# An Experimental Investigation of Alternatives to Expected Utility Using Pricing Data 

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#### Abstract

: Experimental research on decision making under risk has until now always employed choice data in order to evaluate the empirical performance of expected utility and the alternative nonexpected utility theories. The present paper performs a similar analysis which relies on pricing data instead of choice data. Since pricing data lead in many cases to a different ordering of lotteries than choices (e.g. the preference reversal phenomenon) our analysis may have fundamental different results than preceding investigations. We elicit three different types of pricing data: willingness-to-pay, willingness-to-accept and certainty equivalents under the Becker-DeGroot-Marschak (BDM) incentive mechanism. One of our main result shows that the comparative performance of the single theories differs significantly under these three types of pricing data.


Key words: expected utility, non-expected utility, experiments, WTP, WTA, BDM. JEL-classification: C91, D81.

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## 1 Introduction

Since its axiomatization by von Neumann and Morgenstern (1944), the expected utility model has been the dominant framework for analyzing decision problems under risk and uncertainty. Starting with the well-known paradox of Allais (1953), however, a large body of experimental evidence has been gathered which indicates that individuals tend to violate the assumptions underlying the expected utility model systematically. This empirical evidence has motivated researchers to develop alternative theories of choice under risk and uncertainty able to accommodate the observed patterns of behavior. Nowadays a large number of alternative theories exist (cf. Starmer (2000), Sugden (2004), and Schmidt (2004) for surveys) and naturally the question arises which theory can accommodate observed choice behavior best.

There exist many experimental studies comparing the empirical performance of the single alternatives, most notable seem to be the investigations of Harless and Camerer (1994) and Hey and Orme (1994). All of these existing studies we are aware of use individual choice data in order to evaluate the alternatives, i.e. individuals have mostly to perform pairwise choices between lotteries or, as in Carbone and Hey (1994), (1995), and Morone (2005) a complete ranking of a set of alternatives. However, apart from choices, the preferences of a decision maker can also be assessed by pricing tasks. Moreover, the main application of utility theories is not only to analyze real-world choice behavior but also real-world pricing behavior, for instance on financial markets. Therefore, it is rather striking that neither the validity of expected utility nor the comparative performance of the single alternatives has been systematically investigated with pricing data and the present paper aims to fill this gap.

One could argue that choice and pricing tasks should in principle generate the same preference ordering for one individual and, therefore, it is irrelevant whether choice or pricing tasks are employed in the investigation. However, there is much evidence that pricing tasks yield, in general, different preferences than choice tasks for the same individual (Hey, Morone
and Schmidt (2007)). The most prominent result in this context seems to be the preference reversal phenomenon which was first observed by Lichtenstein and Slovic (1971) and afterwards extensively analyzed in the economics literature. The preference reversal phenomenon employs two lotteries, a safe and a risky one, with roughly the same expected value. The typical pattern observed is that subjects tend to choose the riskless lottery but assign a higher minimal selling price to the risky one. Thus, the preference reversal phenomenon shows clearly that choice and pricing tasks may yield completely different preference orderings. This leads to the question whether the evidence against expected utility observed with choice tasks remains valid if preferences are assessed with pricing tasks. Moreover, the question arises whether alternative theories which perform well under choice tasks have to be rejected if pricing tasks are employed or, vice versa, whether some alternative with a poor performance so far emerges as an acceptable descriptive theory for pricing data.

There exist different pricing tasks which can be employed in our analysis. The most prominent concepts are the willingness to pay (WTP), i.e. the maximal buying price for a lottery, and the willingness to accept (WTA), i.e. the minimal selling prices. The empirical literature has clearly shown that both concepts yield in general different results. More precisely, the WTA is in experimental studies in general much higher than the WTP (cf. e.g. Knetsch and Sinden (1984)). This disparity motivated us to use both concepts in our investigation. A third concept which has often been employed in order to elicit certainty equivalents for lotteries is the so called BDM mechanism (cf. Becker, DeGroot and Marschak (1963)). Although this mechanism is closely related to the WTA it may cause different responses. Therefore, we also integrated the BDM in our analysis.

Altogether, our study aims to investigate the empirical performance of expected utility and some of its alternatives by employing three different pricing tasks: WTP, WTA, and the BDM mechanism. Besides the BDM mechanism we also assessed the WTP and WTA with incentive compatible mechanisms, i.e. second-price auctions.

The experimental design will be discussed in the next section. Section 3 explains our estimation procedure and presents the five preference functionals employed in the analysis, i.e. risk neutrality, expected utility, the theory of disappointment aversion, and two variants of rank-dependent utility. Section 4 presents our results and, finally, section 5 contains a concluding discussion.

## 2 Experimental Design

The experiment was conducted at the Centre of Experimental Economics at the University of York with 24 participants. Each participant had to attend five separate occasions, A, B, C, D, and E, but occasions A and B are irrelevant for the present analysis as they involved only pairwise choices. During five days of one week one of each five different occasions was offered on every single day with varying chronological order. Consequently, 20 occasions were offered altogether and the participants could choose on which days they attended which occasions.

