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on real commodity prices

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Keywords: Commodity prices, monetary policy, inflation, the 1970s.

JEL classification: E31, E52, E65, Q00.

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The effect of inflation on real commodity prices

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Abstract:

Recent research has shown that economic conditions have an important effect on real commodity prices. We quantify the contribution of fluctuations in inflation to this particular link. In the data, a temporary rise in inflation causes real commodity prices to rise, as does a rise in trend inflation. We find that a simple dynamic equilibrium model of commodity supply and demand gives a realistic response of real commodity prices to inflation. Based on historical simulations, shocks to inflation played an important role in commodity price dynamics during the 1970s, but they have contributed negligibly to commodity price movements since then.

The effect of inflation on real commodity prices

1. Introduction and motivation

In this paper, we document the effects of inflation on real commodity prices in the United States both in the data and in theory. The relationship between inflation and commodity prices is the subject of an ongoing public debate, especially in light of large real commodity price movements during the 2000s and 2010s. Recent research has shown that shocks to economic activity may have an important effect on commodity prices, and we wish to quantify the contribution of nominal disturbances to such fluctuations.[1] First we run a VAR with long-run restrictions and show that inflation has a robust effect on real commodity prices. Then we formulate a parsimonious forward-looking dynamic general equilibrium model of real commodity prices where the economy is subject to nominal disturbances. We find that the model can replicate the behavior of commodity prices found in the data. We further explore the role of shocks to inflation in the historical evolution of commodity prices. Shocks to inflation played a large role in the 1973-74 commodity price runup, but in general, runups in commodity prices have *not* signaled impending inflationary pressure since nominal shocks in general are small. Our evidence supports the claim by Barsky and Kilian (2002) that nominal shocks had a large effect on real commodity prices during the 1970s, but we present an alternative mechanism of transmission which is not based on the systematic misperception of trend inflation.

Commodity prices have been volatile in recent years; one spike of volatility occurred in 2008. A debate has gone on as to how much monetary policy is to blame for volatile real commodity prices. On June 20, 2008, Guillermo Calvo wrote in VoxEU, "Today's explosion of commodity prices is the result of a very real global financial storm associated with large excess liquidity in several non-G7 countries and nourished by the low interest rates set by G7 central banks. This price explosion could be a leading indicator of future inflation driven by fundamentals." Taylor (2009) echoes Calvo's view, blaming deviations of Fed policy from the eponymous Taylor rule for the runup in oil prices at that time. Erceg, Guerrieri, and Kamin (2011) disagree; they argue that strong global commodity demand unrelated to monetary policy led to the 2008 commodity price runup and that Fed policy was not to blame.

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[1] Kilian (2009), for instance, presents convincing evidence that movements in real global economic activity are an important driver of oil prices. Hamilton (2009a) discusses the role of global oil demand in the context of the 2007-08 runup in oil prices. Barsky and Kilian (2004) further discuss the idea that oil prices are endogenous.

With the runup in commodity prices in 2010-11, the possible role of monetary policy is again subject to debate, especially since the Fed has undertaken massive easing operations in response to the late 2000s financial crisis.

Surprisingly little work has gone into closely evaluating the quantitative role of inflationary disturbances in determining commodity prices. Most work has gone in the opposite direction, seeking to understand how commodity prices affect inflation. Causation almost certainly runs both ways. We disentangle the two directions of causation using a VAR with long-run restrictions, focusing on the role of shocks to long-run inflation and to nominal spending conditional on long-run inflation in the movements of a real commodity price index. Our analysis shows that the discussion regarding the relationship between inflation and commodity prices is well-motivated. In the data, high inflation raises real commodity prices, and nominal commodity prices tend to adjust more quickly in response to shocks than core prices. We find a statistically and quantitatively robust relationship running from inflation to real commodity prices; a one percent permanent increase in the core price level leads to between a three and four percent increase in real commodity prices along the transition path. A shock to trend inflation also shows up in higher real commodity prices, though we later argue that most of this effect comes through the relationship between trend inflation and short-run inflation.

To explain the relationship which we see in the data, we develop a simple structural macroeconomic model where core prices are sticky and commodity prices are flexible. The model performs well; given realistic supply and demand elasticities for commodities, it generates realistic movements in commodity prices. A one-off increase in nominal spending should cause producers to move out along their aggregate supply curves, increasing output. As output increases, final demanders seek to spend more on primary commodities—energy, food, and raw materials—since commodities and sticky-price output are complements. Since commodity supply and demand are fairly price-inelastic, real commodity prices should increase in response to the increase in demand. We show that theory and data agree on the magnitude of the effect of inflation on commodity prices at short horizons. We also discuss the long-run implications of an increase in trend inflation; high trend inflation by itself introduces distortions into the price-setting process and should be neutral or mildly contractionary to both output and real commodity prices.

