Kiel Institute of World Economics
Duesternbrooker Weg 120
24105 Kiel (Germany)

Kiel Working Paper No. 970

Noise Traders' Trigger Rates, FX Options, and Smiles

by

Christian Pierdzioch

March 2000

Noise Trader's Trigger Rates, FX-Options, and Smiles*

Abstract:
A contingent claims valuation model which allows to highlight the implications of program trading in spot markets for the pricing of European-style foreign currency options and for the volatility strike structure implicit in these contracts is devoloped. The curvature of the volatility strike structure is explained by focusing attention on the expected aggregate net volume and direction of standing orders executed when the exchange rate reaches certain implicit price barriers triggering program traders to reallocate financial wealth. The valuation framework allows to endogenously reproduce the characteristic convex shape of volatility strike structures documented in the empirical literature. A volatility-based test for implicit price barriers in foreign exchange markets is employed to examine whether empirical evidence supports the barriers hypothesis of the volatility strike structure proposed in the paper.

JEL Classification: F31, G13
Keywords: Foreign Currency Options, Volatility Smile, Noise Trading, Implicit Price Barriers, GARCH model

Christian Pierdzioch
Kiel Institute of World Economics
24100 Kiel, Germany
Telephone: (0)49–431–8814–269
Fax: (0)49–431–8814–525
Email: c.pierdzioch@ifw.uni–kiel.de

* Financial support by the Deutsche Forschungsgemeinschaft is gratefully acknowledged. I thank Jörg Döpke and Georg Stadtmann for valuable comments on an earlier version of the manuscript. All remaining errors are my own.

Table of Contents

1 Introduction

The central idea underlying the present paper is to develop a foreign currency option valuation model which allows to highlight the implications of noise trading in the spot market for the pricing of FX contingent claims and for the volatility strike structure implicit in these contracts. Following Hung (1997: 781), noise traders can be characterized as "....investors whose demand for currencies is influenced by beliefs or sentiments that are not fully consistent with economic fundamentals". The adoption of this research strategy is motivated by the well known stylized fact that the volatility of exchange rates can hardly be attributed to fluctuations in macroeconomic fundamentals alone (see e.g. Rose (1994), Flood and Rose (1995), and Jeanne and Rose (1999)). Confronted with the failure of classical fundamentals-based structural models of exchange rate determination to explain the erratic movements of real-life floating exchange rates (Meese and Rogoff (1983); Boughton (1987)), recent research in international economics has begun to elaborate on the microstructure of FX markets and on noise trading as a potential source of exchange rate volatility. Because the volatility structure of the underlying economy is a key-factor in determining the premia of options on foreign currency, the impact of technical position taking of traders in the spot market on exchange rate dynamics is also an issue of central importance for the pricing of FX options. In this paper, we address this question by focusing on the implications of market sentiment induced implicit price barriers in spot markets for the dynamic properties of exchange rates and for arbitrage-free premia of the corresponding foreign currency options.

In the first part of the paper, a continuous-time noise trader model of exchange rate determination developed by Krugman and Miller (1993) and by Krugman (1987) is employed to discuss theoretically how the placing of stop-loss and standing orders affects actual and expected exchange rate fluctuations. Provided a non–negligible fraction of spot market participants adheres to such a trading rule, a large amount of selling or buying orders reaches the market when the exchange rate crosses certain implicit trigger levels and program traders are induced either to enter or to exit the market for domestic assets. Rational traders anticipate the impact of this type of non-fundamental FX trading on the exchange rate path and this, in turn, affects both the volatility of the current spot rate and the international interest rate differential even before the exchange rate hits an implicit trigger point resulting from the specific timing of orders by technical traders.

Utilizing this framework of analysis, it is examined how the effect of program trading in the spot market on the dynamics of the exchange rate and on the domestic and the foreign interest rates transmits onto the arbitrage-free prices of foreign currency options. Results obtained by implementing a Monte Carlo simulation approach indicate that implicit exchange rate thresholds due to the placing of limit orders can give rise to the type of smile and smirk effects in the volatility strike structure often observed in real-life FX options markets. Thus, the model to be developed in section 2 below offers an economically attractive new alternative explanation for the pricing errors resulting when the baseline FX option valuation frameworks are implemented to price real-life foreign currency options.

The second part of the present study is devoted to a closer examination of one of the empirically testable implications of the theoretical framework. The model implies that options premia reflect that the actual instantaneous variability of exchange rates increases (decreases) as the distance of the actual exchange rate to destabilizing (stabilizing) implicit threshold spot rates at which a non-negligible amount of standing orders reaches the market declines. Starting from this central insight of the theoretical study, we draw on ideas presented in i.a. Donaldson and Kim (1993) and De Grauwe and Decupere (1992) and use a modified version of a volatility-based empirical methodology recently suggested by Cyree et al. (1999) to test for implicit price barriers in actual foreign exchange markets. The testing procedure allows to recover generally not immediately observable implicit trigger exchange rates at which a non-negligible number of traders is perceived either to buy or to sell foreign currency. The test uses a time-series model to trace out a time-varying conditional exchange rate volatility and then examines the behavior of exchange rate volatility near potential trading induced implicit price barriers. The volatility-based test also allows to detect the perceived sign of the impact of non-fundamental FX trading on the anticipated volatility of exchange rates in actual foreign currency markets. Thus, the empirical methodology utilized in section 3 renders it possible to directly address the questions whether implicit market-sentiment induced price barriers do exist and whether such thresholds are perceived to stabilize or to destabilize actual exchange rates. Moreover, as the theoretical analysis presented in section 2 underscores that implicit trigger rates can exert a significant impact on foreign currency option premia and can even account for smiles and smirks in the volatility strike structure, the empirical study should provide further insights into the issue of whether or not taking care of the microstructure of the spot market can help firms and financial institutions to hedge currency risks more efficiently.

The plan of the analysis is as follows. The theoretical part of the study is contained in section 2. The underlying stochastic continuous time model of exchange rate determination is reviewed, the option pricing framework is developed, and the implications of implicit trading induced price barriers in foreign exchange markets for the volatility strike structure backd out of the corresponding FX options are analyzed. Section 3 is utilized to lay out the empirical part of the paper. The volatility-based test for implicit price barriers in FX markets is discussed, the empirical model is estimated, and the economic implications of the findings of this exercise for the pricing of foreign currency options are analyzed. The final section offers some concluding remarks and provides suggestions for future research.

2 Implicit Price Barriers, Exchange Rate Dynamics, and Smiles

In the present theoretical section, a stylized noise trader model of exchange rate determination advanced by Krugman and Miller (1993) is employed to shed light on the consequences of implicit price barriers in FX markets for the valuation and hedging of foreign currency options.[1] Discretely spaced implicit support or resistance levels can show up in actual exchange rate paths when a non-negligible group of FX market participants tends to rearrange the share of domestic and foreign assets in their portfolios infrequently at certain spot rate levels. Constantinides (1986) and Davis and Norman (1990) argue that such a trading strategy can be optimal if transaction costs induce economic agents to discretely rebalance their portfolios only if special asset price levels are reached or passed. A similar trading pattern can emerge if e.g. information gathering costs force relatively uninformed traders to rationally cluster in certain markets and to simultaneously submit orders (cf. Admati and Pfleiderer 1988; see also the arguments discussed in Balduzzi, Foresi and Hait (1997: pp. 139)). Yet an alternative point has been emphasized by Grossman and Zhou (1996) who argue that certain convex portfolio insurance strategies might be optimal if market participants intend to shelter their budget from falling below a pre-specified fraction of the originally invested wealth.

---

1 The framework of Krugman and Miller (1993) stands in the tradition of models developed by i.a. Grossman (1988), Brennan and Schwartz (1989), and Genotte and Leland (1990). The main objective behind these theoretical frameworks is to shed light on the consequences of programme trading and portfolio insurance for the returns and the volatility of asset prices.

With respect to the trading pattern in actual foreign exchange markets, De Grauwe and Decupere (1992) find empirical evidence in favor of implicit price barriers in the US-dollar/Yen but not in the US-dollar/DM spot rate. Skeptical notes regarding the existence of implicit price barriers in stock market data can be found in Ceuster et al. (1998) and in Ley and Varian (1994). Other empirical studies including Donaldson (1990), Donaldson and Kim (1993), Koedjik and Stork (1994), and Cyree et al. (1999), however, document evidence supporting the hypothesis of market sentiment induced psychological price barriers in stock market data. The results reported in these studies indicate that the option valuation model developed below might not only be a fruitful instrument to explain smile and smirk effects implicit in FX options but might also contribute to a better understanding of the shapes of volatility strike structures implicit in options on stocks and stock indices.

The spot rate process utilized in the option pricing framework to be developed below is derived by setting up a stochastic version of the continuous-time flex-price monetary model of exchange rate determination. Following Krugman and Miller (1993) and Krugman (1987), the economy features a class of portfolio managers committed to buy or to sell domestic interest bearing assets when the exchange rate reaches certain prespecified critical threshold levels. One implication of the model is that market participants' rational expectations regarding the timing and the aggregate net volume of asset swaps discretely carried out by portfolio manager alter the dynamics of the spot rate even before the exchange rate reaches a level triggering portfolio managers to enter or to leave the market. Assuming that domestic and foreign assets are imperfect substitutes and that portfolio managers buy (sell) domestic interest bearing securities if the domestic currency price of foreign currency falls (rises) below (above) a certain critical level, it can be shown that the presence of technical traders makes exchange rates excessively volatile as compared to economic fundamentals. In line with other models of asset pricing under implicit price barriers (see e.g. Balduzzi, Foresi and Hait (1997) and Balduzzi, Bertola, and Foresi (1995)), the functional form of the non–linear exchange rate mapping solving the model implies that the endogenously determined stochastic volatility of the spot rate process raises as the exchange rate reaches its implicit price barriers.

The implications of rational investors' anticipation of technical asset swaps of portfolio managers for both the endogenously computed stochastic exchange rate volatility and the international interest rate differential are then used to construct a foreign currency option valuation model. It is demonstrated that the FX option valuation framework laid out below can be utilized to generate the smile and smirk effects most often characterizing real-life implied volatility strike structures. However, rather than pointing to purely technical arguments, the FX option valuation framework provides a rich setting which renders it possible to interpret a smiling volatility strike structure economically. Results obtained by simulating the model numerically indicate that important factors influencing the specific shape of the *endogenously* derived volatility smiles and smirks implicit in FX option premia are the width of the interval of inaction of portfolio managers, the net volume of standing orders, and the magnitude of the corresponding exchange rate risk premia. The noise trader FX option valuation model predicts that the convexity of the volatility smile depends upon the distance between the exchange rate levels inducing portfolio managers to purchase and to sale domestic assets. The model can also be employed to generate volatility strike structures which resemble a skewed smirk rather than a symmetric smile. Such a smirk arises if, for example, the net volume of purchases of domestic assets reaching the market at the entry threshold set by portfolio managers tends to be of a different magnitude than the net sales of this trader group triggered when the spot rate crosses its exit threshold. The central insight motivating these findings is that a variation of one or more of the mentioned variables alters the overall shape of the non-linear exchange rate function which, in turn, is a main factor in determining the volatility of the stochastic spot rate process utilized to price foreign exchange contingent claims.

An important aspect of the FX option pricing framework is that the volatility of the spot rate and the differential between domestic and foreign interest rates are derived endogenously. This is in contrast to the assumptions made in many of the stochastic volatility models discussed in the options pricing literature (see e.g. Stein and Stein (1991), Heston (1993), and Scott (1997) and the references therein). The general idea underlying the more traditional apporach to option pricing is to calculate theoretical option premia which allow to eliminate the empirically observed cross-sectional pricing biases of the baseline foreign currency option valuation models advanced by Garman and Kohlhagen (1983) (hereafter GK), Grabbe (1983), Giddy (1983), and Biger and Hull (1983). In general, this is achieved by composing the stochastic environment in a way such that the dynamics of the spot rate match important statistical properties of real-life foreign exchange rate returns. With the exception of general equilibrium option pricing models (see i.a. Bick (1987) and Bakshi and Chen (1997), Nielson and Saá-Requejo (1993)), however, the stochastic differential equations suggested to generate spot rate dynamics which give rise e.g. to an U-shaped volatility strike structure are most often specified in a rather ad hoc manner.

The analysis contained in this paper departs from this modeling strategy. A foreign currency option valuation model featuring non-fundamental FX trading is constructed which renders it possible to interpret the pricing errors produced by the first generation foreign currency option valuation set ups in an economically meaningful way. Hence, rather than exogenously introducing a separate stochastic differential equation for the volatility of the exchange rate, the formation of exchange rate expectations and the concomitant trading behavior of spot market participants are utilized to figure out the dynamics of the exchange rate and of its volatility. This important feature implies that the model can be compared to recent contributions to the options pricing literature stressing the role of feedback effects from hedging (actually existing or synthesized) derivatives for the dynamics of the price of the underlying financial security. Examples for this area of the noise trader literature can be found i.a. in Frey and Stremme (1997), in Sircar and Papanicolaou (1998), and in Platen and Schweizer (1998). Setting up microeconomic equilibrium frameworks as in Föllmer and Schweizer (1993), these authors analyze the impact of technical demand induced by option traders implementing dynamic Delta hedging strategies for the evolution of the underlying financial security. Program traders are assumed to rely on a fictitious reference model given by the Black and Scholes (1973) framework for European options to figure out hedge ratios required to implement this dynamic trading strategy. The consequences of this trading behavior for the stochastic volatility of the underlying asset price are derived endogenously upon invoking a market clearing condition. It is shown that the endogenously determined volatility of the price of the underlying security is inflated for far-from the money options due to the fact that the Gamma of the fictitious reference model shows up in the denominator of the endogenous diffusion term of the stochastic differential equation driving the asset price. Platen and Schweizer (1998) show that this class of models can be employed to theoretically generate smile and smirk effects in the volatility strike structure. The volatility smile arises due to the fact that the feedback effects from options hedging amplify asset price volatility as the underlying moves away from its at-the-money level. Skewness in the smile arises if out-of-the-money options are more heavily traded than in-the-money options, et vice versa.

The contribution contained in the present section differs from the approach taken by Platen and Schweizer (1998) in (at least) two important respects:

- Exchange rate dynamics in our noise trader options pricing model are influenced by the presence of discretely spaced implicit price barriers due to the existence of standing orders of portfolio managers rather than by feedback effects from derivatives hedging. Furthermore, the framework draws attention to other factors which might be important in influencing the shape of real-life volatility strike structures. The model suggested by Platen and Schweizer (1998) relies on the quantitative importance of feedback effects from options hedging and on the relative trading volume of options with a different moneyness. The set up outlined in the present section, in contrast, emphasizes the role of the width of the band of inaction of portfolio managers and of the expected aggregate net volume of buying and selling orders reaching the market when the exchange rate reaches a threshold level triggering technical traders to rebalance their portfolios.

- A critical assumption underlying the model of Platen and Schweizer (1998) is that option writers resort to a fictitious reference model formed by the Black and Scholes (1973) framework to compute hedge ratios needed to implement dynamic portfolio insurance strategies. This option pricing formula is derived by presuming that the underlying asset price is driven by an exogenously given geometric Brownian motion. The impact of feedback effects from options hedging, however, implies that this assumption does not hold for the endogenously derived asset price process in the model of Platen and Schweizer (1998). Therefore, the decision to resort to the baseline option valuation function to model program traders' hedging behavior is somewhat arbitrary.[2] As the noise trader option pricing framework outlined in the present section does not feature feedback effects from options hedging, we do not need to invoke this critical presupposition to derive smile and smirk effects in the volatility strike structure.

The plan of the theoretical analysis is as follows. The underlying model of exchange rate determination and the implications of implicit price barriers for the dynamics of the spot rate and its volatility are discussed in subsection 2.1. Subsection 2.2 is devoted to the presentation of the foreign currency option valuation model. As it is not possible to derive closed–form analytical expressions for FX option premia, the model is solved numerically by resorting to Monte Carlo simulation techniques. This technique has often been used in the options pricing literature. Subsection 2.2 therefore also provides a brief description of the prin-

---

2 As discussed in Platen and Schweizer (1998), the ideal solution to account for feedback from options hedging would be to follow a three–step procedure: (i) Employ first the market clearing exchange rate process to derive an option pricing model for a given hedging function. (ii) Proceed to use this new option valuation model to neatly modify market participants hedging plans. (iii) Search for a kind of rational expectations equilbrium by trying to find the point of convergence (provided it exists) of this algorithm. Such a procedure, of course, gives rise to complex theoretical problems (see the discussion in Frey (1998)). For this reason, Platen and Schweizer (1998) prefer to invoke the simplyfying assumption that programme traders resort to the baseline option valuation framework when implementing dynamic trading strategies.

ciples on which this strategy to simulate option premia rests. In subsection 2.3, the results of the performed numerical simulations are discussed and the implications of the theoretical study for the shape of the volatility strike structure are elucidated.