Each of the occasions lasted between 25 and 40 minutes. The time varied not only between the single occasions but also across the subjects since they were explicitly encouraged to proceed at their own pace. After a subject had completed all five occasions one question of one occasion was selected randomly and played out for real. The average payment to the subjects was $£ 34.17$ with $£ 80$ being the highest and $£ 0$ being the lowest payment.

On each of the five occasions the subjects were presented the same 56 lotteries presented in Table 1. The lotteries were presented as segmented circles on the computer screen. Figure 1 presents an example in which there is a $50 \%$ chance of getting $£ 10$, a $20 \%$ chance of getting $£ 30$, and a $30 \%$ chance of getting $£ 40$. If a subject received a particular lottery as reward he or she had to spin a wheel on the corresponding circle. The amount won was then determined by the segment of the circle in which the arrow on the wheel stopped.

Table 1: The Lotteries

| No. | $£ 0$ | $£ 10$ | $£ 30$ | $£ 40$ | $\mathbf{N o}$. | $£ 0$ | $£ 10$ | $£ 30$ | $£ 40$ | $\mathbf{N o .}$ | $£ 0$ | $£ 10$ | $£ 30$ | $£ 40$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\mathbf{1}$ | .000 | .000 | 1.000 | .000 | $\mathbf{2 0}$ | .000 | .200 | .700 | .100 | $\mathbf{3 9}$ | .000 | .500 | .000 | .500 |
| $\mathbf{2}$ | .750 | .000 | .250 | .000 | $\mathbf{2 1}$ | .000 | .000 | .500 | .500 | $\mathbf{4 0}$ | .500 | .250 | .000 | .250 |
| $\mathbf{3}$ | .300 | .600 | .100 | .000 | $\mathbf{2 2}$ | .500 | .000 | .500 | .000 | $\mathbf{4 1}$ | .200 | .000 | .400 | .400 |
| $\mathbf{4}$ | .000 | .600 | .100 | .300 | $\mathbf{2 3}$ | .250 | .500 | .250 | .000 | $\mathbf{4 2}$ | .100 | .000 | .200 | .700 |
| $\mathbf{5}$ | .000 | 1.000 | .000 | .000 | $\mathbf{2 4}$ | .000 | .500 | .000 | .500 | $\mathbf{4 3}$ | .800 | .000 | .000 | .200 |
| $\mathbf{6}$ | .000 | .500 | .500 | .000 | $\mathbf{2 5}$ | .500 | .250 | .000 | .250 | $\mathbf{4 4}$ | .400 | .000 | .500 | .100 |
| $\mathbf{7}$ | .500 | .500 | .000 | .000 | $\mathbf{2 6}$ | .000 | .250 | .500 | .250 | $\mathbf{4 5}$ | .400 | .000 | .000 | .600 |
| $\mathbf{8}$ | .000 | .000 | .700 | .300 | $\mathbf{2 7}$ | .000 | .000 | .750 | .250 | $\mathbf{4 6}$ | .700 | .000 | .000 | .300 |
| $\mathbf{9}$ | .800 | .000 | .140 | .060 | $\mathbf{2 8}$ | .250 | .250 | .500 | .000 | $\mathbf{4 7}$ | .200 | .000 | .000 | .800 |
| $\mathbf{1 0}$ | .200 | .000 | .740 | .060 | $\mathbf{2 9}$ | .200 | .000 | .000 | .800 | $\mathbf{4 8}$ | .200 | .000 | .400 | .400 |
| $\mathbf{1 1}$ | .000 | .200 | .800 | .000 | $\mathbf{3 0}$ | .800 | .000 | .000 | .200 | $\mathbf{4 9}$ | .100 | .000 | .000 | .900 |
| $\mathbf{1 2}$ | .500 | .100 | .400 | .000 | $\mathbf{3 1}$ | .320 | .600 | .000 | .080 | $\mathbf{5 0}$ | .600 | .000 | .000 | .400 |
| $\mathbf{1 3}$ | .000 | .200 | .600 | .200 | $\mathbf{3 2}$ | .020 | .600 | .000 | .380 | $\mathbf{5 1}$ | .300 | .500 | .000 | .200 |
| $\mathbf{1 4}$ | .000 | .100 | .300 | .600 | $\mathbf{3 3}$ | .700 | .000 | .000 | .300 | $\mathbf{5 2}$ | .200 | .200 | .000 | .600 |
| $\mathbf{1 5}$ | .200 | .800 | .000 | .000 | $\mathbf{3 4}$ | .350 | .000 | .500 | .150 | $\mathbf{5 3}$ | .600 | .100 | .000 | .300 |
| $\mathbf{1 6}$ | .100 | .400 | .500 | .000 | $\mathbf{3 5}$ | .850 | .000 | .000 | .150 | $\mathbf{5 4}$ | .000 | .350 | .000 | .650 |
| $\mathbf{1 7}$ | .000 | .400 | .600 | .000 | $\mathbf{3 6}$ | .150 | .000 | .000 | .850 | $\mathbf{5 5}$ | .000 | .100 | .250 | .650 |
| $\mathbf{1 8}$ | .500 | .200 | .300 | .000 | $\mathbf{3 7}$ | .830 | .000 | .000 | .170 | $\mathbf{5 6}$ | .250 | .350 | .000 | .400 |
| $\mathbf{1 9}$ | .000 | .200 | .300 | .500 | $\mathbf{3 8}$ | .230 | .000 | .600 | .170 |  |  |  |  |  |



