Credit Frictions (dashed line, $\mu = 0.25$, model with $\omega_{m,t} = 1$). Monetary policy rule: $\ln\left(\frac{1+R_t}{1+R}\right) = 1.5\ln\left(\frac{\pi_t}{\pi}\right)$.
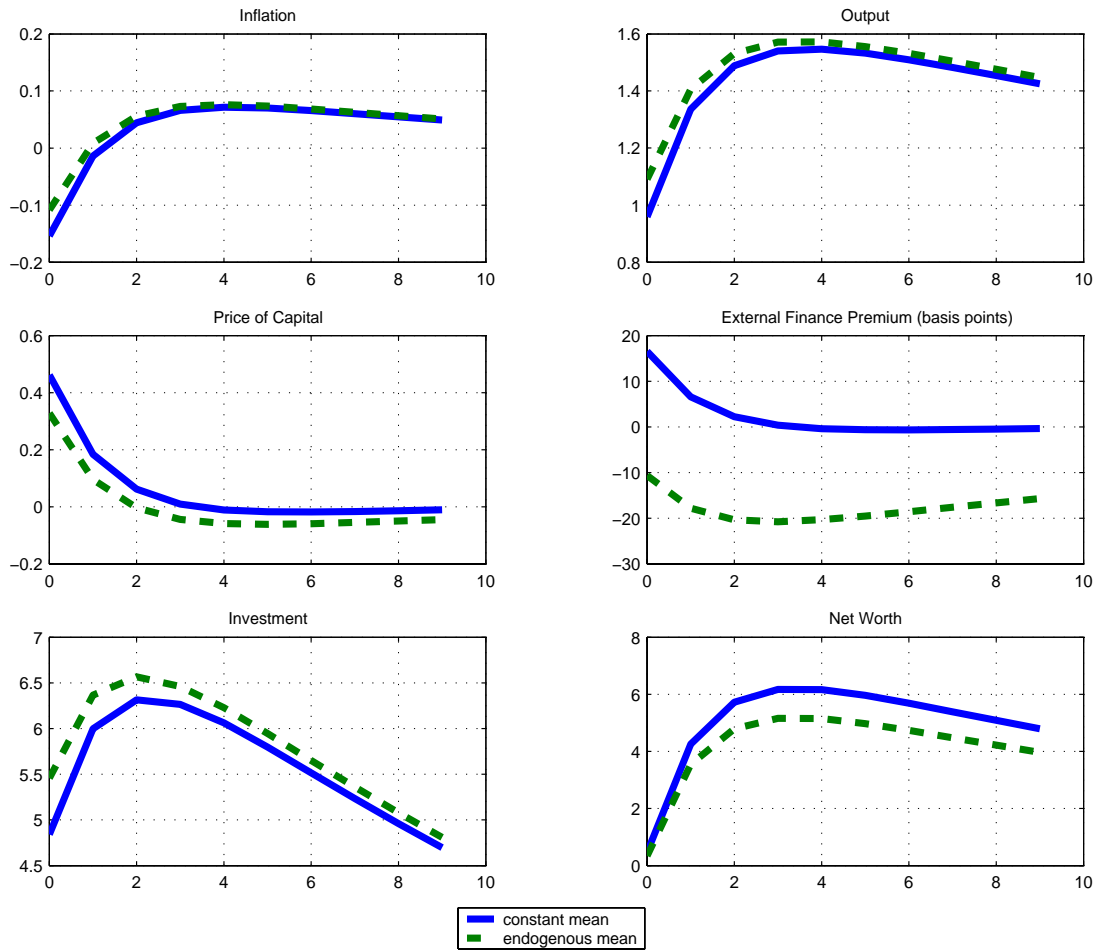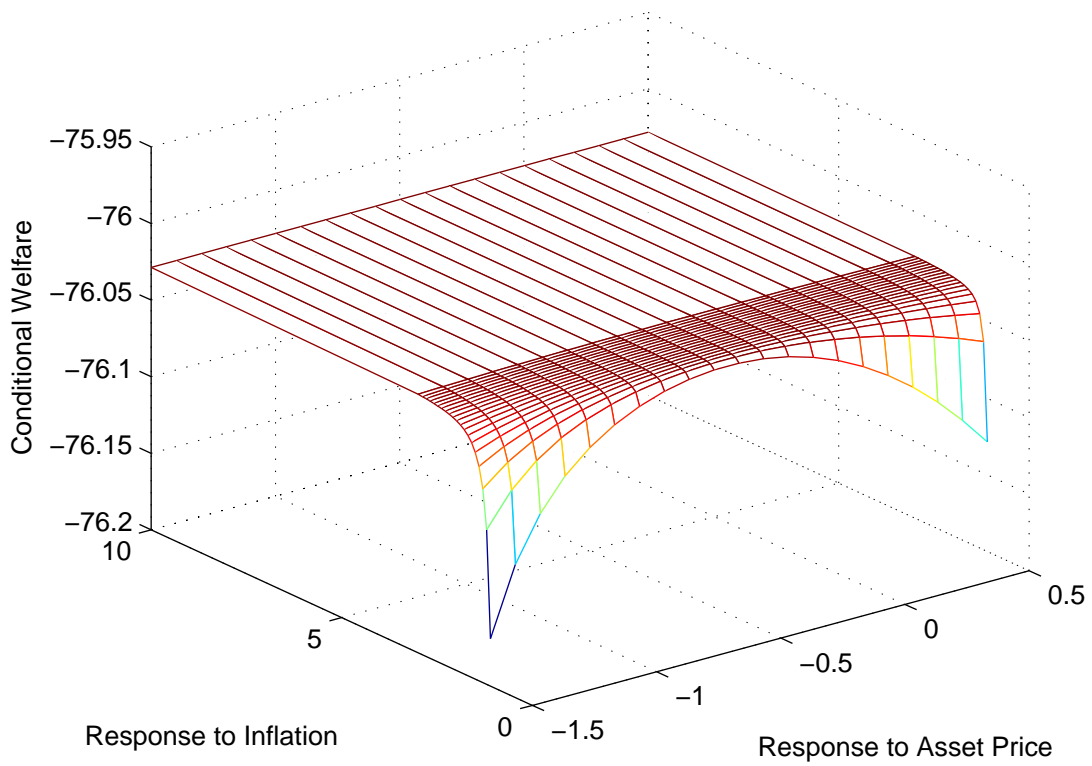
Figure 2: Impulse Responses to a Total Factor Productivity Shock: Model with $\omega_{m,t} = 1$ (solid line) vs. Model with endogenous $\omega_{m,t}$ (dashed line). Monetary policy