2.1 Implicit Price Barriers and Exchange Rate Dynamics

The flex-price monetary model formulated in continuous time t exerts that the logarithm of the exchange rate $e(t)$ defined as domestic currency units expressed in terms of a foreign currency unit is equal to the sum of a set of general economic fundamentals $f(t)$ and the differential between the domestic r and the foreign $r*$ interest rates premultiplied by an interest–semi elasticity of money demand υ:

(1) $$e = f + \upsilon(r - r^*)$$

Economic fundamentals are assumed to evolve according to a driftless stochastic differential equation:

(2) $$\mathrm{d}f = \sigma\,\mathrm{d}W$$

with σ denoting a constant diffusion coefficient and $\mathrm{d}z$ symbolizing the differential of a standard Gauss-Wiener process with expected value zero and unit variance.

To determine the interest rate differential, we follow Krugman and Miller (1993) and assume that the foreign exchange market is populated by two groups of price taking agents. The first group of market participants consists of a continuum of continuously speculating economic agents with rational expectations. The second trader group is formed by a non-negligible number of portfolio managers adhering to a stop-loss type investment strategy. The latter are assumed to be foreign traders holding an internationally diversified portfolio consisting of domestic and foreign interest bearing assets. Exchange rate fluctuations are the only source of risk affecting the performance of the investments undertaken by the members of this trader group. Instead of rebalancing the composition of their asset holdings continuously, these managers do change the fraction of domestic assets in their portfolios only if the spot rate reaches one of the boundaries of a pre-specified band of inaction. Let the threshold exchange rates triggering the execution of portfolio managers' standing orders be denoted by $(\underline{e}, \bar{e})$ with $\underline{e} < \bar{e}$. Program traders are presumed to buy (to sell) foreign assets once the exchange rate reaches the trigger point $\bar{e}$ **(**$\underline{e}$**)**. It is further assumed that technical investors change the composition of their portfolios only once. This presupposition implies that once the exchange rate has hit the trigger threshold $\bar{e}$ ($\underline{e}$) and program traders have been induced to sell (buy) domestic assets, the members of this group of FX market participants do not become active again and all other trigger points cease to exist thereafter. While the set up can be generalized to the case of repeated transactions of program traders (see Krugman and Miller (1993)), we invoke this assumption to simplify the numerical calculation of option premia performed below. As the results of these simulations will demonstrate, focusing attention on a "one shot" scenario does not hamper the capability of the theoretical framework to generate smile and smirk effects in the volatility strike structure implicit in foreign currency option premia.

The order flow due to program traders' transactions reaching the market at the edges of the interval $\left(\underline{e},\bar{e}\right)$ is equivalent to a sterilized intervention against the domestic currency and will, thus, only affect the path of the spot rate if capital is internationally imperfectly mobile. To construct a meaningful model allowing to highlight the impact of program trading on exchange rate dynamics, it is therefore assumed that economic agents are risk averse and that, in the context of an international capital asset pricing model, there exists a market price of risk γ_e accounting for systematic risk between domestic and foreign assets. Let this risk–premium be expressed as:

$$(3) \qquad \gamma_e = \frac{\mu_e + r^* - r}{\sigma_e} \qquad \text{or} \qquad r - r^* = \mu_e - \gamma_e \sigma_e$$

Equation (3) states that the international differential between domestic and foreign interest rates is equivalent to the difference between the expected rate of depreciation of the domestic currency and the standard deviation of the logarithm of the exchange rate. The latter is weighted by the market price of taking non-diversifiable risk stemming from holding foreign currency denominated assets.

The market price of risk γ_e is constant as long as program traders hold the structure of their portfolios constant. However, when the standing orders of program traders are executed, the risk premium adjusts in order to restore capital market equilibrium. Taking the trading behavior of technical investors into account, it follows that the expected excess returns from holding foreign currency denominated assets must decrease (increase) when program traders are induced to sell off (buy in) domestic assets as the spot rate reaches the critical level $\bar{e}$ ($\underline{e}$). To formalize this notion, units are scaled such that the market price of risk γ_e is defined in terms of the following piecewise linear mapping:

(4) $$\gamma_e = \begin{cases} \bar{\gamma} & for\ \bar{e} \leq e\ and\ for\ all\ e(t)\ iff\ e(t') \geq \bar{e}\ for\ t' \leq t \\ 0 & for\ \underline{e} < e(t') < \bar{e}\ \forall\ t' \leq t \\ \underline{\gamma} & for\ e \leq \underline{e}\ and\ for\ all\ e(t)\ iff\ e(t') \leq \underline{e}\ for\ t' \leq t \end{cases} \qquad \text{with } \bar{\gamma} < 0 < \underline{\gamma}$$

Upon using the international capital asset pricing model to re-express the differential between domestic and foreign interest rates, the baseline exchange rate equation (1) can be written as:

(5) $$e = f + \upsilon\left[E_t^g(\mathrm{d}e)/\mathrm{d}t - \gamma_e\sigma_e\right]$$

where E_t^g denotes the conditional expectations operator applying under the objective probability measure g. To solve the model, define a continuous twice differentiable mapping $e(f)$ to express the logarithm of the exchange rate as a deterministic function of economic fundamentals. Focusing first on the case that $\underline{e} \leq e(t') \leq \bar{e}$ for all $t' \leq t$ and applying the rules of stochastic calculus, the following inhomogenous second-order ordinary differential equation describing the dynamics of the exchange rate obtains:

(6) $$e = f + \upsilon\sigma^2 e_{ff}$$

Resorting e.g. to the method of undetermined coefficients, the general solution to equation (6) can be pinned down as:

(7) $$e = f + A_1\exp(\xi_1 f) + A_2\exp(\xi_2 f) \qquad \text{with} \qquad \xi_{1,2} = \pm\sqrt{\frac{2}{\upsilon\sigma^2}}$$

where A_1 and A_2 are constants of integration to be determined by imposing appropriate boundary conditions and $\xi_{1,2}$ are the positive and negative roots of the characteristic equation of (6).

To determine the set of boundary conditions, the first step is to invoke transversality conditions stipulating that the exchange rate is equal to its intrinsic value once the spot rate settles outside the interval $(\underline{e},\bar{e})$ and no further standing orders exist. Upon imposing these side–conditions and utilizing the definition of the market price of exchange rate risk, it follows that the regime–dependent exchange rate function is given as below:

(8) $$e(t) = \begin{cases} f(t) - \upsilon\bar{\gamma}\sigma & for\ \bar{e} \leq e\ and\ for\ all\ e(t)\ iff\ e(t') \geq \bar{e}\ for\ t' \leq t \\ f(t) + \upsilon E_t(\mathrm{d}e)/\mathrm{d}t & for\ \underline{e} < e(t') < \bar{e}\ \forall\ t' \leq t \\ f(t) - \upsilon\underline{\gamma}\sigma & for\ \bar{e} \leq e\ and\ for\ all\ e(t)\ iff\ e(t') \leq \bar{e}\ for\ t' \leq t \end{cases}$$

While equation (8) already contains the complete solution of the model for $\bar{e} \leq e$ and for $\bar{e} < e$, it remains to find two boundary conditions allowing to uniquely identify the constants of integration A_1 and A_2 and, thus, the solution of the model for the case that $\underline{e} \leq e(t') \leq \bar{e}$ for all $t' \leq t$.

These boundary conditions obtain by recognizing that continuously trading rational investors will anticipate the impact of program traders' portfolio reshuffling on the path of the spot rate. Rational investors' awareness of the behavior of technical traders participating in FX trading rules out the occurrence of discrete step-jumps in the exchange rate at the boundaries of the implicit range of inaction of program traders. To see this, suppose that the spot rate reaches the sell off trigger $\bar{e}$ from below. In this case, investors with rational expectations will anticipate that during the next infinitesimal instant of time, movements in fundamentals will either induce a small step of the exchange along the continuous spot rate function in the direction of the central parity of its implicit fluctuation band or a discrete non–negligible shift of the spot rate due to the execution of program traders' standing orders. Clearly, the resulting asymmetric magnitude of the potential infinite rate of trading profits as opposed to the limited size of appreciation losses would stimulate investors to go short in domestic assets. The additional supply of domestic assets would drive up the exchange rate until no further „bets" with limited downside risk are traded in FX markets and the exchange rate function does no longer exhibit a discrete jump at the edges of the interval $(\underline{e}, \bar{e})$. Taking this line of argumentation into account, it is possible to invoke the following set of boundary conditions (see Krugman and Miller (1993)):

(9a) $\quad \bar{e} = e(\bar{f})$

(9b) $\quad \underline{e} = e(\underline{f})$

Note that $\bar{f}$ and $\underline{f}$ are defined implicitly by $\bar{e} + \upsilon\sigma^2 = \bar{f}$ and $\underline{e} + \upsilon\sigma^2 = \underline{f}$, respectively. The two no-jump boundary conditions formalized in equation (9) can thus be utilized to determine the two unknown constants of integration. Upon carrying out the necessary manipulations, one obtains:

(10) $$A_1 = -\frac{\upsilon\bar{\gamma}\sigma}{\exp\left[\xi_1\left(\bar{e}+\upsilon\bar{\gamma}\sigma\right)\right]} +$$

$$\frac{\exp\left[\xi_1\left(\bar{e}+\upsilon\bar{\gamma}\sigma\right)+2\xi_2\left(\underline{e}+\upsilon\underline{\gamma}\sigma\right)\right]\left(-\upsilon\bar{\gamma}\sigma\right)+\exp\left[2\xi_1\left(\bar{e}+\upsilon\bar{\gamma}\sigma\right)+\xi_2\left(\underline{e}+\upsilon\underline{\gamma}\sigma\right)\right]\upsilon\underline{\gamma}\sigma}{\exp\left[2\xi_1\left(\bar{e}+\upsilon\bar{\gamma}\sigma\right)\right]\left[\exp\left[2\xi_1\left(\bar{e}+\upsilon\bar{\gamma}\sigma\right)\right]-\exp\left[2\xi_1\left(\underline{e}+\upsilon\underline{\gamma}\sigma\right)\right]\right]}$$

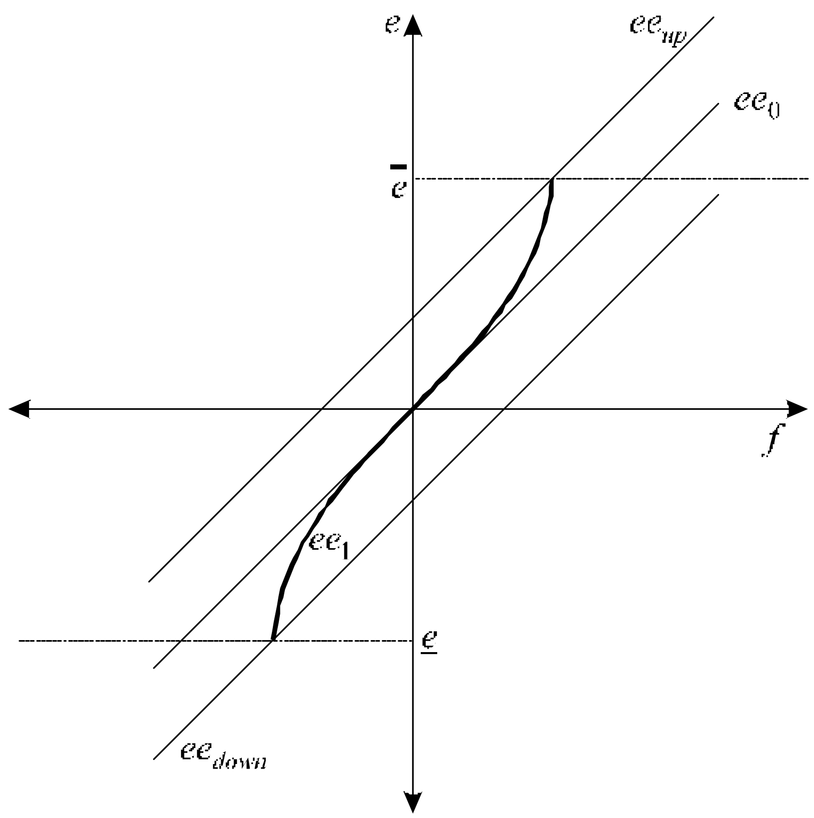
(11) $$A_2 = \frac{\exp\left[\xi_1\left(\bar{e}+\upsilon\bar{\gamma}\sigma\right)+2\xi_2\left(\underline{e}+\upsilon\underline{\gamma}\sigma\right)\right]\upsilon\bar{\gamma}\sigma+\exp\left[2\xi_1\left(\bar{e}+\upsilon\bar{\gamma}\sigma\right)+\xi_2\left(\underline{e}+\upsilon\underline{\gamma}\sigma\right)\right]\upsilon\underline{\gamma}\sigma}{\exp\left[2\xi_1\left(\bar{e}+\upsilon\bar{\gamma}\sigma\right)\right]-\exp\left[2\xi_1\left(\underline{e}+\upsilon\underline{\gamma}\sigma\right)\right]}$$

Figure 1 illustrates exchange rate mapping resulting in the Krugman and Miller (1993) model. The exhibit plots the logarithm of the exchange rate on the vertical axis as a function of economic fundamentals f on the horizontal axis. Ruling out extrinsic bubbles and starting in a world with $\gamma_e = 0$, the graph ee_0 depicts the solution to the model for all exchange rate realizations if program traders are in general absent from FX markets.[3] In such an economy, rational foreign exchange traders recognize that the stochastic process chosen to depict the dynamics of economic fundamentals does not feature a predictable component and form static expectations regarding future changes of the exchange rate. This, in turn, implies that the constants of integration in equation (7) are identically equal to zero and that the set up degenerates to the baseline stochastic representative agent monetary model of exchange rate determination.

---

3 The mapping ee_0 also applies in a situation characterized by $\underline{e} \leq e(t') \leq \bar{e}$ for all $t' \leq t$ if continuously trading agents have myopic expectations regarding the trading behavior of programme traders. See Krugman and Miller (1993) for a discussion.

Figure 1 — Implicit Price Barriers and Exchange Rate Dynamics



The function ee_{up} represents the exchange rate path applying in the noise trader scenario once the spot rate has reached the upper threshold level $\bar{e}$ triggering program traders to irreversibly exit the market for domestic assets. This portfolio reallocation in favor of foreign assets causes the world market price of exchange rate risk to decline to $\bar{\gamma}$ and results in a one-time upward shift of magnitude $-\upsilon\bar{\gamma}$ of the long-run exchange rate function. Presuming $\underline{e} = -\bar{e}$ and $\underline{\gamma} = -\bar{\gamma}$, the linear mapping ee_{down} summarizes the symmetric scenario of a downward shift of the exchange rate path of size $-\upsilon\underline{\gamma}$ caused by a one-time asset swap of technical traders from foreign to domestic assets induced by $e(t') \leq \bar{e}$ for some $t' \leq t$.

While the mappings ee_{up} and ee_{down} merely parallel the linear representative agent solution ee_0, the most interesting exchange rate dynamics can be observed in FX markets featuring standing orders and implicit price barriers. The non-linear function ee_1 gives an example for an equilibrium exchange rate solution applying in such an economy. To explain the specific shape of this function capturing exchange rate dynamics in FX markets with heterogeneous agents, suppose that the process driving economic fundamentals forces the exchange rate to reach the sell off trigger $\bar{e}$ from below. Continuously trading agents rationally recognize that this development increases the probability that the standing orders of program traders will be executed during the next infinitesimal instant of time. Rational traders embed this information in current exchange rate expectations by weighting the depreciation pressure on the domestic currency resulting from this simultaneous portfolio rebalancing carried out by a non–negligible group of FX market participants with the increased probability of such an event. These depreciation expectations are taken into consideration by rational agents when currently pricing foreign exchange. As can be seen by checking equation (5), this implies that for any given realization of economic fundamentals in the upper half of the implicit exchange rate fluctuation band the depreciation of the domestic currency is always an edge stronger than the corresponding depreciation observed in a world without noise traders. As the exchange rate approaches the sell off trigger $\bar{e}$ from below, continuously trading FX market participants bit up the spot rate until the resulting additional supply of domestic assets is large enough to rule out a discrete jump of the exchange rate function at the upper boundary of program traders' no-trade interval.