Figure 1: The Presentation of Lotteries

Recall that occasions A and B are irrelevant for the present analysis. In occasions C, D, and $E$ the 56 lotteries appeared in randomized order on screen and subjects were asked for each lottery:

- to state their maximal buying price (WTP) in occasion C,
- to state their minimal selling price in occasion D , and
- to state their certainty equivalent under the BDM mechanism in occasion E .

Let us describe the single occasions more detailed now. In occasion C the following question appeared under each lottery: "Submit your bid for this lottery in a second-price sealedbid auction." That is subjects were asked to assume they did not have the lottery and had to bid to get it. They had to type in their bid and confirm it by pressing the return key. At the beginning of the experimental session subjects received a three-page instruction sheet. Then an audio-tape of these instructions was played which took approximately ten minutes. The instructions explained clearly the rules and the incentive compatibility of second-price sealed-bid auctions. If a question of occasion $C$ was selected for the reward, the subject received a payment of $£ \mathrm{y}$ where y is the highest amount in the corresponding lottery. Moreover, if the subject submitted the highest bid among all subjects in the group with whom she completed occasion C , he or she would additionally play out the lottery and had to pay the second highest bid. Occasion D was identical to occasion C except that for each lottery a different question was asked: "Submit your ask for this lottery in a second-price offer auction". That is subjects were asked to assume that they owned the lottery and had to make an offer to dispose of it. Again subjects received a handout and had to listen to an audio-tape of the three-page instructions which explained clearly the rules and the incentive compatibility of the second-price offer auction. If a question from occasion D was selected for the reward, the subject could play out the corresponding lottery. However, if he or she submitted the lowest offer among all subjects in the group with whom she completed occasion D , he or she received the second lowest offer instead of the lottery. In occasion E the following question appeared under each lottery: "State the amount of money such that you do not care whether you will receive this amount or the lottery". If a question of occasion E was chosen as reward we employed the standard BDM mechanism: A number z was
randomly drawn between zero and y where y is the highest possible prize in the given lottery. If z was greater or equal to the answer, the subject received $£ z$, otherwise she or he could play out the given lottery. Also in occasion E subjects received a handout and had to listen to an audio-tape of the instructions which clearly explained the rules and the incentive compatibility of the BDM mechanism.

Since subjects participated in the experiment at five different occasions it is important to mention that all recruited subjects had to show up for all sessions.

## 3. Estimation Procedure: Two Possible Alternatives

In order to estimate subjects' preference functional we can follow two alternative routes: the "fictitious gamble technique" (FGT) and the "interpolation technique" (IT). On a priori grounds it is difficult to say which technique is superior. In this section we will present both techniques, and their advantages and disadvantages

### 3.1. The Fictitious Gamble Technique (FGT)

FGT is an attempt to apply the estimation technique of Hey and Orme (1994) for choice data to pricing data. In FGT the information gathered in a pricing task experiment is transformed into choice data as follows. Let assume we ask for the certain equivalent (CE) of lottery $\boldsymbol{A}$ and the subject reports $C E(\boldsymbol{A})=34$. From this information FGT concludes that the subject in a pairwise choice would prefer lottery $\boldsymbol{A}$ to a lottery $\boldsymbol{B}$ which pays for sure an amount of 33 , and also would prefer a lottery $\boldsymbol{C}$ which pays for sure 35 to lottery $\boldsymbol{A}$. Obviously, FGT has some disadvantages:

Contra 1: There is a loss of information since the certain equivalent of lottery $\boldsymbol{A}$ contains more information than the observations that $\boldsymbol{A}$ is preferred to $\boldsymbol{B}$, and $\boldsymbol{C}$ is preferred to $\boldsymbol{A}$.

Solution: We can generate more synthetic lotteries than $\boldsymbol{B}$ and $\boldsymbol{C}$ in order to reduce the information loss; in principle, as the number of synthetic lotteries goes to infinity, the information loss would approach zero.

Contra 2: The derived pairwise choices are not real data and do not provide a basis for empirical analysis since people often make errors or choice not compatible with pricing behavior.

Solution: We do not advocate that the information we get from the synthetic lotteries is consistent with actual choice behavior. We just claim that the certainty equivalents provide a ranking of lotteries in the pricing task and the utility functions we estimate will reflect these ranking. If there are errors or biases in the pricing task, they will be reflected in our synthetic data.

Pro: We do not need to make any assumptions concerning the functional form of utility functions.