We then discuss the historical behavior of real commodity prices in the context of the actual shocks to have hit the United States economy since 1971. Using both the VAR and the theoretical model, we find that large temporary disturbances to inflation during the 1973-74 episode could account for a large part of the runup in commodity prices during that period. Outside of that episode, disturbances to inflation are a minor contributor to commodity price movements. Even though the effect of inflation on commodity prices is large and robust, the shocks are usually small, so most commodity price movements reflect real factors which affect the net supply of commodities. Even though high inflation signals higher commodity prices, higher commodity prices do not necessarily signal high inflation.

2. The data and VAR setup
2.1 Measures of trend inflation and real commodity prices

We use data running from the first quarter of 1970 through the fourth quarter of 2010. The dataset used in the different VAR specifications consists of five basic series in the same relative order: A survey-based measure of long-run inflation expectations which we have constructed, the PPI for crude materials (SOP1000) from the BLS deflated by the core (nonfood, nonenergy) PCE deflator, real GDP, core PCE inflation, and the three-month treasury constant maturity rate. In the VAR, the first three series are taken in first differences and the latter two series are detrended using their cointegrating relationships with long-term inflation expectations. All growth rates, inflation rates, and interest rates are quarterly except for in the narrative discussion which follows.

Long-term inflation expectations equal the one-to-ten year forward inflation forecast derived from a composite of median forecast surveys, for the CPI following 1983 and for the old fixed-weight GDP deflator (which mimics the CPI in the long run) before that. After 1990, the longer-term forecasts come directly from the Survey of Professional Forecasters, and all shorter-term forecasts come from that survey. From the end of 1979 to 1990, the longer-term forecasts come from a composite of the Blue Chip Survey and the Livingston Survey provided by the Philadelphia Fed. Before 1979 and during missing quarters, we interpolate by regressing CPI inflation on the forward interest rate. The regression coefficient equals 0.688, which yields about the same cointegrating relationship as that which Crowder and Hoffman (1996) find between interest rates and CPI inflation. We interpolate inflation expectations using that rate and coefficient. The spreadsheets provided by the Philadelphia

Fed give the details of the data underlying the different surveys. In general our measure tracks that of Clark and Nakata (2008) extremely well, though we think that more historical scholarship is needed if we wish to firm up our estimates of trend inflation beyond the early 1970s.

The top panel of Figure 1 shows the behavior of the composite measure of expected inflation throughout the sample along with two other alternative measures, all at annual rates. At the beginning of the 1970s, long-term interest rates and inflation expectations were not especially high; trend CPI inflation was near four percent. In the first quarter of 1973, monetary policy suddenly became unanchored. The trend inflation rate jumped discretely by about one percentage point from below four percent to near five percent, creeping up another half point by the third quarter. After the runup in oil prices during the fourth quarter, measured trend inflation crept up another point and then settled into the five to six percent range. Most of the original 1970s runup in trend inflation seems to have occurred in the quarters before the late-1973 oil shock.

Throughout 1978 and 1979, inflation expectations crept upward again. At the end of 1977, trend inflation stood at just under five percent, where it had stood in mid-1973. It began to creep upward in 1979; it rose to over six and a half percent at the end of 1979, which is when Volcker announced a change in operating procedures. Trend inflation continued to rise through late 1980 and then declined precipitously through 1982 back toward 1970s levels. It fell further at the beginning of 1985 and has since declined gradually toward the two to three percent range, where it has remained. From about late 1986 onward, long-term inflation expectations appear remarkably stable, and monetary policy has appeared exceptionally well-anchored in the long run.

Figure 1 also shows the 10-20 year forward treasury rate, calculated using constant maturity data from the FRED database. For the period when no data exist for 20-year treasuries, we linearly project quarterly changes in the 20-year rate using changes in the 10-year and 30-year rates and then correct linearly for the error of closure. Other interpolation methods give almost the exact same results. The composite forward rate broadly tracks changes in inflation expectations but shows more short-run volatility, particularly during the early to mid 1980s. There are also important low-to-medium-frequency movements in real interest rates (particularly during the mid 1980s) which show up in the forward interest rate series but do

not show up as strongly in the forward inflation rate series. This is why we choose our composite inflation indicator as our measure of trend inflation; there do appear to be factors which affect long-term interest rates in the medium run apart from trend inflation.

Because of these concerns about using interest rates to project inflation expectations back through the 1970s, we also create a regression-based expected inflation indicator broadly similar to that developed by Kozicki and Tinsley (2006). The Livingston Survey goes back before 1979 but it lacks explicit information on long-term inflation forecasts. Kozicki and Tinsley estimate a state-space model in order to extract trend and temporary components to inflation forecasts. We do something simpler which delivers a similar result to theirs. We take the Livingston Survey median forecasts for the CPI and regress our post-1980 forward inflation indicator on forecasted two-year-ahead inflation (based on the two year forecast and the one year forecast) and on fourteen-month inflation (based on the twelve-month forecast and base-period levels which are reported with a lag of two months).