Of course, the situation is just reversed as the spot rate reaches the buy in trigger $\underline{e}$ from above. The additional demand for domestic assets puts a strong appreciation pressure on the domestic currency in the lower half of the interval $(\underline{e},\bar{e})$. This, in turn, rules out arbitrage opportunities in FX markets by eliminating the possibility of a one-time discrete jump of the exchange rate function at the threshold level $\underline{e}$.

2.2 The Foreign Currency Option Valuation Framework

In this subsection, a valuation framework is developed which allows to discuss the implications of infrequent noise trading and implicit price barriers in FX spot markets for the pricing of foreign currency options and, thus, for the shape of the volatility strike structure implicit in these price contingent contracts. The model applies in the case of European style options. We focus on call options and denote the premium of this financial security by a continuous valuation function $C(E,t)$ twice-differentiable in the anti-log E of the exchange rate and once in t.

The first step in the analysis is to apply the rules of stochastic calculus to trace out the process driving the anti-log of the spot rate under measure g. Noting that

(12) $$\mathrm{d}e = \frac{1}{2} e_{ff} \sigma^2 \,\mathrm{d}t + e_f \sigma \,\mathrm{d}W$$

one obtains for $E \equiv \exp(e)$ the stochastic process:

(13) $$\mathrm{d}E = \frac{1}{2}\left(e_f^{\,2} + e_{ff}\right)\sigma^2 E\,\mathrm{d}t + e_f \sigma E\,\mathrm{d}W$$

Two features of the stochastic differential equation derived in equation (13) are worth mentioning. Firstly, both the drift rate and the diffusion term of the exchange rate process are path dependent. Starting in the interior of the interval $\left(\underline{e},\bar{e}\right)$, the dynamics of the exchange rate irreversibly change when the spot rate crosses one of the thresholds triggering program traders to reshuffle their portfolios. Secondly, the volatility of the anti-log of the spot rate is an increasing function of the first derivative of the natural logarithm of the exchange rate with respect to economic fundamentals. In a world without noise traders and in FX markets in which program traders already have undertaken an irreversible portfolio reshuffling, this derivative assumes the value one. However, as highlighted in figure 1, one obtains for the slope of the exchange rate function the inequality $e_f \geq 1$ if $\underline{e} \leq e(t') \leq \bar{e}$ for all $t' \leq t$. Economically speaking, this formal result shows that the presence of a non-negligible group of program traders rebalancing their internationally diversified portfolios only discretely at certain trigger thresholds inflates the instantaneous volatility of exchange rates. Moreover, the shape of the function $e(f)$ implies that the impact of program trading on the volatility of the exchange rate tends to become stronger as the spot rate moves closer to the trigger rates fixed by program traders. As will be shown when discussing the results of numerical simulations of the model, it is this latter property of the present valuation framework which plays a crucial role for the shape of the volatility strike structure implicit in foreign currency options.

Let μ_C and σ_C denote the expected rate of return and standard deviation of holding an European FX call. Applying the rules of stochastic calculus to $C(E,t)$, it follows that

(14) $$\mu_C C = \frac{1}{2} C_E \left(e_f^{\,2} + e_{ff}\right)\sigma^2 E\,\mathrm{d}t - C_\tau\,\mathrm{d}t + \frac{1}{2} C_{EE}\left(e_f \sigma E\right)^2 \mathrm{d}t$$

(15) $$\sigma_C C = C_E e_f \sigma E$$

The premium on holding a European FX call can be expressed as:

(16) $$\gamma = \frac{\mu_C - r}{\sigma_C}$$

Applying the standard no-arbitrage argument, the fundamental contingent claims valuation equation can be derived as:

(17) $$\frac{1}{2}C_{EE}\sigma^2 {e_f}^2 E^2 + (r - r^*)EC_E - rC = C_\tau$$

where $\tau \equiv T - t$ denotes the time to expiry of the contract. From now on, domestic discount bonds are taken as numeraire. Upon resorting to the underlying structural exchange rate model to re-express the differential between domestic and foreign interest rates, it is then possible to rewrite equation (17) as:

(18) $$\frac{1}{2}C_{EE}\sigma^2 {e_f}^2 E^2 + \left(\frac{1}{2}\sigma e_{ff} - \gamma e_f\right)\sigma EC_E = C_\tau$$

which must be solved subject to the standard set of terminal and boundary conditions applying in the case of European plain vanilla call options:

(19) $$C(E,0) = [E - X]^+$$

(20) $$\lim_{E\to 0} C(E,\tau) = 0$$

(21) $$C(E,\tau) \le E$$

The European call option valuation model formalized in equation (18) to (21) degenerates to the GK model when either $e(s) \le \underline{e}$ or $e(t') \ge \underline{e}$ for some $t' \le t$, or $\gamma_e = 0 \ \forall \ e(f)$, or the band of inaction defined by program traders becomes infinitely large.

An important point to note from the fundamental contingent claims valuation equation is that FX options are priced as if all economic agents were risk–neutral. In an arbitrage-free international setting, it is therefore possible to value FX options under an equivalent martingale measure $\tilde{g}$. The spot rate process discounted for the forward report/ discount martingalizes under this measure. Upon utilizing Ito’s lemma, the exchange rate process applying under the equivalent martingale measure can be expressed as:

(22) $$dE = -r^* E\,\mathrm{d}t + e_f \sigma\,\mathrm{d}\tilde{W}$$

The Wiener process $\left\{\tilde{W}(t), t \ge 0\right\}$ is a martingale under measure $\tilde{g}$ while the process $\{W(t), t \ge 0\}$ martingalizes with respect to measure g. The two processes are related through:

(23) $$d\tilde{W} = dW + dZ \qquad \text{with} \qquad dZ = \left(\gamma_e + 0.5 e_f \sigma\right) dt$$

Also note that the change of the probability measure has not altered the structure of the diffusion term of the exchange rate process.

Equation (22) is used in the Monte Carlo simulations performed to numerically calculate FX currency option prices in the presence of program traders and implicit price barriers in foreign exchange markets. The Monte Carlo valuation technique has been introduced into the options pricing literature by Boyle (1977). The technique is particularly useful to address options pricing problems which cannot be solved in terms of elementary mathematical functions or to verify that closed-form option pricing formulae utilized in numerical simulations have been coded up correctly. A comprehensive discussion of the ideas underlying various alternative designs of Monte Carlo simulations can be found in Hammersley and Handscomb (1964). An analysis of these techniques in the context of options pricing theory is provided by Schäfer (1993). A discussion of recent applications can be found in Boyle et al. (1997).

To illustrate the central idea motivating the application of Monte Carlo techniques to address options pricing problems, recall from of the risk–neutral valuation approach advanced by Cox and Ross (1976) and further examined by Harrison and Kreps (1979) that the current premium of a European style foreign exchange option can be expressed as the expected terminal payout of the contract discounted at the risk-free rate of interest. Expectations are taken under the equivalent martingale measure $\tilde{g}$. Noting that domestic discount bonds have been chosen as numeraire, this result can be translated into a mathematical language by writing:

(24) $$C(E(t),\tau) = E_t^{\tilde{g}}\left[(E(T) - X)^+ | E(t)\right]$$

$$= \int_0^{\infty} (E(T) - X)^+ \tilde{g}(E(T), E(t), T, t) dE(T)$$

Equation (24) states that the current value of the European FX call is equal to the area under the risk-neutral probability density function of the terminal exchange rate considered for all $E(T) \geq X$. A frequently encountered central problem when applying the risk–neutral valuation technique to value price contingent claims is that it is often not possible to derive closed-form analytical expressions for the integral appearing on the right-hand side of equation (24). The important value–added of the Monte Carlo approach to the pricing of European style FX options is that this simulation methodology provides technically elegant instruments allowing to calculate this integral and, thus, the premium of the derivative security numerically.

To compute an unbiased estimate $\hat{C}(E(t),\tau)$ of $C(E(t),\tau)$, the discrete time analogue of the exchange rate process provided in equation (22) obtained by subdividing the time to maturity τ of the option into $m = \tau / \Delta t$ disjoint subintervals of length Δt is simulated n times. In the following analysis, an Euler approximation is utilized to obtain a discretization of the stochastic differential equation describing the evolution of the spot rate. A number of n sampled values is drawn from the probability density function of the terminal exchange rate $E(T)$. Following Hammersley and Handscomb (1964: pp. 51), the respective estimate $\hat{C}(E(t),\tau)$ and the variance of the sampling distribution of this estimator are calculated by computing:

(25) $$\hat{C}(E(t),\tau) = \frac{1}{n}\sum_{i=1}^{n} C(E_i(T),0) = \frac{1}{n}\sum_{i=1}^{n}\left[(E(T)-X)^{+}|E(t)\right]$$

(26) $$\hat{C}^2(E(t),\tau) = \frac{1}{n-1}\sum_{i=1}^{n}\left(C(E_i(T),0) - \hat{C}(E(t),\tau)\right)^2$$

To generate confidence bounds for the estimate of the option price, Boyle (1977: 325) suggests to replace $n-1$ in (26) by n and to exploit the result that due to the central limit theorem the standardized variable $\left[\hat{C}(E(t),\tau) - C(E(t),\tau)\right] / \sqrt{\hat{C}^2(E(t),\tau)/n}$ converges to a standard normal distribution if the number of performed simulation runs is reasonably large.

Finally, it should be mentioned that, for a given number of simulation runs, the standard deviation of the estimate of the option price can be reduced by implementing a variance reduction technique as discussed i.a. in Hammersley and Handscomb (1964). Though technically appealing, enriching the programs used to simulate the model by coding up one of these approaches, in general, inflates the computer-time needed to obtain an estimate of the option price. As the foreign currency option pricing model discussed in the present section is relatively demanding in terms of the processing time needed for the simulations even when the crude Monte Carlo method described above is implemented, we refrain from using a variance reduction technique. The relative complexity of the present FX option valuation framework stems from the fact that the underlying structural exchange rate model itself is relatively complicated. To underscore the latter aspect, note that the risk premium, the interest rate differential, and the volatility of the stochastic spot rate process are all price- and path-dependent.[4]

2.3 Implicit Price Barriers and the Volatility Strike Structure

The following paragraphs are devoted to the discussion of results of numerical simulations of the valuation model laid out in the preceding subsection. The results demonstrate that the presence of program traders and implicit price barriers can give rise to the kind of smile and smirk effects most often exhibited by real-life volatility strike structures implicit in foreign currency options premia.

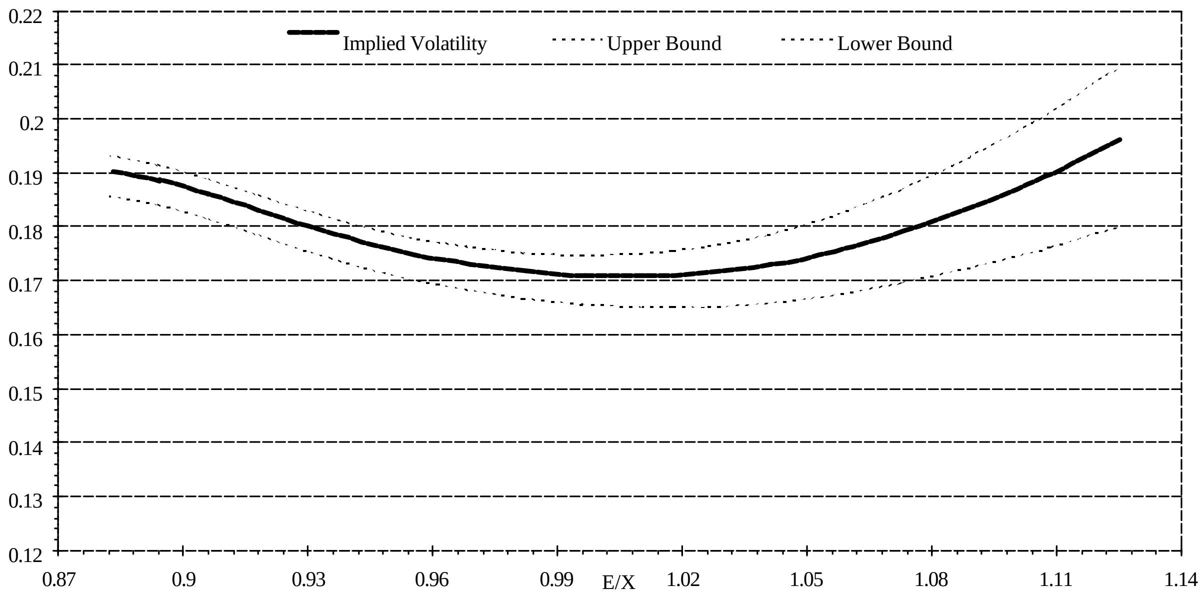
Figure 2 plots an example for an endogenously determined volatility strike structure. The exhibit depicts volatility quotes implicit in simulated foreign currency option premia on the vertical axis as a function of the exchange rate/ strike price ratio on the horizontal axis. In this scenario, program traders standing orders reach the FX market either at an implicit sell off or at an implicit buy in exchange rate. The graphed volatility smile was obtained by utilizing a symmetric band of inaction $\underline{e} = -\bar{e}$ to describe the actions of program traders. It was further assumed that the respective rise and downshift of the exchange rate risk-premium taking place at these spot rate levels are equal to each other in absolute value ($\underline{\gamma} = -\bar{\gamma}$). The resulting symmetric exchange rate function utilized to price FX contingent claims therefore resembles the mapping ee_1 depicted in figure 1. To start the simulation, it was presumed that the natural logarithm of the current exchange rate has settled in the middle of the implicit spot rate band and that $\underline{e} \leq e(t') \leq \bar{e}$ for all $t' \leq t$.

Visual inspection of figure 2 shows that volalilities implicit in near- and far-from-the-money options tend to be higher than volatility quotes backed out of at-the-money options. The implied volatility mapping reaches a minimum for at-the-money contracts and rises as the exchange rate/ strike price ratio moves closer to the critical spot rate realizations inducing program traders either to step in or to exit the market for domestic asset. Furthermore, the mapping $(E/X) \mapsto \sigma_{implied}$ is almost perfectly symmetrically spaced around the at-the-money point $(E/X) \approx 1$.

---

4 The qualitative validity of the results of the numerical analyses of the impact of implicit price triggers on the shape of the volatility strike structure presented below has been confirmed in repeated simulations of the Monte Carlo experiments. Notwithstanding, if a financial institution or a trader intended to apply the present noise trader foreign currency option pricing model to the pricing or the hedging of actual FX options, it would certainly be desirable to resort to a variance reduction technique to generate narrower confidence bands.