rule: $\ln\left(\frac{1+R_t}{1+R}\right) = 1.5 \ln\left(\frac{\pi_t}{\pi}\right)$.

Figure 3: Effect on Conditional Welfare of Varying the Response to Inflation (ϕ_π) and to the Asset Price (ϕ_q). No Interest Rate Smoothing ($\phi_r = 0$).
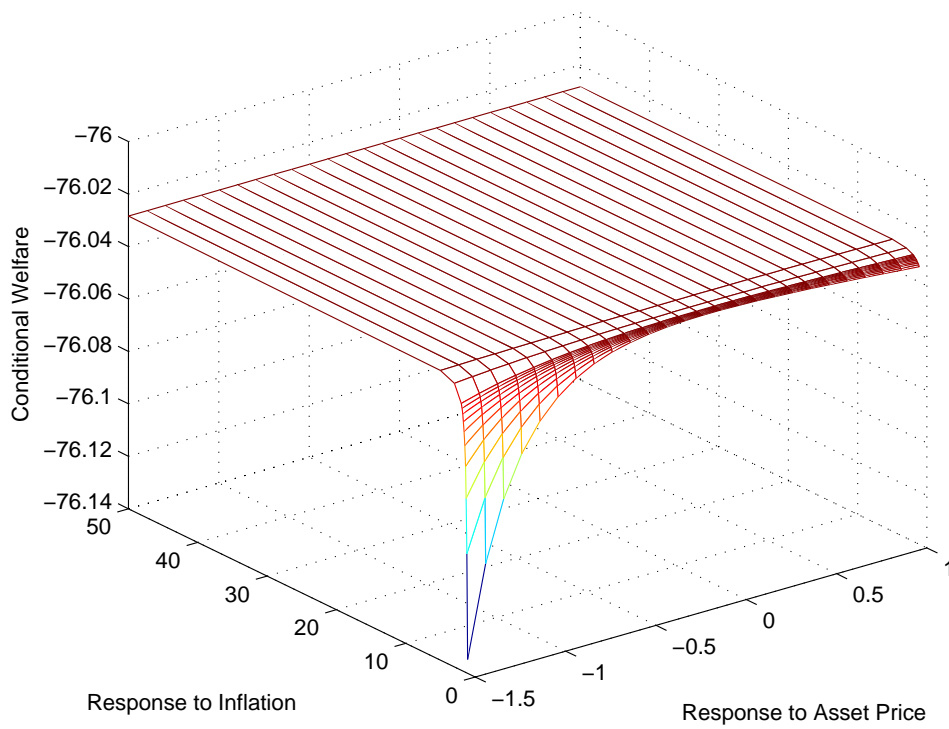
Figure 4: Effect on Conditional Welfare of Varying the Response to Inflation (ϕ_π) and to the Asset Price (ϕ_q). Interest Rate Smoothing ($\phi_r = 0.9$).