Figure 1 also shows the results of that projection. During the overlap period, the only strange-looking residual occurs in 2004. The projected inflation forecast measure continues to track our measure before 1979. The broad similarity between our version of the Kozicki-Tinsley measure and our interest-rate-based measure gives us some confidence that our trend inflation rate series actually describes the behavior of trend inflation. Clark and Nakata (2008) give a good description of the evolution of inflation expectations over our sample, using a composite measure which is also used in the FRB/US model. The lack of good data on the expected term structure of inflation before 1974 causes us some concern, so we regard our measure of inflation expectations with a certain degree of caution before that date. For that reason, and because of the role of the Texas Railroad Commission in setting key commodity prices, we do not venture before 1970 in our analysis.

The bottom panel of Figure 1 shows trend inflation alongside actual CPI inflation and PCE core inflation. In the medium to long run, these measures track each other extremely well. As one might expect, PCE core inflation is much smoother than CPI inflation, since the CPI includes food and energy prices. It is worth noting that core PCE inflation tracks long-run CPI expectations very closely, with one large deviation during the 1973-74 episode.

Figure 2 shows three measures of real commodity prices. It shows the BLS PPI for crude commodities deflated by the core PCE deflator (our preferred measure), and it shows the prices of noncore PCE (food and energy) deflated by the core PCE deflator. It also shows the CRB commodity index deflated by the core PCE deflator. All three measures move together substantially at a high frequency, with noncore PCE prices lagging commodity prices somewhat and CRB prices showing more volatility and a pronounced downward trend. The lag of noncore PCE prices makes sense because consumer expenditure on food and energy (e.g. for a steak in the supermarket) covers a broader range of things than the pure material costs of food or energy (e.g. transportation, refrigeration, and butchering). It takes time, labor, and intermediate inputs to market, store, transport, and process commodities for final use, and as noncommodity prices are more sluggish than commodity prices, prices for final food and energy goods will lag the prices for raw goods. Therefore we prefer to use the PPI measure as our commodity price measure since PPI prices are more flexible than finished goods prices. Real commodity prices do not appear to have a strong trend since 1970, though they are prone to sudden large movements. The PPI crude materials price shows upward spikes beginning in the first quarter of 1973 (three quarters before the OPEC oil shock), a more gradual runup in the late 1970s, a precipitous decline centered on the beginning of 1985, obvious spikes driven by the Persian Gulf Wars and Hurricane Katrina, and three more runups peaking in 2001, 2008, and beginning in 2010. The first three episodes coincide with large movements in inflation discussed above, while the latter episodes do not coincide with such movements.

2.2 Previous studies on the effects of inflation

There is a large literature on the effects of commodity prices on inflation, but there is only a small literature on the effect of inflation on commodity prices. Most empirical evidence so far on the short-run effects of inflation has come through the use of short-run identifying restrictions in a VAR framework, though Bordo (1980) presents some early evidence using distributed lags. Gordon and Leeper (1994) and Christiano, Eichenbaum, and Evans (1999) look into commodity prices as part of a larger system; the former find a slight expansionary effect of monetary policy on commodity prices, while the latter find an ambiguous effect. Scrimgeour (2010) uses futures market data to identify "policy surprises" and finds a negative effect of a surprise movement in interest rates on commodity prices. Anziuni, Lombardi, and Pagano (2010) look at the relationship between interest rate disturbances and commodity

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prices, and they find a negative effect of a positive interest rate disturbance. A close look at Anziuni et al.'s impulse responses shows that their use of short-run restrictions can lead to unusual results. In their VAR, a monetary policy shock has no effect on the price level, which is an unusual result; in all three studies, the effect of an expansionary nominal shock on the general price level is not unambiguously positive. Any economically plausible identification scheme must have the price level rise in response to an expansionary nominal shock.

There is another issue with the short-run approach which leads us to use long-run restrictions. Monetary policy at a quarterly frequency is almost certainly endogenous and it is difficult to extract an economically plausible structural shock from reduced-form VAR disturbances. It is difficult to justify a story where the Fed simply decides to adjust interest rates in order to shock the economy; instead, it responds to disturbances which hit the economy, some of which come in the form of commodity supply shocks. We therefore follow Lastrapes (2006) and Lastrapes and Selgin (1995) and implement an economically plausible long-run identification scheme, and we additionally take variations in trend inflation into account since the inflation rate itself appears to be nonstationary.[2] The shocks which they and we identify correspond with the classical thought experiment which posits the long-run neutrality (but not necessarily superneutrality) of money and prices. We take the nonstationarity of inflation into account by separately estimating the effects of shocks to long-run inflation expectations and short-run inflation. Our findings add considerable strength to the vague consensus that nominal aggregates are not neutral with respect to the level of real commodity prices.