### 3.1.1 Some Technical Details on the FGT Estimation

The estimation of the parameters of the utility function from pairwise choice data follows the procedure adopted in Hey and Orme (1994). Let us restrict attention to expected utility and denote the two lotteries in the pairwise choice by $\boldsymbol{L}$ and $\boldsymbol{R}$. Expected utility for the lotteries is denoted by by $E U(\mathbf{L})$ and $E U(\mathbf{R})$ respectively. Then, if there is no noise or error in the subject's responses, he or she will report $\mathbf{L} \succ \boldsymbol{R}$ or $\boldsymbol{R} \succ \boldsymbol{L}$, if and only if $E U(\mathbf{L})>E U(\mathbf{R})$ or $E U(\mathbf{R})>E U(\boldsymbol{L})$, respectively. This is equivalent to saying that $L$ or $R$ is reported as preferred if and only if $C E(\boldsymbol{L})>C E(\boldsymbol{R})$ or $C E(\boldsymbol{R})>C E(\boldsymbol{L})$ respectively. However, as we know from the existing literature, subjects' responses are typically affected by noise. We assume that this noise also affects certainty equivalents. Let us denote the error in measuring the difference between the certainty equivalents by $\varepsilon$. With this error the subject will report a preference for $\boldsymbol{L}$, if and only if $C E(\boldsymbol{L})-C E(\boldsymbol{R})+\varepsilon>0$, that is, if and only if $\varepsilon>C E(\boldsymbol{R})-C E(\boldsymbol{L})$; he or she will report a preference for $\boldsymbol{R}$, if and only if $C E(\boldsymbol{L})-C E(\boldsymbol{R})+\varepsilon<0$, that is, if and only if $\varepsilon<$ $C E(\boldsymbol{R})-C E(\boldsymbol{L})$. We can now write the probability that the subject reports a preference for $\boldsymbol{L}$ as $\operatorname{Prob}\{\varepsilon>C E(\boldsymbol{R})-C E(\boldsymbol{L})\}$, and the he or she probability that the subject reports a preference for $\boldsymbol{R}$ as $\operatorname{Prob}\{\varepsilon<C E(\boldsymbol{R})-C E(\boldsymbol{L})\}$.

To proceed to the estimation of the parameters using maximum likelihood methods, we need to specify the distribution of the measurement error. We assume this to be normally distributed with mean 0 and variance $s^{2}$. The magnitude of $s$ measures the noisiness of the subject's responses: if $s=0$ then the subject makes no mistakes - as $s$ increases, the noise gets larger and larger. In the limit, when $s$ is infinite, there is no information content in the subject's responses. There is a slight complication when the subject reports indifference. Following Hey and Orme (1994) we assume that those subjects expressing indifference do so when $-\tau<C E(\boldsymbol{L})-C E(\boldsymbol{R})+\varepsilon<\tau$ where $\tau$ is some threshold. We estimate $\tau$ along with the other parameters.

### 3.2. The Interpolation Technique (IT)

The IT uses the certainty equivalents directly fror estimation. If $W$ is the preference functional and $C E(\boldsymbol{A})$ the certainty equivalent of lottery $\boldsymbol{A}$ one could simply take the equation $C E=$ $W^{I}(W(A))$ as basis for the estimation. Compared to FGT, here the problem occurs that the stated certainty equivalent will in general lie between two of the outcomes used for the estimation which makes interpolation necessary. This leads to the following disadvantages:

Contra 1: We need to prespecify a functional form for the utility function of each subject.
Solution: We could repeat the estimation using many different functional forms and choose (for each subject) the one that fit data best.

Contra 2: We need a utility function which is invertible, otherwise we cannot apply IT.
Solution: We can restrict attention to functional forms which are invertible.
Pro: We are using all the information contained in the data.

### 3.2.1 Some Technical Details on the IT Estimation

For the certainty equivalent methods, we follow the same route as above. If the subject is asked to provide his or her certainty equivalent for some gamble $\boldsymbol{A}$, we assume that the
subject calculates the expected utility of the gamble, $E U(\boldsymbol{A})$, according to his or her utility function, and then calculates the certainty equivalent $V$ - that is, certain amount of money that yields the same utility. We can now write $V=u^{-1}(E U(\mathbf{A}))$. Incorporating errors as above, we get $V=u^{-1}(E U(\mathbf{A}))+\varepsilon$. Hence, the probability density of $V$ being reported as the certainty equivalent of the gamble is given by $f\left[V-u^{-1}(E U(\mathbf{A}))\right]$, where $f($.$) is the probability density$ function of $\varepsilon$. If we now make the same assumption about the distribution of the measurement error $\varepsilon$ - namely that it is $N\left(0, s^{2}\right)$ - we can now proceed to the estimation of the parameters of the utility function.