Figure 2 — Implicit Trigger Rates and a U–shaped Implied Volatility Strike Structure



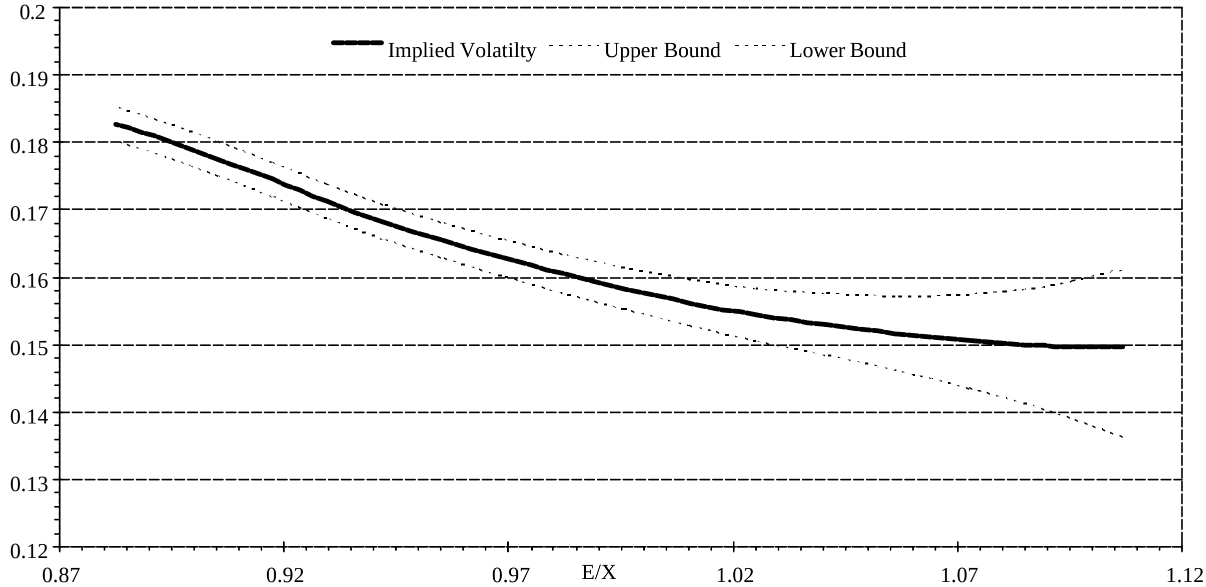
Note: The exhibit plots volatility quotes implicit in simulated foreign currency option premia on the vertical axis as a function of the exchange rate/ strike price ratio on the horizontal axis. The figure is based on the following set of numerical parameter values: $\sigma = \upsilon = 0.15$, $\bar{e} = -\underline{e} = 0.15$, $e(0) = 0$, $\tau = 0.25$, and $\gamma = -\underline{\gamma} = -1$. For the Monte Carlo analysis, the number of simulation runs was chosen to be 20.000 and the length of a time–step was fixed at 0.001. The upper and lower bounds are used to visualize the 90% confidence interval for simulated implied volatilities.

To elucidate the economic intuition behind these findings, note that continuously trading rational spot market participants anticipate the portfolio reshuffling of technical traders triggered at the boundaries of the interval $(\underline{e},\bar{e})$. The expectations that a significant number of standing orders will reach the spot market at these critical exchange rate realizations enlarges traders' information set. Economic agents acting in a competitive environment take this information regarding the present discounted value of the perceived movement of the exchange rate induced by the demand or supply of technical traders into account when currently trading foreign exchange. In contrast to the linear homogenous agents GK case, the resulting spot rate function mimics an inverted *S* and the endogenously determined volatility of the exchange rate increases as the stochastic system moves closer to one of the trigger rates associated with program traders' portfolio rebalancing. This rise in the instantaneous variability of the exchange rate exerts an increasing impact on the premia of options on this asset price. Because the volatility of the underlying increases as the spot rate reaches its critical threshold levels $\underline{e}$ and $\bar{e}$, the premia of in-the-money (out-of-the-money) options tend to increase (decrease) by more (by less) as compared to the GK case than at-the-money contracts. The result is the convex volatility strike structure depicted in the figure.

The moderate asymmetry of the smile can be attributed to the fact that in the present setting the international interest rate differential is closely related to rational investors exchange rate expectations. In the upper (lower) half of the implicit exchange rate band $\left(\underline{e},\bar{e}\right)$, the likelihood that program traders will exit (enter) the market for domestic assets is larger than the likelihood of a portfolio reallocation in the opposite direction. This fosters economic agents depreciation (appreciation) expectations which must be covered by a corresponding positive (negative) interest rate differential. From this it follows that the price of foreign discount bonds expressed in terms of the price their domestic counterparts increases for $\left(\bar{e}-\underline{e}\right)/2 \leq e(t) \leq \bar{e}$, et vice versa. As is known from the baseline foreign currency valuation models that this, in turn, implies that the premia of FX options as forward looking instruments are stimulated to increase (to decline) if the contract is currently in-the-money (out-of-the-money). Thus, the interest rate effect explains the slight asymmetry of the volatility smile depicted in figure 2 arising in the case of relatively far-from-the-money options.

To further highlight the impact of implicit price barriers on the shape of the smile, figure 3 depicts a skewed volatility strike structure obtained by abstracting from the lower spot rate threshold. To simulate this case, the buy in exchange rate risk-premium was set equal to zero ($\underline{\gamma}=0$). In this stop-loss trading scenario, program traders are invariant to an appreciation of the domestic currency but restructure their portfolios when the exchange rate reaches a critical upper level $\bar{e}$. As a result, the spot rate path eventually converges to the solution of the baseline stochastic monetary model of exchange rate determination with homogenous traders if the domestic currency sharply appreciates. However, if the spot rate is pushed into the direction of programme traders' exit trigger, the likelihood of a further depreciation of the domestic currency increases and continuously trading rational investors take short positions in domestic assets. This, in turn, increases the demand for foreign assets and the exchange rate shows a more pronounced tendency to depreciate than in a foreign exchange market without technical traders. As e.g. in the stochastic volatility model of Heston (1993), the resulting positive correlation between the spot rate process and the endogenously determined exchange rate volatility raises the premia of out-of-the-money foreign currency options. Note, however, that the present noise trader FX option pricing set up departs from standard stochastic volatility option valuation frameworks in that our set up allows to derive these characteristics of options prices endogenously. This property of the present model renders it possible to interpret the volatility smile in economic terms rather than in terms of a superimposed correlation structure between the innovation terms driving exogenously specified stochastic processes for the underlying asset price and its volatility.

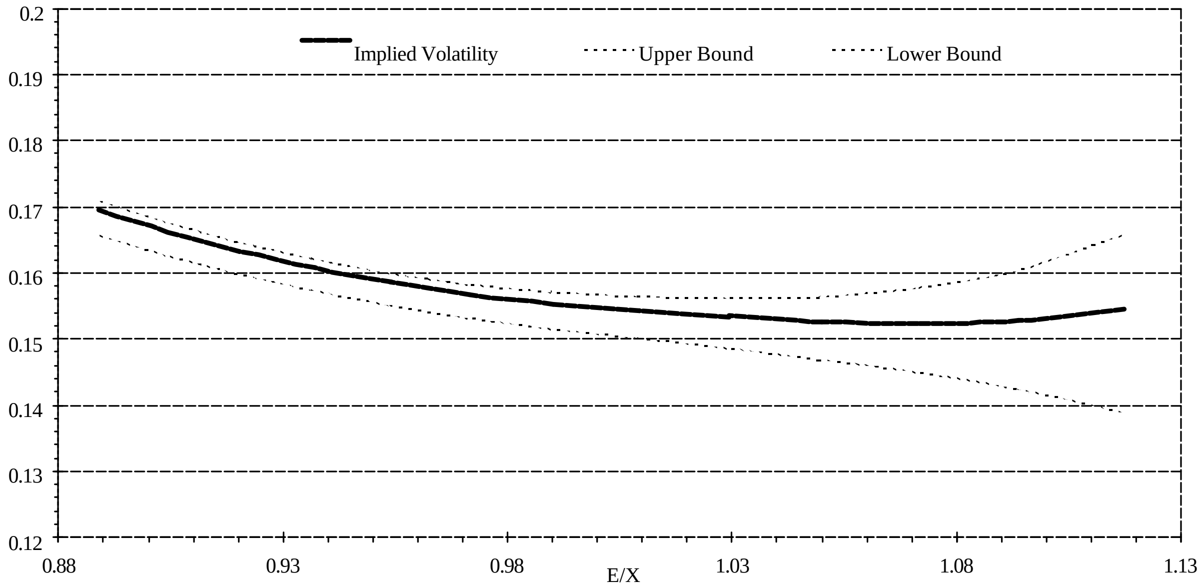
Figure 3 — Exit Triggers and Skewness in the Smile



Note: The exhibit plots volatility quotes implicit in simulated foreign currency option premia on the vertical axis as a function of the exchange rate/ strike price ratio on the horizontal axis. The numerical parameter values assigned to the structural parameters of the model are the same as in figure 2 except that $\underline{\gamma} = 0$. For the Monte Carlo analysis, the number of simulation runs was chosen to be 20.000 and the length of a time–step was fixed at 0.001. The upper and lower bounds are used to visualize the 90% confidence interval for simulated implied volatilities.

Finally, note that under the presumed parameter constellation the endogenous positive correlation between the exchange rate and its volatility does not result in a clear-cut decline of the premia of in-the-money FX options. The economic intuition motivating this finding can be understood by noticing that the relative decline of the foreign interest rate required to cover rational traders depreciation expectations in the wake of program traders stop-loss orders tends to stabilize the premia of in-of-the-money contracts. The volatilities backed out of these option prices capture these depreciation expectations and the concomitant effect on the international interest rate differential. Therefore, in the present model, the relative underpricing of out-of-the-money foreign currency options resulting when utilizing the GK framework goes not necessarily in line with a corresponding significant overpricing of in-the-money contracts. This resembles the option pricing implications of the jump-diffusion models employed e.g. by Borenzstein and Dooley (1987) to explain the underpricing of out-of-money FX option contracts for major dollar exchange rates during the mid-eighties.

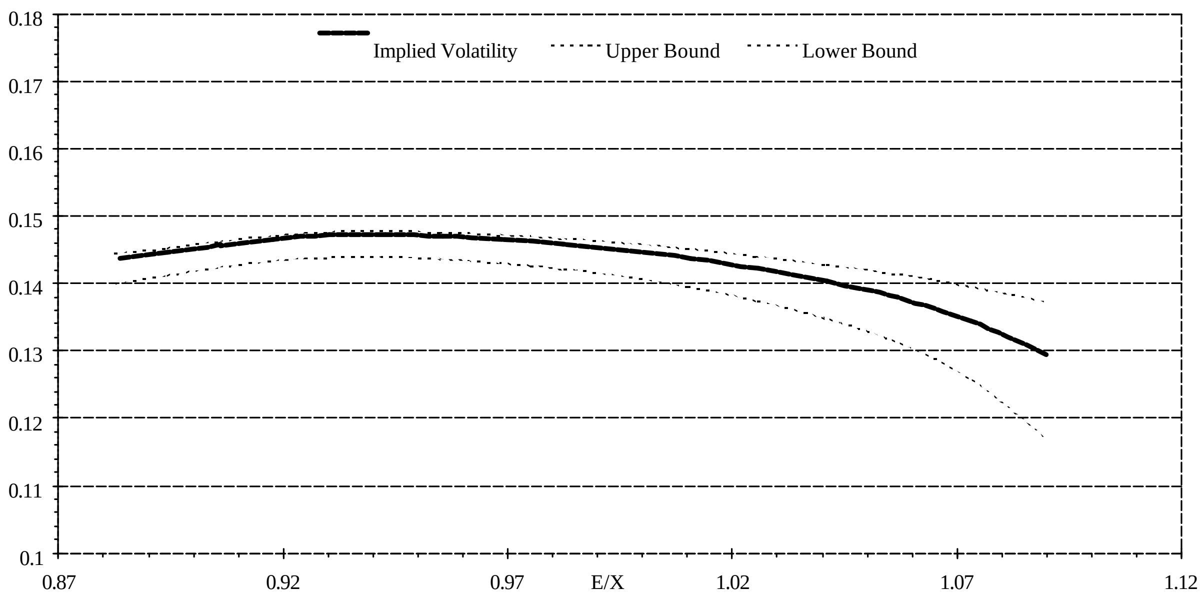
Figure 4 — A Distant Exit Trigger and a Skewed Smile



Note: The exhibit plots volatility quotes implicit in simulated foreign currency option premia on the vertical axis as a function of the exchange rate/ strike price ratio on the horizontal axis. The numerical parameter values assigned to the structural parameters of the model are the same as in figure 3 except that $\bar{e} = 0.2$. For the Monte Carlo analysis, the number of simulation runs was chosen to be 20.000 and the length of a time–step was fixed at 0.001. The upper and lower bounds are used to visualize the 90% confidence interval for simulated implied volatilities.

Figure 4 serves to examine the impact of increasing the distance between the current exchange rate realization and program traders' implicit exit trigger on the shape of the volatility strike structure. As can be seen by visually inspecting the graph, the mapping $(E / X) \mapsto \sigma_{implied}$ plotted in the exhibit is less skewed than its counterpart shown in figure 3. Thus, the impact of program trading in the spot market on the curvature of the smile declines as the current exchange rate wanders farther away from technical traders' trigger thresholds. This finding reflects that the present discounted value of the shift in the exchange rate level caused by the execution of the standing exit orders of program traders shrinks as the distance $\bar{e} - e$ is increased. As compared to the economy depicted in figure 3, the likelihood of a sharp depreciation of the domestic currency is set on the decrease and this, in turn, abates the pressure on the domestic currency tracing back to the trading behavior of continuously acting FX market participants. Consequently, the endogenously determined instantaneous exchange rate volatility implied by the underlying structural noise trader exchange rate model increases less rapidly than in figure 3 as the exchange rate moves in the direction of the critical exit threshold placed by program traders. This lowers the positive correlation between the spot rate and its volatility and results in the mildly smiling volatility strike structure depicted in figure 4.

Figure 5 — Stabilizing Spot Market Speculants and Frowns in the Volatility Strike Structure



Note: The exhibit plots volatility quotes implicit in simulated foreign currency option premia on the vertical axis as a function of the exchange rate/ strike price ratio on the horizontal axis. The numerical parameter values assigned to the structural parameters of the model are the same as in figure 3 except that $\bar{\gamma} = -\underline{\gamma} = 1$. For the Monte Carlo analysis, the number of simulation runs was chosen to be 20.000 and the length of a time–step was fixed at 0.001. The upper and lower bounds are used to visualize the 90% confidence interval for simulated implied volatilities.

As a final exercise, the model was simulated by assuming that program traders exert a stabilizing rather than a destabilizing impact on the dynamics of the exchange rate. Such an economy can be considered by assuming that the world market price of exchange rate risk can be described by a piecewise linear function of the format formalized in equation (4) with $\underline{\gamma} < 0 < \bar{\gamma}$. This assumption reflects that stabilizing program traders adhere to a contrarian trading philosophy and increase (decrease) the fraction of financial wealth invested in domestic assets when the spot rate reaches the upper (lower) trigger rate $\bar{e}$ $(\underline{e})$. The resulting additional demand for (supply of) domestic assets which reaches the market at the buy in (sell off) trigger requires an increase (decrease) of the exchange rate risk premium for capital market equilibrium to hold. Continuously trading spot market traders anticipate the appreciation (depreciation) pressure on the domestic currency arising as the exchange rate is driven in the direction of the buy in (sell off) trigger and no-jump conditions similar to those provided in equation (9) require that the current spot rates $\underline{e} \leq e(t') \leq \bar{e}$ for all $t' \leq t$ already incorporates these stabilizing exchange rate expectations. Contrasting the situation characterized by destabilizing program trading, the exchange rate mapping now exhibits an S-shaped rather than an inverted S-shaped functional form. Starting in the interior of the implicit band $\left(\underline{e},\bar{e}\right)$, this, in turn, implies that the instantaneous volatility of the spot rate decreases as the stochastic system forces the exchange rate to move in the direction of one of the threshold levels inducing program traders to reallocate financial wealth.

Figure 5 depicts the implications of two-sided stabilizing program trading for the volatility strike structure implicit in foreign currency options. The exhibit shows that FX option premia tend to decline as the exchange rate reaches program traders implicit stabilizing trigger points so that the volatility strike structure now mimics a frown rather than a smile or a smirk. Similar frown effects have been reported by Dumas et al. (1995) and by David and Veronesi (1999), albeit in differently motivated papers. Also note, that the volatility frown plotted in figure 5 shows a moderately asymmetric shape. In fact, the model predicts that in-the-money contracts tend to have a lower implied volatility than both at-the-money and out-of-the-money options. Consistent with the line of argumentation put forward to describe the volatility smile graphed in exhibit 2, this asymmetry reflects that stabilizing program trading fosters appreciation (depreciation) expectations for the domestic currency in the upper half of its implicit fluctuation band. The corresponding increasing effect on the price of foreign discount bonds (in terms of domestic assets) raises the opportunity costs of being long the option and requires its premium to decline. The fact that the absolute value of the Rho taken with respect to the relative numerical value assumed by the foreign interest rate of the contract is an increasing function of the moneyness of the security accounts for the magnitude of this effect. In perfect analogy, the impact of stabilizing exchange rate expectations on the international interest rate differential can be viewed as a factor countervailing the impact of the decline in exchange rate volatility on the premia of out-of-the-money contracts.

