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Technology Choice and International Trade

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Gabriela Schmidt

Abstract:

This paper develops two extensions of the dynamic model presented in Melitz (2003). The first extension consists in the introduction of technology choice between three alternative production technologies: L, M and H. L is assumed to be the same as Melitz's single production technology, while M and H are assumed to be superior production technologies, stemming this superiority from the fact these technologies substitute the more primitive capital goods used in technology L with newer, updated versions which embody technological advances, and also from the fact that M and H are more skill-intensive than L. Technologies M and H are equally skill-intensive, but H still is superior to M because it incorporates world-technology-frontier capital goods, while the capital goods used in M are below such frontier. The second extension consists in the introduction of two different exporting profiles: "Low-Commitment Exporters" –who make the minimum possible investment required to penetrate export markets- and "High-Commitment Exporters" –who are ready to make additional trade-related investments to gain additional export sales. In order to preserve the General Equilibrium approach, an intermediate educational sector is introduced following an approach similar to that of Kugler and Verhoogen (forthcoming).

The present model shares Melitz's result that when the economy opens up to trade increased competition reallocates market shares towards firms with higher idiosyncratic productivity, forcing the least productive ones out of the market, with the consequence that the productivity threshold to enter the industry rises and therefore so does average productivity "at the factory gate". However, other factors influencing static welfare –variety, additional trade-related fixed investments, transport costs and the impact of technology choice on average variable production cost- lead to an ambiguous outcome, which is another difference with the Melitz (2003) model, in which static welfare undoubtedly increased. The present model also allows for the evaluation of the impact of trade on dynamic welfare –which increases as the absorption of new technical progress increases-, but yields an indefinite result in this respect unless additional assumptions are made regarding the distribution of idiosyncratic productivities and the value of parameters.

It is worth noting that despite the model presented in this paper does not require that the superior technologies employ imported capital goods –it is only required that the quality of the capital goods they employ be superior-, if we think of it in terms of the empirics the interpretation that the country of origin of the intermediate capital goods is determinant of their quality comes naturally –being this higher the shorter the distance between the technology frontier belonging to the country where the capital good in question was produced and the world-technology-frontier. This highlights the crucial influence that a country's choice of its trading partners may exert over the productivity it will be able to achieve, and consequently over its growth trajectory.

Keywords: technology choice – heterogeneous firms – export profiles – embodied technology– monopolistic competition

JEL classification: O14, O33

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1. Introduction

Recent empirical studies on the subjects of growth and convergence have reached the conclusion that nowadays most of the world's technical progress originates in a handful of rich leading countries: the United States, Germany, France, Japan and Great Britain[1]. Thus, for the great majority of countries foreign sources of technology account for most of the increase in domestic productivity[2], being its contribution estimated in 90% of the total increase or more[3]. All this reveals that the path of technical change at world-wide level is determined to a great extent by international technology diffusion.

However, despite technical progress diffuses rapidly from the countries where it originates towards the rest of the world through different channels[4], it is not immediately absorbed. This observation has given rise to an abundant literature which intends to explain what causes the different levels and speeds of absorption of technical progress shown by potential receptors, both at the macroeconomic and microeconomic levels. This literature has coined the term "barriers to technology adoption" to reflect the concept that technology transfer is a process which takes place between a source and a receptor and whose intensity and speed are determined not only by the transmissibility of technological progress itself, but also by the characteristics of both the source and the receptor of the technology flow. These barriers can be modeled in many ways and taking different approaches, but nevertheless the basic underlying idea remains that the potential receptor must possess certain characteristics to be in grade of taking advantage from such innovations.

Getting to individualize which are these key characteristics is a crucial step towards identifying the specific mechanisms through which international technology transfer actually occurs and towards uncovering the reasons why different potential receptors benefit so unevenly from it. In particular, one of the most important questions posed is why are there so many low and medium income countries in the world which are permanently lagging in technology, while others satisfactorily follow up the rhythm of expansion of the world-technology-frontier and remain as a consequence on the technology lead, even if they are getting most of their technological progress from foreign sources[5].

Technology adoption barriers can be modeled in many different ways. One option which has received considerable attention in the literature relates this concept with a problem which may emerge when it is allowed that different alternative production technologies coexist. The basic idea behind this approach is that the technology choice compatible with achieving static efficiency –such that resources are efficiently allocated in the short run– may not coincide with the technology choice that would lead to dynamic efficiency –the highest achievable growth–. As a consequence of this, short run objectives would be in conflict with long run objectives, causing an inefficient equilibrium to materialize. This suboptimal situation would tend to worsen as time passes and the world-technology-frontier continues its expansion, widening the gap between itself and the technologically lagging country. This hypothesis has been explored in the literature both from macroeconomic –country level studies- and microeconomic –firm level studies- perspectives and some contributions include Caselli and Coleman (2000), Basu and Weil (1998), Restuccia (2004) and Zeira (1998).

[1] Eaton and Kortum (1995b). Keller (2001) also include Italy and Canada in this group, so long they are counted among the seven major R&D producers in the world.
[2] Keller (2000, 2004), Eaton and Kortum (1995a, 1995b, 1997), among others.
[3] Keller (2004), Eaton and Kortum (1995b).
[4] One of the most important technology diffusion channels mentioned in the literature is international trade, particularly trade in capital goods, whose relevance has been confirmed by various empirical studies, such as Eaton and Kortum (1997, 2001) and Coe, Helpman and Hoffmaister (1995). The underlying idea when considering trade as a vehicle for the transmission of technical progress is that goods embody the best technology available in the place and time of their production, which in turn determines that goods produced in countries whose national technology frontier is closer to the world-technology-frontier be superior in quality (compared to goods produced in countries whose national technology frontier is farther from the world-technology-frontier). When referring to intermediate capital goods in particular, superior quality is usually interpreted as greater efficiency when used in production.
[5] Eaton and Kortum (1995a, 1995b) find that, except for the United States, all OECD countries derive almost all of their productivity growth from foreign sources of technical progress. According to their estimations, even the United States, which is the world's main R&D producer, derives more than 40% of its productivity growth from innovations occurred abroad.

The monopolistic competition model presented in Melitz (2003) –which will be used as a departing point for the construction of the present model-, is not itself a model "of appropriate technology" as it assumes the existence of a single production technology common to all firms. Nevertheless, it provides an innovative element to approach the concept of barriers to technology adoption: the idiosyncratic productivity of every firm. In Melitz's model all firms use the same production technology but are heterogeneous in terms of their idiosyncratic productivity, being this heterogeneity modeled through an exogenous productivity distribution, of which every individual idiosyncratic productivity parameter is a realization. As a consequence of this heterogeneity, both in the closed as well as in the open economy settings, the model reaches an equilibrium in which differently productive firms obtain different results, corresponding in both cases higher market shares and profits to more productive firms. The concept of heterogeneity in the idiosyncratic productivity taken from Melitz (2003) will be key to modeling the choice between alternative production technologies and the barriers to the absorption of technical progress in the present model. It will also be central to modeling firms' market strategy choice.

An important thing to consider in more detail on reaching this point is the role of human capital and the specific ways in which it is incorporated into models with technology choice and barriers to technology adoption. The idea of new technologies being complementary with human capital has been extensively explored in the literature[6], and has had an impact on the approach to thinking and modeling human capital in the present context, growing farther from the quantification and analysis of the impact of human capital in itself –attempting to isolate it-, and paying increasing attention to the interaction between human capital and technical progress. Bustos (2005) incorporates this concept into a model of monopolistic competition with firms that are heterogeneous in their idiosyncratic productivities à la Melitz. In her model firms must choose between two alternative production technologies: a modern technology –which uses updated capital goods and is intensive in skilled labor- and a traditional technology –which uses more primitive capital goods and is intensive in unskilled labor-. Firms also have to decide whether to export or serve only the domestic market. Working with data from Argentinean industrial firms during the trade and capital account liberalization process undertaken by the country in the early 1990s, she finds empirical support for the hypothesis that technical change is skewed towards technologies that are complementary with human capital.

2. Basic Assumptions of the Model[7]

There are two sectors in the economy: a final goods sector and an intermediate educational sector.

2.1 Educational Sector

In the context of the model, education is conceived as an intermediate good, which is produced in Perfect Competition to be used as an input in the production of the final good. The producers of this intermediate good are the unskilled workers, in response to the relative demand for skilled and unskilled labor from the final good producers. It is assumed that all workers are equally capable for studying, and that this activity does not yield the worker any additional utility or disutility, beyond income gain and loss.

The education production function, which transforms unskilled labor into skilled labor, is:

$$F_s(u,c) = \frac{u}{c} \qquad (1)$$

where u is the number of unskilled labor hours devoted to studying that are required to produce an hour of skilled labor, and $c>1$ represents the level of difficulty of the learning process. Thus, in order to produce an hour of skilled labor with skilled level c , c hours of unskilled labor are required.

[6] Bartel and Source (1987), Acemoglu (1998, 2003) and Krusell et. al. (2000) among others.
[7] This model builds on Melitz (2003) and also incorporates some key elements from Bustos (2005).

Let $w_{s,c}(c)$ be the price of an hour of labor with skill level **c**. Assuming the producers of the final good are price takers in the skilled labor market, in equilibrium its price equals its marginal production cost:

$$c = w_s$$

For simplicity, the existence of a single level of attainable qualification is assumed, so that the division of labor according to its quality reduces to two categories (skilled and unskilled). This allows to simplify notation by defining $w_{s,c}(c) = c$.

2.2 Final Goods Sector

Demand

It is assumed that the demand side is characterized by a representative consumer with CES preferences over a continuum of varieties of good q:

$$U = \left[\int_0^N q(i)^\rho di\right]^{\frac{1}{\rho}} \qquad (2)$$

where N is the number of available varieties indexed by i. These varieties of good q are substitutes, implying $0 < \rho < 1$ and an elasticity of substitution between any two goods of $\sigma = \frac{1}{(1-\rho)} > 1$.

Using the well known derivation by Dixit and Stiglitz (1977), the set of available varieties is modeled as an aggregate good $Q \equiv U$ whose aggregate price is:

$$P = \left[\int_0^N p(i)^{1-\sigma} di\right]^{\frac{1}{1-\sigma}} \qquad (3)$$

As usual, consumers maximize their utility subject to the budget constraint:

$$\int_0^N p(i)q(i)di = E \qquad (4)$$

where $p(i)$ is the price of variety i and E is total expenditure in good q. This process yields the demand for each variety:

$$q(i) = \frac{E}{P}\left(\frac{p(i)}{P}\right)^{-\sigma} \qquad (5)$$

Optimal consumption and expenditure decisions for individual varieties are then given by:

$$q(i) = Q\left(\frac{p(i)}{P}\right)^{-\sigma} \qquad (6)$$

$$r(i) = R\left(\frac{p(i)}{P}\right)^{1-\sigma} \qquad (7)$$

where $R = PQ = \int_0^N r(i)di$ denotes aggregate expenditure.

Production

The market structure is Monopolistic Competition, with a free entry condition and à la Melitz (2003) heterogeneous firms[8], each of whom produce a different variety. Technology choice is modeled following the approach in Bustos (2005), but here the set of alternatives is extended to include three

[8] Melitz's heterogeneity can be interpreted either in terms of "quantitative differences" –producing a symmetric variety at lower marginal cost- or in terms of "qualitative differences" –producing a higher quality variety at equal cost-.

distinct options, with the purpose of making it possible to evaluate the impact of different-quality technologies while holding skill-intensity constant. Each technology features a constant marginal cost (c), which reflects the payments to two types of labor used in fixed proportions, skilled (S) and unskilled (U), and a fixed cost (f), which in turn reflects the cost of the machinery needed for production.

Entering production with technology L involves the lowest fixed cost because it implies the usage of an inferior (older, more primitive) technology, embodied in machines that are assumed to be therefore cheaper than the higher-quality machines used in the other two technologies, M and H. On the other hand, using technology L implies facing a higher marginal cost than those corresponding to technologies M and H. The reason is that technology L employs the highest proportion of unskilled labor, which despite earning a lower salary than skilled labor, is also less productive, which brings about the overall result of a higher marginal cost in this technology relative to technologies M and H.

Adopting technology M involves facing a fixed cost which is higher than that needed to begin production with technology L, but lower than the fixed cost needed to acquire technology H, as the machines it uses for production embody an intermediate-quality technology, not as primitive as the one employed in L, nor as advanced as the one required by H[9]. Technology M is more skill-intensive than technology L, which means that it has a lower associated marginal cost.

Finally, adopting technology H implies paying the highest fixed cost, as this technology employs machines which embody world-technology-frontier technology, which are assumed to be the most efficient and expensive. Regarding the marginal cost, technology H features a novelty: despite having the same degree of skill-intensity as technology M, it still achieves a lower marginal cost due to a "productivity enhancing effect" brought about by the superiority of the machines used. Put in other words, equally skilled workers exhibit higher efficiency when working with H-type machines (compared to M-type machines), because these allow for a more efficient use of labor in general.

With a minor modification, Bustos (2005) Total Cost function allows to accommodate the three above outlined technologies:

$$TC_T(\varphi) = f_T + c_T \frac{q}{\varphi} \quad , \qquad T = L, M, H \qquad (8)$$[10]

where:

- $\varphi > 0$ indexes idyosincratic productivity
- $f_L < f_M < f_H$
- $c_L > c_M > c_H$
- $c_L = a_{LU} + \frac{w_s}{w_u} a_{LS}$
  where w_u is the salary paid to unskilled workers and w_s is the salary paid to skilled workers.
- $c_M = a_{MU} + \frac{w_s}{w_u} a_{MS}$
  being $\frac{a_{MS}}{a_{MU}} > \frac{a_{LS}}{a_{LU}}$ as technology M is more skill-intensive than technology L.
- $c_H = \left[a_{HU} + \frac{w_s}{w_u} a_{HS}\right] = \left[a_{MU} + \frac{w_s}{w_u} a_{MS}\right](1 - \alpha_H)$
  because $a_{HU} = a_{MU}$ and $a_{HS} = a_{MS}$. That is, technologies M and H are equally skill-intensive, remaining the only difference between them the "productivity enhancing effect" $0 < \alpha_H < 1$, which stems from the superior quality of the technology embodied in H-type machines and has an

[9] The underlying idea is that technology M employs updated capital goods, but coming from countries which are below the world-technology-frontier.

[10] The Total Cost function could be alternatively reformulated as $TC_T(\varphi) = f_T + \frac{q}{\varphi(1+\lambda_T)}$, where $\lambda_T \geq 0$ represents the proportion in which the quality of the inputs used in technology T enhances idiosyncratic productivity and thus contributes to an increase in overall productivity (and to a decrease in total cost). Normalizing $c_L = 1$ and $\lambda_L = \frac{1}{c_L} - 1 = 0$ (meaning that for firms using technology L overall productivity equals idiosyncratic productivity), we have $\lambda_L < \lambda_M < \lambda_H$ because $c_H < c_M < c_L$.

homogeneous enhancing effect on the productivity of both skilled and unskilled labor[11]. This results in c_H being lower than c_M. Normalizing $w_u = 1$ like in Melitz yields $\frac{w_s}{w_u} = w_s$.

3. Closed Economy Setting

The assumption of CES preferences leads to all firms facing residual demand curves with constant elasticity σ and consequently choosing the same profit maximizing constant markup $\frac{\boldsymbol{\sigma}}{\boldsymbol{\sigma-1}} = \frac{\mathbf{1}}{\boldsymbol{\rho}}$ over marginal cost. This results in the following exprssions for technology-specific price, quantity sold, revenue and profits[12]:

$$p_d^T(\varphi) = \frac{1}{\rho}\frac{c_T}{\varphi} \tag{9}$$

$$q_d^T(\varphi) = EP^{\sigma-1}\left(\rho\frac{\varphi}{c_T}\right)^{\sigma} = EP^{\sigma-1}\left(\frac{1}{p_d^T(\varphi)}\right)^{\sigma} \tag{10}$$

$$r_d^T(\varphi) = p_d^T(\varphi)q_d^T(\varphi) = E\left(P\rho\frac{\varphi}{c_T}\right)^{\sigma-1} \tag{11}$$

$$\pi_d^T(\varphi) = \frac{1}{\sigma}r_d^T(\varphi) - f_T = \frac{1}{\sigma}\,E\left(P\rho\frac{\varphi}{c_T}\right)^{\sigma-1} - f_T \tag{12}$$

It is important to note that the ratios of any two firms' outputs and revenues depend both on their respective idiosyncratic productivities and on the production technology used by each:

$$\frac{q_d^T(\varphi_1)}{q_d^{T'}(\varphi_2)} = \left(\frac{\frac{\varphi_1}{c_T}}{\frac{\varphi_2}{c_{T'}}}\right)^{\sigma} \quad \text{and} \quad \frac{r_d^T(\varphi_1)}{r_d^{T'}(\varphi_2)} = \left(\frac{\frac{\varphi_1}{c_T}}{\frac{\varphi_2}{c_{T'}}}\right)^{\sigma-1} \tag{13}$$

The following conclusions are readily obtained from the above expression:

- For any two firms producing with the same technology ($T = T'$), the most productive one will charge a lower price, consequently achieve larger output and revenues, and earn higher profits than the other, less productive firm[13].
- For any two equally productive firms ($\varphi_1 = \varphi_2$), the one using a superior technology will charge a lower price and thus achieve larger output and revenues than the other, less productive firm.

3.1 Entry, Exit and Technology Upgrading in the Closed Economy

Prospective entrants to the industry are required to make an initial fixed investment $f_e^T > 0$ in order to learn what their idiosyncratic productivity is. This is modeled like in Melitz (2003) as firms drawing an exogenous productivity parameter φ from a common distribution $g(\varphi)$ which has positive support over $(0, \infty)$ and has a continuous cumulative distribution $G(\varphi)$. Entry sunk costs are technology specific, that is, a firm must pay f_e^T to begin production with technology T, being $f_e^L < f_e^M < f_e^H$. However, firms do not know their productivity parameter by the time they have to pay the entry cost, and so they do not know which of these entry costs will correspond to them, not even if they will successfully enter any of the three available technologies at all. Nevertheless, even though they do have to pay "an" entry cost in order to be able to learn what their idiosyncratic productivity is, they are allowed to pay "any" entry cost, so long when they finally discover which technology they will

[11] If the "productivity enhancing effect" is zero then there is no difference between technologies M and H and thus the model reduces to Bustos' case, with only two distinct technologies. If the "productivity enhancing effect" is one then then it is so strong that drives the variable cost in technology H to zero.
[12] The subindex "d" stands for "domestic market".
[13] This conclusion is shared with Melitz (2003).

effectively enter, they will pay up the amount needed to cover any resulting cost gap –if such technology happens to have a higher entry cost than that already paid by the firm. Thus, the rational thing to do on the part of any prospective entrant is to sink the lowest possible entry cost (f_e^L), because if it does not enter the industry, it suffers the minimum possible loss, and at the same time, if it becomes a successful entrant, it does not risk ending up paying a greater entry cost than necessary[14]. Unsuccessful entrants are those whose idiosyncratic productivity is too low to be compatible with their making nonnegative profits, and consequently they will immediately exit without ever producing. On the other hand, successful entrants will face from the moment they start production a probability δ in every period of being hit with a bad shock which will put them out of the market, which is constant across productivity levels. Summing up, a firm will either exit immediately upon entry or otherwise produce and earn $\pi_d^T(\varphi) > 0$ in every period until it is hit with the bad shock and forced out of the market, which yields the following firm's value function:

$$v_d^T(\varphi) = max\left\{0\,,\sum_{t=0}^{\infty}(1-\delta_T)^t\,\pi_d^T(\varphi)\right\} = max\left\{0\,,\frac{1}{\delta}\pi_d^T(\varphi)\right\} \quad (14)$$

The threshold $\varphi_T^{**} = inf\{\varphi : v_d^T(\varphi) > 0\}$ identifies the lowest idiosyncratic productivity level (Melitz's "zero cutoff productivity level") firms need to have in order to make nonnegative profits when producing with technology T. It is also possible to define some $\varphi_T^* \geq \varphi_T^{**}$ which stands for the minimum idiosyncratic productivity level for which it is convenient for the firm (in terms of profitability) to use technology T. Therefore, φ_T^* is the lowest idiosyncratic productivity level of firms actually using technology T (the "effective cutoff productivity level"). Note that for T=L we have $\varphi_T^* = \varphi_T^{**}$. Because the only exit process affecting the equilibrium idiosyncratic productivity distribution $\mu(\varphi)$ is that occurring immediately upon entry, such distribution $\mu(\varphi)$ is just the original distribution $g(\varphi)$ conditional on successful entry:

$$\mu(\varphi) = \begin{cases} \frac{g(\varphi)}{1-G(\varphi_L^*)}\, if\ \varphi \geq \varphi_L^* \\ 0 \qquad if\ \varphi < \varphi_L^* \end{cases} \quad (15)$$

The ex ante probability of successful entry to the industry –that is, entry into technology L "or superior"[15]- is denoted $p_{in}^{L+M+H} \equiv 1 - G(\varphi_L^*)$ and defines the industry average idiosyncratic productivity level $\tilde{\varphi} = \tilde{\varphi}^L$ (the mean idiosyncratic productivity of all producing firms, no matter if they are using technology L, M or H) as a function of the cutoff level φ_L^* :

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[14] For example, if the firm decided to sink f_e^M, it not only risks a greater loss if it never successfully enters the industry at all, but also risks paying ($f_e^M - f_e^L$) extra if enters technology L. The same reasoning is valid (and intensified) if the firm decided to sink f_e^H. Therefore, if the firm decides to take the chance to try to enter the industry, it will always sink f_e^L. However, the "real" entry cost it will pay if it turns to be a successful entrant is $f_e = f_e^L + p_{in}^T(f_e^T - f_e^L)$ because once the firm has entered the market and drawn its productivity parameter from the distribution $g(\varphi)$, it will *immediately* decide one course of action out of four:

- Exit immediately and never produce (if $\varphi < \varphi_L^*$), in which case it loses the amount f_e^L it had already paid.
- Start production with technology L (if $\varphi_L^* < \varphi < \varphi_M^*$), in which case the fixed (entry) cost remains the already paid amount f_e^L.
- Switch immediately to technology M (if $\varphi_M^* < \varphi < \varphi_H^*$), in which case the fixed (entry) cost escalates from f_e^L to f_e^M (the firm must add up to its initial payment the amount $f_e^M - f_e^L$).
- Switch immediately to technology H (if $\varphi > \varphi_M^*$), in which case the fixed (entry) cost escalates from f_e^L to f_e^H(the firm must add up to its initial payment the amount $f_e^H - f_e^L$).