2.3 Long-run identification and ordering

We identify a Bayesian VAR using long-run restrictions in order to get an economically plausible identification of shocks.[3] The baseline VAR contains long-run inflation expectations (which we assume is an exogenous random walk), real commodity prices, and the deviation of PCE inflation from long-run CPI expectations, in that order. To check for robustness, we also look at larger systems which have real GDP growth ordered third and three-month treasury rates multiplied by 0.688, minus trend inflation, ordered last. All variables are at a quarterly rate. The variables follow a natural economic ordering. First, we

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[2] Lastrapes (2006) looks at the dispersion of individual commodity prices over time in response to inflation. Lastrapes and Selgin (1995) do not look at commodity prices, but they use long-run restrictions to investigate the liquidity effect.
[3] We use a Bayesian VAR primarily to get exact confidence bands for our impulse responses, not because we have any interesting prior information.

treat inflation as primarily a monetary phenomenon and view the long-run inflation target as substantially under the Fed's control. Long-run inflation is therefore an exogenous random walk. We order commodity prices second, in order to allow for some effect of commodity prices on output (where appropriate) and on short-run monetary policy. We put output growth next. Output can respond in the long run to trend inflation and to the price of commodities. One can imagine situations where the price of commodities in the long run responds to output as well—rapid growth in China and India has coincided with a runup in global commodity prices. Since we do not directly analyze commodity price shocks or long-run output shocks, the ordering of these two variables does not affect our results, which concentrate on the effects of monetary policy.

We order detrended core inflation next. Conditional on the other shocks, we define a shock which affects only the price level in the long run as a short-run inflation shock. In the short run, monetary authorities can and do respond substantially to the condition of the economy, and they may also respond directly or indirectly to commodity prices or shifts in their inflation target. But, we assume that the price level has no long run effect on output, commodity prices, or the long run inflation target. Our inflation shock resembles a classical monetary shock in its long-run effects. We put the short-run interest rate last; its residual has no obvious interpretation but it fills out the matrix of shocks. We concentrate mostly on a three-variable baseline VAR containing the long-run inflation target, commodity prices, and short-run inflation. We choose to concentrate on the three-variable VAR because it gives similar results to the other systems but is more parsimonious. Also, when we look at shorter subsamples, the three-variable VAR is inherently less noisy and provides fewer issues with unstable MCMC draws than the larger systems. It seems to capture the same information as the larger systems but more precisely since there are fewer parameters to estimate. Appendix A contains a lengthy description of the estimation procedure for the VAR, which we estimate by MCMC. All coefficients have an uninformative normal prior distribution and all variances have an uninformative inverse Wishart prior distribution.

3. Estimation results
3.1 Main results: The effects of shocks to inflation, 1970-2010.

Figure 3 shows the impulse responses of the baseline three-variable VAR to a one percentage point shock to long-run quarterly inflation, and Table 1 shows summary statistics for the

impulse responses. The baseline VAR has long-run inflation ordered first, followed by real commodity prices and short-run inflation. The figure shows exact 95% and 90% confidence bands bounded by the 2.5th and 97.5th percentiles and the 5th and 95th percentiles, respectively. Long-run inflation expectations have a large posterior median effect on both real commodity prices and on short-run inflation, though it is impossible to firmly say that the median effect is positive. It is clearer that a shock to long-run inflation affects commodity prices in the short run. Two quarters after a 1% quarterly long-run inflation shock, commodity prices rise by 35%, and then they revert incompletely back toward trend. The positive short-run effect of trend inflation on commodity prices occurs with a 99.0% probability. The estimates come with a fairly wide standard error, but they do indicate that Barsky and Kilian (2002, 2004) are on firm statistical ground when they claim that the unanchoring of monetary policy during the 1970s may have contributed to the rise in commodity prices.

With regard to short-run monetary shocks, the evidence is somewhat stronger. Figure 4 shows the impulse responses to a short-run inflation shock which results in a cumulative increase in prices of 1%. This type of shock corresponds with the classical thought experiment that traces out the transition path of the economy following a one-off injection of money. Core prices rise gradually; the process of inflation takes some time and inflation rates are positively autocorrelated. The real price of commodities peaks quickly the following quarter, registering a rise of 3.4%. The posterior probability that this response is positive starts at 91.0% and peaks at 94.5% four quarters out. These impulse responses imply a large degree of overshooting in the short run—nominal commodity prices rise quickly in response to an inflationary shock while core prices rise slowly. The statistical evidence that both types of inflation shocks affect short-run commodity prices is stronger than the evidence that long-run inflation affects long-run commodity prices.

In general, the baseline results suggest that both long-run inflation and short-run inflation affect real commodity prices in the short run, with the total long run effect of a rise in the inflation rate remaining much more ambiguous. The response of real commodity prices to a one-off burst of inflation is fast and dies out within a year and a half of the initial shock. We will argue that the short-run relationship between inflation and commodity prices makes sense theoretically, while long-run inflation should not have a large direct effect in theory. However, this view is still compatible with the idea that the combination of stagflation and

high oil prices in the 1970s may have partially arisen from monetary policy, since short-run inflation shocks are not in general orthogonal to long-run inflation shocks.