It is evident that we need some assumptions on the utility function: it has to be invertible. We could assume that subjects have a constant relative risk aversion (CRRA) utility function and adopt the following specific form, which embodies the normalisation that $u(0)=0$ and $u(40)=1:$

$$
u(x)=(x / 40)^{r}
$$

We need to estimate only the parameter $r$ (the relative risk aversion coefficient) as it fully describes the utility function of the individual. As noted above, we assume that the standard deviation of the noise - that is, the magnitude of $s$ - is different for the different elicitation methods (i.e. choice and price), and we estimate them individually.

### 3.3. A Comparison of FGT and IT

To explore the non-trivial problem which of the two estimation techniques is superior we run a Montecarlo simulation. The simulation technique is as follows: we chose a particular utility function (i.e. $u(x)=x^{1 / 2}$ ) and calculated the preferences between the lotteries used in our experimental design as well as the certainty equivalents resulting from this utility function in the expected utility framework. Then we estimated the resulting utility values for both estimation techniques.

Table 2: The Simulation

|  | FGT |  | IT |  |
| :--- | :---: | :---: | :---: | :---: |
|  | $u(10)$ | $u(30)$ | $u(10)$ | $u(30)$ |
| Mean | 3.1206 | 5.5081 | 3.4500 | 5.5756 |
| Variance | 0.0313 | 0.0848 | 0.0030 | 0.0014 |
| Bias | 0.0017 | 0.0009 | 0.0828 | 0.0096 |
| MSE | 0.0330 | 0.0857 | 0.0858 | 0.0111 |

Table 2 reports mean and variance of the estimated utility values for both techniques. The bias of each estimator is given by the estimated utility value minus the true utility value, i.e. $x^{1 / 2}$. It turns out that IT estimators have a higher bias but a smaller variance. Comparing the mean squared error (MSE) of the two estimators we have to prefer FGT to estimate $u(10)$ and IT to estimate $u(30)$. We can, therefore, conclude that no method is strictly superior to the other.

Since in contrast to our simulation we do not know in general the right functional form of utility functions, we decided to use the FGT to estimate preference functionals. We derived from our pricing data 504 pairwise preference statements for each subject and each occasion. These preference statements are the data basis for our estimation. More precisely, we used the maximum likelihood method to estimate the parameters of the single preference functionals. The estimation was performed by a special program we wrote using the GAUSS software package ${ }^{2}$.

### 3.4. The preference functionals

Now we want to present the preference functionals used in our analysis. Let $\mathbf{x}=\left\{x_{1}, x_{2}, \ldots, x_{9}\right\}$ be the extended vector of outcomes, i.e. ( $£ 0, £ 5, £ 10, £ 15, £ 20, £ 25, £ 30, £ 35, £ 40)$. Since we used the certainty equivalents to derive pairwise preference statements our data involve always two lotteries which are represented by two probability vectors denoted by $\mathbf{p}=\left\{p_{1}, p_{2}, \ldots, p_{9}\right\}$ and $\mathbf{q}=\left\{q_{1}, q_{2}, \ldots, q_{9}\right\}$. Let $W$ denote the subject's preference functional and $V(\mathbf{p}, \mathbf{q}):=W(\mathbf{p})-W(\mathbf{q})$ the relative evaluation or net preference functional. If a particular subject actually prefers $\mathbf{p}$ to $\mathbf{q}$
then her or his net preference functional obviously will be positive. On the other hand, if she or he actually prefers $\mathbf{q}$ to $\mathbf{p}, V(\mathbf{p}, \mathbf{q})$ will be negative. Finally, we have $V(\mathbf{p}, \mathbf{q})=0$ in the case of indifference. In reality subjects' derived preferences are determined by:
$V^{*}(\mathbf{p}, \mathbf{q})=V(\mathbf{p}, \mathbf{q})+\varepsilon$,
where $\varepsilon$ is an error term. We assume that $\varepsilon$ is symmetric and has a mean of zero.
The first model we have estimated is risk neutrality (RN) given by
$\mathrm{RN}: \quad V^{*}(\mathbf{p}, \mathbf{q})=k \sum_{i=1}^{9} r_{i} x_{i}+\varepsilon$.
For RN we have to estimate only the parameter $k$ which is the relative magnitude of subjects' errors. For expected utility (EU) we have
$E U: \quad \quad V^{*}(\mathbf{p}, \mathbf{q})=\sum_{i=2}^{9} r_{i} u\left(x_{i}\right)+\varepsilon$.
For EU we estimated $u\left(x_{i}\right), i=2,3,4,5,6,7,8,9$. We normalized $u\left(x_{1}\right)$, i.e. utility of zero, to zero, and the variance of the error term to one. We did the same also for the three alternative theories presented below. Under this procedure a subject who makes relatively small errors will have relatively large values for $u\left(x_{i}\right)$ whereas a subject who makes relatively large errors will have relatively small values for $u\left(x_{i}\right)$.