[15] This stems from the assumption that technology L is equivalent to the single production technology specified in Melitz (2003). This means that this model preserves that technological option, while it introduces two additional options, technologies M and H, which are "more advanced" and require higher entry productivity levels. As a consequence of this, a firm which is unable to produce profitably using technology L, will be equally unable to produce profitably using technologies M or H as well, while the converse is not true: a firm that can produce profitably using technology H will also be able to produce profitably using either technology M or L, and a firm that can produce profitably using technology M will also be able to produce profitably using technology L, while we cannot assure that it will be equally able to produce profitably using technology H (it may or may not be able to produce profitably using technology H).
Therefore, we can affirm that $p_{in}^{L+M+H} \equiv 1 - G(\varphi_L^*)$ is the probability of successful entry into production with one of the available production technologies: either L or M or H, and thus it is the probability of successful entry to the industry. This p_{in}^{L+M+H} is conceptually equivalent to the probability of successful entry in Melitz (2003).

$$\tilde{\varphi} = \tilde{\varphi}^L(\varphi_L^*) = \left[\frac{1}{1-G(\varphi_L^*)}\int_{\varphi_L^*}^{\infty} \varphi^{\sigma-1} g(\varphi) d\varphi\right]^{\frac{1}{\sigma-1}} \quad (16)^{16}$$

Analogously, it is possible to determine the mean idiosyncratic productivity of the group of firms using technology "M or superior" (that is, M or H), denoted by $\tilde{\varphi}^M$, as a function of the threshold for the adoption of technology M, φ_M^*, as well as the mean idiosyncratic productivity of the group of firms using technology "H or superior" (that is, H), denoted by $\tilde{\varphi}^H$, as a function of the threshold for the adoption of technology H, φ_H^*:

$$\tilde{\varphi}^M(\varphi_M^*) = \left[\frac{1}{1-G(\varphi_L^*)}\int_{\varphi_M^*}^{\infty} \varphi^{\sigma-1} g(\varphi) d\varphi\right]^{\frac{1}{\sigma-1}} \quad (17)$$

and

$$\tilde{\varphi}^H(\varphi_H^*) = \left[\frac{1}{1-G(\varphi_L^*)}\int_{\varphi_H^*}^{\infty} \varphi^{\sigma-1} g(\varphi) d\varphi\right]^{\frac{1}{\sigma-1}} \quad (18)$$

Calculation of the average idiosyncratic productivity level corresponding to each technology T requires taking into account both the threshold for the profitability of the adoption of such technology as well as the threshold for the profitability of the adoption of the immediately superior technology:

$$\tilde{\varphi}_L(\varphi_L^*, \varphi_M^*) = \left[\frac{1}{1-G(\varphi_L^*)}\int_{\varphi_L^*}^{\varphi_M^*} \varphi^{\sigma-1} g(\varphi) d\varphi\right]^{\frac{1}{\sigma-1}} \quad (19)$$

$$\tilde{\varphi}_M(\varphi_M^*, \varphi_H^*) = \left[\frac{1}{1-G(\varphi_L^*)}\int_{\varphi_M^*}^{\varphi_H^*} \varphi^{\sigma-1} g(\varphi) d\varphi\right]^{\frac{1}{\sigma-1}} \quad (20)$$

$$\tilde{\varphi}_H(\varphi_H^*) = \tilde{\varphi}^H(\varphi_H^*) = \left[\frac{1}{1-G(\varphi_L^*)}\int_{\varphi_H^*}^{\infty} \varphi^{\sigma-1} g(\varphi) d\varphi\right]^{\frac{1}{\sigma-1}} \quad (21)$$

The ex ante probabilities of successful and profitable entry to technologies L, M and H are respectively $p_{in}^L \equiv G(\varphi_M^*) - G(\varphi_L^*)$, $p_{in}^M \equiv G(\varphi_H^*) - G(\varphi_M^*)$ and $p_{in}^H \equiv 1 - G(\varphi_H^*)$.[17]

In order for the thresholds for adopting each of the three available production technologies to lie in the desired order ($\varphi_L^* < \varphi_M^* < \varphi_H^*$), it is required that the gain obtained by a firm with a given productivity level φ when switching to a superior technology be smaller "in proportion" to the increase in the fixed cost it simultaneously faces. That is, the neat gain a firm experiences when switching from M to H must be smaller "in proportion" to the gain it experiences when switching from L to M (see Appendix A).

Since each of these average idiosyncratic productivity levels $\tilde{\varphi}_T$ is completely determined by the minimum idiosyncratic productivity level for which it is convenient for the firm (in terms of profitability) to use technology T (φ_T^*) and that corresponding to the profitability of adoption of the

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[16] Exactly as it happens in the Melitz (2003) model, the assumption of a finite $\tilde{\varphi}$ imposes certain restrictions on the size of the upper tail of the distribution $g(\varphi)$: the $(\sigma-1)$th uncentered moment of the upper tail must be finite.

[17] Note that $\tilde{\varphi}_T(\varphi_T^*, \varphi_{T+1}^*) = \tilde{\varphi}^T(\varphi_T^*)$ only when T=H, while the same is not true for technologies L and M. The reason is that H is the best technology available (there is not a technology "superior" to H, that is, when T=H there is not a T+1 technology, and consequently we assume in that case $G(\varphi_{T+1}^*) = 1$, meaning we have reached the upper tail of the productivity distribution. Also note that these ex-ante probabilities of successful entry to each technology (p_{in}^T, $T = L, M, H$) are calculated directly from the original distribution $g(\varphi)$ instead of the modified distribution $\mu(\varphi)$ because at this stage (attempt of entry to the industry) all firms (including those who will eventually not succeed) are taken into account. Thus, for the sake of calculating these probabilities the relevant distribution is $g(\varphi)$, not $\mu(\varphi)$.

immediate superior technology (φ_{T+1}^{*})[18], then the average profit and revenue levels corresponding to each production technology are also linked to these thresholds:

$$\bar{r}_d^T = r_d^T(\tilde{\varphi}_T) = \left(\frac{\tilde{\varphi}_T(\varphi_T^*,\varphi_{T+1}^*)}{\varphi_T^*}\right)^{\sigma-1} r_d^T(\varphi_T^*) \qquad T = L, M, H \tag{22}$$

$$\bar{\pi}_d^T = \pi_d^T(\tilde{\varphi}_T) = \left(\frac{\tilde{\varphi}_T(\varphi_T^*,\varphi_{T+1}^*)}{\varphi_T^*}\right)^{\sigma-1} \frac{r_d^T(\varphi_T^*)}{\sigma} - f_T \qquad T = L, M, H \tag{23}$$ [19]

But these thresholds φ_T^* are in turn linked to the cutoff idiosyncratic productivity levels corresponding to technology T (φ_T^{**}):

$$\varphi_T^* = \begin{cases} \varphi_T^{**} & if\ T = L \\ \varphi_T^{**} + \varepsilon & if\ T = M, H \end{cases} \qquad \varepsilon > 0$$

This allows a generalization of a central relation in Melitz (2003) to also hold in the technology choice framework, namely that the "Effective Cutoff overall Profit condition for technology T" (ECP^T) – which in the autarky setting coincides with the "Effective Cutoff domestic Profit condition for technology T" (ECP_d^T)- pins down the revenue gained by each technology's effective cutoff firm[20] and consequently implies a relationship between the average profit per firm using technology T and the cutoff idiosyncratic productivity level for the adoption of technology T:

$$\pi_d^T(\varphi_T^*) = A_d^T \quad \leftrightarrow \quad r_d^T(\varphi_T^*) = \sigma\left(A_d^T + f_T\right) \quad \leftrightarrow \quad \bar{\pi}_d^T = A_d^T h_d^T(\varphi_T^*) + f_T k_d^T(\varphi_T^*) \tag{24}$$

where $h_d^T = \left[\frac{\tilde{\varphi}_T(\varphi_T^*,\varphi_{T+1}^*)}{\varphi_T^*}\right]^{\sigma-1}$, $k_d^T(\varphi_T^*) = \left[\frac{\tilde{\varphi}_T(\varphi_T^*,\varphi_{T+1}^*)}{\varphi_T^*}\right]^{\sigma-1} - 1$ and $A_d^T > 0$ is the profit gained by the least idiosyncratically productive firm using technology T (a constant).[21] Note that when T=L, then $\varphi_T^* = \varphi_T^{**}$ and so $\pi_d^T(\varphi_T^*) = 0$ (that is: $A_d^T = 0$). Consequently in such case $r_d^T(\varphi_T^*) = \sigma f_T$ and $\bar{\pi}_d^T = f_T k_d^T(\varphi_T^*)$.

The other Melitz (2003) central relation, the "Free Entry Condition" is also generalized for the technology choice framework. Even though its formulation is more complex now because of the existence of alternative production technologies, its essence does not change: the expected value of profits in the market must be zero.

The expected value of the profits a firm will earn if it successfully enters the market is a weighted average of the average profits it would obtain in each technology T, with the probability of entering each of the different available technologies –conditional to having successfully entered the market-acting as weights.

The present value of the average profit flow of the firm using technology T in the closed economy conditional on successful entry is also the average value of firms using technology T in the closed economy, and is given by:

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[18] If T=L then T+1=M, if T=M then T+1=H, if T=H there is no T+1 technology available (H is the best available technology).

[19] For easier derivation of equation (22) recall equation (11): $\pi_d^T(\varphi) = \frac{1}{\sigma} r_d^T(\varphi) - f_T$.

[20] The "effective cutoff firm" for technology T is the least idiosyncratically productive firm actually using technology T.

[21] Since $\varphi_T^* > \varphi_T^{**}$ and $\pi_d^T(\varphi_T^{**}) = 0$, then we know $\pi_d^T(\varphi_T^{**}) > 0$. We already know as well that $\bar{\pi}_d^T = \pi_d^T(\tilde{\varphi}_T) = \left(\frac{\tilde{\varphi}_T(\varphi_T^*,\varphi_{T+1}^*)}{\varphi_T^*}\right)^{\sigma-1} \frac{r_d^T(\varphi_T^*)}{\sigma} - f_T$. Replacing $r_d^T(\varphi_T^*) = \sigma\left(A_d^T + f_T\right)$ in this equation we obtain $\bar{\pi}_d^T = \pi_d^T(\tilde{\varphi}_T) = A_d^T \left(\frac{\tilde{\varphi}_T(\varphi_T^*,\varphi_{T+1}^*)}{\varphi_T^*}\right)^{\sigma-1} + f_T \left(\frac{\tilde{\varphi}_T(\varphi_T^*,\varphi_{T+1}^*)}{\varphi_T^*}\right)^{\sigma-1} - f_T =$
$= A_d^T \left(\frac{\tilde{\varphi}_T(\varphi_T^*,\varphi_{T+1}^*)}{\varphi_T^*}\right)^{\sigma-1} + f_T \left[\left(\frac{\tilde{\varphi}_T(\varphi_T^*,\varphi_{T+1}^*)}{\varphi_T^*}\right)^{\sigma-1} - 1\right] = A_d^T h_d^T(\varphi_T^*) + f_T k^T(\varphi_T^*)$.

$$\bar{v}_d^T = \sum_{t=0}^{\infty}(1-\delta)^t\, \bar{\pi}_d^T = \frac{1}{\delta}\bar{\pi}_d^T \quad (25)$$

The net value of entry to the market is then the average profit the firm will earn if it successfully enters the market (the expression between braces) multiplied by the probability of being a successful entrant:

$$v_e = p_{in}^{L+M+H}\left\{\frac{p_{in}^L}{p_{in}^{L+M+H}}\bar{v}_d^L + \frac{p_{in}^M}{p_{in}^{L+M+H}}\left(\bar{v}_d^M - \left(f_e^M - f_e^L\right)\right) + \frac{p_{in}^H}{p_{in}^{L+M+H}}\left(\bar{v}_d^H - \left(f_e^H - f_e^L\right)\right)\right\} - f_e^L \quad (26)$$

If this value were negative, no firm would ever enter the market. If it were positive, entry would be infinite. Consequently, the Free Entry Condition requires that $v_e = 0$.

After entering the market, firms whose idiosyncratic productivity is high enough to adopt technologies M ($\varphi \geq \varphi_M^*$) or H ($\varphi \geq \varphi_H^*$) will only do so if this upgrading allows them to obtain larger profits than producing with technology L, after paying up the difference in the entry cost. Taking into account that profits in each technology T are increasing in φ, the two "Technology Upgrading Conditions" (for M and H, respectively) are given by:

$$\pi_d^M\left(\varphi_M^*\right) - \pi_d^L\left(\varphi_M^*\right) = f_e^M - f_e^L \quad \text{(UP1)}$$

$$\pi_d^H\left(\varphi_H^*\right) - \pi_d^L\left(\varphi_H^*\right) = f_e^H - f_e^L \quad \text{(UP2)}$$

These two conditions, which did not appear in Melitz (2003), are represented in Figure 1. The profits of the individual firm in each technology T for each idiosyncratic productivity level φ are in turn represented in Figure 2.

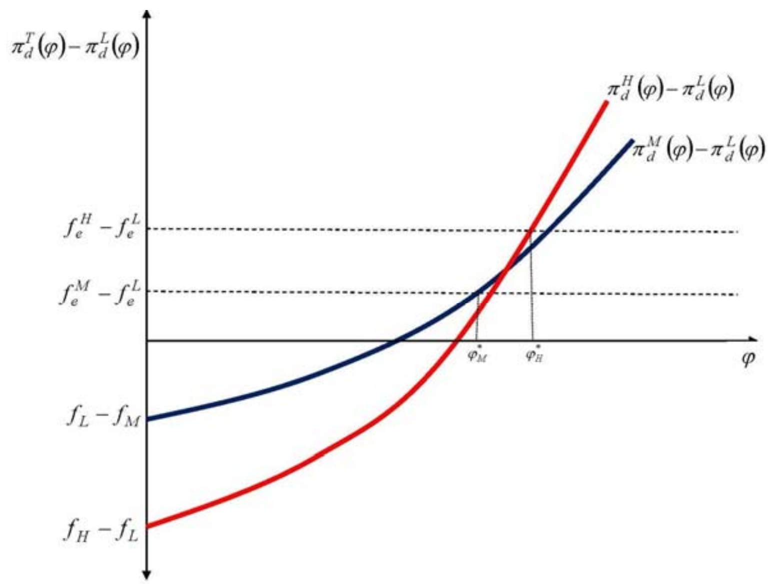

Figure 1

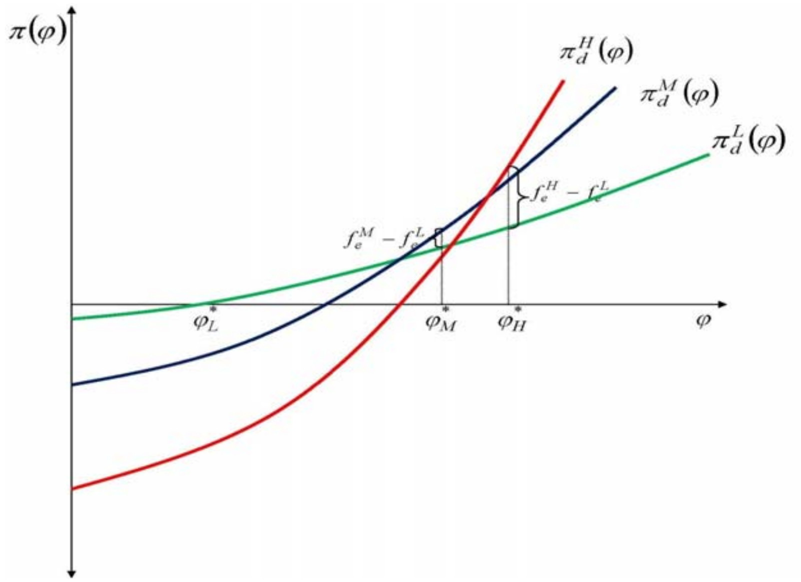

Figure 2

3.2 Aggregation Conditions in the Closed Economy

Assuming the industry is comprised of Z firms, there will be a proportion of $\frac{L}{Z}$ firms using technology L, a proportion of $\frac{M}{Z}$ using technology M and a proportion of $\frac{H}{Z}$ firms using technology H. $L+M+H=Z$, meaning all incumbent firms in the industry must use one –and one only- of the three available production technologies. Industry average idiosyncratic productivity $\tilde{\varphi}$ is a weighted average of the firms' idiosyncratic productivity levels –with firms' output shares as weights- and is independent both from total firm population Z and from the proportions of firms using each of the three available technologies, though it will be useful to disaggregate the integral into three smaller ones, according to the thresholds for upgrading technology[22]:

$$\tilde{\varphi}^{\sigma-1} = \int_0^\infty \varphi^{\sigma-1}\mu(\varphi)d\varphi = \frac{L}{Z}\int_{\varphi_L^*}^{\varphi_M^*} \varphi^{\sigma-1}\mu(\varphi)d\varphi + \frac{M}{Z}\int_{\varphi_M^*}^{\varphi_H^*} \varphi^{\sigma-1}\mu(\varphi)d\varphi + \frac{H}{Z}\int_{\varphi_H^*}^{\infty} \varphi^{\sigma-1}\mu(\varphi)d\varphi =$$

$$= \frac{L}{Z}\tilde{\varphi}_L{}^{\sigma-1} + \frac{M}{Z}\tilde{\varphi}_M{}^{\sigma-1} + \frac{H}{Z}\tilde{\varphi}_H{}^{\sigma-1} \quad (27)$$

where φ_L^* is the idiosyncratic productivity threshold upon which it becomes profitable for the firm to have positive production with technology L, φ_M^* is the idiosyncratic productivity threshold upon which it becomes profitable for the firm to drop technology L and adopt technology M instead, and φ_H^* is the idiosyncratic productivity threshold upon which it becomes profitable for the firm to drop technology M and adopt technology H instead.

In such autarkic equilibrium, the aggregate price, quantity, revenue and profits can be re expressed as (proof in Appendix B):

$$P = \left[Lp_d^L(\tilde{\varphi}_L)^{1-\sigma} + Mp_d^M(\tilde{\varphi}_M)^{1-\sigma} + Hp_d^H(\tilde{\varphi}_H)^{1-\sigma}\right]^{\frac{1}{1-\sigma}} \quad (28)$$

$$Q = \left[Lq_d^L(\tilde{\varphi}_L)^{\rho} + Mq_d^M(\tilde{\varphi}_M)^{\rho} + Hq_d^H(\tilde{\varphi}_H)^{\rho}\right]^{\frac{1}{\rho}} \quad (29)$$

$$R = Lr_d^L(\tilde{\varphi}_L) + Mr_d^M(\tilde{\varphi}_M) + Hr_d^H(\tilde{\varphi}_H) = R_L + R_M + R_H \quad (30)$$

[22] Recall that knowing these proportions is not a necessary condition for the calculation of $\tilde{\varphi}$, which had already been obtained solely from $g(\varphi)$ in combination with φ_L^*.

$$\Pi = L\pi_d^L(\tilde{\varphi}_L) + M\pi_d^M(\tilde{\varphi}_M) + H\pi_d^H(\tilde{\varphi}_H) = \Pi_L + \Pi_M + \Pi_H \tag{31}$$

The average price, quantity, revenue and profits in this industry in the closed economy setting are obtained as a weighted average of the price, quantity, revenue and profits of each technological group:[23]

$$\bar{p}_d = \frac{P}{Z^{\frac{1}{1-\sigma}}} = \left[\frac{L}{Z}p_d^L(\tilde{\varphi}_L)^{1-\sigma} + \frac{M}{Z}p_d^M(\tilde{\varphi}_M)^{1-\sigma} + \frac{H}{Z}p_d^H(\tilde{\varphi}_H)^{1-\sigma}\right]^{\frac{1}{1-\sigma}} \tag{32}$$

$$\bar{q}_d = \frac{Q}{Z^{\frac{1}{\rho}}} = \left[\frac{L}{Z}q_d^L(\tilde{\varphi}_L)^{\rho} + \frac{M}{Z}q_d^M(\tilde{\varphi}_M)^{\rho} + \frac{H}{Z}q_d^H(\tilde{\varphi}_H)^{\rho}\right]^{\frac{1}{\rho}} \tag{33}$$

$$\bar{r}_d = \frac{R}{Z} = \frac{L}{Z}r_d^L(\tilde{\varphi}_L) + \frac{M}{Z}r_d^M(\tilde{\varphi}_M) + \frac{H}{Z}r_d^H(\tilde{\varphi}_H) \tag{34}$$

$$\bar{\pi}_d = \frac{\Pi}{Z} = \frac{L}{Z}\pi_d^L(\tilde{\varphi}_L) + \frac{M}{Z}\pi_d^M(\tilde{\varphi}_M) + \frac{H}{Z}\pi_d^H(\tilde{\varphi}_H) \tag{35}$$

3.3 Determination of the Equilibrium in the Closed Economy

The three "Effective Cutoff Profit Conditions" (ECP^T) can be substituted into the equation of the average value of firms using technology T, which in turn enter the "Free Entry Condition" (FE). This last condition is therefore expressed as a function of the idiosyncratic productivity thresholds a firm needs to reach to effectively enter each technology T:

$$\boxed{p_{in}^L\bar{v}_d^L\left(\varphi_L^*, \varphi_M^*\right) + p_{in}^M\left[\left(\bar{v}_d^M\left(\varphi_M^*, \varphi_H^*\right) + f_e^L - f_e^M\right)\right] + p_{in}^H\left[\left(\bar{v}_d^H\left(\varphi_H^*\right) + f_e^L - f_e^H\right)\right] - f_e^L = 0} \tag{36}$$

Equation (36) toghether with the two "Technology Upgrading Conditions" (UP1 and UP2) form a system with three equations and three unknowns, whose resolution yields market equilibrium. However, it is not possible to determine in this general formulation whether the system reaches an equilibrium (or multiple equilibria). In order to answer that question, a particularization of the problem is required, by making additional assumptions regarding the shape of the productivities distribution $g(\varphi)$.