3.2 Robustness: Larger and smaller systems and a higher lag order

Estimates for the other sized systems (also shown in Table 1) look almost the same as for the three-variable VAR, with the larger systems naturally showing a lesser degree of statistical confidence because of the large number of parameters to be estimated. The impulse responses to the two sets of monetary shocks for inflation and commodity prices look almost the same quantitatively. In all cases, commodity prices show mild signs of overshooting in response to trend inflation, and all of them give very similar quantitative estimates of the response of commodity prices to short-run inflation. The four-variable and five-variable VARs do not show any consistent behavior with respect to output, except to show that inflation may be mildly stimulative in the short run. Either way, it does not seem to matter much whether one includes output or interest rates in the VAR when discussing the dynamics of commodity prices. The last column of Table 1 shows the results of extending the baseline VAR to six lags. The six-lag VAR delivers the same basic behavior as the baseline VAR. In response to short run inflation shocks, the six-lag specification shows a slightly smaller but still large endogenous commodity price response than the four-lag VAR. An eight-lag VAR (not shown) also shows similar responses, though naturally even less precisely measured. In short, the estimated effect of shocks to inflation is robust to changes in the size and order of the VAR.

3.3 Stability across subsamples

One natural concern would be to see if the estimates are stable across time. We therefore look to see if the VAR has similar predictions for the effects of shocks through the end of 1984 and after the beginning of 1985. We choose these dates because McConnell and Perez-Quiros (2000) and Kim and Nelson (1999) place the beginning of the erstwhile Great Moderation around 1984 or 1985, and these dates are close enough to the middle of the sample to allow us to estimate coefficients on both sides of the break. Also, as discussed above, inflation expectations became much more firmly anchored in the mid-1980s and core inflation has deviated relatively little from trend. We report the results for the VAR for both subsamples in Table 2.   The estimated response of commodity prices to long-run inflation is large across

both subsamples but imprecisely measured. However, in terms of magnitude, there is no evidence that the response of commodity prices to inflation has changed over time—if anything the response may have increased, though it is not possible to say by how much with any degree of statistical confidence.

In response to short-run inflation, the VAR shows that inflation increases commodity prices across both subsamples, with a reasonable degree of statistical confidence. The relationship between inflation and commodity prices is not confined to the pre-1985 period; on the contrary, the evidence indicates that the effect of inflation on real commodity prices may have increased—though the change in the effect is not precisely estimated. The relationship between inflation and commodity prices is a robust feature of the data; it is not a statistical artifact from looking at the 1970s.

3.4 Results using an alternative measure of food and energy prices

The second to last column in Table 2 shows the results using the price of noncore PCE (that is, PCE food and energy) deflated by the core PCE deflator. In comparison with the baseline results, the effects of a shock to long run inflation on commodity prices are reduced somewhat. This is not surprising since PCE food and energy prices measure the cost of food and energy to the consumer. Consumers pay for both the material costs of what they consume and any distribution and processing costs incurred along the way. These intermediate goods and services constitute a large share of consumers' final expenditure, with material costs constituting a smaller share. The degree to which nonmaterial costs contribute to food and energy expenditure will show up as reduced passthrough of commodity prices into food and energy prices. The estimates from the VAR indicate overwhelmingly that real food and energy prices respond positively to trend inflation. The response of noncore PCE prices takes longer to respond to trend inflation than PPI raw commodities, peaking at five quarters out instead of four. If production and distribution costs lag commodity prices, one would expect to see exactly this type of lag.

Real noncore PCE prices show much less sensitivity to a short-run bout of inflation than do raw commodity prices. They peak later and increase by only a quarter as much as do raw commodity prices. This is further evidence that noncore PCE prices contain a significant

sticky component. The response of noncore PCE prices looks like a dampened and delayed version of the response of PPI commodity prices; this is exactly what one would expect.

3.5 Granger causality – the statistical exogeneity of trend inflation

The final column of Table 2 shows what happens under an unrestricted VAR when trend inflation is not constrained to follow a random walk. Loosening that constraint involves estimating 13 more coefficients and results in a noticeable increase in the (still small) share of MCMC draws rejected due to instability. The estimated response of commodity prices to long run inflation tracks that from the baseline VAR rather well; the exogeneity assumption does not seem to matter much in practice. Impulse responses suggest a tiny degree of autocorrelation in the change in inflation expectations from quarter to quarter.[4] Eyeballing Figure 1 would lead one to the same conclusion since trend inflation shows what appears to be a long rise followed by a long descent. Short-run inflation still has a stimulative effect on commodity prices to a similar degree as before. It does not seem that constraining trend inflation to follow an exogenous random walk has much of an effect on our estimates.

To formally test the constraint that all of the coefficients in the first line of the VAR equal zero, it is perhaps most intuitive to do a classical F test. Comparing the total sum of squares of the change in inflation expectations with the residual sum of squares from the first line in the VAR yields an F statistic of 1.5555, which is distributed according to an $F(13,147)$ distribution.[5] The p value of the test against the null hypothesis of an exogenous random walk using that F distribution equals 0.3182. The data do not under any reasonable circumstance reject the null hypothesis that inflation expectations follow an exogenous random walk without drift. Since both the VAR evidence and an F test indicate that the exogenous random walk assumption is reasonable, we choose to impose that assumption since it is implied by rational expectations and it improves the efficiency of our estimates. In general, none of our results seems sensitive to the exact assumptions which we use.