The third model is the theory of disappointment aversion (DA) introduced by Gul (1991). The main psychological motivation of this theory is the hypothesis that choice behavior tries to avoid disappointment where disappointment occurs if the actual outcome of the lottery is lower than the certainty equivalent. In our framework, DA is characterized by the following equation (see also Hey and Orme (1994))
$D A: \quad V *(\mathbf{p}, \mathbf{q})=\min _{j=0,1, \ldots, 8}\left(\frac{\sum_{i=2}^{9-j} p_{i} u\left(x_{i}\right)+(1-\beta) \sum_{i=2}^{8-j} p_{i} u\left(x_{i}\right)}{1+\beta \sum_{i=1}^{8-j} p_{i}}-\frac{\sum_{i=2}^{9-j} q_{i} u\left(x_{i}\right)+(1-\beta) \sum_{i=2}^{8-j} q_{i} u\left(x_{i}\right)}{1+\beta \sum_{i=1}^{8-j} q_{i}}\right)+\varepsilon$.

[^1]For DA we estimated $u\left(x_{i}\right), i=2,3,4,5,6,7,8,9$, and $\beta$. The parameter $\beta$ is the additional parameter which determines the degree of disappointment aversion. If $\beta=0 \mathrm{DA}$ reduce to EU.

We now turn to rank-dependent utility which is nowadays the most prominent alternative to EU. Note that rank-dependent utility is in our analysis equivalent to cumulative prospect theory since our outcome set does not involve losses. As Hey and Orme (1994) we estimate two variants of rank-dependent utility, one with a power weighting function and one with the weighting function proposed by Quiggin (1982).

For rank-dependence with power function (RP) the weighting function $w$ is given by $w(r)=r^{\nu}$ and we have

RP: $\quad V^{*}(\mathbf{p}, \mathbf{q})=\sum_{j=2}^{9} u\left(x_{j}\right)\left\{\left[\left(\sum_{i=j}^{9} p_{i}\right)^{\gamma}-\left(\sum_{i=j+1}^{9} p_{i}\right)^{\gamma}\right]-\left[\left(\sum_{i=j}^{9} q_{i}\right)^{\gamma}-\left(\sum_{i=j+1}^{9} q_{i}\right)^{\gamma}\right]\right\}+\varepsilon$
We have to estimate $u\left(x_{i}\right), i=2,3,4,5,6,7,8,9$, and $\gamma$. Note that if $\gamma=1$ RP reduce to EU.

For rank-dependence with 'Quiggin` weighting function (RQ) the weighting function is given by $w(r)=r^{\eta} /\left[r^{\nu}+(1-r)^{\nu}\right]^{I / \gamma}$, which yields
$\mathrm{RQ}: \quad V^{*}(\mathbf{p}, \mathbf{q})=$


RQ reduces to EU if $\boldsymbol{\gamma}=1$. In the case of RQ we have to estimate $u\left(x_{i}\right)$ for $i=2,3,4,5,6,7,8,9$ and $\gamma$.

## 4. Results

In our analysis we can distinguish 15 different models given by the combination of the five preference functionals with the three different elicitation methods. Table A1 in the appendix is concerned with the question which model represents individual preference best and reports for all of the 24 subjects the precise ranking of the models in terms of their goodness of fit (as measured by the Akaike criterion). Since it is difficult to observe a clear structure in this table it supports the hypothesis that "people are different". In order to get a clearer picture, we calculated the average rankings ${ }^{3}$ of all 15 models in order to evaluate their performance. Table 3 lists the single models ordered according to increasing average rank. The first conclusion which emerges from this table is the fact that BDM performs rather well since the models on the first three ranks are all based on BDM. Secondly, it seems to be obvious that RN has a rather poor performance since all models with RN are on the last ranks. The third and possibly the most important conclusion from Table 3 is the fact that the performance of a preference functional depends crucially on the employed elicitation method. RQ is for instance, as for choice data analyzed in the study of Hey and Orme (1994), the best preference functional in terms of average rank. However, in the present study this is only true if RQ is combined with BDM. In contrast, combined with WTP or WTA, RQ turns out to be the worst model apart from RN. This clearly shows that there does not exist one "best" preference functional for all tasks but instead for different tasks different preference functional perform better. The last conclusion from Table 3 which is also in line with the results of Hey and Orme (1994) is the fact that EU does not seem to perform substantially worse than the alternative preference functionals.

Table 3: Average ranks of the single models

| RQ | RP | EU | DA | DA | RP | DA | EU | EU | RP | RQ | RQ | RN | RN | RN |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| BDM | BDM | BDM | WTP | BDM | WTP | WTA | WTP | WTA | WTA | WTP | WTA | BDM | WTA | WTP |

[^2]Since the performance of the single preference functionals depends on the employed elicitation method we analyzed in Table 4 each elicitation method separately. More precisely, the first row of Table 4 reports the average rank of each preference functional if we elicit preferences using WTP; the second row reports the average rank of each preference functional if we elicit preferences using WTA and the last row refers to BDM.