3.4 Analysis of the Equilibrium in the Closed Economy

In case an equilibrium exists, as we are focusing in steady-state equilibriums, not only the number of successful new entrants $p_{in}^{L+M+H}Z_e$ –where Z_e is the total number of new entrants- must exactly replace the δZ firms who are hit by a bad shock and exit (this is the Aggregate Stability Condition: $p_{in}^{L+M+H}Z_e = \delta Z$), but furthermore, the number of successful new entrants into each technological group needs to exactly replace the number of failing incumbents amongst the same technological group, so that all aggregate variables remain constant over time:

$$p_{in}^L Z_e = \delta L \leftrightarrow [G(\varphi_M^*) - G(\varphi_L^*)]Z_e = \delta L \tag{37}$$

$$p_{in}^M Z_e = \delta M \leftrightarrow [G(\varphi_H^*) - G(\varphi_M^*)]Z_e = \delta M \tag{38}$$

$$p_{in}^H Z_e = \delta H \leftrightarrow [1 - G(\varphi_H^*)]Z_e = \delta H \tag{39}$$

where $p_{in}^L Z_e + p_{in}^M Z_e + p_{in}^H Z_e = p_{in}^{L+M+H}Z_e$ and $\delta L + \delta M + \delta H = \delta Z$.

Because both the successful entrants and failing incumbents draw their idiosyncratic productivity from the same exogenous distribution $g(\varphi)$, and besides the probability of suffering the negative shock that

[23] These "theoretical" averages are calculated just in order to provide a rough measure of the industry's "per firm performance". They need not coincide with the price, quantity, revenue and profits of any particular firm.

will put the firm out of the market is the same for all active firms (whichever their idiosyncratic productivity and production technology), neither the equilibrium distribution $\mu(\varphi)$ nor the proportions of firms belonging to each technological group are affected by this firms' turnover.

In each technology, a fraction of total labor employed –skilled and unskilled- is used for production purposes –by all active firms- and the rest is used for "setting up the business" –by new entrants-:

$$(U_T + S_T) = (U_T + S_T)_p + (U_T + S_T)_e \qquad T = L, M, H \tag{40}$$

where $(U_T + S_T)_p$ and $(U_T + S_T)_e$ represent, respectively, the aggregate labor used in production technology T for regular production (by all incumbents producing with T) and setting up the bussiness (by new entrants to technology T).

Because the unskilled labor wages are normalized to 1, the payments received by such workers coincide with the number of hours of unskilled labor used in production. On the other hand, the educational sector establishes a fixed correspondence between both types of labor and their wages, depending on how many hours of unskilled labor are required to produce an hour of skilled labor.[24] Finally, all fixed costs are expressed in labor units. Because of all this, all costs can be expressed in terms of unskilled labor hours.

Let Z_e be the number of new potential entrants, who pay the lowest sunk market entry cost, f_e^L, and $p_{in}^{L+M+H} Z_e = Z = L + M + H$ be the total number of firms active in the market. Of these last ones, L is the number of active firms who produce using technology L, M is the number of active firms who produce using technology M and H is the number of active firms who produce using technology H. Therefore, like in Melitz (2003) total effective utilization of unskilled labor (or equivalent) by the mass of active firms ($Z = L + M + H$) equals the difference between total income and profits by the final good producers:

$$U_p = \left(L\bar{r}_d^L + M\bar{r}_d^M + H\bar{r}_d^H\right) - \left(L\bar{\pi}_d^L + M\bar{\pi}_d^M + H\bar{\pi}_d^H\right) = \bar{R} - \bar{\Pi} \tag{41}$$

The Free Entry Condition and the Aggregate Stability Condition ($p_{in}^{L+M+H} Z_e = \delta Z$) together yield the result that the sum of total profits in the final good market must match the total amount paid for entry by all potential entrants (both successful and unsuccessful), which in turn equals the total number of unskilled labor hours (or equivalent) used in business setting up activities (proof in Appendix C):

$$U_e = Z_e f_e^L + p_{in}^M Z_e (f_e^M - f_e^L) + p_{in}^H Z_e (f_e^H - f_e^L) = \bar{\Pi} \tag{42}$$

Thus, the unskilled labor market clearing condition is that the total effective unskilled labor hours used (directly or indirectly) by final goods producers plus the effective unskilled labor hours used (directly or indirectly) in business setting up activities equal its total offer:

$$U = \left[\bar{R} - \bar{\Pi}\right] + \left[Z_e f_e^L + p_{in}^M Z_e (f_e^M - f_e^L) + p_{in}^H Z_e (f_e^H - f_e^L)\right] = \bar{R} \tag{43}$$

This is also the final good market clearing condition: workers' total income (which they spend entirely in consumption of the final good) equals total income of final good producers.

On other grounds, welfare per worker is given by: $W = \frac{1}{P} = \frac{1}{Z^{\frac{1}{1-\sigma}} \bar{p}}$. Therefore, the present model shares with Melitz's the result that the number of incumbent firms –and thus, of available varieties-, increases proportionally with country size, resulting in higher aggregate welfare:

$$\frac{U}{\bar{r}_d} = \frac{U_L}{\bar{r}_d^L} + \frac{U_M}{\bar{r}_d^M} + \frac{U_H}{\bar{r}_d^H} = \frac{\bar{R}_d^L}{\bar{r}_d^L} + \frac{\bar{R}_d^M}{\bar{r}_d^L} + \frac{\bar{R}_d^H}{\bar{r}_d^L} = L + M + H = Z \tag{44}$$

[24] In other words: total payments of the final good producers for skilled labor are equal to the cost incurred by the skilled workers in terms of unskilled labor hours dedicated to studying.

Another important result of Melitz's model which continues to hold in the present context is that the rest of the key variables are independent from country size, that is: φ_T^*, $\tilde{\varphi}_T$, $\bar{r}_d^L$ and $\bar{\pi}_d^T$ (T=L, M, H), as well as $\tilde{\varphi}$, $\bar{r}_d$ and $\bar{\pi}_d$ do not vary with country size. However, even if technology L were identical to Melitz's single production technology and the idiosyncratic productivities distribution $g(\varphi)$ were also the same in both models, the threshold for entry to the market will still be higher here than in Melitz (2003), because of the additional fixed costs firms face when they upgrade technology. This will determine a higher idiosyncratic productivity. Besides, even if average idiosyncratic productivity were the same in both models, average revenues and profits would still be larger in the present model, due to the productivity gains stemming from technological improvements and labor skills.[25]

4. Open Economy Setting

Opening the economy to trade implies both modeling exporting behavior and taking into account the competition coming from foreign goods sold in the domestic market. The basic traits of the Melitz open economy environment are maintained: the home country can trade with $n \geq 1$ other countries, all of them assumed identical to it so as to ensure they all share the same input costs for each technology T and the same aggregate variables. Exporting firms face two types of trade related costs: a variable "iceberg" cost –capturing mainly fleet and tariff- and a fixed cost –representing the investment needed to penetrate export markets-. The decision regarding export status choice takes place once the firm has already drawn its productivity parameter, and there is no additional uncertainty.

We depart from this benchmark setting by dividing exporting firms into two subgroups according to their "level of commitment" with the export markets. Firms can now choose between three alternative market strategies: they can serve the domestic market only, or they can otherwise self-select into the export markets in two different ways, one "more accessible" and the other "more demanding"[26]. The "accessible" export status is achieved by incurring the minimum fixed cost indispensable for the firm to enter the foreign market. Firms who follow this strategy are called "Low-Commitment Exporters". Achieving the "demanding" export status involves making an additional investment –beyond the minimum fixed cost indispensable to begin exporting-, in order to gain a greater market share in the foreign market[27]. Firms who follow this strategy are called "High-Commitment Exporters"[28]. The per

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[25] It can be proved that $\bar{r}_d$ will always be higher in the present model, because the average variable cost is undoubtedly lower than in Melitz (2003). Recall that in the Melitz (2003) model every incumbent firm –whatever its productivity φ- uses production technology L, obtaining profits $\pi_d^L(\varphi)$. Thus, average profits in such model are given by $\bar{\pi}_{Melitz} = \frac{L}{Z}\pi_d^L(\tilde{\varphi}_L) + \frac{M}{Z}\pi_d^L(\tilde{\varphi}_M) + \frac{H}{Z}\pi_d^L(\tilde{\varphi}_H)$. Instead, in the current model any firm whose productivity surpasses the threshold φ_M^* will find $\pi_d^L(\varphi) < \pi_d^M(\varphi)$and will consequently switch to technology M, while any firm whose productivity surpasses the threshold φ_H^* will find $\pi_d^M(\varphi) < \pi_d^H(\varphi)$ and will consequently switch to technology H. Thus, in equilibrium only those incumbent firms with productivity below φ_M^* (a proportion $\frac{L}{Z}$ of total firm population) will be getting profits $\pi_d^L(\varphi)$, while those for whom $\varphi_M^* \leq \varphi < \varphi_H^*$ (a proportion $\frac{M}{Z}$ of total firm population) will be getting profits $\pi_d^M(\varphi)$, and those with $\varphi \geq \varphi_H^*$ (a proportion $\frac{H}{Z}$ of total firm population) will be getting profits $\pi_d^H(\varphi)$ -bear in mind that the proportions $\frac{L}{Z}$, $\frac{M}{Z}$ and $\frac{H}{Z}$ depend solely on the equilibrium productivity distribution, $\mu(\varphi)$, and the value of parameters-. Consequently, in the current model average profits are given by $\bar{\pi}_{Schmidt} = \frac{L}{Z}\pi_d^L(\tilde{\varphi}_L) + \frac{M}{Z}\pi_d^M(\tilde{\varphi}_M) + \frac{H}{Z}\pi_d^H(\tilde{\varphi}_H)$ which readily yields $\bar{\pi}_{Schmidt} > \bar{\pi}_{Melitz}$.

[26] Because of the presence of simplifying assumptions, in the model the individual firm can always expand its sales if it is capable of reducing its production cost or rising the quality of its variety enough. Both things are achievable by means of technology upgrading. However, in empirical analysis (to whose aid the model is intended) limitations in domestic market size pose a problem which is necessary to bear in mind in relatively small economies (like Argentina). Taking this into account, the purpose of modeling two distinct export strategies is to capture the intuition that a firm that faces a reduced domestic market can overcome the limitation this imposes to the expansion of its sales by engaging into trade.

[27] This additional investment can be understood in terms of extra advertising expenditure, the creation of better distribution channels, better postsale service, etc.

[28] On reaching this point it will be useful to remember that differences in productivity, both idiosyncratic and overall, can be interpreted either as differences in the costs associated to the production of goods of similar quality, or as differences in the quality of goods produced at the same cost. For convenience in the current context, the second interpretation (quality differences) will be adopted. We will then consider that "High-Commitment Exporters" incur a fixed cost which is superior to the minimum required fixed cost to begin exporting, in order to carry out certain activities in the export market with the purpose to induce potential buyers to choose their variety from among all the available varieties. As in terms of the model, sales –given the price- only increase as product quality increases –which in this context is the same as saying "as firm's

unit trade iceberg cost captures mainly fleet and tariffs and is the same for both types of exporting firms.

Analytically, the trade costs faced by exporting firms are:

- **"Low-Commitment Exporters":** the fixed export cost faced by these firms is denoted by $f_{elcx} > 0$, while the per unit export cost is $\tau > 1$.
- **"High-Commitment Exporters":** the fixed export cost faced by these firms is $f_{ehcx} > f_{elcx}$, while the per unit export cost is once again $\tau > 1$.

Domestic price remains a constant mark-up over marginal cost, and firms pass on the additional variable costs incurred in export sales to foreign consumers. Thus, the firm's pricing rule for the export markets is given by:

$$p_{ix}^{T} = \frac{\tau}{\rho}\frac{c_T}{\varphi(1+\beta_i)} = \frac{\tau}{(1+\beta_i)}p_d^T \qquad i = lc, hc \tag{45}$$

where β_i stands for the "effects of the additional trade-related fixed investments" made by type i exporters. Therefore $\beta_{lc} = 0$ –because "Low-Commitment Exporters" do not make any additional investments to foster export sales- and $0 < \beta_{hc} < A$, being A a positive finite number –because "High-Commitment Exporters" do make some additional investments to increase their exports, which are assumed not sterile, but nevertheless neither unlimited-. This means each firm may simultaneously set two different prices:

- Domestic price (p_d^T)
- Export price (p_{ix}^T)

It is important to note that since export costs –fixed and variable- for each export profile are assumed equal across countries, then a firm will either export to all countries –and with the same level of "commitment"- in every period or never export at all.

Quantity sold in the domestic market still is $q_d^T(\varphi) = EP^{\sigma-1}\left(\rho\frac{\varphi}{c_T}\right)^{\sigma} = EP^{\sigma-1}\left(\frac{1}{p_d^T(\varphi)}\right)^{\sigma}$ while quantity sold in the foreign markets is $q_{ix}^T(\varphi) = EP^{\sigma-1}\left(\rho\frac{\varphi(1+\beta_i)}{\tau c_T}\right)^{\sigma} = EP^{\sigma-1}\left(\frac{1}{p_{ix}^T(\varphi)}\right)^{\sigma}$ per export destination. Therefore, total quantity sold depends on export status:

$$q^T(\varphi) = \begin{cases} q_d^T(\varphi) & \textit{if the firm is a "Non-Exporter"} \\ q_d^T(\varphi) + nq_{lcx}^T(\varphi) = \left[1 + n\left(\frac{\tau}{(1+\beta_{lc})}\right)^{-\sigma}\right]q_d^T(\varphi) = \left[1 + n\tau^{-\sigma}\right]q_d^T(\varphi) & \textit{if the firm is a "Low-Commitment Exporter"} \\ q_d^T(\varphi) + nq_{hcx}^T(\varphi) = \left[1 + n\left(\frac{\tau}{(1+\beta_{hc})}\right)^{-\sigma}\right]q_d^T(\varphi) = \left[1 + n\left(\frac{\tau}{(1+\beta_{hc})}\right)^{-\sigma}\right]q_d^T(\varphi) & \textit{if the firm is a "High-Commitment Exporter"} \end{cases} \tag{46}$$ [29]

overall productivity increases"-, the specific purpose of this additional fixed investment is to make potential buyers in the foreign market perceive the firm's variety as better quality. Such increase in quality may be real –e.g.: due to better distribution channels or postsale service- or immaginary –e.g.: due to smart advertising-, but its effect in terms of the model is nevertheless "as if" the overall productivity of the firm was increased by these additional investments. This way, the firm is able to expand its sales in the foreign market beyond what its true overall productivity would determine. Therefore, given its overall productivity level, a firm will achieve larger export sales –and thus, larger total sales- if it follows the "High-Commitment Exporter" strategy than if it follows the "Low-Commitment Exporter" strategy. In exchange for such larger sales, the High-Commitment Exporter faces a higher fixed export cost.

[29] Total sales for the exporting firm are given by :

$$q^T(\varphi) = q_d^T(\varphi) + nq_{ix}^T(\varphi) = EP^{\sigma-1}\left(\frac{1}{p_d^T(\varphi)}\right)^{\sigma} + nEP^{\sigma-1}\left(\frac{1}{p_{ix}^T(\varphi)}\right)^{\sigma} = EP^{\sigma-1}\left(\rho\frac{\varphi}{c_T}\right)^{\sigma} + nEP^{\sigma-1}\left(\rho\frac{\varphi(1+\beta_i)}{\tau c_T}\right)^{\sigma}$$
$$= EP^{\sigma-1}\left(\rho\frac{\varphi}{c_T}\right)^{\sigma} + nEP^{\sigma-1}\left(\rho\frac{\varphi}{c_T}\right)^{\sigma}\left(\frac{\tau}{(1+\beta_i)}\right)^{-\sigma} = EP^{\sigma-1}\left(\rho\frac{\varphi}{c_T}\right)^{\sigma}\left[1 + n\left(\frac{\tau}{(1+\beta_i)}\right)^{-\sigma}\right] = \left[1 + n\left(\frac{\tau}{(1+\beta_i)}\right)^{-\sigma}\right]q_d^T(\varphi).$$

The revenue earned by a firm from its domestic sales and from its export sales per export destination (either “Low-Commitment” or “High-Commitment”) are:

- $r_d^T(\varphi) = E\left(P\rho\frac{\varphi}{c_T}\right)^{\sigma-1}$ (recall equation (11))
- $r_{lcx}^T(\varphi) = E\left(P\rho\frac{\varphi(1+\beta_{lc})}{\tau c_T}\right)^{\sigma-1} = E\left(P\rho\frac{\varphi}{c_T}\right)^{\sigma-1}\left(\frac{1}{\tau}\right)^{\sigma-1} = \tau^{1-\sigma}E\left(P\rho\frac{\varphi}{c_T}\right)^{\sigma-1} = \tau^{1-\sigma}r_d^T(\varphi)$.
- $r_{hcx}^T(\varphi) = E\left(P\rho\frac{\varphi(1+\beta_{hc})}{\tau c_T}\right)^{\sigma-1} = E\left(P\rho\frac{\varphi}{c_T}\right)^{\sigma-1}\left(\frac{(1+\beta h_c)}{\tau}\right)^{\sigma-1} = \left(\frac{\tau}{(1+\beta_{hc})}\right)^{1-\sigma}E\left(P\rho\frac{\varphi}{c_T}\right)^{\sigma-1} = \left(\frac{\tau}{(1+\beta_{hc})}\right)^{1-\sigma}r_d^T(\varphi)$

Which yields:

$$r^T(\varphi) = \begin{cases} r_d^T(\varphi) \quad if\ the\ firm\ is\ a\ "Non-Exporter" \\ r_d^T(\varphi) + nr_{lcx}^T(\varphi) = [1+n\tau^{1-\sigma}]r_d^T(\varphi) \\ \qquad if\ the\ firm\ is\ a\ "Low-Commitment\ Exporter" \\ r_d^T(\varphi) + nr_{hcx}^T(\varphi) = \left[1+n\left(\frac{\tau}{(1+\beta_{hc})}\right)^{1-\sigma}\right]r_d^T(\varphi) \\ \qquad if\ the\ firm\ is\ a\ "High-Commitment\ Exporter" \end{cases} \tag{47}$$

4.1 Firm Entry, Exit and Market Strategy in the Open Economy

While the general setting of the economy is the same as in autarky –thus entry and exit dynamics remain unchanged-, an additional choice comes by with trade: the necessity that firms choose their export status. As in Meliz (2003), the simplifying assumption of no additional uncertainty concerning the export markets determines isomorphy between modelling the sunk investment cost assotiated to exporting, f_{eix}, as such –thus, paid all at once when the firm begins exporting- or as a fixed cost incurred in every period –equivalent to the amortized per period portion of this cost $f_{ix} = \delta f_{eix}$-.[30] Also, because the variable profit from domestic sales $\frac{1}{\sigma}r_d^T(\varphi)$ is always positive, and the fixed production cost f_T is paid on entering production –before choosing export status-, then no firm will ever export and not also produce for its domestic market, which allows separation of total profits according to their source –domestic or export markets:

$$\begin{cases} \pi_d^T(\varphi) = \frac{1}{\sigma}r_d^T(\varphi) - f_T \\ \pi_{xi}^T(\varphi) = \frac{1}{\sigma}r_{xi}^T(\varphi) - f_{xi} \end{cases} \tag{48}$$

Consequently, total profits for a firm using technology T are given by:

$$\pi^T(\varphi) = \pi_d^T(\varphi) + max\{0, n\pi_{lcx}^T(\varphi), n\pi_{hcx}^T(\varphi)\} \tag{49}$$

Firm value is once again given by $v(\varphi) = max\{0, v^L(\varphi), v^M(\varphi), v^H(\varphi)\}$ and $\varphi^{T*} = inf\{\varphi: v^T(\varphi) > v^{T-1}(\varphi)\}$[31] identifies the cutoff idiosyncratic productivity level for profitable entry to production with technology T in the open economy setting. Exporting productivity thresholds are determined similarly: $\varphi_{lcx}^{T*} = inf\{\varphi: \varphi \geq \varphi^{T*} and\ \pi_{lcx}^T(\varphi) > 0\ for\ some\ technology\ T\}$ is the

[30] The logic of this reasoning is not altered by the introduction of technology choice. Here again, if such cost is modeled as sunk, then only new exporters will pay it and all at once (new Low-Commitment Exporters will pay f_{elcx}, and new High-Commitment Exporters will pay f_{ehcx}). If instead it is modeled as a per period fixed cost, then all exporting firms will spend resources to cover the smaller amortized portion of the corresponding cost $f_{ix}^T = \delta f_{eix}$. Because in equilibrium the ratio of new type i exporters to all type i exporters (i = Low-Commitment, High-Commitment) in each technology T is δ (see Appendix D), it follows that the same aggregate labor resources are spent in either case.
resources to cover the smaller amortized portion of the corresponding cost $f_{ix} = \delta f_{eix}$. Because in equilibrium the ratio of new type i exporters to all type i exporters (i = Low-Commitment, High-Commitment) in each technology T is δ (see Appendix D), it follows that the same aggregate labor resources are spent in either case.

[31] For T=L there is no T-1 technology, and thus the condition reduces to $\varphi^{L*} = \varphi^{L**} = inf\{\varphi: v^L(\varphi) > 0\}$ –analogously to what happened in the closed economy case-.

cutoff idiosyncratic productivity level that in combination with technology T determines an overall productivity high enough for firms to find it profitable to enter the export market as "Low-Commitment Exporters", while $\varphi_{hcx}^{T*} = inf\{\varphi: \varphi \geq \varphi^{T*} and\ \pi_{hcx}^{T}(\varphi) > \pi_{lcx}^{T}(\varphi) for\ some\ technology\ T\}$ is the cutoff idiosyncratic productivity level that in combination with technology T determines an overall productivity high enough for firms to find it profitable to enter the export market as "High-Commitment Exporters".[32]

Idiosyncratic Productivity Threshold to become a "Low-Commitment Exporter" for users of technology T (φ_{lcx}^{T*}):

If $\varphi_{lcx}^{T*} = \varphi^{T*}$ then all firms using technology T or superior export (either as "Low-Commitment Exporters" or as "High-Commitment Exporters") while no firm using technology T-1 or inferior exports at all. In this case, the effective cutoff exporting firm (with idiosyncratic productivity level $\varphi^{T*} = \varphi_{lcx}^{T*}$) earns nonnegative total profit ($\pi^{T}(\varphi^{T*}) = \pi_{d}^{T}(\varphi^{T*}) + \pi_{lcx}^{T}(\varphi^{T*}) \geq 0$) and nonnegative export profit ($\pi_{lcx}^{T}(\varphi^{T*}) \geq 0$). If $\varphi_{lcx}^{T*} > \varphi^{T*}$ then some firms using technology T (with idiosyncratic productivity levels between φ^{T*} and φ_{lcx}^{T*}) do not export, as well as all firms using technology T-1 or inferior. Meanwhile, some firms using technology T (those with idiosyncratic productivity levels equal to or above φ_{lcx}^{T*}) do export, either as "Low-Commitment" or as "High-Commitment"exporters, as well as all firms using technology T+1 or superior[33].