All results considered, the numerical simulations underscore that the foreign currency option valuation framework outlined in subsection 2.2 contributes to the strand of research aiming at explaining empirically observed shapes of volatility strike structures implicit in FX options premia theoretically. The curvature of the volatility strike structure is explained by focusing attention on the expected aggregate net volume and direction of standing orders executed when the exchange rate reaches a threshold level triggering program traders to reallocate financial wealth. A central finding is that volatility quotes backed out of foreign currency option premia embed valuable information regarding implicit support or resistance levels in FX spot markets. Foreign currency options implied volatilities tend to increase (decrease) as the underlying exchange rate moves in the direction of a destabilizing (stabilizing) trigger threshold reflecting the extrapolative (contrarian) program trading of a non-negligible fraction of market participants.

The interpretation of the results of the various numerical simulations further indicates that the *endogenously* derived predictions of the present noise trader contingent claims valuation framework for the premia of foreign currency options tend to resemble those obtained from more classic jump-diffusion models originated by Merton (1976). FX option pricing settings belonging to this latter class of price contingent claims valuation frameworks resort to an *exogenously* given discontinuous spot rate process to generate a leptokurtic exchange rate returns distribution. Implied volatilities backed out of the resulting foreign currency option premia reproduce the empirically observed convexity of the volatility smile. Moreover, upon increasing the probability of a discontinuous exchange rate movement in one particular direction, the jump models can be utilized to compute skewed volalatility strike structures.

Though in the context of the present model the expectations formation of continuously acting spot market participants rules out discontinuous exchange rate movements, program traders trigger investment strategies give rise to relatively large perceived exchange rate movements which can be interpreted to mimic the effect of a jump process on asset price dynamics. Moreover, similar to the effects at work in option pricing models featuring an asymmetric jump process, the present set up predicts a skewed volatility strike structure if technical traders are assumed to adhere to an asymmetric asset allocation strategy. In analogy to jump and jump-diffusion scenarios, a one-way program trading strategy raises the likelihood of a large exchange rate movement in one particular direction and implies a specific correlation between spot rate changes and exchange rate volatility.

However, despite the fact that the implications for the pricing of foreign currency options are similar to each other, the present noise trader FX contingent claims valuation framework differs from the well-established jump and jump-diffusion models in at least one important respect. While in jump and jump-diffusion set ups an *exogenously* given stochastic differential equation is employed to describe discontinuous exchange rate dynamics, the present option valuation framework allows to derive the dynamics of the underlying spot rate and, thus, smile and smirk effects in the volatility strike structure *endogenously*. Rather than relying on a superimposed formal mathematical interpretation, our model renders it possible to interpret regularities in the volatility strike structure documented in the empirical literature in terms of the trading behavior of economic agents in the spot market. For this reason, the noise trading approach to option pricing developed in the present sections offers an *economically meaningful new alternative explanation* for the pricing errors resulting when the baseline FX option valuation frameworks are implemented to price real-life foreign currency options.

The results of the theoretical analysis also indicate that both the instantaneous variability of the spot rate and volatilities embedded in FX options might help to identify implicit price barriers in actual foreign currency spot markets. In the next section, the former issue will be discussed in more detail and a simple volatility-based empirical test will be utilized to test for implicit support or resistance thresholds in real-life FX market.

3 Implicit Trigger Prices and Exchange Rate Dynamics: Volatility–Based Empirical Evidence

The noise trader foreign currency option pricing model laid out in the preceding section stipulates that discretely spaced implicit trigger spot rate levels at which a substantial fraction of market participants swaps between domestic and foreign assets are an important determinant of the shape of the volatility strike structure implicit in FX options premia. The core intention of the following empirical analysis is to unearth whether such generally not immediately observable trading induced implicit trigger exchange rates do exist in real-life foreign exchange markets. As the theoretical analysis presented in the preceding section predicts that implicit trigger rates can exert a quantitatively substantial impact on foreign currency option prices, the empirical analysis helps to verify whether real-life exchange rate data do or do not support the barriers hypothesis of the volatility strike structure. The empirical study utilizes the result derived in the previous section that the instantaneous volatility of spot rates reflects the impact of program trading on the dynamics of the underlying exchange rate process.

Early tests for implicit price barriers in financial markets can be found in Donaldson and Kim (1993) and De Grauwe and Dewachter (1992).[5] The central idea underlying the testing approach used in these contributions is to subdivide the real line between asset price levels suspected to hold special significance for investors into a certain number of disjoint subintervals. Upon constructing a unique mapping between these subintervals and the level of the asset price under investigation, it is then examined whether either the relative frequency distribution of the categorized returns or asset price series show a shape in line with the predictions of models of asset pricing under regulated fundamentals. Consistent with the implications of the model of Krugman and Miller (1993) laid out in the previous section or of the framework of e.g. Bertola and Caballero (1992), a hump-shaped functional form of the respective relative frequency distributions is interpreted as evidence for the presence of destabilizing price barriers in the market. The formal reasoning underlying this line of argumentation exploits the fact that the existence of trading behavior induced price barriers in real-life FX markets implies that closing spot prices will spend less time near the barriers and will, therefore, be found significantly less frequently in those subintervals closest to the respective implicit price thresholds.

While Donaldson and Kim (1993) and De Grauwe and Dewachter (1992) resort to the relative frequency distribution of categorized returns and asset prices to test the barrier hypothesis, the central input to a new empirical testing strategy recently suggested by Cyree et al. (1999) are estimated time-varying conditional volatilities of the returns of financial market prices. While Cyree et al. (1999) use the volatility-based test to examine potential links between implicit price barriers and the variability of stock market returns, the intention behind the present analysis is to study whether systematic patterns in the volatility of major exchange rates point to the existence of trading induced price thresholds in the respective spot markets. The testing approach suggested by Cyree et al.l (1999) is modified to account for the statistical properties of the exchange rate returns series under investigation and to ensure that our results are comparable to those presented by De Grauwe and Decupere (1992) in an earlier study on implicit price barriers in FX markets. Inspired by the predictions of the noise trader model developed above, the idea motivating the test is to categorize exchange rates between certain reference values and to test whether the estimated conditional volatility of exchange rate returns in those classes close to the suspected barriers is significantly higher or lower than the conditional volatility in the other

5 For empirical studies using a similar testing procedure, cf. Donaldson (1990), Ley and Varian (1994), and Koedjik and Stork (1994).

classes. Using volatilities in tests for implicit price barriers is convenient as these data provide an economically intuitive terminology to examine (i) whether implicit price barriers do exist in real–life FX spot markets, and, (ii) whether such prices barriers can be perceived to stabilize or to destabilize exchange rates.

A technical advantage of volatility based tests for implicit price barriers in financial market price data is that such a quantitative research techniques allows to circumvent a critical facet of relative frequency based testing strategies stressed by Ceuster et al. (1998) and also noted in Ley and Varian (1994). The former show that the assumption underlying the tests conducted by e.g. Donaldson and Kim (1993) and De Grauwe and Dewachter (1992) that under the null hypothesis of no implicit price barriers the trailing digits' distribution of the asset price under investigation should resemble a uniform distribution is not correct. Thus, testing for implicit price barriers by plotting the relative frequencies of price realizations against categorized asset prices and testing whether or not the obtained mapping approaches a uniform distribution can lead to spurious results. Volatility-based testing approaches do not rely on the frequency distributions of asset prices or returns and are therefore not subject to the criticism of Ceuster et al. (1998).

The remainder of the following empirical section is organized as follows. Subsection 3.1 is employed to introduce an empirically testable concept of implicit price barriers in FX markets. The empirical model estimated to trace out the impact of implicit trading thresholds on the time–varying conditional volatility of exchange rates is formalized in subsection 3.2. The data used in the empirical analysis are described in subsection 3.3. The results of implementing the volatility-based test for implicit price barriers in exchange rates are reported and discussed in subsection 3.4.

3.1 Identification of Potential Trading Triggers in FX Markets

The first step in setting up a volatility-based test for implicit price barriers in FX markets is to identify a set of potential exchange rate trading triggers which are suspected to hold special significance for market participants. Scrutinizing the financial press and the investor guides published by large financial institutions propounds that agents involved in FX trading perceive such trading thresholds to show up whenever exchange rates reach certain rounded numbers. For example, philosophizing on profitable FX strategies, Merill Lynch (2000) explains that "The US dollar is close to a 10 year high against the Deutschemark and DEM 2.00

is proving to be a tough barrier....." (p.5) and, in the same booklet, it is emphasized that the "Euro/yen has the potential for a significant recovery and after failing to breach the 110 level in early January." (p.7). Taking this anecdotal evidence into consideration, we follow the literature and test whether or not the last trailing digits of the price of a currency hold special significance for agents participating in spot market trading. To be more specific, it is analyzed whether or not the volatility-based testing technique laid out below provides evidence for the presence of implicit price barriers at specific rounded exchange rate realizations ending with zero at the second and third digit after the decimal point. To offer an example, it is examined whether an implicit price barrier can be detected at a spot rate of 1.800 Deutsche Mark/US-Dollar or, in the reverse case, at a ratio of 0.700 US-Dollar/Deutsche Mark. In a similar vein, it is tested whether implicit trading trigger thresholds are reached when the $1000\times$US-Dollar/Yen exchange rate moves toward e.g. 6.100 or when the Yen/$100\times$US-Dollar parity can be found in the neighborhood of 1.600.

Based on the theoretical analysis outlined in the previous subsection, the economic idea behind the volatility-based test for implicit trading triggers in FX markets is to examine whether exchange rate volatility is significantly higher or lower in the neighborhood of such an implicit support or resistance barrier. Having defined a set of potential implicit exchange rate trading thresholds, the next step is therefore to identify the position of the spot rate relative to these suspected trigger levels. To accomplish this exercise, the real line between two potential price barriers is subdivided into 100 disjoint intervals of same length. Adopting the notation introduced by Donaldson and Kim (1993) and De Grauwe and Decupere (1992), the respective interval numbers are called *M*–values with $M = 1,....,100$. With this concept, it is possible to assign to every spot rate realization a unique interval number or *M*–value summarizing the distance of the current exchange rate from the two nearest potential trigger rates. Whenever the exchange rate can be found next to an implicit barrier price, his *M*–value is either very close to unity or to 100. Given this property, the *M*–values can be utilized to define the following set of dummy variables allowing to further operationalize the proposition that an exchange rate can be found in the „neighborhood“ of an implicit trading trigger:

(27a) $$dummy_1 = \begin{cases} 1 & if \quad M \in \{1,....,15,85,....,100\} \\ 0 & else \end{cases}$$

(27b) $$dummy_2 = \begin{cases} 1 & if \quad M \in \{1,....,20,80,....,100\} \\ 0 & else \end{cases}$$

(27c) $$dummy_3 = \begin{cases} 1 & if \quad M \in \{1,\ldots,25,75,\ldots,100\} \\ 0 & else \end{cases}$$

The three dummy variables are similar to those employed by De Grauwe and Decupere (1992) to define "neighborhood" intervals around the potential price barriers. This guarantees that the results reported in the present paper can be compared to those in the existing literature on implicit price barriers in FX markets. Moreover, given that the specific choice of the neighborhood of a potential trigger price is always to some extent arbitrary, utilizing three alternative different dummy variable to construct such intervals around an implicit price barrier provides an impression of the robustness of the results with respect to the definition of the "neighborhood" of implicit price barriers. The first dummy variable $dummy_1$ provided in equation (3.27a) defines a relatively narrow symmetric interval around a potential implicit support or resistance level. In contrast, the dummy variable $dummy_3$ formalized in equation (3.27c) is unity whenever an exchange rate settles in a relatively wide band around the suspected trigger rates. Finally, the variable named $dummy_2$ defined in equation (3.27b) can be employed to select a medium–sized range of M–values around the threshold spot rates suspected to hold special significance for FX spot market participants. As defined, roughly 1/3 (approximately ½) of the data can be found in the narrow (wide) interval placed around the potential implicit price barriers.

The set of dummy variables defined in equation (27) can now be utilized in a quantitative model of conditional asset price variability to test whether exchange rate volatility increases (or decreases) significantly as the distance of the exchange rate from a potential implicit spot market trading trigger tends to decline.

3.2 Modeling the Link Between Trading Triggers and Exchange Rate Volatility

A variety of competing concepts have been proposed in the empirical literature to measure the variability of financial market prices (see Pagan and Schwert (1990) for a survey). Among the most popular frameworks designed to estimate conditional financial market volatility are the numerous models belonging to the class of Generalized Autoregressive Conditional Heteroskedasticity (GARCH) frameworks pioneered by Engle (1982) and extended by Bollerslev (1986). Surveys of the strand of the literature dealing with issues related to the construction and the estimation of these types of models include Bollerslev et al. (1992), Bera and Higgins (1993), and Diebold and Lopez (1995). Early evidence for autoregressive conditional heteroskedasticity in daily exchange rate data is reported in Hsieh (1988) and in Diebold and Nerlove (1989).

To construct a GARCH model of conditional exchange rate volatility, the first step is to define the rate of return $\Delta \tilde{e}_t$ of the spot rate E_t from time $t-1$ to time t as:

(28) $$\Delta \tilde{e}_t \equiv 100 \times \left[\ln(E_t) - \ln(E_{t-1})\right]$$

with $\tilde{e}_t$ being the natural logarithm of E_t and Δ denoting the first-difference operator. To rule out that possible systematic day–of–the–week effects distort the results of the volatility-based test for implicit price barriers in FX markets, the following regression for the conditional mean of the raw data is estimated:

(29) $$\Delta \tilde{e}_t = \sum_{i=1}^{5} \beta_i d_{i,t} + \varepsilon_t$$

where $d_{i,t}$, $i = 1,....,5$ represent day-of-the-week dummies. Let the residuals obtained from estimating equation (29) by ordinary least squares be written as $\hat{\varepsilon}_t \equiv \Delta e_t$. Adopting a two–step methodology similar to the ones employed by Pagan and Schwert (1990), Amin and Ng (1997), and Alexander (1998), the input data employed in the estimation of the GARCH models used to trace out the time-path of conditional exchange rate volatility are taken from the series Δe_t. The general specification of a GARCH process of order $p \geq 0$ and $q \geq 0$, abbreviated as GARCH(p,q), can then be formulated as follows:

(30) $$\Delta e_t = u_t \qquad \text{with} \qquad u_t | \Omega_{t-1} \sim D(0, h_t)$$

(31) $$u_t = \eta_t \sqrt{h_t} \qquad \text{with} \qquad \eta_t \sim_{\text{i.i.d.}} D(0,1)$$

(32) $$h_t = \omega + \alpha_p \sum_{i=1}^{p} u_{t-i}^2 + \beta_q \sum_{i=1}^{q} h_{t-i} + \delta dummy_{i,t}$$

where $D(0, h_t)$ is some density function with mean zero and variance h_t, Ω_{t-1} is the information set including the realizations of all relevant variables up to and including time $t-1$, and η_t is an independently identically distributed (i.i.d.) white–noise process independent of u_{t-i} with zero mean and unit variance density D. Equation (30) implies that the conditional and the unconditional mean of the serially uncorrelated exchange rate returns are equal to zero. The unconditional variance of the exchange rate returns is constant and finite provided the roots of the characteristic polynom of the difference equation (32) lie outside the unit circle (see e.g. Bera and Higgins (1993)).

Though not linearly related, the model in equation (30)–(32) implies that exchange rate returns are not statistically independent because the variance of the data generating process conditioned on the information set Ω_{t-1} is given by $E_t\left(\Delta e_t^2 | \Omega_{t-1}\right) = h_t$. It follows that the conditional variance of exchange rate returns is a function of the information set Ω_{t-1} and is, thus, in general time–dependent. A sufficient condition ensuring that the conditional variance assumes only positive values is that $\omega > 0$, $\omega + \delta > 0$, $\alpha_i \geq 0$, and $\beta_j \geq 0$ for all $i = 1,\ldots.,p$ and $j = 1,\ldots.,q$.[6] According to this model, the time-dependent conditional variance of exchange rate returns depends on a constant mean level ω, on the lagged squared residuals u_{t-i}^2, $i = 1,\ldots.,p$ from the mean equation, and on a set of past forecasted variances h_{t-i}^2, $i = 1,\ldots.,q$ (the GARCH-terms). The GARCH model is equivalent to an ARCH model of infinite order featuring a rational lag structure with $p \rightarrow \infty$ and $\beta_q = 0$ for all q. The specific autoregressive functional form of the variance equation (32) implies that a volatility increase in a past period also raises the estimate of current volatility where the magnitude of the impact of a disturbance in period $t' < t$ on h_t declines geometrically in the distance $t - t'$.