Table 4: Ranking of the preference functionals

|  | RN | EU | DA | RQ | RP |
| :---: | :---: | :---: | :---: | :---: | :---: |
| WTP | 3.958 | 2.592 | 2.230 | 2.417 | 2.833 |
| WTA | 4.292 | 2.792 | 2.417 | 2.625 | 2.667 |
| BDM | 4.417 | 2.708 | 2.917 | 2.458 | 1.833 |

Table 4 show that RN is for all elicitation methods the worst preference functional in terms of average ranks. For WTP and WTA DA turns out to be best while it performs rather poorly under BDM where RP turns out to be best. Since in DA a reference point plays a prominent role the bad performance of DA under BDM may possibly due to the fact that the reference point receives less attention under BDM as compared to WTP and WTA.

The fact that RP is the best preference functional under BDM but performs rather poorly under WTP reinforces our conclusion from above that the performance of the single preference functionals depends crucially on the elicitation method. Altogether, Table 4 also show that EU does not perform substantially worse than its alternatives.

Finally we are interested in the question which elicitation method is best for the single preference functionals. Corresponding information is provided in Table 5. For instance the first row of Table 5 reports the average rank of each elicitation method for RN. Subsequent rows contain the same information for EU, DA, RQ, and RP respectively.

Table 5: Ranking of the elicitation methods

|  | WTP | WTA | BDM |
| :--- | :--- | :--- | :--- |
| RN | 2.417 | 2.042 | 1.417 |
| EU | 1.958 | 2.125 | 1.708 |
| DA | 1.875 | 2.042 | 1.833 |
| RQ | 1.958 | 2.042 | 1.792 |
| PR | 1.958 | 2.125 | 1.958 |

It turns out that BDM is always the best elicitation method both in terms of average rank (and also in terms of number of subjects for which a given elicitation method is best). Additionally, according to these two criteria, WTP is except for RN always better than WTA. The latter result may have some conclusions for the contingent valuation method. Until now it is an open question whether contingent valuation surveys should rely on WTP or WTA. Some authors have argued in favor of WTP since WTA usually decreases during experiments whereas WTP remains relatively constant during the single rounds. Our results seem to provide additional support for WTP.

## 5 Conclusions

In this paper we have analyzed the empirical performance of several preference functionals. The main difference with existing studies in the literature is the fact that we used pricing data instead of choice data. Our main results can be summarized as follows:

- The performance of the single preference functionals depends crucially on the elicitation method.
- EU does not perform substantially worse than its alternatives
- DA turns out to be the best preference functional under WTP and WTA while RP is best under BDM.
- BDM seems to be the best elicitation method and WTA the worst.