Idiosyncratic Productivity Threshold to become a "High-Commitment Exporter" for users of technology T (φ_{hcx}^{T*}):

If $\varphi_{hcx}^{T*} = \varphi^{T*}$ then all firms using technology T or superior are "High-Commitment Exporters", while no firm using technology T-1 or inferior is so (firms using technology T-1 or inferior may be exporters, but only of the "Low-Commitment" type). In this case, the effective cutoff "High-Commitment" exporting firm (with idiosyncratic productivity $\varphi^{T*} = \varphi_{hcx}^{T*}$) earns nonnegative total profit ($\pi^{T}(\varphi^{T*}) = \pi_{d}^{T}(\varphi^{T*}) + \pi_{hcx}^{T}(\varphi^{T*}) \geq 0$) and export profits equal to or greater than those it would earn as a "Low-Commitment Exporter" ($\pi_{hcx}^{T}(\varphi^{T*}) \geq \pi_{lcx}^{T}(\varphi^{T*})$). If $\varphi_{hcx}^{T*} > \varphi^{T*}$ then some firms using technology T (with idiosyncratic productivity levels between $\varphi^{T*}$and φ_{hcx}^{T*}) are not "High-Commitment Exporters", as well as all firms using technology T-1 or inferior[34]. Meanwhile, some firms using technology T (those with idiosyncratic productivity levels equal to or above φ_{hcx}^{T*}) are "High-Commitment Exporters", as well as all firms using technology T+1 or superior.

The aforementioned implies that the individual firm can achieve the overall productivity level required to export (with low or high commitment) by means of three alternative combinations of idiosyncratic productivity and technological factors, which in turn determines the existence of three idiosyncratic productivity thresholds compatible with entering the export market with low commitment (φ_{lcx}^{L*}, φ_{lcx}^{M*} y φ_{lcx}^{H*}) and another three idiosyncratic productivity thresholds compatible with entering the export market with high commitment (φ_{hcx}^{L*}, φ_{hcx}^{M*} y φ_{hcx}^{H*}). However, in each case only one of these thresholds is relevant. This is because any firm whose idiosyncratic productivity is high enough to upgrade technology does it, as the upgrading allows it to earn higher profits. This way, the relevant threshold of idiosyncratic productivity to export with low commitment, φ_{lcx}^{*}, could be (alternatively)

[32] Note that there is an important difference between the decisions regarding technology strategy and market strategy. Whereas the firm chooses its technology on the basis of idiosyncratic productivity alone, it chooses its market strategy on the basis of overall productivity (which is obtained combining idiosyncratic and technological factors). Thus, a firm possessing a high enough idiosyncratic productivity will be capable of profitably exporting even if it produces with the most basic available technology (L) and does not upgrade it at all. But if the firm's idiosyncratic productivity alone (that is, in combination with technology L, in which case total productivity is considered to coincide with idiosyncratic productivity) is not enough to export profitably, then the firm can still achieve this objective by upgrading its technology, so that its overall productivity indeed surpasses the required threshold.

[33] Their being "Low-Commitment" or "High-Commitment" exporters depends on whether their intrinsic productivity parameter falls between the thresholds corresponding to each exporting category (in which case the firm is a "Low-Commitment Exporter") or surpasses both (in which case the firm is a "High-Commitment Exporter").

[34] These firms are not "High-Commitment Exporters" because they can have higher profits by being "Low-Commitment Exporters" or "Non-Exporters".

one (and one only) of the following: φ_{lcx}^{L*}, φ_{lcx}^{M*} or φ_{lcx}^{H*}. The same is true for the relevant threshold of idiosyncratic productivity to export with high commitment, φ_{hcx}^{*}, which can be (alternatively) one (and one only) of the following: φ_{hcx}^{L*}, φ_{hcx}^{M*} or φ_{hcx}^{H*}.

In general:

- $\pi_d^T(\varphi^{T**}) = 0$ and $\pi_d^T(\varphi^{T*}) \geq 0$ for all T (analogously as in the Closed Economy case)
- $\pi_{lcx}^T(\varphi_{lcx}^*) = 0$ for some T satisfying $\varphi^{T*} \leq \varphi_{lcx}^*$
- $\pi_{hcx}^T(\varphi_{hcx}^*) = \pi_{lcx}^T(\varphi_{hcx}^*)$ for some T satisfying $\varphi^{T*} \leq \varphi_{hcx}^*$

The model does not provide any elements to determine which of the three possible thresholds is the relevant in each case. It provides an order of the idiosyncratic productivity thresholds a firm needs to surpass to adopt the available technologies, and an order of the thresholds of overall productivity (and thus, of idiosyncratic productivity in each technology) it needs to surpass to adopt each of the export profiles, but it does not determine the position of the technology thresholds relative to the export thresholds.[35]

For now the following "plausible" situation will be assumed: that in the industry there are both exporting and nonexporting firms, and that users of technology L can only be found in the second group, while only some firms using technology M and all firms using technology H export. However, "High-Commitment Exporters" can be found only among users of technology H, meaning that any firm using technology M who is an exporter, is necessarily a "Low-Commitment Exporter". If a firm with a certain productivity level can maximize profits by assuming a certain export status (eg: "High-Commitment Exporter"), then all firms whose idiosyncratic productivity is above that level will too. Therefore, because there are some "High-Commitment Exporters" who produce with technology H and none who produces with technology M (or L), it follows that the idiosyncratic productivity threshold to adopt the "High-Commitment Exporter" profile must lie to the right of the threshold to adopt technology H. Besides, because there are users of technology M who are exporters, then all users of technology H are so too. Figure 3 illustrates the ordering of idiosyncratic productivity thresholds resulting from these assumptions:

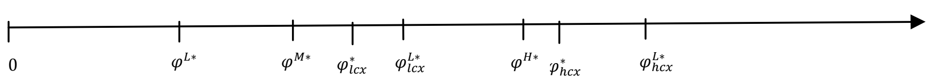

Figure 3

As it was the case in the closed economy, for the assumed order of the five productivity thresholds ($\varphi^{L*} < \varphi^{M*} < \varphi_{lcx}^* < \varphi^{H*} < \varphi_{hcx}^*$) to hold, it is required that, roughly speaking, the gain obtained by a firm when switching from each category to the immediate superior category (ej: from Non-Exporter

[35] An empirical evaluation may be useful to get hints on which of these technology-specific idiosyncratic productivity thresholds is the relevant for the adoption of each export profile. For example, if it is observed in data that some firms who produce with technology M and all firms producing with technology H export, while no firm producing with technology L does so, then the relevant idiosyncratic productivity threshold for exporting with at least low commitment must be $\varphi_{lcx}^{M*} = \varphi_{lcx}^*$. The relevant threshold cannot be $\varphi_{lcx}^{L*} > \varphi_{lcx}^{M*}$, because no firm will continue using technology L if it is idiosyncratically productive enough to adopt M. On the other hand, $\varphi_{lcx}^{H*} < \varphi_{lcx}^{M*}$ cannot be the relevant threshold either, because if there are some exporters who produce with technology M this means their idiosyncratic productivity is not high enough to adopt technology H. Similarly, if it is observed in data that no firm producing with technologies L or M exports with high commitment, but some firms producing with technology H do so, then the relevant idiosyncratic productivity threshold for the adoption of such market strategy must lie to the right of the threshold to adopt technology H. In this case, the relevant threshold to begin exporting with high commitment is $\varphi_{hcx}^{H*} = \varphi_{hcx}^*$. Despite these considerations, looking at raw data can only provide hints. If there are firms upgrading technology and serving only the domestic market (suggesting that the thresholds associated to exporting lie to the right of those associated to technology upgrading) and these coexist with firms who export but do not upgrade technology (which suggests the opposite order), the question remains unresolved, especially if the amounts of firms in both of these groups are similar.

who uses technology M to Low-Commitment Exporter who uses technology M) must be smaller "in proportion" to the increase in the fixed cost it simultaneously faces for a given productivity level.[36]

Because the entry and exit dynamics are unchanged by trade, the equilibrium distribution of idiosyncratic productivity levels for incumbent firms continues to be $\mu(\varphi) = \frac{g(\varphi)}{[1-G(\varphi^{L*})]}\ \forall\varphi \geq \varphi^{L*}$. The ex ante probability that a successful entrant to the industry will become a "Low-Commitment Exporter" is $p_{inlcx}^{L+M+H} = \frac{G(\varphi_{hcx}^{*})-G(\varphi_{lcx}^{*})}{1-G(\varphi^{L*})}$, while $p_{inhcx}^{L+M+H} = \frac{1-G(\varphi_{hcx}^{*})}{1-G(\varphi^{L*})}$ now represents the ex-ante probability that one of these successful entrants will become a "High-Commitment Exporter" and $p_{innx}^{L+M+H} = \frac{G(\varphi_{lcx}^{*})-G(\varphi^{L*})-}{1-G(\varphi^{L*})}$ is the ex-ante probability that one of these successful entrants will become a non exporter. These coincide with the ex post fractions of Low-Commitment Exporters, High-Commitment Exporters and Non Exporters, respectively[37]. We can further define[38]:

- $p_{innx}^{L} = \frac{G(\varphi^{M*})-G(\varphi^{L*})}{G(\varphi^{M*})-G(\varphi^{L*})} = 1$ represents both the ex-ante probability that one of the successful entrants into technology L will be a Non Exporter and the ex-post fraction of firms that use technology L and serve the domestic market only.
- $p_{innx}^{M} = \frac{G(\varphi_{lcx}^{*})-G(\varphi^{M*})}{G(\varphi^{H*})-G(\varphi^{M*})}$ represents both the ex-ante probability that one of the successful entrants into technology M will be a Non Exporter and the ex-post fraction of firms that use technology M and serve the domestic market only.
- $p_{inlcx}^{M} = \frac{G(\varphi^{H*})-G(\varphi_{lcx}^{*})}{G(\varphi^{H*})-G(\varphi^{M*})}$ represents both the ex-ante probability that one of the successful entrants into technology M will be a "Low-Commitment Exporter" and the ex-post fraction of firms that use technology M and are "Low -Commitment Exporters".
- $p_{inlcx}^{H} = \frac{G(\varphi_{hcx}^{*})-G(\varphi^{H*})}{1-G(\varphi^{H*})}$ represents both the ex-ante probability that one of the successful entrants into technology H will be a "Low-Commitment Exporter" and the ex-post fraction of firms that use technology H and are "Low -Commitment Exporters".
- $p_{inhcx}^{H} = \frac{1-G(\varphi_{hcx}^{*})}{1-G(\varphi^{H*})}$ represents both the ex-ante probability that one of the successful entrants into technology H will be a "High-Commitment Exporter" and the ex-post fraction of firms that use technology H and are "High-Commitment Exporters".

[36] The intuition that innovation activities entail risks for the firm could be explicitly captured in the model by introducing a probabilistic element. The basic idea of this extension is that adopting a superior technology is equivalent to participation in a lottery, whose expected value is an increase of a given proportion in firm overall productivity. The expected value of such lottery is assumed to be larger for firms adopting technology H. Like in the deterministic approach, for firms using technology L overall productivity equals idiosyncratic productivity, while firms adopting technologies M and H are are likely to experience a boost in their overall productivity because, along with the idiosyncratic factors, they now also have technological factors contributing to it. However, the positive outcome is not guaranteed. While most firms will experience productivity gains as a consequence of upgrading technology, some will fail to adequately absorb the new technologies and will have no positive impact on the overall productivity.
This probabilistic extension, which is left for future work, is interesting because it provides an explanation for situations that can be observed in data where firms do not follow the expected market strategy given their technology strategy, and vice versa. Specifically, a firm can upgrade technology (to M or H) but fail to absorb the new technologies and consequently not reach the overall productivity required to export (or to export with the expected commitment level). Conversely, it is also possible to think of a risk averse (or financially constrained) firm that may decide not to run the risks associated to upgrading technology even if the expected return of doing so is positive, and still export if its idiosyncratic productivity is high enough (so that it does not need the "help" of a better technology).

[37] Note that, unlike the procedure for the calculation of the ex-ante probabilities of successful entry into each production technology ($p_{in}^{T}, T = L, M, H$), here for the calculation of the ex-ante probabilities of entry into each of the market strategies, the distribution we use is $\mu(\varphi)$, not $g(\varphi)$. This is because the choice of export status occurs after the firm draws its productivity parameter φ, which means only successful entrants must be taken into account.

[38] Continuing with the reasoning in the previous footnote, we know that firms choose their export status after they gain knowledge on their productivity parameter. Thus, only successful entrants to the industry decide whether to become "High-Commitment" or "Low-Commitment" exporters or neither. But firms who face such decision not only already know that they are successful entrants to the industry, but also they know precisely with which production technology: L, M or H. Therefore, the rational thing to do when calculating the probabilities of adopting each exporting profile ("Low-Commitment Exporter", "High-Commitment Exporter") is to take into account all the available information. Because of this, the ex ante probabilities for each firm of becoming each type of exporter are calculated conditional on the production technology they are using.

Denoting the number of incumbent firms in any country by Z_{open}, it is possible to calculate the number of firms belonging to each export satatus: "Low-Commitment Exporters" are $Z_{lcx} = p_{inlcx}^{L+M+H} Z_{open}$, "High-Commitment Exporters" are $Z_{hcx} = p_{inhcx}^{L+M+H} Z_{open}$ and "Non Exporters" are $Z_{nx} = p_{innx}^{L+M+H} Z_{open} = Z_{open} - Z_{lcx} - Z_{hcx}$. To be more specific, $p_{innx}^{L} L = \gamma p_{innx}^{L+M+H} Z_{open}$ (with $0 < \gamma < 1$) is the number of firms that use technology L and serve the domestic market only, $p_{innx}^{M} M = (1-\gamma) p_{innx}^{L+M+H} Z_{open}$) is the number of firms that use technology M and serve the domestic market only, $p_{inlcx}^{M} M = \theta p_{inlcx}^{L+M+H} Z_{open}$ (with $0 < \theta < 1$) is the number of firms that use technology M and are "Low-Commitment Exporters", $p_{inlcx}^{H} H = (1-\theta) p_{inlcx}^{L+M+H} Z_{open}$ is the number of firms that use technology H and are "Low-Commitment Exporters" and $p_{inhcx}^{H} H = p_{inhcx}^{L+M+H} Z_{open}$ is the number of incumbent firms that use technology H and are "High-Commitment Exporters".[39]

Using these definitions it is possible to calculate the total mass of varieties available to consumers in any country –or alternatively, the total mass of firms competing in any country-:

$$\boldsymbol{Z_t} = Z_{open} + np_{inlcx}^{L+M+H} Z_{open} + np_{inhcx}^{L+M+H} Z_{open} = Z_{open}\left(1 + np_{inlcx}^{L+M+H} + np_{inhcx}^{L+M+H}\right) =$$

$$= L_{open} + M_{open} + H_{open} + np_{inlcx}^{M} M_{open} + np_{inlcx}^{H} H_{open} + np_{inhcx}^{H} H_{open} =$$

$$= \boldsymbol{L_{open}} + \boldsymbol{M_{open}}(\boldsymbol{1} + \boldsymbol{np_{inlcx}^{M}}) + \boldsymbol{H_{open}}(\boldsymbol{1} + \boldsymbol{np_{inlcx}^{H}} + \boldsymbol{np_{inhcx}^{H}}) \tag{50}$$

Unless the contrary is explicitly stated, from now on we will refer to Z_{open} simply as Z, L_{open} as L, M_{open} as M and H_{open} as H.

4.2 Aggregation Conditions in the Open Economy

As in autarky, it is possible to calculate the average idiosyncratic productivity level across all incumbent firms as a function of the new threshold for entering the industry:

$$\tilde{\varphi} = \tilde{\varphi}^{L}(\varphi^{L*}) = \left[\frac{1}{1-G(\varphi^{L*})}\int_{\varphi^{L*}}^{\infty} \varphi^{\sigma-1} g(\varphi)d\varphi\right]^{\frac{1}{\sigma-1}} \tag{51}$$

The average idiosyncratic productivity levels corresponding to each Market Strategy ("Non-Exporter", "Low-Commitment Exporter", "High-Commitment Exporter") are:

$$\tilde{\varphi}_{nx}(\varphi^{L*}, \varphi_{lcx}^{*}) = \left[\frac{1}{1-G(\varphi^{L*})}\int_{\varphi^{L*}}^{\varphi_{lcx}^{*}} \varphi^{\sigma-1} g(\varphi)d\varphi\right]^{\frac{1}{\sigma-1}} \tag{52}$$

$$\tilde{\varphi}_{lcx}(\varphi_{lcx}^{*}, \varphi_{hcx}^{*}) = \left[\frac{1}{1-G(\varphi^{L*})}\int_{\varphi_{lcx}^{*}}^{\varphi_{hcx}^{*}} \varphi^{\sigma-1} g(\varphi)d\varphi\right]^{\frac{1}{\sigma-1}} \tag{53}$$

$$\tilde{\varphi}_{hcx}(\varphi_{hcx}^{*}) = \left[\frac{1}{1-G(\varphi^{L*})}\int_{\varphi_{hcx}^{*}}^{\infty} \varphi^{\sigma-1} g(\varphi)d\varphi\right]^{\frac{1}{\sigma-1}} \tag{54}$$

We can further calculate the average idiosyncratic productivity corresponding to each Market Strategy and production technology:

[39] $\gamma = \frac{G(\varphi^{M*})-G(\varphi^{L*})}{G(\varphi_{lcx}^{*})-G(\varphi^{L*})}$ is the proportion of "Non Exporters" who produce using technology L, $(1-\gamma) = \frac{G(\varphi_{lcx}^{*})-G(\varphi^{M*})}{G(\varphi_{lcx}^{*})-G(\varphi^{L*})}$ is the proportion of "Non Exporters" who produce using technology M, $\theta = \frac{G(\varphi^{H*})-G(\varphi^{M*})}{G(\varphi_{hcx}^{*})-G(\varphi_{lcx}^{*})}$ is the proportion of "Low-Commitment Exporters" who produce using technology M and $(1-\theta) = \frac{G(\varphi_{hcx}^{*})-G(\varphi^{H*})}{G(\varphi_{hcx}^{*})-G(\varphi_{lcx}^{*})}$ is the proportion of "Low-Commitment Exporters" who produce using technology H. Note that all "High-Commitment Exporters" produce using technology H. Besides this, we also know $p_{in}^{L+M+H} Z_e = Z$. Using this information together with the definition of p_{inix}^{L+M+H} and p_{inix}^{T} the equivalences stated above can be easily derived.

$$\tilde{\varphi}_{Lnx}(\varphi^{L*}, \varphi^{M*}) = \left[\frac{1}{1-G(\varphi^{L*})}\int_{\varphi^{L*}}^{\varphi^{M*}} \varphi^{\sigma-1} g(\varphi)d\varphi\right]^{\frac{1}{\sigma-1}} \quad (55)$$

$$\tilde{\varphi}_{Mnx}(\varphi^{M*}, \varphi^{*}_{lcx}) = \left[\frac{1}{1-G(\varphi^{L*})}\int_{\varphi^{M*}}^{\varphi^{*}_{lcx}} \varphi^{\sigma-1} g(\varphi)d\varphi\right]^{\frac{1}{\sigma-1}} \quad (56)$$

$$\tilde{\varphi}_{Mlcx}(\varphi^{*}_{lcx}, \varphi^{H*}) = \left[\frac{1}{1-G(\varphi^{L*})}\int_{\varphi^{*}_{lcx}}^{\varphi^{H*}} \varphi^{\sigma-1} g(\varphi)d\varphi\right]^{\frac{1}{\sigma-1}} \quad (57)$$

$$\tilde{\varphi}_{Hlcx}(\varphi^{H*}, \varphi^{*}_{hcx}) = \left[\frac{1}{1-G(\varphi^{L*})}\int_{\varphi^{H*}}^{\varphi^{*}_{hcx}} \varphi^{\sigma-1} g(\varphi)d\varphi\right]^{\frac{1}{\sigma-1}} \quad (58)$$

$$\tilde{\varphi}_{Hhcx}(\varphi^{*}_{hcx}, \infty) = \left[\frac{1}{1-G(\varphi^{L*})}\int_{\varphi^{H*}}^{\infty} \varphi^{\sigma-1} g(\varphi)d\varphi\right]^{\frac{1}{\sigma-1}} \quad (59)$$

Because all these averages are constructed in the same way as in the autarky setting, they only take into account domestic market share differences between firms, and ignore the additional sales more productive firms are now gaining in the export markets, as well as the fraction of resources consumed by transportation when selling abroad (and thus no longer available for consumption).[40] In order to provide a measure of average idiosyncratic productivity more adequate for the open economy setting, these factors must be incorporated.[41] The comprehensive average idiosyncratic productivity measure $\tilde{\varphi}_t$ is constructed on the basis of the combined market share of all firms, taking into account the transport costs faced by both types of exporters and the export-sales-boosting effect of the additional fixed investments undertaken by High-Commitment Exporters:

$$\tilde{\varphi}_t = \left\{\frac{1}{Z_t}\left[Z\tilde{\varphi}^{\sigma-1} + np^{L+M+H}_{inlcx} Z\left(\frac{\tilde{\varphi}_{lcx}}{\tau}\right)^{\sigma-1} + np^{L+M+H}_{inhcx} Z\left(\frac{\tilde{\varphi}_{hcx}(1+\beta_{hc})}{\tau}\right)^{\sigma-1}\right]\right\}^{\frac{1}{\sigma-1}} \quad (60)$$

Expression (61) is equivalent to:

$$\tilde{\varphi}_t^{\sigma-1} = \frac{1}{Z_t}\left[Z\int_0^{\infty} \varphi^{\sigma-1}\mu(\varphi)d\varphi + np^{L+M+H}_{inlcx} Z\int_{\varphi^{*}_{lcx}}^{\varphi^{*}_{xc}} \left(\frac{\varphi}{\tau}\right)^{\sigma-1} \mu(\varphi)d\varphi + np^{L+M+H}_{inhcx} Z\int_{\varphi^{*}_{hcx}}^{\infty} \left(\frac{\varphi(1+\beta_{hc})}{\tau}\right)^{\sigma-1} \mu(\varphi)d\varphi\right] \quad (61)$$

Taking explicitly into account the production technology used by every firm (domestic and foreign), comprehensive aggregate productivity $\tilde{\varphi}_t$ can be rewritten as the following (derivation in Appendix E):

$$\tilde{\varphi}_t = \left\{\frac{1}{Z_t}\left[L[\tilde{\varphi}_L{}^{\sigma-1}] + M[\tilde{\varphi}_M{}^{\sigma-1} + np^{M}_{inlcx}\frac{1}{\tau^{\sigma-1}}\tilde{\varphi}_{Mlcx}{}^{\sigma-1}] + H[\tilde{\varphi}_H{}^{\sigma-1} + np^{H}_{inlcx}\frac{1}{\tau^{\sigma-1}}\tilde{\varphi}_{Hlcx}{}^{\sigma-1} + np^{H}_{inhcx}\left(\frac{(1+\beta_{hc})}{\tau}\right)^{\sigma-1}\tilde{\varphi}_{Hhcx}{}^{\sigma-1}]\right]\right\}^{\frac{1}{\sigma-1}} \quad (62)$$

Because of the symmetry assumption, $\tilde{\varphi}_t$ is also the weighted average adjusted idiosyncratic productivity of all firms –domestic and foreign- competing in each country.