The important economic feature of the empirical model formalized in equation (32) to capture the volatility dynamics of exchange rate returns is that the conditional variance h_t depends also on the position of the spot rate relative to the suspected implicit price barriers as measured by the variables $dummy_i$ defined in equation (27). The marginal significance of the coefficient δ allows to examine whether implicit price barriers in FX markets do exist. Moreover, as three different dummy variables have been defined, it is possible to examine the robustness of the findings regarding implicit price barriers in FX markets with respect to the specification of the width of the intervals defining the neighborhood of such a trading threshold.

To close the model formalized in equation (30) to (32), one has to select a specific functional form for the density D. In his seminal paper, Engle (1982) suggested to allow the disturbance term η_t to be standard normally distributed. This assumption implies that the innovation term u_t is conditionally normally distributed with conditional variance h_t. In the context of the modeling of financial market data, however, the assumption of a normally distributed error term has

---

6 Though these conditions are sufficient, weaker conditions allowing individual coefficients of the variance equation to assume negative numerical values can be derived from the inverted representation of the conditional variance. See Nelson and Cao (1992) and Drost and Nijman (1993) for a discussion.

often been found to be insufficient to account for the significant leptokurtosis characterizing the sample distributions of asset price returns in general and of daily and high-frequency returns data in particular. This finding is confirmed by the summary statistics offered in table 3 for the daily exchange rate returns used in the empirical analysis below. For this reason, Bollerslev (1986), Baillie and Bollerslev (1987), and Hsieh (1989) have suggested to resort to a *t*–distribution to describe the properties of u_t in the context of the modeling of exchange rate data. Lim et al. (1998) have advocated a Generalized *t*–distribution to model the distribution of daily exchange rate returns. As an alternative, Lui and Brorsen (1995) have suggested GARCH models featuring stable Paretian distributions introduced into the finance literature by Mandelbrot (1963) and Fama (1965). While Mittnik et al. (1997) and Paolella (1999) provide further examples for models belonging to this strand of the ARCH literature, the focus of Hafner (1998) is on an application of alternative non–parametric GARCH frameworks to the modeling of the dynamics of the conditional volatility of (high-frequency) exchange rate returns.

In the present study, an alternative modeling strategy advanced by Nelson (1991) is adopted. Nelson (1991) proposed to model asset returns with a GARCH framework featuring a Generalized Error Distribution $\eta_t \sim g(0,1,\kappa)$ normalized to have zero mean and unit variance. The positive parameter κ governs the thickness of the tails of the distribution. The parameter κ is not known in advance but must be estimated together with the other structural parameters of the model. In the case of $\kappa < 2$ ($\kappa > 2$), the distribution exhibits fatter (thinner) tails than the normal distribution. In the special case of $\kappa = 2$, the generalized error distribution degenerates to the standard normal density (see Hamilton (1994: ch. 21), Box (1953), or McDonald and Newey (1988)). A discussion of the properties of the framework for the special case of the Exponential GARCH (EGARCH) specification for the conditional variance equation can be found in Nelson (1991). For a general discussion, confer Hamilton (1994: pp. 668). The density of the innovation term η_t is given by:

(33) $$g(0,1,\kappa) = \frac{\kappa \exp\left[-0.5\left|\eta_t / \varpi\right|^{\kappa}\right]}{\varpi 2^{((\kappa+1)/\kappa)}\Gamma(1/\kappa)} \qquad \text{with} \qquad \varpi = \left[\frac{2^{(-2/\kappa)}\Gamma(1/\kappa)}{\Gamma(3/\kappa)}\right]^{1/2}$$

where $\Gamma(\cdot)$ represents the Gamma function:

$$\Gamma(y) = \int_0^{\infty} \exp(-s)s^{y-1}ds \qquad , \qquad y > 0$$

The factorized log likelihood function is then given by (cf. Nelson 1991: 355):

(34) $$LL = N\left\{\ln\left(\frac{\kappa}{\varpi}\right) - \left(\frac{1+\kappa}{\kappa}\right)\ln(2) - \ln\left[\Gamma\left(\frac{1}{\kappa}\right)\right]\right\} - \frac{1}{2}\sum_{t=1}^{N}\left|\frac{\varepsilon_t}{\kappa\sqrt{h_t}}\right|^{\kappa} - \frac{1}{2}\sum_{t=1}^{N}\ln(h_t)$$

The letter N denotes the number of observations. The procedure described in Berndt et al. (1974) is utilized to maximize the log likelihood function over the set of structural parameters of the model.

3.3 Exchange Rate Data

Daily spot rate quotes for the US-dollar/Deutsche Mark (US/DM), the US-Dollar/Yen (US/YEN), the Deutsche Mark/Yen (DM/YEN), the US-dollar/Pound Sterling (US/UK), and for the US-dollar/Canadian dollar (US/CAN) are utilized to perform the volatility-based test for implicit price barriers in foreign exchange markets. The sample period covers eight years of spot trading in these markets and ranges from 1/2/1990 to 12/31/1997. The exceptional influences stemming from both the turmoils in foreign exchange markets during the so-called Asian crises and the preparations for the introduction of the euro in Europe have led to the decision to exclude the year 1998 from the sample.

Table 1 provides information regarding the overall fluctuation range of the exchange rates observed over the sample period under investigation. The table further contains the respective number of exchange rate observations found in the three intervals around the potential implicit price barriers introduced in subsection 3.1. The figures presented in the second column of the table show that roughly 1/3 of the observation can be found in the narrow interval barrier (*Interval* 1) placed around the suspected implicit exchange rate thresholds. The figures summarized in the third column indicate that approximately ½ of the observations are an element of the widest interval (*Interval* 3) surrounding the potential implicit price barriers.

Table 1 — Descriptive Statistics for the Levels of the Currency Pairs under Investigation

Currency pair	*Interval* 1	*Interval* 2	*Interval* 3	*Maximum*	*Minimum*	*Observations*
DM/US	672	887	1133	1.8822	1.3530	2088
US/DM	707	913	1171	0.7391	0.5313	2088
DM/YEN	475	658	877	0.0169	0.0104	2088
YEN/DM	617	839	1020	95.9360	59.3252	2088
YEN/US	572	756	956	0.0124	0.0063	2088
US/YEN	611	832	1034	159.6424	80.6322	2088
CAN/US	415	593	812	1.4395	1.1199	2088
US/CAN	271	426	627	0.8929	0.6947	2088
UK/US	494	671	890	0.7052	0.4983	2088
US/UK	631	841	1045	2.0067	1.4180	2088

Note: The column headed *Interval* 1 (*Interval* 2, *Interval* 3) presents for each currency pair the number of exchange rate realizations falling into the respective intervals around the potential implicit price triggers as defined in equation (27a) (equation (27b), equation (27c)).

Before pushing on to present the results of the empirical study, table 2 presents some diagnostic statistics for the returns series Δe_t. The mean and the median of the series are approximately equal to zero. Furthermore, the returns data exhibit a kurtosis larger than that implied by the normal distribution. The coefficient of skewness is approximately equal to zero for the DM/US returns and is very low for the CAN/US, the DM/YEN, and the UK/US returns series. Visual inspection of the YEN/US returns indicated that the slight negative skewness of the sample distribution of this time-series can be attributed to a few influential outliers. To take account of these outliers, the day-of-the-week effect regressions in equation (27) were enriched in the case of YEN/US with a dummy variable forced to equal unity whenever the returns deviate more than four standard deviations from their sample mean. The results of conducting the portmanteau test developed by Ljung and Box (1978) provide no evidence for autocorrelation in the spot rate returns. However, the Langrange multiplier tests formalized in Engle (1982) for ARCH effects in the squared returns series indicate that the null hypothesis of no conditional heteroskedasticity in second moments can be rejected at all commonly used significance levels. Both visual inspection of the autocorrelation and the partial autocorrelation functions of the squared series Δe_t^2 and tests for remaining ARCH effects in the residual series obtained by estimating the model suggested in equations (30) to (32) under the assumption that $p = q = 1$ indicate that a parsimonious GARCH(1,1) model suffices to account for the conditional heteroskedasticity in the daily exchange rate returns under investigation.

Table 2 — Diagnostic Statistics for the Returns of the Exchange Rates Under Investigation

Currency pair	*DM/US*	*DM/YEN*	*YEN/US*	*CAN/US*	*UK/US*
Mean	0.0000	0.0000	0.0000	0.0000	0.0000
Median	0.0150	-0.0074	0.0252	-0.0092	0.0098
Maximum	3.4939	3.4825	4.1958	1.2396	4.2407
Minimum	-3.5182	-3.6955	-5.4014	-1.4070	-3.3813
Std. deviation	0.6971	0.6745	0.6909	0.2701	0.6546
Skewness	0.0600	0.2310	-0.6790	0.1053	0.2499
Kurtosis	5.3385	5.1480	9.1740	5.2677	6.7227
$Q(1)$	0.0095	0.967902	2.5456	1.5963	0.0396
$Q(4)$	2.1715	3.07574	4.1076	2.1745	4.4636
$ARCH(1)$	18.5993	14.1517	9.2267	15.6835	10.8389
$ARCH(4)$	39.5799	3.0757	13.0390	2.1745	4.4636

Note: For a definition of exchange rate returns, cf. equation (3.28). Under the null of conditional homoskedasticity, the Ljung and Box (1978) $Q(k)$–statistic and the $ARCH(k)$ test of Engle (1982) are asymptotically χ^2 distributed with k degrees of freedom.

A final more institutional aspect which should be taken into consideration when specifying a model for the conditional volatility of UK/US returns is that the crisis of the European Monetary System (EMS) in 1992 has marked an important change in the exchange rate regime. Economically, this is equivalent to a change in the overall stance of monetary policy. As emphasized by Lastrapes (1989), neglecting such policy changes in setting up GARCH models implies that the variance equation is misspecified, resulting in a spuriously integrated conditional volatility process. To take this critique into consideration, a dummy variable is included in the variance equations estimated to figure out the conditional volatility of UK/US and US/UK exchange rate returns. The dummy series assumes the value one before the crisis of the EMS and zero afterwards.

3.4 Conditional Exchange Rate Volatility and the Barriers Hypothesis

Table 3 presents the results of the volatility-based test for implicit price barriers in FX markets suggested above. The test is performed by analyzing whether or not conditional exchange rate volatility depends on the distance of the spot rate from the potential implicit price barriers. This issue is addressed by investigating the marginal significance of the coefficient δ in the variance equation of the empirical models employed to trace out the time-varying variability of the spot rates under investigation.

The table presents the estimation results for three alternative models which differ from each other in the way the variable $dummy_i$, $i = 1,2,3$ is defined. Figures in parentheses below the estimated coefficients give the ratio of the respective parameters and their standard errors. For ease of exposition only, the dummy variable defined to account for the impact of the substantial widening of the EMS target zone on the volatility of the UK/US and the US/UK returns volatility is not presented in the table. The coefficient of the dummy series is significant in both cases.

Table 3 reveals that the coefficient δ is significant in the case of the DM/YEN, the US/UK, the CAN/US, and the US/CAN returns series. It further turns out that the significance of the respective price barrier coefficients is robust with respect to a variation in the interval defining the neighborhood of an implicit price barrier. The coefficient of the trigger threshold dummy is insignificant in the frameworks estimated to model the conditional volatility of DM/US, YEN/US, and UK/US exchange rate returns. Furthermore, it can be seen in the table that the data provide only very weak evidence in favor of implicit trading triggers in the YEN/DM spot market. The respective barrier coefficient δ is significant at the 10% level only if the relatively narrow interval 1 is used to define the neighborhood of potential trigger prices. The parameter is found to be insignificant in the competing models featuring wider bands around the suspected implicit price barriers. Thus, as regards the YEN/DM spot market, the results are rather sensitive to the concrete specification of the model and provide no strong support in favor of the barriers hypothesis.

The figures documented in the table confirm the results of De Grauwe and Decupere (1992) who also find no clear-cut evidence for implicit price barriers in the US/DM exchange rate. Contrasting the findings of these authors, however, the results of the present empirical study do not support the hypothesis of implicit barriers in the US/Yen spot rate, too.

From the viewpoint of exchange rate and of options pricing theory, it is also interesting to note that those barriers coefficients found to be significantly different from zero have a positive sign. This empirical result indicates that the detected implicit trading induced price barriers exert an increasing impact on conditional exchange rate volatility as the spot rate reaches such a trigger rate. In the context of the options pricing model laid out above, this finding implies that the volatility strike structures implicit in these currencies should exhibit an U–shaped functional form resembling either a smile or a smirk. This is consistent with the stylized fact that volatility strike structures backed out of real-life FX options assume much more often a convex rather than a concave shape.

Table 3 — Implicit Price Barriers and Exchange Rate Volatility

Currency pair	*DM/US*	*US/DM*	*YEN/US*	*US/YEN*	*DM/YEN*
Model I	$dummy_1 = 1$ iff $M_i \in \{1,\ldots,15,85,\ldots,100\}$ for $i = 1,\ldots, N$ and zero else				
ω	0.0030 (1.5300)	0.0039 (1.9403)	0.0093 (2.1554)	0.0099 (2.035)	0.0092 (2.5134)
α	0.0347 (4.2068)	0.0343 (4.2621)	0.0327 (3.5240)	0.00328 (3.5038)	0.0521 (4.8301)
β	0.9557 (90.2120)	0.9586 (94.5864)	0.9453 (66.1831)	0.9453 (60.9014)	0.9229 (58.6352)
δ	*0.0056* *(1.1664)*	*-0.0007* *(-0.3096)*	*0.0007* *(0.1998)*	*-0.0013* *(-0.1192)*	*0.0112* *(1.7108)**
κ	1.1508 (23.2827)	1.1535 (22.8264)	1.0641 (31.7439)	1.0640 (31.7007)	1.2919 (27.1536)
LL	-2044.61	-2045.21	-1945.97	-1945.99	-2019.35
Model II	$dummy_2 = 1$ iff $M_i \in \{1,\ldots,20,80,\ldots,100\}$ for $i = 1,\ldots, N$ and zero else				
ω	0.0029 (1.3695)	0.0037 (1.8879)	0.0089 (2.2475)	0.0066 (1.4363)	0.0090 (2.2880)
α	0.0346 (4.2025)	0.0346 (4.2702)	0.0317 (3.6460)	0.0301 (3.6536)	0.05380 (4.8329)
β	0.9566 (92.47023)	0.9581 (92.9112)	0.9485 (68.1955)	0.9503 (71.4675)	0.9212 (56.9528)
δ^c	*0.0038* *(0.9802)*	*0.0001* *(0.0420)*	*-0.00001* *(-0.0046)*	*0.0057* *(0.5633)*	*0.0091* *(1.8113)**
κ	1.1512 (23.2673)	1.1522 (22.8879)	1.0623 (31.0705)	1.0627 (30.9069)	1.2937 (26.8128)
LL	-2044.82	-2045.26	-1979.95	-1979.84	-2019.24
Model III	$dummy_3 = 1$ iff $M_i \in \{1,\ldots,25,75,\ldots,100\}$ for $i = 1,\ldots, N$ and zero else				
ω	0.0031 (1.2767)	0.0039 (1.9267)	0.010 (2.3662)	0.0119 (1.9258)	0.0087 (2.1079)
α	0.0351 (4.2691)	0.0343 (4.2872)	0.0317 (3.7014)	0.0331 (3.6646)	0.05514 (4.9002)
β	0.9570 (92.9255)	0.9587 (95.5949)	0.9485 (69.5782)	0.9459 (65.3708)	0.9200 (56.7005)
δ	*0.0017* *(0.4762)*	*-0.0005* *(-0.2619)*	*-0.0019* *(-0.6029)*	*-0.0050* *(-0.5129)*	*0.0074* *(1.6720)**
κ	1.1521 (23.2529)	1.1531 (22.8698)	1.0621 (31.2050)	1.0620 (31.1439)	1.2943 (26.5138)
LL	-2045.16	-2045.23	-1979.80	-1979.87	-2019.56

Table 3 continued ...