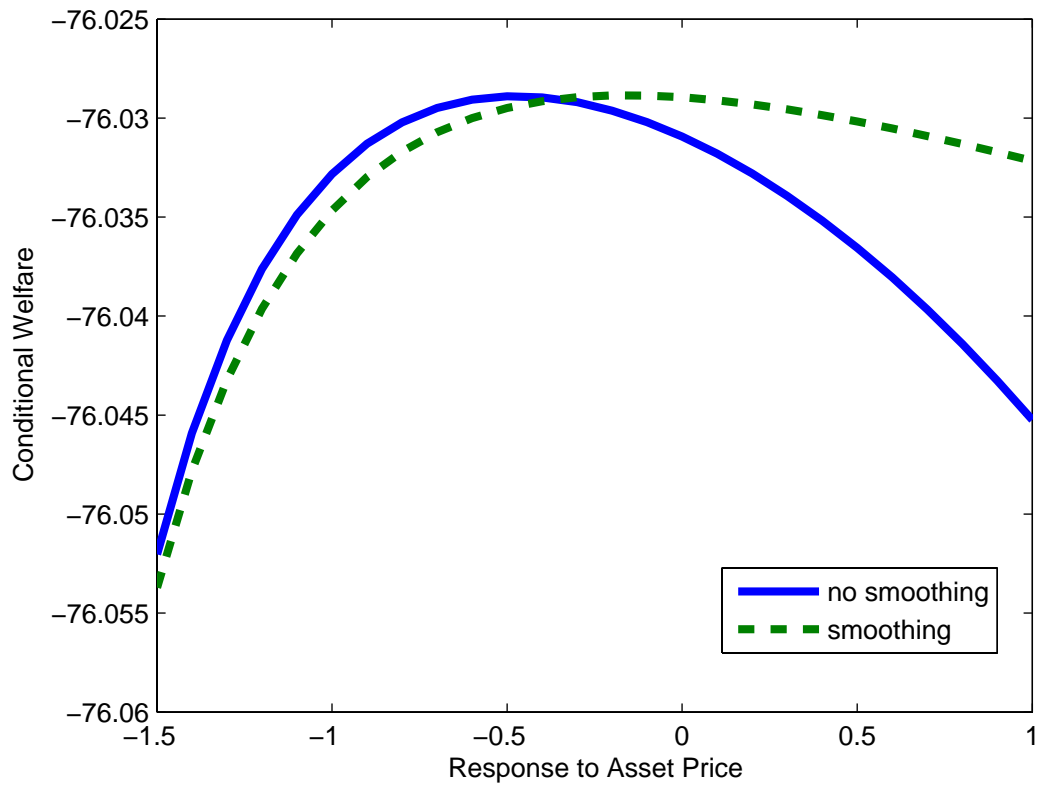
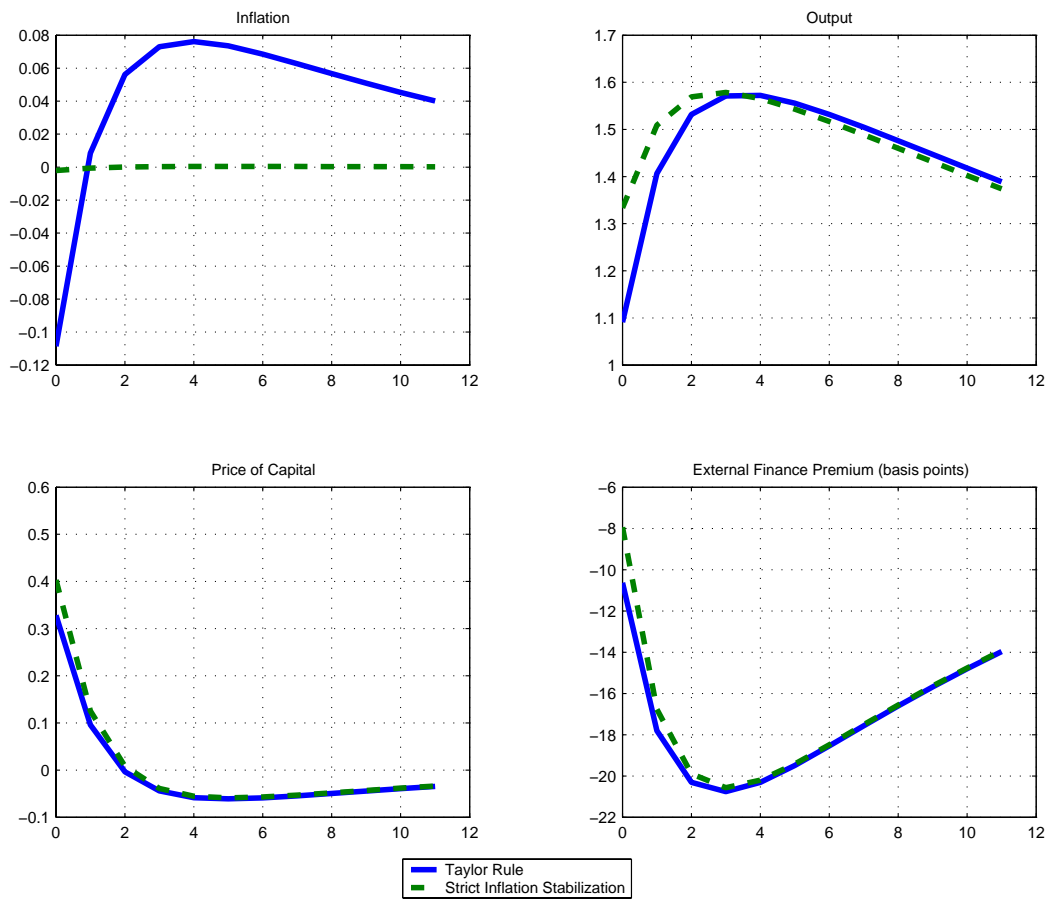Figure 5: Effect on Welfare of Responding to the Asset Price in a Standard Taylor Rule $(\phi_\pi = 1.5)$.