The aggregate price index P_t, quantity Q_t, expenditure level R_t and profits Π_t in the open economy setting are given by:

$$P_t = \left[Lp^{L}_{d}(\tilde{\varphi}_L)^{1-\sigma} + Mp^{M}_{d}(\tilde{\varphi}_M)^{1-\sigma} + Hp^{H}_{d}(\tilde{\varphi}_H)^{1-\sigma} + np^{M}_{inlcx}Mp^{M}_{lcx}(\tilde{\varphi}_{Mlcx})^{1-\sigma} + np^{H}_{inlcx}Hp^{H}_{lcx}(\tilde{\varphi}_{Hlcx})^{1-\sigma} + np^{H}_{inhcx}Hp^{H}_{hcx}(\tilde{\varphi}_{Hhcx})^{1-\sigma}\right]^{\frac{1}{1-\sigma}} =$$

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[40] They also ignore, as their counterparts in the Closed Economy, the technological factors that affect firm overall productivity, which are later incorporated when calculating the prices charged by firms in each technology (in the domestic and export markets). Thus, only idiosyncratic productivity is taken into account, separating it completely from technological factors. Again, the purpose of this is to facilitate comparison with the results obtained by Melitz –and to maintain a methodological coherence in both settings (Closed Economy and Open Economy).

[41] Because neither the introduction of technology choice nor the diversification of market strategies affect this aspect of the original Melitz (2003) model, we stick to its approach for doing so, making only minor amendments when necessary.

$$\left[\begin{matrix} Lp_d^L(\tilde{\varphi}_L)^{1\text{-}\sigma}+Mp_d^M(\tilde{\varphi}_M)^{1\text{-}\sigma}+Hp_d^H(\tilde{\varphi}_H)^{1\text{-}\sigma}+ \\ np_{inlcx}^M Mp_d^M(\tilde{\varphi}_{lcxM})^{1\text{-}\sigma}+np_{inlcx}^H Hp_d^H(\tilde{\varphi}_{lcxH})^{1\text{-}\sigma}+np_{inhcx}^H Hp_d^H(\tilde{\varphi}_{hcxH})^{1\text{-}\sigma}\end{matrix}\right]^{\frac{1}{1\text{-}\sigma}} \quad (63)^{42}$$

$$Q_t = \left[Lq_d^L(\tilde{\varphi}_L)^\rho+Mq_d^M(\tilde{\varphi}_M)^\rho+Hq_d^H(\tilde{\varphi}_H)^\rho+np_{inlcx}^M Mq_{lcx}^M(\tilde{\varphi}_{Mlcx})^\rho+np_{inlcx}^H Hq_{lcx}^H(\tilde{\varphi}_{Hlcx})^\rho+np_{inhcx}^H Hq_{hcx}^H(\tilde{\varphi}_{Hhcx})^\rho\right]^{\frac{1}{\rho}}$$

$$= \left[Lq_d^L(\tilde{\varphi}_L)^\rho+Mq_d^M(\tilde{\varphi}_M)^\rho+Hq_d^H(\tilde{\varphi}_H)^\rho+np_{inlcx}^M Mq_d^M(\tilde{\varphi}_{lcxM})^\rho+ +np_{inlcx}^H Hq_d^H(\tilde{\varphi}_{lcxH})^\rho+np_{inhcx}^H Hq_d^H(\tilde{\varphi}_{hcxH})^\rho\right]^{\frac{1}{\rho}} \quad (64)^{43}$$

$$R_t = Lr_d^L(\tilde{\varphi}_L)+Mr_d^M(\tilde{\varphi}_M)+Hr_d^H(\tilde{\varphi}_H)+np_{inlcx}^M Mr_{lcx}^M(\tilde{\varphi}_{Mlcx})+np_{inlcx}^H Hr_{lcx}^H(\tilde{\varphi}_{Hlcx})$$

$$+np_{inhcx}^H Hr_{hcx}^H(\tilde{\varphi}_{Hhcx})=R_{Ld}+R_{Md}+R_{Hd}+R_{Mlcx}+R_{Hlcx}+R_{Hhcx}=$$

$$=Lr_d^L(\tilde{\varphi}_L)+Mr_d^M(\tilde{\varphi}_M)+Hr_d^H(\tilde{\varphi}_H)+np_{inlcx}^M Mr_d^M(\tilde{\varphi}_{lcxM})+np_{inlcx}^H Hr_d^H(\tilde{\varphi}_{lcxH})+np_{inhcx}^H Hr_d^H(\tilde{\varphi}_{hcxH}) \quad (65)^{44}$$

$$\Pi_t = L\pi_d^L(\tilde{\varphi}_L)+M\pi_d^M(\tilde{\varphi}_M)+H\pi_d^H(\tilde{\varphi}_H)+np_{inlcx}^M M\pi_{lcx}^M(\tilde{\varphi}_{Mlcx})$$

$$+np_{inlcx}^H H\pi_{lcx}^H(\tilde{\varphi}_{Hlcx})+np_{inhcx}^H H\pi_{hcx}^H(\tilde{\varphi}_{Hhcx})=$$

$$=L\left[\frac{1}{\sigma}r_d^L(\tilde{\varphi}_L)\text{-}f_L\right]+M\left[\frac{1}{\sigma}r_d^M(\tilde{\varphi}_M)\text{-}f_M\right]+H\left[\frac{1}{\sigma}r_d^H(\tilde{\varphi}_H)\text{-}f_H\right]+$$

$$np_{inlcx}^M M\left[\frac{1}{\sigma}r_{lcx}^M(\tilde{\varphi}_{Mlcx})\text{-}f_{lcx}\right]+np_{inlcx}^H H\left[\frac{1}{\sigma}r_{lcx}^H(\tilde{\varphi}_{Hlcx})\text{-}f_{lcx}\right]+np_{inhcx}^H H\left[\frac{1}{\sigma}r_{hcx}^H(\tilde{\varphi}_{Hhcx})\text{-}f_{hcx}\right]=$$

$$= \Pi_{Ld}+\Pi_{Md}+\Pi_{Hd}+\Pi_{Mlcx}+\Pi_{Hlcx}+\Pi_{Hhcx} =$$

$$= L\left[\frac{1}{\sigma}r_d^L(\tilde{\varphi}_L)\text{-}f_L\right]+M\left[\frac{1}{\sigma}r_d^M(\tilde{\varphi}_M)\text{-}f_M\right]+H\left[\frac{1}{\sigma}r_d^H(\tilde{\varphi}_H)\text{-}f_H\right]+np_{inlcx}^M M\left[\frac{1}{\sigma}r_d^M(\tilde{\varphi}_{lcxM})\text{-}f_{lcx}\right]+$$
$$np_{inlcx}^H H\left[\frac{1}{\sigma}r_d^H(\tilde{\varphi}_{lcxH})\text{-}f_{lcx}\right]+np_{inhcx}^H H\left[\frac{1}{\sigma}r_d^H(\tilde{\varphi}_{hcxH})\text{-}f_{hcx}\right] \quad (66)^{45}$$

The average price, quantity, revenue and profits in the industry in the Open Economy setting are obtained as a weighted average of the price, quantity, revenue and profits of each group of firms:[46]

$$\bar{p}_t = \frac{P_t}{Z_t^{\frac{1}{1\text{-}\sigma}}} = \left[\frac{L}{Z_t}p_d^L(\tilde{\varphi}_L)^{1\text{-}\sigma}+\frac{M}{Z_t}p_d^M(\tilde{\varphi}_M)^{1\text{-}\sigma}+\frac{H}{Z_t}p_d^H(\tilde{\varphi}_H)^{1\text{-}\sigma}+\frac{np_{inlcx}^M M}{Z_t}p_d^M(\tilde{\varphi}_{lcxM})^{1\text{-}\sigma}+\frac{np_{inlcx}^H H}{Z_t}p_d^H(\tilde{\varphi}_{lcxH})^{1\text{-}\sigma}+\frac{np_{inhcx}^H H}{Z_t}p_d^H(\tilde{\varphi}_{hcxH})^{1\text{-}\sigma}\right] =$$

$$= \left[\frac{L}{Z_t}p_d^L(\tilde{\varphi}_L)^{1\text{-}\sigma}+\frac{M}{Z_t}p_d^M(\tilde{\varphi}_M)^{1\text{-}\sigma}+\frac{H}{Z_t}p_d^H(\tilde{\varphi}_H)^{1\text{-}\sigma}+\frac{np_{inlcx}^M M}{Z_t}p_{lcx}^M(\tilde{\varphi}_{Mlcx})^{1\text{-}\sigma}+\frac{np_{inlcx}^H H}{Z_t}p_{lcx}^H(\tilde{\varphi}_{Hlcx})^{1\text{-}\sigma}+\frac{np_{inhcx}^H H}{Z_t}p_{hcx}^H(\tilde{\varphi}_{Hhcx})^{1\text{-}\sigma}\right]^{\frac{1}{1-\sigma}} \quad (67)$$

$$\bar{q}_t=\frac{Q_t}{Z_t^\rho} = \left[\frac{L}{Z_t}q_d^L(\tilde{\varphi}_L)^\rho+\frac{M}{Z_t}q_d^M(\tilde{\varphi}_M)^\rho+\frac{H}{Z_t}q_d^H(\tilde{\varphi}_H)^\rho+\frac{np_{inlcx}^M M}{Z_t}q_d^M(\tilde{\varphi}_{lcxM})^\rho+\frac{np_{inlcx}^H H}{Z_t}q_d^H(\tilde{\varphi}_{lcxH})^\rho+\frac{np_{inhhcx}^H H}{Z_t}q_d^H(\tilde{\varphi}_{hcxH})^\rho\right]^{\frac{1}{\rho}} =$$

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[42] Recall from expression (47) that $p_{ix}^T(\varphi) = \frac{\tau}{\rho}\frac{c_T}{\varphi(1+\beta_i)} = \frac{\tau}{(1+\beta_i)}p_d^T(\varphi),\ i = lc, hc$.

[43] Recall from expression (48) that $q_{ix}^T(\varphi) = \left(\frac{(1+\beta_i)}{\tau}\right)^\sigma q_d^T(\varphi)$. Therefore, $q_{ix}^T(\varphi) = EP^{\sigma-1}\left(\frac{1}{p_{ix}^T(\varphi)}\right)^\sigma = EP^{\sigma-1}\left(\frac{(1+\beta_i)}{\tau p_d^T(\varphi)}\right)^\sigma$, just as $q_d^T(\varphi) = EP^{\sigma-1}\left(\frac{1}{p_d^T(\varphi)}\right)^\sigma$. Also note that $q_{ix}^T(\tilde{\varphi}_{Tix}) = EP^{\sigma-1}\left(\frac{\rho\tilde{\varphi}_{Tix}(1+\beta_i)}{c_T\tau}\right)^{\sigma-1} = EP^{\sigma-1}\left(\frac{\rho\tilde{\varphi}_{ixT}}{c_T}\right)^\sigma = q_d^T(\tilde{\varphi}_{ixT})$.

[44] Recall from expression (49) that $r_{xi}^T(\varphi) = E\left(\frac{P\rho\varphi(1+\beta_i)}{c_T\tau}\right)^{\sigma-1} = \left(\frac{(1+\beta_i)}{\tau}\right)^{\sigma-1} r_d^T(\varphi)$. Note that $r_{ix}^T(\tilde{\varphi}_{Tix}) = E\left(\frac{P\rho\tilde{\varphi}_{Tix}(1+\beta_i)}{c_T\tau}\right)^{\sigma-1} = E\left(\frac{P\rho\tilde{\varphi}_{ixT}}{c_T}\right)^{\sigma-1} = r_d^T(\tilde{\varphi}_{ixT})$.

[45] Recall from expression (50) that $\pi_d^T(\varphi) = \frac{1}{\sigma}r_d^T(\varphi) - f_T$ and $\pi_{ix}^T(\varphi) = \frac{1}{\sigma}r_{ix}^T(\varphi) - f_{ix}^T$.

[46] Once again, as it was the case in the closed economy setting, these "theoretical" averages are calculated just in order to provide a rough measure of the industry's per firm performance. They need not coincide with the price, quantity, revenue and profits of any particular firm.

$$\left[\frac{L}{Z_t} q_d^L(\tilde{\varphi}_L)^\rho + \frac{M}{Z_t} q_d^M(\tilde{\varphi}_M)^\rho + \frac{H}{Z_t} q_d^H(\tilde{\varphi}_H)^\rho + \frac{np_{inlcx}^M M}{Z_t} q_{lcx}^M(\tilde{\varphi}_{Mlcx})^\rho + \frac{np_{inlcx}^H H}{Z_t} q_{lcx}^H(\tilde{\varphi}_{Hlcx})^\rho + \frac{np_{inhcx}^H H}{Z_t} q_{hcx}^H(\tilde{\varphi}_{Hhcx})^\rho\right]^{\frac{1}{\rho}} \quad (68)$$

$$\bar{r}_t = \frac{R_t}{Z_t} = \frac{L}{Z_t} r_d^L(\tilde{\varphi}_L) + \frac{M}{Z_t} r_d^M(\tilde{\varphi}_M) + \frac{H}{Z_t} r_d^H(\tilde{\varphi}_H) + \frac{np_{inlcx}^M M}{Z_t} r_d^M(\tilde{\varphi}_{lcxM}) + \frac{np_{inlcx}^H H}{Z_t} r_d^H(\tilde{\varphi}_{lcxH}) + \frac{np_{inhcx}^H H}{Z_t} r_d^H(\tilde{\varphi}_{hcxH}) =$$

$$\frac{L}{Z_t} r_d^L(\tilde{\varphi}_L) + \frac{M}{Z_t} r_d^M(\tilde{\varphi}_M) + \frac{H}{Z_t} r_d^H(\tilde{\varphi}_H) + \frac{np_{inlcx}^M M}{Z_t} r_{lcx}^M(\tilde{\varphi}_{Mlcx}) + \frac{np_{inlcx}^H H}{Z_t} r_{lcx}^H(\tilde{\varphi}_{Hlcx}) + \frac{np_{inhcx}^H H}{Z_t} r_{hcx}^H(\tilde{\varphi}_{Hhcx}) \quad (69)$$

$$\bar{\pi}_t = \frac{\Pi_t}{Z_t} = \frac{L}{Z_t} \pi_d^L(\tilde{\varphi}_L) + \frac{M}{Z_t} \pi_d^M(\tilde{\varphi}_M) + \frac{H}{Z_t} \pi_d^H(\tilde{\varphi}_H) + \frac{np_{inlcx}^M M}{Z_t} \pi_d^M(\tilde{\varphi}_{lcxM}) + \frac{np_{inlcx}^H H}{Z_t} \pi_d^H(\tilde{\varphi}_{lcxH}) + \frac{np_{inhcx}^H H}{Z_t} \pi_d^H(\tilde{\varphi}_{hcxH}) =$$

$$= \frac{L}{Z_t} \pi_d^L(\tilde{\varphi}_L) + \frac{M}{Z_t} \pi_d^M(\tilde{\varphi}_M) + \frac{H}{Z_t} \pi_d^H(\tilde{\varphi}_H) + \frac{np_{inlcx}^M M}{Z_t} \pi_{lcx}^M(\tilde{\varphi}_{Mlcx}) + \frac{np_{inlcx}^H H}{Z_t} \pi_{lcx}^H(\tilde{\varphi}_{Hlcx}) + \frac{np_{inhcx}^H H}{Z_t} \pi_{hcx}^H(\tilde{\varphi}_{Hhcx}) \quad (70)$$

In each country, firms using technology T get "in average" the following revenue and profits:

$$\bar{r}^L = r_d^L(\tilde{\varphi}_L) \quad (71)$$

$$\bar{r}^M = r_d^M(\tilde{\varphi}_M) + np_{inlcx}^M r_{lcx}^M(\tilde{\varphi}_{Mlcx}) \quad (72)$$

$$\bar{r}^H = r_d^H(\tilde{\varphi}_H) + np_{inlcx}^H r_{lcx}^H(\tilde{\varphi}_{Hlcx}) + np_{inhcx}^H r_{hcx}^H(\tilde{\varphi}_{Hhcx}) \quad (73)$$

$$\bar{\pi}^L = \pi_d^L(\tilde{\varphi}_L) \quad (74)$$

$$\bar{\pi}^M = \pi_d^M(\tilde{\varphi}_M) + np_{inlcx}^M \pi_{lcx}^M(\tilde{\varphi}_{Mlcx}) \quad (75)$$

$$\bar{\pi}^H = \pi_d^H(\tilde{\varphi}_H) + np_{inlcx}^H \pi_{lcx}^H(\tilde{\varphi}_{Hlcx}) + np_{inhcx}^H \pi_{hcx}^H(\tilde{\varphi}_{Hhcx}) \quad (76)$$

4.3 Equilibrium Conditions in the Open Economy

The conditions for Free Entry (FE^T) and Technology Upgrading (UP1 and UP2) are the same as in the Closed Economy setting. However, ECP^T is now different because average profit per firm using technology T in the Open Economy setting includes a "domestic profit component" and an "export profit component", which in turn subdivides into two categories: a "Low-Commitment export profit component" and a "High-Commitment export profit component". Thus, in order to construct the "Effective Cutoff overall Profit Condition for technology T" (ECP^T) it is necessary to first relate the average profit level for each exporting category to the minimum idiosyncratic productivity level required to enter such category, with which it will be obtained the "Effective Cutoff type *i* export Profit condition for technology T" (ECP_{ix}^T), *i: lc, hc*. That is, a relation must be established between:

- $\bar{\pi}_{lcx}^M = \pi_{lcx}^M(\tilde{\varphi}_{Mlcx})$ with φ_{lcx}^*
- $\bar{\pi}_{lcx}^H = \pi_{lcx}^H(\tilde{\varphi}_{Hlcx})$ with φ^{H*}
- $\bar{\pi}_{hcx}^H = \pi_{hcx}^H(\tilde{\varphi}_{Hhcx})$ with φ_{hcx}^*

The expression of the "Effective Cutoff Low-Commitment export Profit condition for technology M" (ECP_{lcx}^M) is:

$$\boxed{\pi_{lcx}^M(\varphi_{lcx}^*) = 0 \quad \leftrightarrow \quad r_{lcx}^M(\varphi_{lcx}^*) = \sigma f_{lcx} \quad \leftrightarrow \quad \bar{\pi}_{lcx}^M = \pi_{lcx}^L(\tilde{\varphi}_{Mlcx}) = f_{lcx} k_{lcx}^M(\varphi_{lcx}^*)}$$

where $k_{lcx}^M(\varphi_{lcx}^*) = \left[\frac{\tilde{\varphi}_{Mlcx}(\varphi_{lcx}^*, \varphi^{H*})}{\varphi_{lcx}^*}\right]^{\sigma-1} - 1.$

The expression of the "Effective Cutoff Low-Commitment export Profit condition for technology H" (ECP_{lcx}^H) is:

$$\boxed{\pi^H_{lcx}(\varphi^{H*}) = A^H_{lcx} \quad \leftrightarrow \quad r^H_{lcx}(\varphi^{H*}) = \sigma\left(A^H_{lcx} + f_{lcx}\right) \quad \leftrightarrow \quad \bar{\pi}^H_{lcx} = \pi^H_{lcx}(\tilde{\varphi}_{Hlcx}) = A^H_{lcx}h^H_{lcx}(\varphi^{H*}) + f_{lcx}k^H_{lcx}(\varphi^{H*})}$$

where $h^H_{lcx}(\varphi^{H*}) = \left[\frac{\tilde{\varphi}_{Hlcx}(\varphi^{H*},\varphi^*_{hcx})}{\varphi^{H*}}\right]^{\sigma-1}$, $k^H_{lcx}(\varphi^{H*}) = \left[\frac{\tilde{\varphi}_{Hlcx}(\varphi^{H*},\varphi^*_{hcx})}{\varphi^{H*}}\right]^{\sigma-1} - 1$ and $A^H_{lcx} > 0$ is the profit gained in each export destination by the least idiosyncratically productive firm who is a "Low-Commitment Exporter" and uses technology H (a constant).[47]

The expression of the "Effective Cutoff High-Commitment export Profit condition for technology H" (ECP^H_{hcx}) is:

$$\boxed{\pi^H_{hcx}(\varphi^*_{hcx}) = A^H_{hcx} \quad \leftrightarrow \quad r^H_{hcx}(\varphi^*_{hcx}) = \sigma\left(A^H_{hcx} + f_{hcx}\right) \quad \leftrightarrow \quad \bar{\pi}^H_{hcx} = \pi^H_{hcx}(\tilde{\varphi}_{Hhcx}) = A^H_{hcx}h^H_{hcx}(\varphi^*_{hcx}) + f_{hcx}k^H_{hcx}(\varphi^*_{hcx})}$$

where $h^H_{hcx}(\varphi^*_{hcx}) = \left[\frac{\tilde{\varphi}_{Mhcx}(\varphi^*_{hcx},\infty)}{\varphi^*_{hcx}}\right]^{\sigma-1}$, $k^H_{hcx}(\varphi^*_{hcx}) = \left[\frac{\tilde{\varphi}_{Mhcx}(\varphi^*_{hcx},\infty)}{\varphi^*_{hcx}}\right]^{\sigma-1} - 1$ and $A^H_{hcx} > 0$ is the profit gained in each export destination by the least idiosyncratically productive firm who is a "High-Commitment Exporter" and uses technology H (a constant).[48]

Now we can finally write down the "Effective Cutoff overall Profit Condition for technology T" (ECP^T), which implies a relationship between the average overall profit per firm using technology T ($\bar{\pi}^T$) and the effective cutoff idiosyncratic productivity level for the adoption of technology T (φ^{T*}).