Table 3 (continued)

Currency pair	*YEN/DM*	*UK/US*	*US/UK*	*CAN/US*	*US/CAN*
Model I	$dummy_1 = 1$ iff $M_i \in \{1,\ldots,15,85,\ldots,100\}$ for $i = 1,\ldots,N$ and zero else				
ω	0.0109 (2.8328)	0.0019 (1.82199)	0.0012 (1.473)	0.0006 (2.3839)	0.0005 (2.0218)
α	0.0513 (4.8843)	0.0374 (4.4687)	0.0414 (4.5009)	0.0408 (4.7596)	0.0400 (4.9023)
β	0.9295 (60.4376)	0.9495 (103.0950)	0.9473 (90.1352)	0.9485 (91.8329)	0.9526 (100.540)
δ	*-0.0058* *(-1.7298)**	*0.0018* *(1.7614)*	*0.0059* *(1.9018)**	*0.0012* *(2.0124)***	*0.0010* *(1.7305)**
κ	1.2950 (26.3836)	1.0484 (24.5345)	1.0483 (24.8164)	1.2119 (26.0996)	1.2119 (1.7305)
LL	-2019.74	-1779.66	-1778.32	-82.50	-83.89
Model II	$dummy_2 = 1$ iff $M_i \in \{1,\ldots,20,80,\ldots,100\}$ for $i = 1,\ldots,N$ and zero else				
ω	0.0111 (2.8109)	0.00202 (1.8487)	0.0016 (1.3845)	0.0006 (2.4270)	0.0004 (1.9393)
α	0.0524 (4.8776)	0.0378 (4.4755)	0.0415 (4.508)	0.0408 (4.7436)	0.0378 (4.7967)
β	0.9276 (58.2437)	0.9544 (101.6494)	0.9465 (89.1968)	0.9473 (88.9990)	0.9549 (104.3984)
δ	*-0.0039* *(-1.2634)*	*0.0013* *(0.8029)*	*0.0053* *(2.0181)***	*0.0011* *(2.0954)***	*0.0008* *(2.0317)***
κ	1.2923 (26.8085)	1.0482 (24.6037)	1.0486 (24.7498)	1.2120 (26.0722)	1.2133 (26.2606)
LL	-2020.29	-1779.75	-1778.23	-82.20	-83.37
Model III	$dummy_3 = 1$ iff $M_i \in \{1,\ldots,25,75,\ldots,100\}$ for $i = 1,\ldots,N$ and zero else				
ω	0.0110 (2.7520)	0.0021 (1.8549)	0.0013 (1.1214)	0.0006 (2.4044)	0.0004 (1.5861)
α	0.0530 (4.8385)	0.0384 (4.4744)	0.0422 (4.501)	0.0415 (4.7972)	0.0389 (4.8847)
β	0.9256 (56.3953)	0.9531 (98.7271)	0.9447 (87.3756)	0.9454 (87.4032)	0.9544 (102.9607)
δ	*-0.0016* *(-0.5293)*	*0.0015* *(0.9159)*	*0.0057* *(2.3790)***	*0.0011* *(2.3141)***	*0.0005* *(1.7253)**
κ	1.2902 (26.9747)	1.0485 (24.5907)	1.0498 (24.7717)	1.2139 (25.9505)	1.2129 (26.0205)
LL	-2020.85	-1779.65	-1777.51	-81.62016375	-84.19

Note: Figures in brackets represent the ratio of the respective coefficient and its standard error. In the case of the coefficient δ, an asterisk * (**) is utilized to denote significance at the 10% (5%) level.

Before proceeding with the economic interpretation of these results, it should be diagnosed whether the estimated models provide an acceptable representation of the main features of exchange rate returns volatility. The outcomes of the various conducted specification tests are summarized in table 4. For the sake of brevity, only results for Model I are reported. Inspired by Nelson (1991: p. 361), the second, third, and fourth columns of the table have been reserved to present the results of three conditional moments based specification tests. The statistic $M1$ is used to test the null that the standardized residuals η_t have mean zero ($E(\eta_t) = 0$). In a similar vein, the figures presented in the column headed $M2$ summarize the findings of tests of the null that the variance of the scaled residuals is not significantly different from unity ($E(\eta_t^2) = 1$). The intention behind the third conditional moments based specification test $M3$ is to check whether the standardized residuals are symmetrically distributed as required by the Generalized Error Distribution ($E(\eta_t |\eta_t|) = 0$). The marginal significance levels reported below the respective test statistics indicate that for all currencies investigated the null hypotheses of a correct specification cannot be rejected. To further challenge the estimated frameworks for exchange rate returns, the fifth and sixth columns of table 4 present the outcomes of tests for remaining autocorrelation and ARCH effects in the standardized and squared standardized residuals. As was the case with the moments based specification tests, the results provide support for the proposed set ups.

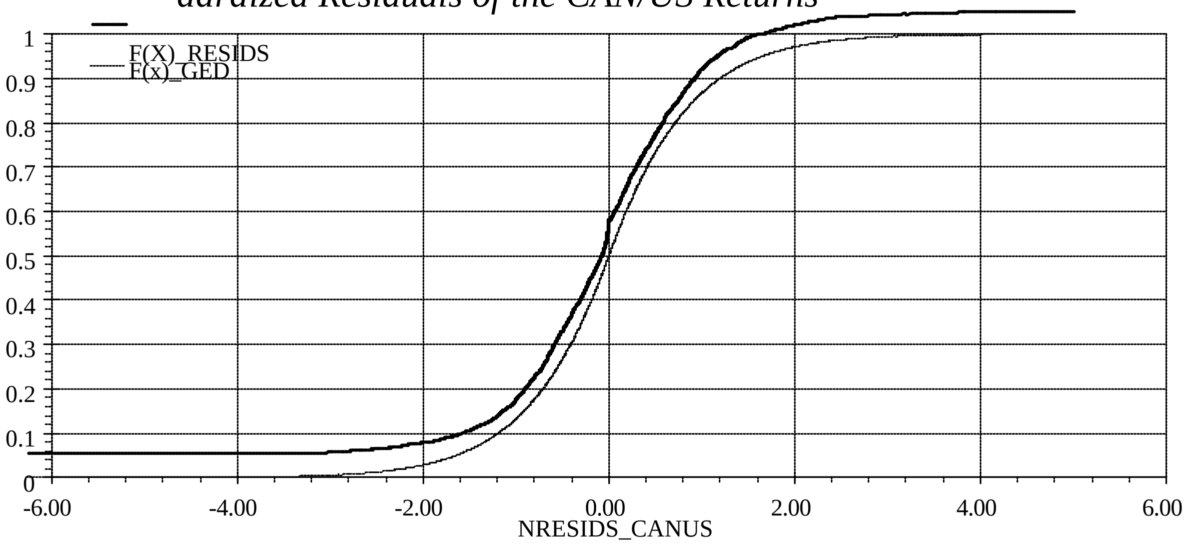
As a further test, the Kolmogorov–Smirnov statistic defined as the maximum of the absolute distance between the cumulative distribution functions of the fitted GED densities of the GARCH models (obtained by numerical integration) and the sample distribution of the standardized resiudals weighted with the square root of the number of observations has been computed (see e.g. De Groot (1989) for further comments on the K+S test). Critical values (reported in the note to table 4) for this test are available for the case that the parameters of the distribution function prevailing under the null hypothesis are known. As the parameter governing the thickness of the tails of the GED density assumed to describe the innovation term of the GARCH models has been estimated, these critical values should not be taken literally but should be used as a first benchmark providing an impression of the overall fit of the model.

Table 4 — Diagnostic Statistics for the Models of Implicit Price Barriers and Exchange Rate Volatility (Model I)

Currency pair	*M*1	*M*2	*M*3	*ARCH*(4)	*Q*(4)	*K+S*
DM/US	-0.2011 (-0.2011)	2084.006 (0.4980)	-0.2544 (0.7992)	3.8740 (0.4233)	2.3290 (0.6755)	0.76679
US/DM	0.1608 (0.8722)	2082.295 (0.4874)	0.1202 (0.9043)	3.4642 (0.4833)	2.2871 (0.6831)	0.7481
YEN/US	-0.2915 (0.7707)	2136.946 (0.2095)	-0.3567 (0.7214)	0.3137 (0.9889)	2.2271 (0.6941)	1.1760
US/YEN	0.2955 (0.7676)	2136.739 (0.2104)	0.3605 (0.7185)	0.3209 (0.9884)	2.2517 (0.6896)	1.1759
DM/YEN	-0.177 (0.8592)	2093.507 (0.4436)	0.9436 (0.3455)	7.3830 (0.1170)	1.8365 (0.7658)	1.3306
YEN/DM	0.3136 (0.7539)	2091.135 (0.4581)	-0.7320 (0.4637)	7.3624 (0.1179)	1.2849 (0.8639)	1.3377
UK/US	-0.7108 (0.4773)	2072.90 (0.4296)	-0.065 (0.9480)	2.6822 (0.6123)	4.8099 (0.3074)	1.2591
US/UK	0.6867 (0.4923)	2077.134 (0.4556)	0.01196 (0:9905)	0.905 (0.9238)	3.7829 (0.4361)	1.2523
CAN/US	-0.2606 (0.7944)	2106.708 (0.3649)	0.7079 (0.7079)	3.0789 (0.3618)	4.3408 (0.5447)	1.9596
US/CAN	0.2890 (0.7726)	2107.399 (0.3609	-0.6807 (0.4962)	4.1536 (0.3856)	3.7040 (0.4476)	1.9596

Note: *M*1, *M*2, and *M*3 denote moments based specification tests explained in the text. *ARCH*(4) and *LM*(4) are Langrange multiplier tests for remaining autocorrelation and ARCH effects in the standardized and the squared standardized residuals, respectively. Figures in parentheses represent marginal significance levels. In the column headed *K+S*, the Kolmogorov–Smirnov statistic is presented. Benchmark critical values for the Kolmogorov–Smirnov test under the assumption that the parameters of the distribution function under the null hypothesis are known are 1.22, 1.36, and 1.63 at the 10%, 5%, and the 1% significance level. See e.g. De Groot (1989).

Figure 6 — Actual versus Fitted Cumulative Distribution Function for the Standardized Residuals of the CAN/US Returns



All in all, the results of the Kolmogorov–Smirnov test indicate that the model provides, at least for our purposes, a fairly acceptable representation of the distribution of the scaled residuals.[7] A remarkable exception arises in the case of the returns of the Canadian dollar/ US dollar parity. However, in view of the magnitude of the estimated tail parameter κ, it was suspected that the high value of the *K+S* tests do not reflect a mismatch in the tails of the distributions of the scaled residuals and the estimated GED densities. This impression is confirmed by figure 6 which confronts the sample (F(x)_NRESIDS) and the estimated (F(x)_GED) cumulative distribution function of the standardized residuals (labeled NRESIDS_CANUS) for the CAN/US exchange rate. Eyeballing the graph reveals that the most pronounced differences between the sample and the estimated distribution function occur when the standardized residuals assume moderately negative numerical values. This suggests that, despite the high *K+S* test, the model still provides a reasonable characterization of the fat-tails property of daily CAN/US and US/CAN exchange rate returns. For this reason and in view of the fact that the other specification tests support the estimated models, it was decided to stick to the relatively simple GARCH models as a tool to model the conditional volatility of the Canadian dollar/ US dollar exchange rate returns.

Finally, the maximized log likelihood function of the above GARCH models have been compared with the log likelihoods obtained under an Integrated (I) GARCH restriction. The motivation to carry out this exercise was the observation that the figures provided in table 4 indicate that the sum of α and β typically lies in the vicinity of unity implying that exchange rate returns volatility is highly persistent. This finding might be interpreted to suggest that an IGARCH model in which the ARCH and the GARCH terms sum up to unity so that conditional volatility exhibits a unit root could be an alternative framework to capture the variability of exchange rate returns. The respective likelihood ratios, however, indicated that the baseline GARCH models are superior to their integrated alternatives. This result further corroborates the impression obtained by implementing and interpreting the results of the other specification tests that the GARCH frameworks provide an acceptable representation of conditional exchange rate volatility.

---

7 Though beyond the scope of the present section, further insights into the ability of the GED assumption to accurately account for the shape of the respective exchange rate returns distributions could be obtained by comparing the results reported in table 3.5 with *K+S* test results obtained for competing models relying on alternative distributional assumptions. In the context of the modeling of returns of financial market prices, examples for model selection strategies involving this criterion can be found in e.g. Mittnik and Rachev (1993), Mettnik et al. (1997), and Paolella (1999).

Taken together, the findings of the empirical analysis indicate that the data support the barriers hypothesis in the case of the DM/YEN, the US/UK, and the CAN/US exchange rates. Recognizing the predictions of the options valuation model developed in section 3.1, these results imply that implicit trading induced price barriers might be, at least to some extent, at the root of smiling volatility strike structures implicit in the FX options on these currencies. Thus, combing the results of the theoretical and the empirical analyses, financial institutions or economic agents participating in the trading of options on one or the other of the mentioned currencies should take care of implicit price barriers when it comes either to the pricing or to the hedging of positions involving these contracts. As regards the US/DM and the US/YEN spot markets, the empirical results do not serve to support the notion that trading induced implicit price barriers do exist at the spot rate levels suspected in the present empirical study to hold special significance for market participants. These finding, of course, do not allow us to reject the barriers hypothesis of the volatility smile *per se*. Indeed, the empirical findings only imply that such implicit price barriers presumably do not exist at the trigger thresholds defined in subsection 3.1. It would, therefore, not conflict with the results of the present empirical study if a researcher or a practitioner claimed that trading induced implicit price barriers might show up when the US/DM or US/YEN spot rates reach other critical levels holding special significance for market participants. It is left for future empirical research to investigate whether such critical spot rate realizations can, for example, be deduced by resorting to more refined techniques of technical chart analysis or be confronting the actual exchange rate realization with the historical extrema of the spot rate path.

4 Summary

The first section of the present paper has been devoted to the construction and the simulation of a continuous-time noise trader foreign currency option valuation framework. Based on a model of exchange rate determination originated by Krugman and Miller (1993), it has been highlighted how the impact of the placing of standing orders on both the volatility of the current spot rate and the international interest rate differential transmits onto the arbitrage–free prices of European style plain vanilla FX options. It has been demonstrated that the noise trader approach to foreign currency option pricing provides a rich and flexible setting allowing to interpret smile and smirk effects most often characterizing real–life implied volatility strike structures in economic rather than in purely technical terms. A Monte Carlo simulation study has unearthed that in our theoretical model the shape of the *endogenously* derived volatility smiles and smirks implicit in FX option premia depends upon (i) the width of the interval of inaction of portfolio managers, (ii) the net volume of standing orders, and, (iii) the magnitude of the corresponding exchange rate risk premia. Emphasizing the link between the trading behavior of economic agents in the spot market and the convexity of the volatility strike structure, the framework of analysis offers an *economically attractive new alternative explanation* for the pricing errors resulting when the first–generation FX option valuation approaches are applied to price and to hedge real–life foreign currency options.

An important result of the theoretical study is that both the instantaneous exchange rate volatility and volatilities implicit in foreign currency option premia embed valuable information regarding implicit support or resistance levels in FX spot markets. The variability of the underlying exchange rate as well as of foreign currency options implied volatilities tend to increase (decrease) as the spot rate reaches a destabilizing (stabilizing) trigger threshold inducing program traders to swap between domestic and foreign assets. In the empirical analysis contained in section 3, this result has been exploited to perform a conditional exchange rate volatility-based test for implicit price barriers in actual FX spot markets. Adopting a strategy suggested by Cyree et al. (1999), it has been examined whether or not the volatility-based test points to the presence of implicit trading induced price barriers at exchange rate realizations ending with zero at the second and third digit after the decimal point. Focusing on a sample period beginning in January 1990 and ending in December 1997, evidence supporting the barriers hypothesis has been found in the case of the DM/YEN, the US/UK, the CAN/US, and the US/CAN returns series. On the background of the theoretical analysis laid out in section 2, the findings indicate that generally not immediately observable implicit trigger thresholds might contribute to explain the occurrence of smiles in the volatility strike structures implicit in the FX options contracts on these exchange rates.