We view the lack of Granger Causality of long-run inflation expectations as providing evidence against the mechanism proposed by Barsky and Kilian (2002), though we do corroborate their hypothesis of a statistical relationship between inflation and commodity prices. Their model relies upon inflation expectations which systematically lag actual

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[4] The autocorrelation of the change in inflation expectations is +0.15 in the raw data.
[5] There are 13 coefficients and 160 quarters in the data once the four initial quarters are accounted for.

inflation, which should show up as Granger Causality of trend inflation in our VAR system—in particular, inflation expectations should lag the deviation of inflation from long-run inflation expectations. Since changes to inflation expectations do not actually lag the deviation of inflation from long-run inflation expectations, a misperceptions model does not seem like a promising way to explain the coexistence of high trend inflation, low output, and high commodity prices. Instead, we argue that the change in commodity prices may have been a direct effect of the monetary policies that the Fed chose to pursue in the 1970s, based on more conventional channels of monetary transmission. In particular, the Fed tried to stimulate the economy by letting inflation overshoot its long run value at the same time that monetary policy became unanchored. High inflation in the long run exerts a drag on the economy and on commodity demand, but in the short run it can be very stimulative and can lead to a runup in commodity demand and hence commodity prices.

4. Theoretical results
4.1 A model economy with some flexible and sticky prices

We set up a simple dynamic general equilibrium model of commodity supply and demand which we then shock to see how it can match the patterns which we see in the data. Commodity prices are flexible and other prices are sticky, and monetary policy can affect output through an aggregate supply channel. Since commodities are not supplied elastically, an economic boom (inflation-driven or otherwise) will result in higher demand for commodities, which will in turn push up their real price. As a result, our model will feature "overshooting" behavior for commodity prices but for different reasons than those described in the original class of overshooting models. We feed through long-run inflation shocks and short-run spending shocks and see to what degree the model can replicate the patterns which we see in the data. We find that it can replicate the effects of inflation on commodity prices very well.

Our emphasis on flexible versus sticky prices echoes theoretical work by Ohanian, Stockman, and Kilian (1995), Aoki (2001), and Bodenstein, Erceg, and Guerrieri (2008). The two latter sets of authors build upon earlier work by Gordon (1975, 1984) and Phelps (1978) in order to analyze the proper behavior of monetary policy in response to exogenous oil supply shocks. We instead look at the quantitative effect of monetary policy on commodity prices. To do this, we must extend the basic model in an important way by allowing commodity supply and

demand to be less than perfectly elastic or inelastic. Based on reasonable supply and demand elasticities for commodities, we find that the model fits the VAR estimates reasonably well.

First we look at the effects of a classical nominal spending shock, showing how inflation stimulates output and commodity demand in the short run. A rise in inflation increases aggregate quantity supplied in the sticky-price sector, pushing up real commodity prices as well since demand across sectors is complementary. Then we analyze the long-run behavior of relative prices in response to inflation. We show that a rise in trend inflation is almost superneutral in its theoretical effects. The long-run and short-run features of the model turn out to have radically different implications from each other—the model predicts that money is almost superneutral in the long run at low rates of trend inflation but is nonneutral in the short run. Nonetheless, changes to trend inflation can show up temporarily in commodity prices if they are accompanied by a short burst of inflation as they are in the data.

4.2 The model

The model is a standard textbook New Keynesian model with a flexible-price sector and a sticky-price sector. Our model is very similar to that of Aoki (2001) with two important differences which we note as we introduce them. Consumers seek to maximize the present value of the utility from consumption (which equals output) minus the disutility of labor:

$$E_t \sum_{t=0}^{\infty} \beta^t \left[\ln(C_t) - \frac{l}{1+\chi} N_t^{1+\chi} \right],$$

subject to the budget constraint:

$$C_t + B_{t+1} = Y_t + (1+r_{t-1})B_t. \tag{1}$$

Economywide output is aggregated according to the CES aggregator:

$$Y_t = \left[\alpha^{\frac{1}{\gamma}} (Y_t^F)^{\frac{\gamma-1}{\gamma}} + (1-\alpha)^{\frac{1}{\gamma}} (Y_t^S)^{\frac{\gamma-1}{\gamma}} \right]^{\frac{\gamma}{\gamma-1}}, \tag{2}$$

with the budget constraint:

$$P_t Y_t = P_t^F Y_t^F + P_t^S Y_t^S .$$

(3)

Quantities and prices with an 'F' in the superscript refer to the flexible-price good, and those with an 'S' superscript refer to the sticky-price good. Price-taking behavior by demanders yields the demand curves:

$$Y_t^F = \alpha \left(\frac{P_t}{P_t^F} \right)^\gamma Y_t ,$$

(4)

and

$$Y_t^S = (1-\alpha) \left(\frac{P_t}{P_t^S} \right)^\gamma Y_t .$$

(5)

The CES price index is given by:

$$(P_t)^{1-\gamma} = \alpha (P_t^F)^{1-\gamma} + (1-\alpha)(P_t^S)^{1-\gamma} .$$

(6)

Aoki (2001) uses a Cobb-Douglas aggregator, but we allow the demand for flexible-price goods and sticky-price goods to have an elasticity which differs from one. If the demand for commodities is relatively price-inelastic, then shifts in the demand for commodities should have a greater effect on commodity prices than if commodity demand were price-elastic.