The "Effective Cutoff overall Profit Condition for technology L" (ECP^L) is:

$$\bar{\boldsymbol{\pi}}^{\boldsymbol{L}} = \pi^L_d(\tilde{\varphi}_L) = \boldsymbol{f_L k^L_d(\varphi^{L*})} \tag{77}$$

The "Effective Cutoff overall Profit Condition for technology M" (ECP^M) is:

$$\bar{\boldsymbol{\pi}}^{\boldsymbol{M}} = \pi^M_d(\tilde{\varphi}_M) + p^M_{inlcx}n\pi^M_{lcx}(\tilde{\varphi}_{Mlcx}) =$$

$$= \left[\boldsymbol{A^M_d h^M_d(\varphi^{M*}) + f_M k^M_d(\varphi^{M*})}\right] + \boldsymbol{p^M_{inlcx}n}\left[\boldsymbol{A^M_{lcx}h^M_{lcx}(\varphi^*_{lcx}) + f_{lcx}k^M_{lcx}(\varphi^*_{lcx})}\right] \tag{78}$$ [49]

The "Effective Cutoff overall Profit Condition for technology H" (ECP^H) is:

$$\bar{\boldsymbol{\pi}}^{\boldsymbol{H}} = \pi^H_d(\tilde{\varphi}_H) + p^H_{inlcx}n\pi^H_{lcx}(\tilde{\varphi}_{Hlcx}) + p^H_{inhcx}n\pi^H_{hcx}(\tilde{\varphi}_{Hhcx}) = \left[\boldsymbol{A^H_d h^H_d(\varphi^{H*}) + f_H k^H_d(\varphi^{H*})}\right] + \boldsymbol{p^H_{inlcx}n}\left[\boldsymbol{A^H_{lcx}h^H_{lcx}(\varphi^{H*}) + f_{lcx}k^H_{lcx}(\varphi^{H*})}\right] + \boldsymbol{p^H_{inhcx}n}\left[\boldsymbol{A^H_{hcx}h^H_{hcx}(\varphi^*_{hcx}) + f_{hcx}f^H_{hcx}(\varphi^*_{hcx})}\right] \tag{79}$$ [50]

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[47] We already know that profits in general, and export profits in particular are increasing in φ. Besides, we know that $\pi^M_{lcx}(\varphi^*_{lcx}) = 0$ and $\varphi^{H*} > \varphi^*_{lcx}$. Thus, we know $\pi^M_{lcx}(\varphi^{H*}) > 0$. Finally, we also know that, keeping all other factors constant, using a superior production technology determines higher revenue and consequently higher profits for the firm. As a result, we know $\pi^H_{lcx}(\varphi^{H*}) \geq \pi^M_{lcx}(\varphi^{H*}) > 0$. We pin down this information by writing $\pi^H_{lcx}(\varphi^{H*}) = A^H_{lcx}$, where A^H_{lcx} is a positive constant. On other grounds, we already know that $\bar{\pi}^H_{lcx} = \pi^H_{lcx}(\tilde{\varphi}_{Hlcx}) = \frac{r^H_{lcx}(\tilde{\varphi}_{Hlcx})}{\sigma} - f_{lcx} = \left(\frac{\tilde{\varphi}_{Hlcx}(\varphi^{H*},\varphi^*_{lcx})}{\varphi^{H*}}\right)^{\sigma-1}\frac{r^H_{lcx}(\varphi^{H*})}{\sigma} - f_{lcx}$. Replacing $r^H_{lcx}(\varphi^{H*}) = \sigma\left(A^H_{lcx} + f_{lcx}\right)$in this equation we obtain $\bar{\pi}^H_{lcx} = \pi^H_{lcx}(\tilde{\varphi}_{Hlcx}) = A^H_{lcx}\left(\frac{\tilde{\varphi}_{Hlcx}(\varphi^{H*},\varphi^*_{hcx})}{\varphi^{H*}}\right)^{\sigma-1} + f_{lcx}\left(\frac{\tilde{\varphi}_{Hlcx}(\varphi^{H*},\varphi^*_{hcx})}{\varphi^{H*}}\right)^{\sigma-1} - f_{lcx} =$
$= A^H_{lcx}\left(\frac{\tilde{\varphi}_{Hlcx}(\varphi^{H*},\varphi^*_{hcx})}{\varphi^{H*}}\right)^{\sigma-1} + f_{lcx}\left[\left(\frac{\tilde{\varphi}_{Hlcx}(\varphi^{H*},\varphi^*_{hcx})}{\varphi^{H*}}\right)^{\sigma-1} - 1\right] = A^H_{lcx}h^H_{lcx}(\varphi^{H*}) + f_{lcx}k^H_{lcx}(\varphi^{H*})$.

[48] The same reasoning in footnote 47 applies here.

[49] $\varphi^*_{lcx} = \varphi^{M*}\tau\left(\frac{f_{lcx}}{f_M}\right)^{\frac{1}{\sigma-1}}$ and thus equation (78) is implicitly a function of φ^{M*}.

[50] $\varphi^*_{hcx} = \varphi^{H*}\frac{\tau}{\beta_{hc}}\left(\frac{A^H_{hcx}+f_{hcx}}{A^H_d+f_H}\right)^{\frac{1}{\sigma-1}}$ and thus equation (79) is implicitly a function of φ^{H*}.

The present value of average profits flows for firms using technology T (that is, the value of such firms) remains $\bar{v}^T = \sum_{t=0}^{\infty}(1-\delta)^t \bar{\pi}^T = \frac{\bar{\pi}^T}{\delta}$.

4.4 Determination of the Equilibrium in the Open Economy

Like in the Closed Economy setting, determination of the equilibrium in the Open Economy requires solving a system formed by the two Conditions for Technology Upgrading (UP1 and UP2) and a third equation which is obtained by substituting the three Effective Cutoff Profit Conditions for each technology T (ECP^T) into the equation expressing the value of the average firm in each technology T, the three of which in turn are incorporated into the Free Entry Condition (FE). This way, this last condition ends up expressed as a function of the idiosyncratic productivity thresholds a firm must surpass to enter each group or category of firms. Such categories are no longer defined solely in terms of the production technologies used (L, M, H), but also in terms of the chosen market strategies (nx, lcx, hcx). Nevertheless, even though the number of firm categories and thus the number of apparent unknowns in the system escalates now to five[51], it must be taken into account that the thresholds for the adoption of the different market strategies are implicitly defined in terms of the thresholds for the adoption of the different production technologies. Consequently, the genuine unknowns in the equations system are the same three as in the Closed Economy setting: the thresholds for the adoption of each available production technology.

As it happened in the Closed Economy, here too is not possible to determine generically whether an equilibrium (or several) exist. However, if by particularizing the problem –through making additional assumptions regarding the parameters and the productivities distribution $g(\varphi)$- makes it possible to find values of the thresholds for the adoption of technologies L, M and H that determine an equilibrium, then these in turn trivially determine the thresholds for the adoption of the different market strategies.

A particular case which is of interest due to its analytical simplicity is the following: when all firms using the same production technology T, share as well the same market strategy –whichever this may be.[52] Under this simplifying assumption, some of the thresholds for the adoption of technologies (φ^{L*}, φ^{M*} and φ^{H*}) will coincide with the thresholds to begin exporting with low and high commitment (φ^*_{lcx} and φ^*_{hcx}). Thus, in this special case φ^{L*}, φ^{M*} and φ^{H*} also determine the averages $\tilde{\varphi}$, $\tilde{\varphi}_{lcx}$, $\tilde{\varphi}_{hcx}$ and $\tilde{\varphi}_t$, the ex ante probability of entry to each available production technology (p^L_{in}, p^M_{in} y p^H_{in}) and the ex ante probability of adoption of each market strategy, conditional on the production technology the firm is using (p^M_{inlcx}, p^H_{inlcx} y p^H_{inhcx}). Note that in the proposed special case, each of these probabilities is either 1 or 0. The case in which $\varphi^*_{lcx} = \varphi^{M*}$ and $\varphi^*_{hcx} = \varphi^{H*}$ (and consequently $p^H_{inlcx} = 0$ and $p^M_{inlcx} = p^H_{inhcx} = 1$) can be considered a limiting case preserving the "plausible" order of the idiosyncratic productivity thresholds proposed in preceding sections ($\varphi^{L*} \leq \varphi^{M*} \leq \varphi^*_{lcx} \leq \varphi^{H*} \leq \varphi^*_{hcx}$). Thus, the analysis will focus in this case.

5. Evaluating the Impact of Trade

If a steady state equilibrium exists in the Open Economy, this will share some basic characteristics already described for its Closed Economy counterpart: not only the number of successful entrants to the market $p^{L+M+H}_{in} Z_e$ –where Z_e is the total number of new entrants- must exactly replace the δZ firms that are hit with the bad shock and exit (this is the "Aggregate Stability Condition": $p^{L+M+H}_{in} Z_e = \delta Z$), but also the number of new successful entrants into each production technology T

[51] These three groups are, according to the assumptions made: firms who produce with technology L and do not export, firms who produce with technology M and do not export, firms who produce with technology M and export with low commitment, firms who produce with technology H and export with low commitment and firms who produce with technology H and export with high commitment.

[52] Because φ^*_{lcx} y φ^*_{hcx} are defined as a function of some φ^{T*}, materialization of this special case requires that certain parametric restrictions must hold.

must exactly replace the number of exiting firms in each technology, so that the aggregate variables remain constant over time –see equations (36), (37) and (38). Nevertheless, because market conditions have changed, the actual value of the key variables characterizing the steady state equilibrium in the Open Economy will differ from the values they had in the steady state equilibrium in the Closed Economy.

We have denoted the cutoff idiosyncratic productivity levels for each production technology in autarky as φ_L^*, φ_M^* and φ_H^*, and the cutoff idiosyncratic productivity levels for each production technology in the Open Economy setting as φ^{L*}, φ^{M*} and φ^{H*}. Let as well $\tilde{\varphi}_L^{closed}$, $\tilde{\varphi}_M^{closed}$ and $\tilde{\varphi}_M^{closed}$ denote the average idiosyncratic productivity levels for each technology in autarky, and $\tilde{\varphi}_L^{open}$, $\tilde{\varphi}_M^{open}$ and $\tilde{\varphi}_M^{open}$ in the Open Economy.

Like in Melitz, increased competition in the final goods market is the most intuitive channel to reason the impact of trade. However, such channel is not operative in the present model neither, because the two key assumptions that rule it out in Melitz are maintained: Monopolistic Competition market structure and CES preferences. As a consequence of these assumptions, the price elasticity of demand for each variety does not react in response to changes in the number or prices of competing varieties. This way, here too the mechanism through which trade impacts the economy is the increased competition for fixed labor resources (bear in mind that all costs are expressed in terms of labor). Because entry to the export market is costly, when the economy opens up to trade the demand for labor (skilled and unskilled, being the former reducible to equivalent units of the latter) increases, causing wages to escalate. This increased labor cost forces some former users of technology L out of the market, as they are no longer able to make nonnegative profits.[53] Figure 4 illustrates this:

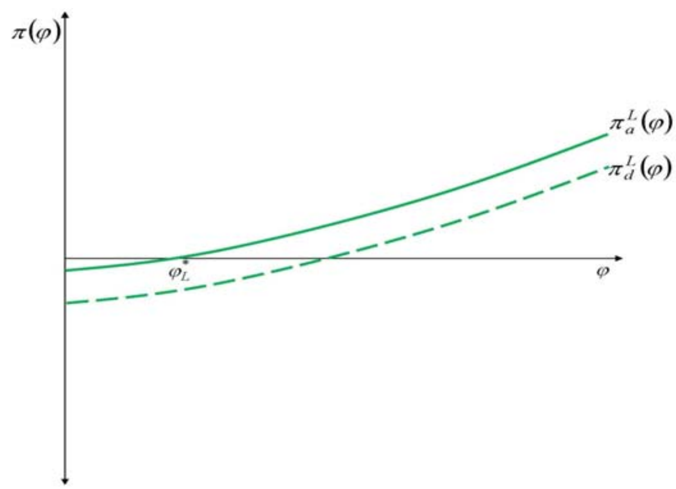
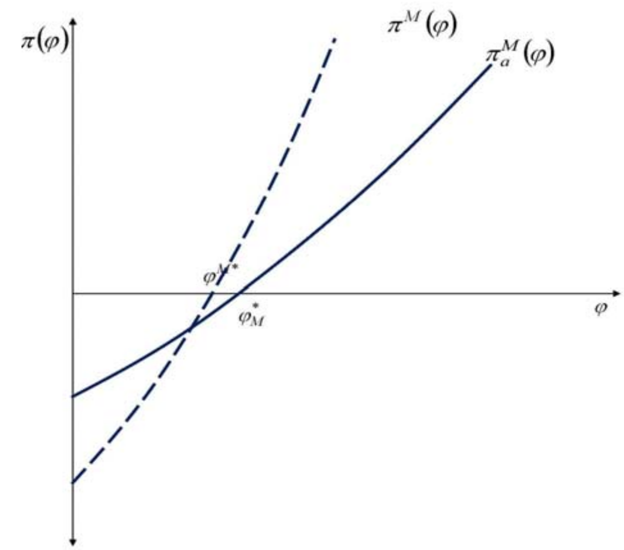


Figure 4

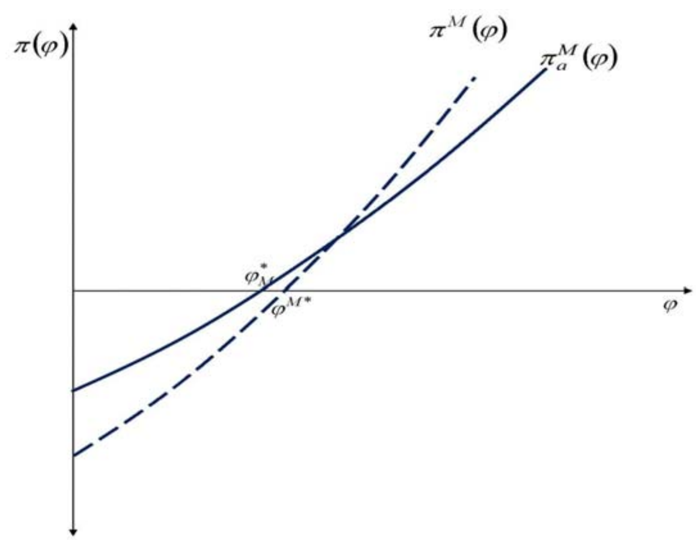
The impact of trade is uncertain for firms that in autarky were producing with technologies M or H, because there are two opposed forces at play. On the one hand, when the economy opens up to trade all active firms in the market will experience an increase in their costs, because in order to cover the fixed costs associated to exporting more labor is needed, and this upward shift in the demand for labor causes wages to go up. However, while for firms using technology L in the Open Economy this negative impact is the only change they experience relative to autarky, the situation is different for firms using technologies M or H. Such firms will simultaneously experience a positive effect consisting in the expansion of their total sales, obtained through participation in the export markets (with low or high commitment). Depending on which of these two opposed effects is stronger, two

[53] Those whose productivity lies between φ_L^* and φ^{L*}.

alternative overall results will come along with trade, as illustrated for technology M in panels (a) and (b) of Figure 5:[54]



(a)



(b)

Figure 5

Panel (a) illustrates the situation in which the magnitude of the positive impact of the greater sales associated to exporting is larger than the negative impact of the increased labor cost. The overall result in this case is a decrease of the idiosyncratic productivity threshold a firm has to surpass to adopt technology M (technology H). Consequently, some firms that in autarky were using technology L (technology M) will now upgrade their technology by adopting technology M (technology H).[55] On the contrary, panel (b) illustrates the situation in which the magnitude of the negative impact of the

[54] The reasoning is in all analogous for technology H.

[55] Those whose productivity lies between φ_M^* y φ^{M*} (Those whose productivity lies between φ_H^* y φ^{H*}).

increased labor cost is stronger, causing the idiosyncratic productivity threshold a firm has to surpass to adopt technology M (technology H) to increase. As a result of this, upgrading technology will be harder in the Open Economy, and some firms that in the Closed Economy were using technology M (technology H) may even downgrade technology and produce with technology L (technology M) in the Open Economy.[56] In both cases, an alternative way to reason these outcomes is to think that when eventually some active firms will be put out of the market by the negative shock δ, their replacements (who in the steady state possess the same idiosyncratic productivity as the falling incumbents) will enter the market with a superior technology –in the case illustrated in panel (a)- or with an inferior technology –in the case illustrated in panel (b).

It is important to note that no matter which of the two scenarios depicted above will materialize, the idiosyncratic productivity threshold associated with entry to the market always rises (as shown in Figure 4), determining that average idiosyncratic productivity increases with trade. If, additionally, the thresholds for the adoption of technologies M and H decrease –as illustrated in panel (a) of Figure 5-, then overall productivity undoubtedly increases, because the quality of the inputs used –capital and labor- raises too, reinforcing the increase in idiosyncratic productivity. This is because the idiosyncratic productivity threshold to enter the market (with technology L) increases, and at the same time the thresholds for the adoption of technologies M and H decrease, so the proportion of firms using these last technologies must increase with certainty. If on the contrary, the productivity thresholds for the adoption of technologies M and H raise –as illustrated in panel (b) of Figure 5-, then the impact of trade on overall productivity is undetermined. On the one hand, it tends to increase because idiosyncratic productivity increases. On the other hand, the fact that all three technology adoption thresholds increase makes it unclear whether the proportion of firms using the superior technologies increases, decreases or remains constant –unless additional assumptions are made. Consequently, in this scenario it is not possible to precise whether technological factors will contribute to an increase in overall productivity in the Open Economy setting (which would be the case if the proportion of firms using technologies M and H increased) or will pull it down (which would be the case if the proportion of firms using technology L increased).[57] In any case, average idiosyncratic productivity always increases, and because only those firms whose productivity lies above the thresholds φ_{lcx}^* and φ_{hcx}^* enter the export market –with low or high commitment, respectively-, this reinforces the reallocation of market shares towards more efficient firms, contributing to the aggregate productivity gain.

The analysis of the steady state equilibrium in the Open Economy, in particular the labor market and final good market clearing conditions, is analogous to what has been already described for the Closed Economy steady state equilibrium. Because the analysis is circumscribed to long term equilibria, it is assumed that labor will reallocate between technologies as needed to guarantee full employment.[58]

5.1 The Impact of Trade on Aggregate Welfare

Static welfare per worker is still given by:

$$W = \frac{1}{P} = \frac{1}{Z_t^{\frac{1}{1-\sigma}} \bar{p}_t}$$

[56] Bear in mind the assumptions of no depreciation and perfect malleability of capital goods.

[57] The result in this respect depends on the value of parameters and the shape of the distribution $g(\varphi)$.

[58] Only long term equilibria are considered in this paper. Consequently, even though each production technology employs skilled and unskilled workers in fixed proportions, the analysis focuses on the result obtained after the necessary adjustments in the qualification of workers has already taken place. If in the Open Economy equilibrium the proportion of firms using technology L is reduced compared to autarky, this means that the excess unskilled labor will be unemployed in the short run until some of this workers acquire the skills necessary to be reallocated into technologies M and H (which are more intensive in skilled labor). If on the contrary, the proportion of firms using technology L increases in the trade equilibrium, then in the short run some skilled workers that in autarky were employed in technologies M and H will be temporarily sub employed in technology L, performing tasks for which they are overqualified. In the long run, the decrease in the incentives to acquire skills (through the educational sector) will lead to an adjustment of the proportions of skilled and unskilled labor to the new economic conditions (there will be fewer skilled workers and more unskilled workers).

Therefore, it once again increases the larger the number of available varieties, but now this result can be achieved not only if country size increases, but also through trade. Even though the number of domestic incumbent firms is lower in the open economy setting than it was in autarky ($Z_{open} < Z_{closed}$), the number of foreign firms who now export to the home country typically overcompensates such reduction in the number of local firms, resulting in increased variety and thus fostering an increase in static welfare ($Z_t = Z_{open}\left(1 + np_{inlcx}^{L+M+H} + np_{inhcx}^{L+M+H}\right) > Z_{closed}$).[59] Besides, static welfare also increases if the average price decreases, which can happen for two reasons. The first is, like in Melitz, the gain in aggregate idiosyncratic productivity, which comes as a result of the increase in the threshold a firm must surpass to enter the market.[60] The second factor which exerts an impact on aggregate welfare –and was precluded by construction in the Melitz (2003) model- is the technology-specific variable cost: the greater the proportion of firms using the superior technologies, the lower the average price will tend to be, not only because such firms are idiosyncratically more productive, but also because they have lower variable production costs, and can therefore charge lower prices (for a given quality). This result is obtained with certainty if the idiosyncratic productivity thresholds for the adoption of technologies M and H decrease when the economy opens up to trade –the situation illustrated in panel (a) of Figure 5-. However, if the thresholds for the adoption of technologies M and H increase –the situation illustrated in panel (b) of Figure 5-, we cannot anticipate how the proportions of firms using each of the available technologies will vary among domestic firms in each country –or even if they will vary at all- without making further assumptions regarding technology parameters and the distribution $g(\varphi)$. In any case, whenever the proportion of firms using technologies M and especially H increases, not only static welfare will increase, but also dynamic welfare, because in the model technical progress is internalized by firms through utilization of the more efficient capital goods such technologies employ.

In the special case in which we are focusing now, with $p_{inlcx}^{M} = p_{inhcx}^{H} = 1$, it can be readily noted that the proportion of firms using technology L among all the firms (domestic and foreign) competing in the home country tends to decrease as the number of trading partners n rises, not only when the idiosyncratic productivity thresholds for the adoption of technologies M are H decrease –panel (a) of Figure 5-, but also when they increase –panel (b) of Figure 5. This is because all the firms exporting to the home country are users of technologies M or H. Consequently, unless the proportion of firms using technology L among surviving domestic firms rises significantly when the economy opens up to trade[61], then the overall outcome will be a decreased participation of technology L users in the total number of firms (domestic and foreign) competing in each country, thus conducing to a lower average price and higher welfare.