In future research, both the theoretical and the empirical work presented in the present paper could be extended in several interesting directions. From the viewpoint of economic theory, it might be worthwhile to build a more complex theoretical set up allowing to elaborate on the impact of stochastic investor sentiment, of uncertainty regarding the concrete spacing of noise traders trigger prices, or of repeated portfolio reallocations of program traders on the pricing and on the hedging of foreign exchange contingent claims. Yet another possibility would be to examine the potential link between the trading behavior in the market for the underlying financial security and the shape of the volatility strike structure by constructing FX option valuation frameworks featuring alternative forms of noisy spot trading. To perform such an exercise, one could e.g. construct a framework featuring heterogeneous trader groups in FX markets by subdividing market participants into fundamentalists and trend chasing chartists. Stylized models of exchange rate determination which utilize this modeling approach and which might, therefore, serve as a good point of departure for such an innovative research project can be found i.a. in Frankel and Froot (1990, 1998), De Grauwe et al. (1993), or Geiger (1996).

An alternative strategy would be to further explore the empirical predictions of the present noise trader option pricing framework. In this respect, it might be interesting to use volatilities implicit in actual foreign currency options instead of estimated conditional returns volatilities to perform volatility-based tests for implicit price barriers in financial markets. An example for such a research approach can be found in a recent contribution of Campa and Chang (1998). These authors utilize volatilities implicit in FX options to analyze the impact of the perforate price barriers which defined the EMS target zone regime on the variability of major European currencies.

Finally, it should be noted that the framework of analysis suggested in the present paper can not only be employed to model the relation between noisy spot trading and the shape of the volatility strike structure implicit in FX options prices but might serve as a useful starting point for analyses of markets for options on other financial securities as well. For example, the empirical evidence reported in i.a. Rubinstein (1994) suggests that the skewed implied volatility smirk which could be backed out of stock options in the aftermath of the tremendous market crash of October 19, 1987 might embody options traders anticipation of the existence of implicit stop-loss trading triggers in the market for the underlying shares. Recent findings documented in Gurdip and Madan (1999) indicate that a closer examination of the link between the dynamics of options prices and potential stock market rallies might be a promising area for future research. The results derived in the present paper should provide a rich and flexible point of departure for carrying out such projects.

5 References

Admati, A. and P. Pfleiderer (1988), A Theory of Intraday Patters: Volume and Price Variability. *Review of Financial Studies* (1): 3–40.

Alexander, V. (1998), Geldpolitik and Volatilitäten auf Finanzmärkten. In E. Baltensberger (ed.): *Spekulation, Preisbildung und Volatilität auf Finanz– und Devisenmärkten.* Berlin: Duncker and Humblot: 95–128.

Amin, K.I. and V.K. Ng (1997), Inferring Future Volatility From the Information in Implied Volatility on Eurodollar Options: A New Approach. *The Review of Financial Studies* (10): 333–367.

Baillie, R.T. and T. Bollerslev (1987), The Message in Daily Exchange Rates: A Conditional Variance Tale. *Journal of Business and Economic Statistics* (7): 297–305.

Bakshi, G.S. and Z. Chen (1997), Equilibrium Valuation of Foreign Exchange Claims. *Journal of Finance* (52): 799–826.

Bakshi, G.S. and D. Madan (1999), Crash Discovery in Stock and Option Markets. Working Paper. University of Maryland.

Balduzzi, P., G. Bertola, and S. Foresi (1995), Asset Price Dynamics and Infrequent Feedback Trades. *Journal of Finance* (50): 1747–1766.

Balduzzi, P., S. Foresi, and D.J. Hait (1997), Price Barriers and the Dynamics of Asset Prices in Equilibrium. *Journal of Financial and Quantitative Analysis* (32): 137–159.

Bera, A.K. and M.L. Higgins (1993), ARCH Models: Properties, Estimation, and Testing. *Journal of Economic Surveys* (7): 305–366.

Berndt, E.K., B.H., R.E. Hall, and J. Hausman (1974), Estimation and Inference in Nonlinear Structural Models. *Annals of Economic and Social Measurement* (4): 653–665.

Bertola, G. and R. Caballero (1992), Target Zones and Realignments. *American Economic Review* (82): 520–536.

Bick, A. (1987), On the Consistency of the Black–Scholes With a General Equilibrium Framework. *Journal of Financial and Quantitative Analysis* (22): 259–275.

Biger, N. and J.Hull (1983), The Valuation of Foreign Currency Options. *Financial Management* (12): 24–28.

Black, F. and M. Scholes (1973), The Pricing of Options and Corporate Liabilities. *Journal of Political Economy* (81): 637–659.

Bollerslev, T. and J.M. Woolridge (1992), Quasi-Maximum Likelihood Estimation and Inference in Dynamic Models with Time-varying Covariances, *Econometric Reviews* 11, 143-179.

Bollerslev, T., R.Y. Chou, and K.F. Kroner (1992), ARCH Modelling in Finance: A Review of the Theory and Empirical Evidence. *Journal of Econometrics* (52): 5–59.

Bollerslev, T., 1986, A Generalized Autoregressive Conditional Heteroscedasticity, *Journal of Econometrics* (31) 307-327.

Borenzstein, E.R. and M.P. Dooley (1987), Options on Foreign Exchange and Exchange Rate Expectations. IMF–Staff Papers (34): 643–680.

Boughton, J.M. (1987), Tests of the Performance of Reduced-Form Exchange Rate Models. *Journal of International Economics* (1): 41-56.

Boyle, P., M. Broadie, and P. Glasserman (1997), Monte Carlo Methods for Security Pricing. *Journal of Economic Dynamics and Control* (21): 1267–1321.

Boyle, P.P. (1977), Options: A Monte Carlo Approach. *Journal of Financial Economics* (4): 323–338.

Box, G.E.P. (1953), A Note on Regions for Tests of Kurtosis. *Biometrika* (40):465–469.

Brennan, M.J. and E.S. Schwartz (1989), Portfolio Insurance and Financial Market Equilibrium. *Journal of Business* (62): 455–476.

Campa, J.M. and P.H.K. Chang (1998), ERM Realignment Risk and Its Economic Determinants as Reflected in Cross–Rate Options. *Economic Journal* (108): 1046–1066.

Ceuster, M.J.K., G. Dhaene, and T. Schatteman (1998), On the Hypothesis of Psychological Barriers in Stock Markets and Benford's Law. *Journal of Empirical Finance* (5): 263–279.

Constantinides, G.M. (1986), Capital Market Equilibrium with Transaction Costs. *Journal of Political Economy* (94): 842–862.

Cox, J.C. and S.A. Ross (1976), The Valuation of Options for Alternative Stochastic Processes. *Journal of Financial Economics* (3): 145–166.

Cyree, K.B., D.L. Domian, D.A. Louton, and E.J. Yobaccio (1999), Evidence of Psychological Barriers in the Conditional Moments of Major World Stock Indices. *Review of Financial Economics* (8): 73–91.

David, A. and P. Veronesi (1999), Option Pricing with Uncertain Fundamentals – Theory and Evidence on the Dynamics of Implied Volatilities. Working Paper. Board of Governors of the Federal Reserve System, Washington, D.C.

Davis, M.H.A. and A.R. Norman (1990), Portfolio Selection with Transaction Costs. *Mathematics of Operations Research* (15): 677–713.

De Grauwe, P., H. Dewachter, and M. Embrechts (1993), *Exchange Rate Theory – Chaotic Models of Foreign Exchange Markets.* Oxford: Blackwell Publishers.

De Grauwe, P. and D. Decupere (1992), Psychological Barriers in the Foreign Exchange Markets. *Journal of International and Comparative Economics* (1): 86–101.

De Groot, M.H. (1989), *Probability and Statistics.* 2.ed. Reading, Massachusetts: Addison–Wesley.

Diebold, F.X. and J.A. Lopez (1995), Modeling Volatility Dynamics. Research Paper No. 9522. Federal Reserve Bank of New York.

Diebold, F.X. and M. Nerlove (1989), The Dynamics of Exchange Rate Volatility: A Multivariate Latent Factor ARCH Model. *Journal of Applied Econometrics* (4): 1–21.

Donaldson, G. and H.Y. Kim (1993), Price Barriers in the Dow Jones Industrial Average. *Journal of Financial and Quantitative Analysis* (28): 313–330.

Donaldson,G. (1990), International Evidence on Psychological Barriers in Asset Prices and the Efficient Market Hypothesis. Financial Research Center Memorandum No. 116. Department of Economics, University of Princeton.

Drost, F.C. and T.E. Nijman (1993), Temporal Aggregation of GARCH Processes. *Econometrica* (61): 909–927.

Dumas, B., P. Jennergren, and B. Näslund (1995), Realignment Risk and Currency Option Pricing in Target Zones. *European Economic Review* (39): 1523 – 1544.

Engle, R.F., 1982, Autoregressive Conditional Heteroscedasticity with Estimates of the Variance of U.K. Inflation. *Econometrica* 50, 987-1008.

Fama, E.F. (1965), The Behavior of Stock Market Prices. *Journal of Business* (38): 34–105.

Flood, R.P. and A.K. Rose (1995), Fixing Exchange Rates: A Virtual Quest for Fundamentals. *Journal of Monetary Economics* (36): 3–37.

Föllmer, H. and M. Schweizer (1993), A Microeconomic Approach to Diffusion Models for Stock Prices. *Mathematical Finance* (3): 1—23.

Frankel, J.A. and K.A. Froot (1990), Chartists, Fundamentalists, and the Demand for Dollars. In A.S. Courakis and M.P. Taylor (eds.): *Private Behaviour and Government Policy in Interdependent Economies*. Oxford: Clarendon Press.

Frankel, J.A. and K.A. Froot (1998), Chartists, Fundamentalists, and the Demand for Dollars. *Greek Economic Review* (10): 49–102.

Frey, R. (1998), Perfect Option Hedging for a Large Trader. *Finance and Stochastics* (2):115–141.

Frey, R. and A. Stremme (1997), Market Volatility and Feedback Effects from Dynamic Hedging. *Mathematical Finance* (7): 351–274.

Geiger, U. (1996), *Heterogene Erwartungen am Devisenmarkt – Das Portfoliomodell der Wechselkursbestimmung unter dem Einfluß autoregressiver Erwartungen.* Tübingen: Francke Verlag.

Genotte, G. and H. Leland (1989), Market Liquidity, Hedging, and Crashes. *American Economic Review* (80): 999–1021.

Giddy, I.H. (1983), Foreign Exchange Options. *Journal of Futures Markets* (3): 143–166.

Grabbe, J.O. (1983), The Pricing of Call and Put Options on Foreign Exchange. *Journal of International Money and Finanace* (2): 239–253.

Grossman, S. and Z. Zhou (1996), Equilibrium Analysis of Portfolio Insurance. *Journal of Finance* (51): 1379-1403.

Grossman, S.J. (1988), An Analysis of the Implications for Stock and Futures Price Volatility of Program Trading and Dynamic Hedging Strategies. *Journal of Business* (61): 275–298.

Hafner, C.M. (1998), *Nonlinear Time Series Analysis with Applications to Foreign Exchange Rate Volatility.* Heidelberg: Physica–Verlag.

Hamilton, J.D. (1994), Time Series Analysis. Princeton: Princeton University Press.

Hammersley, J.M. and D.C. Handscomb (1964), *Monte Carlo Methods.* London, New York: Chapman and Hall.

Harrison, J.M. and D.M. Kreps (1979), Martingales and Arbitrage in Multiperiod Securities Markets. *Journal of Economic Theory* (20): 381–408.

Heston, S. (1993), A Closed–Form Solution for Options with Stochastic Volatility with Applications to Bond and Currency Options. *Review of Financial Studies* (6): 327–343.

Hsieh, D.A. (1988), The Statistical Properties of Daily Foreign Exchange Rates: 1974–1983. *Journal of International Economics* (24): 129–145.

Hsieh, D.A. (1989), Testing for Nonlinear Dependence in Daily Foreign Exchange Rate Changes. *Journal of Business* (62): 339–368.

Hung, J.H. (1997), Intervention Strategies and Exchange Rate Volatility: A Noise Trading Perspective (16): 779 – 793.

Jeanne, O. and A.K. Rose (1999), Noise Trading and Exchange Rate Regimes. National Bureau of Economic Research Working Paper 7104. Cambridge, Massachusetts.

Koedjik, K.G. and P.A. Stork (1994), Should We Care? Psychological Barriers in Stock Markets. *Economics Letters* (44): 427–432.

Krugman, P. and M. Miller (1993), Why Have A Target Zone? Carnegie–Rochester Series on Public Policy (38): 279–314.

Krugman, P.R. (1987), Trigger Strategies and Price Dynamics in Equity and Foreign Exchange Markets. National Bureau of Economic Research Working Paper No. 2459. Cambridge, Massachusetts.

Lastrapes, W.D. (1989), Exchange Rate Volatility and U.S. Monetary Policy: An ARCH Application. *Journal of Money, Credit, and Banking* (21): 66–77.

Ley, E. and H. Varian (1994), Are there Psychological Barriers in the Dow Jones Index? *Applied Financial Economics* (4): 217–224.

Lim, G.C., J.N. Lye, G.M. Martin, and V.L. Martin (1998), The Distribution of Exchange Rate Returns and the Pricing of Currency Options. *Journal of International Economics* (45): 351–368.

Ljung, G. and G. Box (1978), On a Measure of Lack of Fit in Time Series Models. *Biometrika* (65): 297–303.

Lui, S. and B.W. Brorsen (1995), Maximum Likelihood Estimation of a GARCH–Stable Model. *Journal of Applied Econometrics* (10): 273–285.

McDonald, J.B. and W.K. Newey (1988), Partially Adaptive Estimation of Regression Models via the Generalized T Distribution. *Econometric Theory* (4): 428–457.

Mandelbrot, B. (1963), The Variation of Certain Speculative Prices. *Journal of Business* (36): 394–419.

Meese, R.A. and K. Rogoff (1983), Empirical Exchange Rate Models of the Seventies. *Journal of International Economics* (14): 3–24.

Merill Lynch (2000), FX Strategies: Bi–Weekly Themes and Forecasts. February 2000, Global Securities Research and Economics Group; Fixed Income Strategy.

Merton, R.C. (1976), Option Pricing When Underlying Stock Returns are Discontinuous. *Journal of Financial Economics* (3): 125–144.

Mittnik, S. and S.T. Rachev (1993), Modeling Asset Returns with Alternative Stable Distributions. *Econometric Reviews* (12): 261–330.

Mittnik, S., M.S. Paolella, and S.T. Rachev (1997), Modeling the Persistence of Conditional Volatility with GARCH–stable Processes. Working Paper No.

110/1997. Institute of Statistics and Econometrics, Christian–Albrechts–Universität at Kiel.

Nelson, D.B. (1991), Conditional Heteroskedasticity in Asset Returns: A New Approach. *Econometrica* (59): 347–370.

Nelson, D.B. and C.Q. Cao (1992), Inequality Constraints in the Univariate GARCH Model. *Journal of Business and Economic Statistics* (10): 229–235.

Nielson, L. and J. Saá–Requejo (1993), Exchange Rate and Term Structure Dynamics and the Pricing of Derivative Securities. Working Paper 92/72/Fin. INSEAD, Fontainebleau, France.

Pagan A.R. and G.W. Schwert (1990), Alternative Models of Conditional Stock Volatility, *Journal of Econometrics* 45, 267-290.

Paolella, M. (1999), Tail Estimation and Conditional Modeling of Heteroscedastic Time–Series. Berlin: Pro Business.

Platen, E. and M. Schweizer (1998), On Feedback Effects From Hedging Derivatives. *Mathematical Finance* (8): 67–84.

Rose, A.K. (1994), Are Exchange Rates Macroeconomic Phenomena? Federal Reserve Bank of San Francisco Economic Review (1): 19–30.

Rubinstein (1994), Implied Binomial Trees. *Journal of Finance* (49): 771–818.

Schäfer, K. (1993), *Optionsbewertung mit Monte–Carlo–Methoden*. Bergisch–Gladbach, Köln: Verlag Josef Eul.

Scott, L.O. (1997), Pricing Stock Options in a Jump–Diffusion Model with Stochastic Volatility and Interest Rates. *Mathematical Finance* (7): 413–426.

Sircar,K.R. and G. Papanicolaou (1998), General Black–Scholes Models Accounting for Increased Market Volatility from Hedging Strategies. *Applied Mathematical Finance* (5): 45—82.

Stein, E.M. and J.C. Stein (1991), Stock Price Distributions with Stochastic Volatility: An Analytical Approach. *Review of Financial Studies* (4): 727–752.