Figure 6: Impulse Responses to a Total Factor Productivity Shock: Taylor Rule (solid line) vs. Strict Inflation Stabilization ($\pi_t = 0$ all t, dashed line).
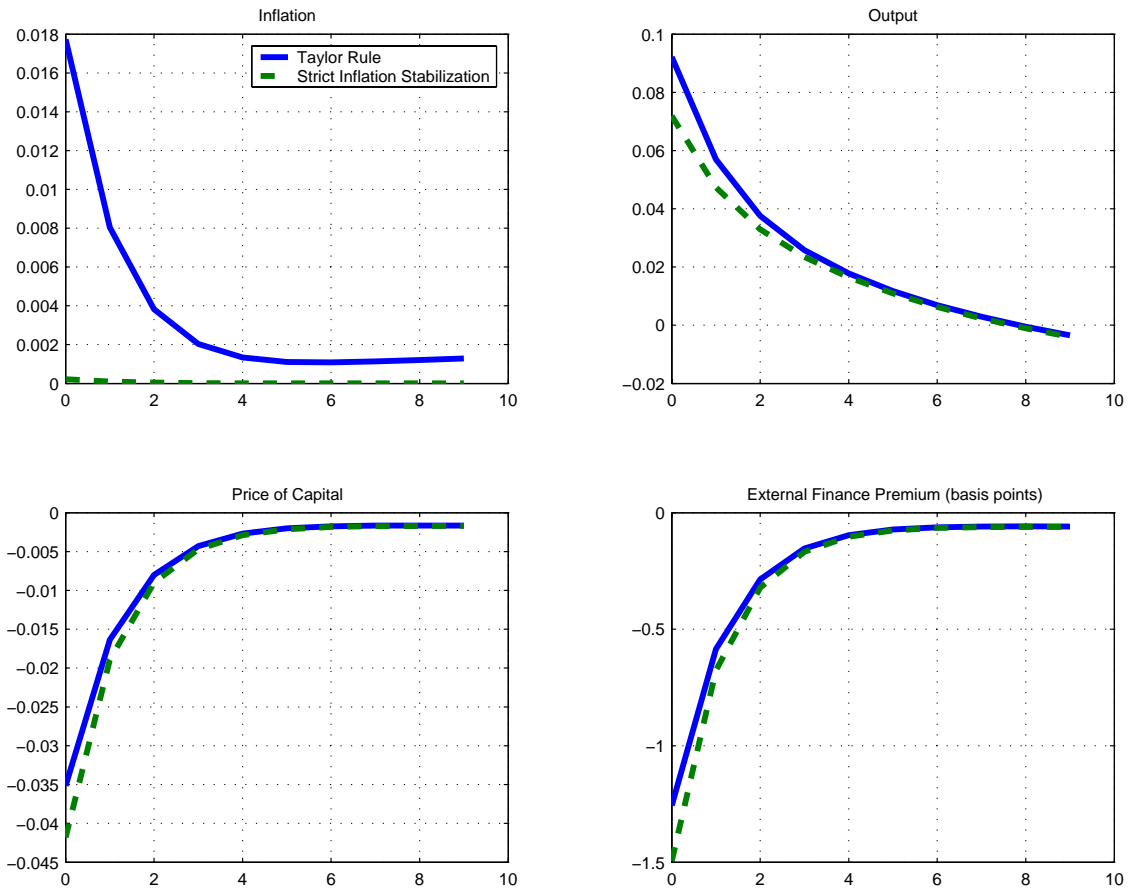
Figure 7: Impulse Responses to a Government Spending Shock: Taylor Rule (solid line) vs. Strict Inflation Stabilization ($\pi_t = 0$ all t, dashed line).