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## Appendix

Table A1: Overall rankings

| Rank | I | II | III | IV | V | VI | VII | VIII | IX | X | XI | XII | XIII | XIV | XV |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Subject 1 | RQ-BDM | EU-BDM | RP-BDM | DA-BDM | RN-BDM | DA-WTP | RP-WTP | EU-WTP | RQ-WTP | RP-WTA | DA-WTA | RQ-WTA | EU-WTA | RN-WTP | RN-WTA |
| Subject 2 | RP-BDM | EU-WTA | RQ-BDM | RQ-WTA | RP-WTA | DA-WTA | EU-BDM | DA-BDM | RP-WTP | DA-WTP | RN-BDM | RQ-WTP | EU-WTP | RN-WTA | RN-WTP |
| Subject 3 | EU-WTA | EU-BDM | DA-BDM | RP-WTA | RP-BDM | RQ-WTA | RQ-BDM | DA-WTA | RN-WTA | RN-BDM | RN-WTP | EU-WTP | RP-WTP | RQ-WTP | DA-WTP |
| Subject 4 | EU-BDM | RQ-BDM | EU-WTA | RQ-WTA | RP-BDM | DA-BDM | DA-WTA | RP-WTA | RN-BDM | RN-WTA | DA-WTP | RP-WTP | RQ-WTP | EU-WTP | RN-WTP |
| Subject 5 | RQ-WTA | RP-WTA | EU-WTA | DA-WTA | RQ-BDM | RP-BDM | EU-BDM | DA-WTP | DA-BDM | RP-WTP | RQ-WTP | EU-WTP | RN-BDM | RN-WTA | RN-WTP |
| Subject 6 | RP-WTA | DA-WTA | RQ-WTA | EU-WTA | RP-WTP | DA-WTP | RQ-WTP | EU-WTP | RP-BDM | DA-BDM | RQ-BDM | EU-BDM | RN-BDM | RN-WTA | RN-WTP |
| Subject 7 | DA-BDM | RP-BDM | RQ-BDM | EU-BDM | RN-BDM | DA-WTA | RN-WTA | RN-WTP | RP-WTA | RQ-WTP | EU-WTP | EU-WTA | DA-WTP | RQ-WTA | RP-WTP |
| Subject 8 | EU-WTA | RQ-WTA | DA-WTA | RP-WTA | RQ-WTP | RP-WTP | EU-WTP | DA-WTP | RN-WTP | RN-WTA | DA-BDM | RP-BDM | RQ-BDM | EU-BDM | RN-BDM |
| Subject 9 | RN-WTP | DA-WTP | EU-WTP | RQ-WTP | RP-WTP | DA-WTA | EU-WTA | RQ-WTA | RP-WTA | RN-WTA | DA-BDM | RQ-BDM | RP-BDM | EU-BDM | RN-BDM |
| Subject 10 | RQ-BDM | EU-BDM | RP-BDM | DA-BDM | RN-BDM | RP-WTA | EU-WTA | DA-WTP | DA-WTA | RP-WTP | RQ-WTA | RN-WTA | EU-WTP | RQ-WTP | RN-WTP |
| Subject 11 | DA-WTP | RP-WTP | RQ-BDM | EU-WTP | RQ-WTP | DA-WTA | RP-WTA | EU-BDM | RP-BDM | DA-BDM | RQ-WTA | EU-WTA | RN-BDM | RN-WTP | RN-WTA |
| Subject 12 | DA-WTP | RQ-WTP | EU-WTP | RP-WTP | RQ-BDM | EU-BDM | DA-BDM | RP-BDM | RQ-WTA | EU-WTA | DA-WTA | RP-WTA | RN-WTA | RN-WTP | RN-BDM |
| Subject 13 | EU-WTP | DA-WTP | RP-WTP | RQ-WTP | RQ-BDM | EU-BDM | RP-BDM | DA-BDM | RN-BDM | DA-WTA | RP-WTA | RQ-WTA | EU-WTA | RN-WTA | RN-WTP |
| Subject 14 | EU-WTP | RP-WTP | RQ-WTP | DA-WTP | DA-WTA | RP-WTA | EU-WTA | RQ-WTA | RP-BDM | RQ-BDM | EU-BDM | DA-BDM | RN-WTA | RN-WTP | RN-BDM |
| Subject 15 | RQ-WTA | RP-WTA | DA-WTA | EU-WTA | RP-WTP | RQ-WTP | DA-WTP | EU-WTP | RQ-BDM | RP-BDM | DA-BDM | EU-BDM | RN-BDM | RN-WTA | RN-WTP |
| Subject 16 | RP-BDM | DA-BDM | RQ-BDM | RP-WTP | EU-BDM | DA-WTP | RQ-WTP | EU-WTP | DA-WTA | RP-WTA | EU-WTA | RQ-WTA | RN-BDM | RN-WTA | RN-WTP |
| Subject 17 | DA-BDM | RQ-BDM | EU-BDM | RP-BDM | RN-BDM | RQ-WTA | RP-WTA | DA-WTA | EU-WTA | RN-WTA | EU-WTP | RQ-WTP | RP-WTP | RN-WTP | DA-WTP |
| Subject 18 | EU-BDM | RQ-BDM | DA-BDM | RP-BDM | RN-BDM | RP-WTP | DA-WTP | RQ-WTP | RQ-WTA | EU-WTP | RP-WTA | EU-WTA | DA-WTA | RN-WTA | RN-WTP |
| Subject 19 | RQ-BDM | EU-BDM | RP-BDM | DA-BDM | RQ-WTP | EU-WTP | RP-WTP | DA-WTP | RN-BDM | RP-WTA | RQ-WTA | EU-WTA | DA-WTA | RN-WTP | RN-WTA |
| Subject 20 | RN-WTP | RN-BDM | RN-WTA | EU-WTP | EU-WTA | EU-BDM | DA-WTP | DA-WTA | DA-BDM | RP-WTP | RP-WTA | RP-BDM | RQ-WTP | RQ-WTA | RQ-BDM |
| Subject 21 | RN-BDM | EU-BDM | DA-BDM | RP-BDM | RQ-BDM | RN-WTA | EU-WTA | RP-WTA | DA-WTA | RQ-WTA | RP-WTP | DA-WTP | EU-WTP | RQ-WTP | RN-WTP |
| Subject 22 | RN-WTP | RN-BDM | EU-WTP | EU-BDM | DA-WTP | DA-WTA | DA-BDM | RP-WTP | RP-BDM | RQ-WTP | RQ-WTA | RQ-BDM | RN-WTA | EU-WTA | RP-WTA |
| Subject 23 | RN-WTP | RQ-WTP | EU-WTP | RP-WTP | DA-WTP | DA-WTA | EU-WTA | RP-WTA | RN-WTA | RQ-WTA | RP-BDM | RQ-BDM | RN-BDM | EU-BDM | DA-BDM |
| Subject 24 | RP-WTP | DA-WTP | RQ-WTP | RQ-BDM | DA-BDM | EU-BDM | RP-BDM | EU-WTP | RN-BDM | DA-WTA | RN-WTA | RQ-WTA | EU-WTA | RP-WTA | RN-WTP |


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[^1]:    ${ }^{2}$ Our estimation program is available upon request.

[^2]:    ${ }^{3}$ When we calculated the average rankings two models got the same rank if they performed identical. If for example two models have the highest Akaike criterion they both get the first rank and the next model gets rank three. For this reason the average of the average ranks may differ from the rank average.