The sticky-price sector follows the standard New Keynesian staggered pricing rule, which results in a relationship between aggregate supply and inflation. Firm-level output within the sticky price sector is aggregated by aggregators according to the CES production function:

$$Y_t^S = \left[\int_0^1 y_{jt}^{\frac{\theta-1}{\theta}} dj \right]^{\frac{\theta}{\theta-1}} ,$$

(7)

with the budget constraint:

$$P_t^S Y_t^S = \int_0^1 p_{jt} y_{jt} dj \, . \tag{8}$$

Price-taking behavior by aggregators yields the demand curve:

$$y_{jt} = \left(\frac{p_{jt}}{P_t^S} \right)^{-\theta} Y_t^S \, , \tag{9}$$

and the CES price index is given by:

$$P_t^S = \left[\int_0^1 p_{jt}^{1-\theta} dj \right]^{\frac{1}{1-\theta}} \, . \tag{10}$$

Individual prices in the sticky sector are allowed to reset in a given quarter with a probability $1 - \omega$ and producers produce according to the production function $y_{it} = z_t N_{it}$. Firms seek to reset prices such that they maximize expected profits conditional on not changing prices and on satisfying whatever demand comes along. This is reflected by their objective function over the reset price P_t^* :

$$E_t \sum_{i=0}^{\infty} \omega^i D_{t+i,t} \left[\frac{P_t^*}{P_{t+i}} y_{it+i}(P_t^*) - W_{t+i} N_{it+i}(P_t^*) \right]$$

$$= E_t \sum_{i=0}^{\infty} \omega^i D_{t+i,t} \left[\frac{P_t^*}{P_{t+i}} \left(\frac{P_t^*}{P_{t+i}^S} \right)^{-\theta} Y_{t+i}^S - \frac{W_{t+i}}{z_{t+i}} \left(\frac{P_t^*}{P_{t+i}^S} \right)^{-\theta} Y_{t+i}^S \right] \, .$$

Taking first derivatives with respect to the reset price gives the first order condition:

$$E_t \sum_{i=0}^{\infty} \omega^i D_{t+i,t} \left[(1-\theta) \frac{P_t^*}{P_{t+i}} \left(\frac{P_t^*}{P_{t+i}^S} \right)^{-\theta} Y_{t+i}^S + \theta \frac{W_{t+i}}{z_{t+i}} \left(\frac{P_t^*}{P_{t+i}^S} \right)^{-\theta} Y_{t+i}^S \right] = 0 \, , \tag{11}$$

so that

$$\frac{(\theta-1)}{\theta}\frac{P_t^*}{P_t}E_t\sum_{i=0}^{\infty}\omega^i D_{t+i,t}\left[\frac{P_t}{P_{t+i}}\left(\frac{P_t^S}{P_{t+i}^S}\right)^{-\theta}Y_{t+i}^S\right]=E_t\sum_{i=0}^{\infty}\omega^i D_{t+i,t}\left[\frac{W_{t+i}}{z_{t+i}}\left(\frac{P_t^S}{P_{t+i}^S}\right)^{-\theta}Y_{t+i}^S\right]. \quad (12)$$

Output aggregates to:

$$N_t^S = \int_{j=0}^{1}\left(\frac{p_{jt}}{P_t^S}\right)^{-\theta}dj\,\frac{Y_t^S}{z_t}, \quad (13)$$

so that production in the sticky-price sector aggregates according to:

$$Y_t^S = \left(\omega\left(\frac{P_{t-1}^S}{P_t^S}\right)^{-\theta} + (1-\omega)\left(\frac{P_t^*}{P_t^S}\right)^{-\theta}\right)^{-1}z_t N_t^S. \quad (14)$$

When prices in the sticky sector do not equal their previous price, a wedge appears between labor input and output in that sector. This phenomenon becomes important in explaining the nonlinear steady-state relationship between inflation and output and in particular why high trend inflation may be contractionary.

The price index for the sticky-price goods follows the law of motion:

$$(P_t^S)^{1-\theta} = (1-\omega)(P_t^*)^{1-\theta} + \omega(P_{t-1}^S)^{1-\theta}. \quad (15)$$

The flexible-price sector is the commodity-producing sector. Commodity producers are price takers and they produce commodities using labor according to a decreasing returns technology. It is not easy to increase the amount of oil pumped or coal mined in the short to medium run. Firms within the flexible price sector have a decreasing-returns Cobb-Douglas production function in labor and maximize profits. They produce according to the production function:

$$Y_t^F = x_t z_t (N_t^F)^{\phi}. \quad (16)$$

Profit maximization gives the first order condition:

$$\frac{P_t^F}{P_t} \phi x_t z_t (N_t^F)^{\phi-1} = W_t. \tag{17}$$

Aoki (2001) has a flexible-price sector which produces according to a constant returns technology. By making commodity supply less than perfectly elastic, it is possible to generate realistic movements in the real price of commodities in response to shocks to commodity demand.