As a result of all this, we cannot assure that welfare per worker, as measured above, will always increase when the economy opens up to trade, which is an important difference with the Melitz (2003) model (see Appendix F). However, what we do know is that if at least one of the three above mentioned factors (variety, aggregate productivity or average variable production cost) is sufficiently "better" in the open economy setting than in autarky (that is, if at least one the first two factors is sufficiently higher, or if the third is sufficiently lower), static welfare per worker will here too increase when trade is allowed.

6. Final Considerations

The model shows that, given its idiosyncratic productivity distribution $g(\varphi)$, the industry will achieve a more favorable aggregate result –in terms of higher aggregate and average revenue and profits and

[59] However the possibility that the number of domestic firms being pushed out of the market be larger than the number of exporting foreign firms entering it is not ruled out, especially if trade costs are high.

[60] This is a likely but not certain result, because even though aggregate productivity "at the factory gate" (that is, before it is corrected to capture transport iceberg costs and the effects of additional trade-related fixed invesments) always increases, once these corrections are made there is a possibility that $\tilde{\varphi}_t < \tilde{\varphi}_{closed}$, if τ is sufficiently large and β_{hc} , f_{lcx}^{T} and f_{hcx}^{T} are sufficiently low.

[61] This could only happen in the case illustrated in panel (b) of Figure 5, never in the case illustrated in panel (a).

also in terms of welfare per worker- the higher the number of firms in it whose idiosyncratic productivity surpasses the thresholds for the adoption of technologies M and H, beyond what can be attributed to the increase in industry average productivity. The reason is that to be in grade of taking advantage from the technical progress embodied in the higher quality intermediate capital goods used in technologies M and H and from the higher relative efficiency of the skilled labor they employ more intensively, the individual firm needs first to adopt such technologies, which will only be able to do if it possesses a high enough idiosyncratic productivity φ. Consequently, if there are in the industry many firms whose productivity φ lies above the thresholds for the adoption of the superior production technologies (that is, if its distribution $g(\varphi)$ is skewed towards the high values in the domain), the introduction of technology choice in the model (which opens up the possibility of adopting a superior production technology if a certain standard is met –that is, if the firm has a sufficiently high $\varphi -$ leads to an increase in revenue and profits, both for the individual firm holding the required φ and for the industry as a whole, as total and average revenue and profits increase. Welfare per worker also increases. On the contrary, if there are in the industry many firms whose productivity φ lies below the above mentioned thresholds (that is, if its distribution $g(\varphi)$ is skewed towards the low values in the domain), then the introduction of technology choice in the model leads only to a slight increase in welfare, revenue and profits, as very few firms will be in grade of adopting the superior technologies. On other grounds, it must also be taken into account that even if technology L is identical to Melitz's single production technology and the distribution $g(\varphi)$ also coincides in both models, the threshold for entry to the market will be higher here than in Melitz (2003), because of the impact of the costs associated to technology upgrading. This causes average idiosyncratic productivity to be higher in this model than in Melitz's. Finally, if no firm in the industry is productive enough to adopt any of the superior technologies (that is, if the corresponding thresholds are too high to be relevant), then the results of the model are identical to those obtained prior to the introduction of technology choice: as no firm is in grade to choose any of the newly introduced technologies, the situation is virtually the same as if technology choice had not been introduced.

The explanation behind these results is that firm overall productivity has two components: on the one hand, idiosyncratic factors (which are captured by the parameter φ and cannot be modified –at least in the context of this model-, and on the other hand, the quality –that is, the efficiency- of the inputs used for production (namely, the quality of the intermediate capital goods and the skill-intensity of the labor employed). If we added to this that in the real world it is reasonable to think that in the medium or long run the quality of the technology used exerts an influence on the firm's idiosyncratic productivity –a possibility that in the present model is excluded-, this would lead to the emergence of a virtuous circle (if the industry is characterized by a distribution $g(\varphi)$ skewed towards the high values in the domain) or a vicious circle (if the industry is characterized by a distribution $g(\varphi)$ skewed towards the low values in the domain).

The present model shares Melitz's result that when the economy opens up to trade, increased competition reallocates market shares toward the more efficient firms, thus forcing the least productive ones to exit, as they can no longer make nonnegative profits. The productivity threshold to enter the industry rises and therefore so does average productivity "at the factory gate". However, because exporters actual productivity when they arrive with their variety in the export destination is corrected downward to capture the effect of transport costs, and in the case of "High-Commitment Exporters" it is also corrected upward to capture the effect of their additional trade-related fixed investments, it cannot be assured that aggregate idiosyncratic productivity $\tilde{\varphi}_t$ will actually increase. However, the higher the fixed export costs f_{lcx}^T, f_{hcx}^T and the effects of additional trade-related fixed investments β_{hc}, and the lower the variable transport and tariff costs τ, the greater the chances that $\tilde{\varphi}_t$ will be higher than the aggregate idiosyncratic productivity in the closed economy setting, thus promoting an increase in aggregate static welfare. On other grounds, even though the number of domestic firms decreases in the open economy, the total number of incumbent firms Z_t is likely to increase, leading as well to higher aggregate static welfare due to increased variety.

Another important result of the model is that trade can also induce changes in the thresholds for the adoption of the superior technologies, M and H, which can become higher or lower. This uncertain outcome comes as a result of two opposed simultaneous effects at play. On the one hand, trade induces an increase in wages, which tends to make technology upgrading costlier (because all costs are expressed in labor) and consequently pushes upward the idiosyncratic productivity thresholds a firm must surpass to adopt technologies M and H. On the other hand, it brings about the possibility of increasing revenue through export sales, which in turn facilitates access to the superior technologies (which have a fixed cost of adoption) for exporting firms. If the positive effect of trade is larger than its negative counterpart, then the final outcome is that when the economy opens up to trade the idiosyncratic productivity thresholds a firm has to surpass to adopt production technologies M and H will be lower than in autarky, and thus a technology upgrading process will take place. If on the contrary, the negative effect of the higher labor cost dominates, then both thresholds will be higher in the Open Economy and upgrading technology will become more difficult. Some firms that in autarky produced with M or H may even downgrade their technology to L or M, respectively.[62] In both cases, an alternative way to reason these outcomes is to think that when eventually some active firms will be put out of the market by the negative shock δ, their replacements (who in the steady state possess the same idiosyncratic productivity as the falling incumbents) will enter the market with a superior technology –if trade induces a decrease in the thresholds for the adoption of M and H- or with an inferior technology –if trade induces an increase in the thresholds for the adoption of M and H.

If the positive effect of trade is stronger, and consequently the thresholds a firm needs to surpass to upgrade technology are lower in the Open Economy, then both the absolute number and the proportion of firms using technology H (which incorporates the best available technical progress and thus possesses the lowest average variable cost) increases in each country, whereas the absolute number and the proportion of firms using technology L decrease.[63] If on the contrary, the negative effect of trade dominates and consequently the thresholds a firm needs to surpass in order to upgrade technology become higher, then the absolute number of firms using technology H is reduced with certainty in each country. However, it is not possible to anticipate –without making additional assumptions about the shape of the distribution $g(\varphi)$ and the cost parameters- whether the proportion of firms using technology H increases, decreases or remains unchanged –which is the relevant thing for the determination of the average variable cost. The same is true for technologies M and L. Consequently, regarding the portion of productive static efficiency that depends on the quality of the inputs used, in this case the result is uncertain if only domestic firms are taken into account. However, if all the firms (domestic and foreign) competing in the local market are considered, then the proportion of users of technology L decreases with certainty as the number of trading partners, n, increases. This is because all the foreign firms exporting their varieties to the domestic economy are users of technologies M or H. Consequently, unless the proportion of users of technology L raises significantly among domestic survivors, the general result will be a decrease in the participation of technology L among all firms (domestic and foreign) competing in each country. This results in a decrease in the average variable production cost and average price.

As a result of all the above discussed, we cannot assure that static welfare per worker will always increase when the economy opens up to trade, which is an important difference with the Melitz model. However, it is indeed certain that if at least one of these three factors (variety, "final" aggregate productivity or average variable production cost) are sufficiently better in the Open Economy setting than in autarky (that is: if at least one of the first two is sufficiently higher, or if the third is sufficiently lower), static welfare per worker will here too increase when trade is allowed. Meanwhile, dynamic welfare (based on the capacity of the country to absorb technical progress –which in the context of this model is done through using better quality capital goods in production) will increase if the proportion of domestic firms using the superior production technologies rises, especially if technology H becomes more widely adopted among domestic firms, and will decrease otherwise. Therefore, if the positive effect of trade is stronger –in which case the thresholds for technology upgrading are lowered-, dynamic welfare will raise undoubtedly, no matter what the value of parameters and the distribution $g(\varphi)$ may be. On the contrary, if the negative effect of trade dominates –in which case the thresholds for technology upgrading raise-, the outcome regarding dynamic welfare will depend exclusively on

[62] Bear in mind the assumptions of no depreciation and perfect malleability capital goods.

[63] It is not possible to know a priori whether the relative participation of technology M will increase or decrease in each country, as the two thresholds that together determine this (the threshold to adopt M and the threshold to adopt H) both shift in the same direction (they are both lowered).

the value of parameters (which may be influenced by trade agreements and policy) and the shape of the distribution $g(\varphi)$.

Finally, it is worth noting that despite the model presented in this paper does not require that the superior technologies employ import capital goods –it is only required that the quality of the capital goods they employ be superior-, if we look at it in the light of the ideas cast in the introduction, then its results can be reinterpreted in the context of the problem of international technology diffusion. More precisely, it is possible to think that the country of origin of the intermediate capital goods employed in each alternative production technology is determinant of such goods' quality, being this higher the shorter the distance between the technology frontier belonging to the country where the capital good in question was produced and the world-technology-frontier. This highlights the crucial influence that a country's trade policy, including the choice of its trading partners, may exert over the productivity level it will be able to achieve, and consequently over its growth trajectory.

Because solving the model requires assuming a specific productivities distribution $g(\varphi)$ and results can vary depending on such choice and on the value of parameters, an interesting extension would be to particularize the analysis by specifying different alternative distributions –with different skews- and parameter values. This could allow to establish if an equilibrium (or several) may be obtained, and if so is the case, to compare the equilibria obtained in different special cases which may be considered of interest.

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8. Appendixes

Appendix A: Order of idiosyncratic productivity thresholds for the adoption of technologies L, M and H in the Closed Economy

As the quality of technology increases, the fixed cost associated to it f_T also increases, but the variable cost c_T simultaneously decreases, enabling the firm to charge a lower price and consequently increase its revenue and variable profit. Firms with high enough idiosyncratic productivity (those surpassing the thresholds φ_M^* and φ_H^*) also increase their total profits.

In the Closed Economy, each successful entrant to the market faces three options:

1) Produce using technology L, and earn profits $\pi_d^L(\varphi) = E(P\rho)^{\sigma-1}\frac{1}{\sigma}c_L^{1-\sigma}\varphi^{\sigma-1} - f_L$
2) Produce using technology M, and earn profits $\pi_d^M(\varphi) = E(P\rho)^{\sigma-1}\frac{1}{\sigma}c_M^{1-\sigma}\varphi^{\sigma-1} - f_M$
3) Produce using technology H, and earn profits $\pi_d^H(\varphi) = E(P\rho)^{\sigma-1}\frac{1}{\sigma}c_H^{1-\sigma}\varphi^{\sigma-1} - f_H$

Applying the approach of Bustos (2005), it is possible to decompose those profits as follows:

1. Profits earned producing with technology L:
$$\pi_d^L(\varphi)$$
2. Increase in revenue experienced when upgrading technology from L to M:
$$dr_d^{LM}(\varphi) = E(P\rho)^{\sigma-1}\frac{1}{\sigma}(c_M^{1-\sigma} - c_L^{1-\sigma})\varphi^{\sigma-1}$$
3. Increase in revenue experienced when upgrading technology from M to H:
$$dr_d^{MH}(\varphi) = E(P\rho)^{\sigma-1}\frac{1}{\sigma}(c_H^{1-\sigma} - c_M^{1-\sigma})\varphi^{\sigma-1}$$

This way, the profits associated to each technology option can be expressed as:

1) $\pi_d^L(\varphi) = \pi_d^L(\varphi)$
2) $\pi_d^M(\varphi) = \pi_d^L(\varphi) + dr_d^{LM}(\varphi) - (f_M - f_L)$
3) $\pi_d^H(\varphi) = \pi_d^L(\varphi) + dr_d^{LM}(\varphi) + dr_d^{MH}(\varphi) - (f_H - f_M) - (f_M - f_L) = \pi_d^L(\varphi) + dr_d^{LM}(\varphi) + dr_d^{MH}(\varphi) - (f_H - f_L)$

Thus, the firm will have incentives to upgrade technology from L to M whenever:

$$\pi_d^L(\varphi) < \pi_d^M(\varphi) \quad \leftrightarrow \quad \pi_d^L(\varphi) < \pi_d^L(\varphi) + dr_d^{LM}(\varphi) - (f_M - f_L) \quad \leftrightarrow \quad 0 < dr_d^{LM}(\varphi) - (f_M - f_L) \quad \leftrightarrow$$

$$\leftrightarrow \quad dr_d^{LM}(\varphi) < -(f_M - f_L) \quad \leftrightarrow \quad dr_d^{LM}(\varphi) > (f_M - f_L) \leftrightarrow E(P\rho)^{\sigma-1}\frac{1}{\sigma}(c_M^{1-\sigma} - c_L^{1-\sigma})\varphi^{\sigma-1} > f_M - f_L \leftrightarrow$$

$$\leftrightarrow \varphi^{\sigma-1} > \frac{\sigma(f_M-f_L)}{E(P\rho)^{\sigma-1}(c_M^{1-\sigma}-c_L^{1-\sigma})} \leftrightarrow \varphi > \left[\frac{\sigma(f_M-f_L)}{E(P\rho)^{\sigma-1}(c_M^{1-\sigma}-c_L^{1-\sigma})}\right]^{\frac{1}{\sigma-1}}.$$ Entonces, $\varphi_M^* = \left[\frac{\sigma(f_M-f_L)}{E(P\rho)^{\sigma-1}(c_M^{1-\sigma}-c_L^{1-\sigma})}\right]^{\frac{1}{\sigma-1}}$

Similarly the, firm will have incentives to upgrade technology from M to H whenever:

$$\pi_d^M(\varphi) < \pi_d^H(\varphi) \leftrightarrow \pi_d^L(\varphi) + dr_d^{LM}(\varphi) - (f_M - f_L) < \pi_d^L(\varphi) + dr_d^{LM}(\varphi) + dr_d^{MH}(\varphi) - (f_H - f_L) \leftrightarrow$$

$$\leftrightarrow -f_M + f_L < dr_d^{MH}(\varphi) - f_H + f_L \leftrightarrow -f_M < dr_d^{MH}(\varphi) - f_H \leftrightarrow f_H - f_M < dr_d^{MH}(\varphi) \leftrightarrow$$

$$\leftrightarrow f_H - f_M < E(P\rho)^{\sigma-1}\frac{1}{\sigma}(c_H^{1-\sigma} - c_M^{1-\sigma})\varphi^{\sigma-1} \leftrightarrow \varphi^{\sigma-1} > \frac{\sigma(f_H - f_M)}{E(P\rho)^{\sigma-1}(c_H^{1-\sigma} - c_M^{1-\sigma})} \leftrightarrow$$

$$\leftrightarrow \varphi > \left[\frac{\sigma(f_H-f_M)}{E(P\rho)^{\sigma-1}(c_H^{1-\sigma}-c_M^{1-\sigma})}\right]^{\frac{1}{\sigma-1}}.$$ Entonces, $\varphi_H^* = \left[\frac{\sigma(f_H-f_M)}{E(P\rho)^{\sigma-1}(c_H^{1-\sigma}-c_M^{1-\sigma})}\right]^{\frac{1}{\sigma-1}}$.

To obtain $\varphi_M^* < \varphi_H^*$, it must be true that $\left[\frac{\sigma(f_M-f_L)}{E(P\rho)^{\sigma-1}(c_M^{1-\sigma}-c_L^{1-\sigma})}\right]^{\frac{1}{\sigma-1}} < \left[\frac{\sigma(f_H-f_M)}{E(P\rho)^{\sigma-1}(c_H^{1-\sigma}-c_M^{1-\sigma})}\right]^{\frac{1}{\sigma-1}} \leftrightarrow$

$$\leftrightarrow \frac{f_M - f_L}{c_M^{1-\sigma} - c_L^{1-\sigma}} < \frac{f_H - f_M}{c_H^{1-\sigma} - c_M^{1-\sigma}} \leftrightarrow \frac{f_M - f_L}{\frac{1}{c_M^{\sigma-1}} - \frac{1}{c_L^{\sigma-1}}} < \frac{f_H - f_M}{\frac{1}{c_H^{\sigma-1}} - \frac{1}{c_M^{\sigma-1}}}$$

Roughly speaking, this means that the neat gain the firm obtains when upgrading technology form M to H must be smaller "in proportion" than the neat gain it obtains when upgrading it from L to M.[64]

Appendix B: Aggregation Conditions in the Closed Economy

The aggregate price, quantity, revenue (or expenditure) and profits can be derived using the expression of average industry productivity $\tilde{\varphi}$ in (29).

Derivation of the aggregate price P:

$$P = \left[\int_{\varphi_L^*}^{\varphi_M^*} p_d^L(\varphi)^{1-\sigma} L\mu(\varphi)d\varphi + \int_{\varphi_M^*}^{\varphi_H^*} p_d^M(\varphi)^{1-\sigma} M\mu(\varphi)d\varphi + \int_{\varphi_H^*}^{\infty} p_d^H(\varphi)^{1-\sigma} H\mu(\varphi)d\varphi\right]^{\frac{1}{1-\sigma}} =$$

$$= \left[\int_{\varphi_L^*}^{\varphi_M^*} \left(\frac{1}{\rho}\frac{c_L}{\varphi}\right)^{1-\sigma} L\mu(\varphi)d\varphi + \int_{\varphi_M^*}^{\varphi_H^*} \left(\frac{1}{\rho}\frac{c_M}{\varphi}\right)^{1-\sigma} M\mu(\varphi)d\varphi + \int_{\varphi_H^*}^{\infty} \left(\frac{1}{\rho}\frac{c_H}{\varphi}\right)^{1-\sigma} H\mu(\varphi)d\varphi\right]^{\frac{1}{1-\sigma}} =$$

$$= \left[\int_{\varphi_L^*}^{\varphi_M^*} \left(\frac{1}{\rho}c_L\right)^{1-\sigma} \frac{1}{\varphi^{1-\sigma}} L\mu(\varphi)d\varphi + \int_{\varphi_M^*}^{\varphi_H^*} \left(\frac{1}{\rho}c_M\right)^{1-\sigma} \frac{1}{\varphi^{1-\sigma}} M\mu(\varphi)d\varphi + \int_{\varphi_H^*}^{\infty} \left(\frac{1}{\rho}c_H\right)^{1-\sigma} \frac{1}{\varphi^{1-\sigma}} H\mu(\varphi)d\varphi\right]^{\frac{1}{1-\sigma}} =$$

$$= \left[L\int_{\varphi_L^*}^{\varphi_M^*} \left(\frac{1}{\rho}c_L\right)^{1-\sigma} \varphi^{\sigma-1}\mu(\varphi)d\varphi + M\int_{\varphi_M^*}^{\varphi_H^*} \left(\frac{1}{\rho}c_M\right)^{1-\sigma} \varphi^{\sigma-1}\mu(\varphi)d\varphi + H\int_{\varphi_H^*}^{\infty} \left(\frac{1}{\rho}c_H\right)^{1-\sigma} \varphi^{\sigma-1}\mu(\varphi)d\varphi\right]^{\frac{1}{1-\sigma}} =$$

$$= \left[L\left(\frac{1}{\rho}c_L\right)^{1-\sigma} \int_{\varphi_L^*}^{\varphi_M^*} \varphi^{\sigma-1}\mu(\varphi)d\varphi + M\left(\frac{1}{\rho}c_M\right)^{1-\sigma} \int_{\varphi_M^*}^{\varphi_H^*} \varphi^{\sigma-1}\mu(\varphi)d\varphi + H\left(\frac{1}{\rho}c_H\right)^{1-\sigma} \int_{\varphi_H^*}^{\infty} \varphi^{\sigma-1}\mu(\varphi)d\varphi\right]^{\frac{1}{1-\sigma}} =$$

$$= \left[L\left(\frac{1}{\rho}c_L\right)^{1-\sigma} \tilde{\varphi}_L{}^{\sigma-1} + M\left(\frac{1}{\rho}c_M\right)^{1-\sigma} \tilde{\varphi}_M{}^{\sigma-1} + H\left(\frac{1}{\rho}c_H\right)^{1-\sigma} \tilde{\varphi}_H{}^{\sigma-1}\right]^{\frac{1}{1-\sigma}} =$$

$$= \left[L\left(\frac{1}{\rho}c_L\right)^{1-\sigma} \left(\frac{1}{\tilde{\varphi}_L}\right)^{-(\sigma-1)} + M\left(\frac{1}{\rho}c_M\right)^{1-\sigma} \left(\frac{1}{\tilde{\varphi}_M}\right)^{-(\sigma-1)} + H\left(\frac{1}{\rho}c_H\right)^{1-\sigma} \left(\frac{1}{\tilde{\varphi}_H}\right)^{-(\sigma-1)}\right]^{\frac{1}{1-\sigma}} =$$

$$= \left[L\left(\frac{1}{\rho}c_L\right)^{1-\sigma} \frac{1}{\tilde{\varphi}_L{}^{1-\sigma}} + M\left(\frac{1}{\rho}c_M\right)^{1-\sigma} \frac{1}{\tilde{\varphi}_M{}^{1-\sigma}} + H\left(\frac{1}{\rho}c_H\right)^{1-\sigma} \frac{1}{\tilde{\varphi}_H{}^{1-\sigma}}\right]^{\frac{1}{1-\sigma}} =$$

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[64] The analytic expression for φ_L^* is easy to obtain as well: $\pi_d^L(\varphi) > 0 \leftrightarrow E(P\rho)^{\sigma-1}\frac{1}{\sigma}c_L^{1-\sigma}\varphi^{\sigma-1} > f_L \leftrightarrow \varphi > \left[\frac{\sigma f_L}{E(P\rho)^{\sigma-1}c_L^{1-\sigma}}\right]^{\frac{1}{\sigma-1}}$. Thus, $\varphi_L^* = \left[\frac{\sigma f_L}{E(P\rho)^{\sigma-1}c_L^{1-\sigma}}\right]^{\frac{1}{\sigma-1}}$. To obtain $\varphi_L^* < \varphi_M^* < \varphi_H^*$, it must be true that $\left[\frac{\sigma f_L}{E(P\rho)^{\sigma-1}c_L^{1-\sigma}}\right]^{\frac{1}{\sigma-1}} < \left[\frac{\sigma(f_M-f_L)}{E(P\rho)^{\sigma-1}(c_M^{1-\sigma}-c_L^{1-\sigma})}\right]^{\frac{1}{\sigma-1}} < \left[\frac{\sigma(f_H-f_M)}{E(P\rho)^{\sigma-1}(c_H^{1-\sigma}-c_M^{1-\sigma})}\right]^{\frac{1}{\sigma-1}} \leftrightarrow \frac{f_L}{c_L^{1-\sigma}} < \frac{f_M-f_L}{c_M^{1-\sigma}-c_L^{1-\sigma}} < \frac{f_H-f_M}{c_H^{1-\sigma}-c_M^{1-\sigma}}$.

$$= \left[L\left(\frac{1}{\rho}\frac{c_L}{\tilde{\varphi}_L}\right)^{1-\sigma} + M\left(\frac{1}{\rho}\frac{c_M}{\tilde{\varphi}_M}\right)^{1-\sigma} + H\left(\frac{1}{\rho}\frac{c_H}{\tilde{\varphi}_H}\right)^{1-\sigma}\right]^{\frac{1}{1-\sigma}} =$$
$$= [Lp_d^L(\tilde{\varphi}_L)^{1-\sigma} + Mp_d^M(\tilde{\varphi}_M)^{1-\sigma} + Hp_d^H(\tilde{\varphi}_H)^{1-\sigma}]^{\frac{1}{1-\sigma}}.$$

Derivation of the aggregate quantity Q:

$$Q = \left[\int_{\varphi_L^*}^{\varphi_M^*} q_d^L(\varphi)^\rho L\mu(\varphi)d\varphi + \int_{\varphi_M^*}^{\varphi_H^*} q_d^M(\varphi)^\rho M\mu(\varphi)d\varphi + \int_{\varphi_H^*}^{\infty} q_d^H(\varphi)^\rho H\mu(\varphi)d\varphi\right]^{\frac{1}{\rho}} =$$
$$= \left[\int_{\varphi_L^*}^{\varphi_M^*} q_d^L(\tilde{\varphi}_L)^\rho \left(\frac{\varphi}{\tilde{\varphi}_L}\right)^{\sigma\rho} L\mu(\varphi)d\varphi + \int_{\varphi_M^*}^{\varphi_H^*} q_d^M(\tilde{\varphi}_M)^\rho \left(\frac{\varphi}{\tilde{\varphi}_M}\right)^{\sigma\rho} M\mu(\varphi)d\varphi + \int_{\varphi_H^*}^{\infty} q_d^H(\tilde{\varphi}_H)^\rho \left(\frac{\varphi}{\tilde{\varphi}_H}\right)^{\sigma\rho} H\mu(\varphi)d\varphi\right]^{\frac{1}{\rho}} =$$
$$= \left[L\int_{\varphi_L^*}^{\varphi_M^*} q_d^L(\tilde{\varphi}_L)^\rho \left(\frac{\varphi}{\tilde{\varphi}_L}\right)^{\sigma-1} \mu(\varphi)d\varphi + M\int_{\varphi_M^*}^{\varphi_H^*} q_d^M(\tilde{\varphi}_M)^\rho \left(\frac{\varphi}{\tilde{\varphi}_M}\right)^{\sigma-1} \mu(\varphi)d\varphi + H\int_{\varphi_H^*}^{\infty} q_d^H(\tilde{\varphi}_H)^\rho \left(\frac{\varphi}{\tilde{\varphi}_H}\right)^{\sigma-1} \mu(\varphi)d\varphi\right]^{\frac{1}{\rho}} =$$
$$= \left[L\int_{\varphi_L^*}^{\varphi_M^*} q_d^L(\tilde{\varphi}_L)^\rho \frac{1}{\tilde{\varphi}_L{}^{\sigma-1}} \varphi^{\sigma-1}\mu(\varphi)d\varphi + M\int_{\varphi_M^*}^{\varphi_H^*} q_d^M(\tilde{\varphi}_M)^\rho \frac{1}{\tilde{\varphi}_M{}^{\sigma-1}} \varphi^{\sigma-1}\mu(\varphi)d\varphi + H\int_{\varphi_H^*}^{\infty} q_d^H(\tilde{\varphi}_H)^\rho \frac{1}{\tilde{\varphi}_H{}^{\sigma-1}} \varphi^{\sigma-1}\mu(\varphi)d\varphi\right]^{\frac{1}{\rho}} =$$
$$= \left[Lq_d^L(\tilde{\varphi}_L)^\rho \frac{1}{\tilde{\varphi}_L{}^{\sigma-1}} \int_{\varphi_L^*}^{\varphi_M^*} \varphi^{\sigma-1}\mu(\varphi)d\varphi + Mq_d^M(\tilde{\varphi}_M)^\rho \frac{1}{\tilde{\varphi}_M{}^{\sigma-1}} \int_{\varphi_M^*}^{\varphi_H^*} \varphi^{\sigma-1}\mu(\varphi)d\varphi + Hq_d^H(\tilde{\varphi}_H)^\rho \frac{1}{\tilde{\varphi}_H{}^{\sigma-1}} \int_{\varphi_H^*}^{\infty} \varphi^{\sigma-1}\mu(\varphi)d\varphi\right]^{\frac{1}{\rho}} =$$
$$= \left[Lq_d^L(\tilde{\varphi}_L)^\rho \frac{1}{\tilde{\varphi}_L{}^{\sigma-1}} \tilde{\varphi}_L{}^{\sigma-1} + Mq_d^M(\tilde{\varphi}_M)^\rho \frac{1}{\tilde{\varphi}_M{}^{\sigma-1}} \tilde{\varphi}_M{}^{\sigma-1} + Hq_d^H(\tilde{\varphi}_H)^\rho \frac{1}{\tilde{\varphi}_H{}^{\sigma-1}} \tilde{\varphi}_H{}^{\sigma-1}\right]^{\frac{1}{\rho}} =$$
$$= [Lq_d^L(\tilde{\varphi}_L)^\rho + Mq_d^M(\tilde{\varphi}_M)^\rho + Hq_d^H(\tilde{\varphi}_H)^\rho]^{\frac{1}{\rho}}.$$

Derivation of the aggregate revenue (or expenditure) R:

$$R = \int_{\varphi_L^*}^{\varphi_M^*} r_d^L(\varphi)L\mu(\varphi)d\varphi + \int_{\varphi_M^*}^{\varphi_H^*} r_d^M(\varphi)M\mu(\varphi)d\varphi + \int_{\varphi_H^*}^{\infty} r_d^H(\varphi)H\mu(\varphi)d\varphi =$$
$$= \int_{\varphi_L^*}^{\varphi_M^*} r_d^L(\tilde{\varphi}_L)\left(\frac{\varphi}{\tilde{\varphi}_L}\right)^{\sigma-1} L\mu(\varphi)d\varphi + \int_{\varphi_M^*}^{\varphi_H^*} r_d^M(\tilde{\varphi}_M)\left(\frac{\varphi}{\tilde{\varphi}_M}\right)^{\sigma-1} M\mu(\varphi)d\varphi + \int_{\varphi_H^*}^{\infty} r_d^H(\tilde{\varphi}_H)\left(\frac{\varphi}{\tilde{\varphi}_H}\right)^{\sigma-1} H\mu(\varphi)d\varphi =$$
$$= L\int_{\varphi_L^*}^{\varphi_M^*} r_d^L(\tilde{\varphi}_L)\left(\frac{\varphi}{\tilde{\varphi}_L}\right)^{\sigma-1} \mu(\varphi)d\varphi + M\int_{\varphi_M^*}^{\varphi_H^*} r_d^M(\tilde{\varphi}_M)\left(\frac{\varphi}{\tilde{\varphi}_M}\right)^{\sigma-1} \mu(\varphi)d\varphi + H\int_{\varphi_H^*}^{\infty} r_d^H(\tilde{\varphi}_H)\left(\frac{\varphi}{\tilde{\varphi}_H}\right)^{\sigma-1} \mu(\varphi)d\varphi =$$
$$= L\int_{\varphi_L^*}^{\varphi_M^*} r_d^L(\tilde{\varphi}_L)\frac{1}{\tilde{\varphi}_L{}^{\sigma-1}}\varphi^{\sigma-1}\mu(\varphi)d\varphi + M\int_{\varphi_M^*}^{\varphi_H^*} r_d^M(\tilde{\varphi}_M)\frac{1}{\tilde{\varphi}_M{}^{\sigma-1}}\varphi^{\sigma-1}\mu(\varphi)d\varphi + H\int_{\varphi_H^*}^{\infty} r_d^H(\tilde{\varphi}_H)\frac{1}{\tilde{\varphi}_H{}^{\sigma-1}}\varphi^{\sigma-1}\mu(\varphi)d\varphi =$$
$$= Lr_d^L(\tilde{\varphi}_L)\frac{1}{\tilde{\varphi}_L{}^{\sigma-1}}\int_{\varphi_L^*}^{\varphi_M^*}\varphi^{\sigma-1}\mu(\varphi)d\varphi + Mr_d^M(\tilde{\varphi}_M)\frac{1}{\tilde{\varphi}_M{}^{\sigma-1}}\int_{\varphi_M^*}^{\varphi_H^*}\varphi^{\sigma-1}\mu(\varphi)d\varphi + Hr_d^H(\tilde{\varphi}_H)\frac{1}{\tilde{\varphi}_H{}^{\sigma-1}}\int_{\varphi_H^*}^{\infty}\varphi^{\sigma-1}\mu(\varphi)d\varphi =$$
$$= Lr_d^L(\tilde{\varphi}_L)\frac{1}{\tilde{\varphi}_L{}^{\sigma-1}}\tilde{\varphi}_L{}^{\sigma-1} + Mr_d^M(\tilde{\varphi}_M)\frac{1}{\tilde{\varphi}_M{}^{\sigma-1}}\tilde{\varphi}_M{}^{\sigma-1} + Hr_d^H(\tilde{\varphi}_H)\frac{1}{\tilde{\varphi}_H{}^{\sigma-1}}\tilde{\varphi}_H{}^{\sigma-1} =$$
$$= Lr_d^L(\tilde{\varphi}_L) + Mr_d^M(\tilde{\varphi}_M) + Hr_d^H(\tilde{\varphi}_H).$$

Derivation of the aggregate profits Π:

$$\Pi = \int_{\varphi_L^*}^{\varphi_M^*} \pi_d^L(\varphi)L\mu(\varphi)d\varphi + \int_{\varphi_M^*}^{\varphi_H^*} \pi_d^M(\varphi)M\mu(\varphi)d\varphi + \int_{\varphi_H^*}^{\infty} \pi_d^H(\varphi)H\mu(\varphi)d\varphi =$$
$$= \int_{\varphi_L^*}^{\varphi_M^*}\left[\frac{1}{\sigma}r_d^L(\varphi) - f_L\right]L\mu(\varphi)d\varphi + \int_{\varphi_M^*}^{\varphi_H^*}\left[\frac{1}{\sigma}r_d^M(\varphi) - f_M\right]M\mu(\varphi)d\varphi + \int_{\varphi_H^*}^{\infty}\left[\frac{1}{\sigma}r_d^H(\varphi) - f_H\right]H\mu(\varphi)d\varphi =$$
$$= L\left[\frac{1}{\sigma}r_d^L(\tilde{\varphi}_L) - f_L\right] + M\left[\frac{1}{\sigma}r_d^M(\tilde{\varphi}_M) - f_M\right] + H\left[\frac{1}{\sigma}r_d^H(\tilde{\varphi}_H) - f_H\right] =$$
$$= L\pi_d^L(\tilde{\varphi}_L) + M\pi_d^M(\tilde{\varphi}_M) + H\pi_d^H(\tilde{\varphi}_H)$$

Appendix C: Equivalence between the Aggregate Entry Cost, Total Unskilled Labor Employed Directly or Indirectly in Setting Up of Business and Aggregate Profits (Final Good Sector

$$U_e = Z_e f_e^L + p_{in}^M Z_e(f_e^M - f_e^L) + p_{in}^H Z_e(f_e^H - f_e^L) =$$
$$= Z_e\left\{p_{in}^{L+M+H}\left[\frac{p_{in}^L}{p_{in}^{L+M+H}}\frac{\bar{\pi}^L}{\delta} + \frac{p_{in}^M}{p_{in}^{L+M+H}}\left(\frac{\bar{\pi}^M}{\delta} - \left(f_e^M - f_e^L\right)\right) + \frac{p_{in}^H}{p_{in}^{L+M+H}}\left(\frac{\bar{\pi}^H}{\delta} - \left(f_e^H - f_e^L\right)\right)\right]\right\} + p_{in}^M\left(f_e^M - f_e^L\right) + p_{in}^H\left(f_e^H - f_e^L\right) =$$
$$= Z_e\left\{p_{in}^L\frac{\bar{\pi}^L}{\delta} + p_{in}^M\frac{\bar{\pi}^M}{\delta} + p_{in}^H\frac{\bar{\pi}^H}{\delta}\right\} = Z_e p_{in}^{L+M+H}\left\{\frac{p_{in}^L}{p_{in}^{L+M+H}}\frac{\bar{\pi}^L}{\delta} + \frac{p_{in}^M}{p_{in}^{L+M+H}}\frac{\bar{\pi}^M}{\delta} + \frac{p_{in}^H}{p_{in}^{L+M+H}}\frac{\bar{\pi}^H}{\delta}\right\} = \delta Z\left\{\frac{L}{Z}\frac{\bar{\pi}^L}{\delta} + \frac{M}{Z}\frac{\bar{\pi}^M}{\delta} + \frac{H}{Z}\frac{\bar{\pi}^H}{\delta}\right\} =$$
$$= L\bar{\pi}^L + M\bar{\pi}^M + H\bar{\pi}^H = \overline{\Pi}^L + \overline{\Pi}^M + \overline{\Pi}^H = \overline{\Pi}$$

Appendix D: Aggregate Labor Resources Used to Cover the Export Costs

The ratio the ratio of new "Low-Commitment Exporters" who use technology M to all "Low-Commitment Exporters" who use technology M is $\frac{p^M_{inlcx} p^M_{in} Z_e}{p^M_{inlcx} M} = \delta$ because the Aggregate Stability Condition for technology M implies $p^M_{in} Z_e = \delta M$. Analogously, the ratio of Low-Commitment Exporters" who use technology H to all "Low-Commitment Exporters" who use technology H is $\frac{p^H_{inlcx} p^H_{in} Z_e}{p^H_{inlcx} H} = \delta$ because the Aggregate Stability Condition for technology H implies $p^H_{in} Z_e = \delta H$. Finally, the ratio of new "High-Commitment Exporters" who use technology H to all "High-Commitment Exporters" who use technology H is $\frac{p^H_{inhcx} p^H_{in} Z_e}{p^H_{inhcx} H} = \delta$ because the Aggregate Stability Condition for technology H implies $p^H_{in} Z_e = \delta H$. In general terms, the ratio of the new exporters with commitment level *i* who produce with technology T to all the exporters with commitment level *i* who produce with technology T is $\frac{p^T_{inix} p^T_{in} Z_e}{p^T_{inix} T} = \delta$.

Appendix E: Comprehensive Measure of Aggregate Productivity in the Open Economy

$$\tilde{\varphi}_t^{\sigma-1} =$$

$$\frac{1}{Z_t}\Bigg[Z\Big(\frac{L}{Z}\int_{\varphi_L^*}^{\varphi_M^*}\varphi^{\sigma-1}\mu(\varphi)\,d\varphi + \frac{M}{Z}\int_{\varphi_M^*}^{\varphi_H^*}\varphi^{\sigma-1}\mu(\varphi)\,d\varphi + \frac{H}{Z}\int_{\varphi_H^*}^{\infty}\varphi^{\sigma-1}\mu(\varphi)\,d\varphi\Big) +$$

$$np_{inlcx}^{L+M+H}Z\left(\theta\int_{\varphi_{lcx}^*}^{\varphi_H^*}\left(\frac{\varphi}{\tau}\right)^{\sigma-1}\mu(\varphi)\,d\varphi + (1-\theta)\int_{\varphi_H^*}^{\varphi_{hcx}^*}\left(\frac{\varphi}{\tau}\right)^{\sigma-1}\mu(\varphi)\,d\varphi\right) + np_{inhcx}^{L+M+H}Z\left(\int_{\varphi_{hcx}^*}^{\infty}\left(\frac{\varphi(1+\beta_{hc})}{\tau}\right)^{\sigma-1}\mu(\varphi)\,d\varphi\right)\Bigg] =$$

$$\frac{1}{Z_t}\Bigg[L\int_{\varphi_L^*}^{\varphi_M^*}\varphi^{\sigma-1}\mu(\varphi)\,d\varphi + M\int_{\varphi_M^*}^{\varphi_H^*}\varphi^{\sigma-1}\mu(\varphi)\,d\varphi + H\int_{\varphi_H^*}^{\infty}\varphi^{\sigma-1}\mu(\varphi)\,d\varphi +$$

$$np_{inlcx}^{L+M+H}Z\left(\theta\frac{1}{\tau^{\sigma-1}}\int_{\varphi_{lcx}^*}^{\varphi_H^*}\varphi^{\sigma-1}\mu(\varphi)\,d\varphi + (1-\theta)\frac{1}{\tau^{\sigma-1}}\int_{\varphi_H^*}^{\varphi_{hcx}^*}\varphi^{\sigma-1}\mu(\varphi)\,d\varphi\right) + np_{inhcx}^{L+M+H}Z\left(\left(\frac{(1+\beta_{hc})}{\tau}\right)^{\sigma-1}\int_{\varphi_{hcx}^*}^{\infty}\varphi^{\sigma-1}\mu(\varphi)\,d\varphi\right)\Bigg] =$$

$$\frac{1}{Z_t}\left[L\tilde{\varphi}_L^{\ \sigma-1} + M\tilde{\varphi}_M^{\ \sigma-1} + H\tilde{\varphi}_H^{\ \sigma-1} + np_{inlcx}^{L+M+H}Z\left(\theta\frac{1}{\tau^{\sigma-1}}\tilde{\varphi}_{Mlcx}^{\ \sigma-1} + (1-\theta)\frac{1}{\tau^{\sigma-1}}\tilde{\varphi}_{Hlcx}^{\ \sigma-1}\right) + np_{inhcx}^{L+M+H}Z\left(\left(\frac{(1+\beta_{hc})}{\tau}\right)^{\sigma-1}\tilde{\varphi}_{Hhcx}^{\ \sigma-1}\right)\right] =$$

$$\frac{1}{Z_t}\left[L\tilde{\varphi}_L^{\ \sigma-1} + M\tilde{\varphi}_M^{\ \sigma-1} + H\tilde{\varphi}_H^{\ \sigma-1} + n\theta p_{inlcx}^{L+M+H}Z\frac{1}{\tau^{\sigma-1}}\tilde{\varphi}_{Mlcx}^{\ \sigma-1} + n(1-\theta)p_{inlcx}^{L+M+H}Z\frac{1}{\tau^{\sigma-1}}\tilde{\varphi}_{Hlcx}^{\ \sigma-1} + np_{inhcx}^{L+M+H}Z\left(\frac{(1+\beta_{hc})}{\tau}\right)^{\sigma-1}\tilde{\varphi}_{Hhcx}^{\ \sigma-1}\right] =$$

$$\frac{1}{Z_t}\left[L\tilde{\varphi}_L^{\ \sigma-1} + M\tilde{\varphi}_M^{\ \sigma-1} + H\tilde{\varphi}_H^{\ \sigma-1} + np_{inlcx}^{M}M\frac{1}{\tau^{\sigma-1}}\tilde{\varphi}_{Mlcx}^{\ \sigma-1} + np_{inlcx}^{H}H\frac{1}{\tau^{\sigma-1}}\tilde{\varphi}_{Hlcx}^{\ \sigma-1} + np_{inhcx}^{H}H\left(\frac{(1+\beta_{hc})}{\tau}\right)^{\sigma-1}\tilde{\varphi}_{Hhcx}^{\ \sigma-1}\right] =$$

$$\frac{1}{Z_t}\left[L[\tilde{\varphi}_L^{\ \sigma-1}] + M[\tilde{\varphi}_M^{\ \sigma-1} + np_{inlcx}^{M}\frac{1}{\tau^{\sigma-1}}\tilde{\varphi}_{Mlcx}^{\ \sigma-1}] + H[\tilde{\varphi}_H^{\ \sigma-1} + np_{inlcx}^{H}\frac{1}{\tau^{\sigma-1}}\tilde{\varphi}_{Hlcx}^{\ \sigma-1} + np_{inhcx}^{H}\left(\frac{(1+\beta_c)}{\tau}\right)^{\sigma-1}\tilde{\varphi}_{Hhcx}^{\ \sigma-1}]\right]$$

Appendix F: The Impact of Trade on Aggregate Welfare

Welfare (W) per worker, both in Autarky and in the Open Economy is measured by real per capita income/expenditure, which is obtained dividing aggregate real income (or revenue), $\frac{R}{P}$, by the number of workers constituting the labor force in the home country, $U + S$. By its definition $R = PQ$ and at the same time aggregate revenue adds up to total payments to the labor force $(R = U + S)$, which yields $W = \frac{1}{P} = \frac{1}{Z^{\frac{1}{1-\sigma}}\tilde{p}}$. Thus:

$$\boldsymbol{W_{open} > W_{closed} \leftrightarrow Z_t^{\frac{1}{\sigma-1}}\frac{1}{\tilde{p}_t} > Z_{closed}^{\frac{1}{\sigma-1}}\frac{1}{\tilde{p}_{closed}}}$$