The shock z_t gives the exogenous portion of net commodity supply. When taken to U.S. data, this shifter gives the behavior of net commodity supply to the United States. For example, a rise in the demand for oil by China driven by growth will show up as a reduction in the supply of oil to the United States, all else equal. A war in the Persian Gulf region would show up as a reduction in the supply of oil for both countries. Both shocks would show up in x_t as a negative supply shock, since they have the same effect from the perspective of U.S. consumers.

Household labor supply follows a standard first order condition:

$$\frac{W_t}{Y_t} = l N_t^\chi. \tag{18}$$

Markets clear so that consumption equals output; labor supply equals labor demand; total bondholdings equal zero; and household budget constraints bind in equilibrium. The equilibrium also contains the laws of motion of expected inflation, nominal spending, and the relative productivity of the commodity-producing sector, which are all purely exogenous loglinear random walks.

4.3 Calibrated parameter values

In order to deliver quantitative predictions we must first calibrate the model to data for the period since 1970. The twenty-year nominal interest rate from the data equals 6.51% per year and PCE inflation equals 3.89% per year for a real interest rate of 2.62%. Growth equals 2.89% per year and population growth is 1.05% per year, bringing growth per capita to

1.83%. All prices are initially normalized to one, as are the levels of real consumption of the sticky and flexible goods. The share of output of the agriculture and mining sectors in the U.S. economy, from the BEA's industry accounts, is 3.52%, which gives α. This should be regarded as a lower bound for the share of flexible-price commodities in expenditure since the United States imports commodities on net, and the measure excludes the flexible-price production of other sectors. Our results are not sensitive to the choice of α. Prices in the sticky sector are updated 25% of the time for a value of ω of 0.75. The demand parameter θ equals 10. We vary the inverse labor supply elasticity χ and the commodity demand and supply elasticities γ and $\phi/(1-\phi)$ in order to show the role of these parameters in determining the macroeconomic behavior of commodity prices.

We derive an exact solution to the model for steady-state analysis along with a linearized model which we use to discuss dynamics. The equations used to do so are described in Appendices B and C. The intuition in the classical thought experiment is straightforward. A shock to nominal spending has an effect on aggregate quantity supplied in the sticky-price sector. Since flexible-price commodities and sticky-price goods are imperfect substitutes (in fact, they are complements), the change in expenditure on sticky-price goods results in a large corresponding change in expenditure on flexible-price goods. Since those goods, in turn, are not elastically supplied, their relative price should move along with inflation and output.

5. The effects of inflation in the long and short run
5.1 The effects of short-run inflation shocks

Figure 5 shows the effects of an unanticipated once-and-for-all one percent increase in nominal spending under a calibration chosen to represent an economy which behaves inelastically. Under this calibration, the supply and demand for commodities are both highly price-inelastic, with elasticities of 0.1. The inverse labor supply elasticity χ is set to 2. As one might expect, output increases and then reverts slowly back to trend after a nominal shock. Inflation also turns positive. Nominal commodity prices, because they are flexible, spike first, causing real commodity prices to rise—they stay elevated so long as output and commodity demand remain elevated. Real commodity prices rise by about 3.9% after an inflationary shock, which is just slightly more than the 3.4% which the VAR indicates. Interestingly, nominal commodity price inflation soon turns slightly negative, which means that nominal commodity prices overshoot a little bit. Our results are analogous to those of

Frankel (1984, 1986, 2008) and Frankel and Rose (2010) who look at commodity futures markets using the setup of Dornbusch (1976). While their results are driven by the effects of real interest rates on futures prices, ours are driven by a combination of inflation-induced increases in output and a price-inelastic supply and demand for commodities. Real interest rates have nothing to do with our story; ours is a classic supply and demand story.

Figure 6 shows the effects of the same shock under a calibration chosen to represent an economy where supply and demand are more price-elastic. The supply and demand elasticities for commodities are set to 0.5 in line with or above the long-run oil supply and demand elasticities presented by Hamilton (2009b), and the inverse labor supply elasticity χ is set to 0. We get the same result as in Figure 5 though much dampened in magnitude. We still get overshooting and a substantial effect of inflation on real commodity prices, but the effect is smaller than under the inelastic calibration—commodity prices rise by about 1.2%. Our point estimates from the VAR imply an effect of inflation on real commodity prices of between three and four percent, with wide error bands which are a natural consequence of the long-run identification scheme. The VAR does give results in between our extreme calibrations (much closer to the inelastic one), so the observed short-run behavior of commodity prices is completely consistent with our theory.

In fact, the theoretical model fits the short-run behavior of the data extremely well so long as the parameters ϕ and γ are not too high. When those two parameters are set to one and χ to 0, then an inflation shock temporarily raises real commodity prices by only 0.8%. Low elasticities of supply and demand for commodities are the most compatible with the observed behavior of commodity prices at the macroeconomic level.

5.2 Trend inflation and commodity prices

Figure 7 shows the equilibrium steady-state relationship among trend inflation, real commodity prices, and real output calculated from the model. The relationship is highly nonlinear and is not monotonic. For steady-state quarterly inflation rates below about one percent, trend inflation has very little effect on output or on real commodity prices. For moderately high inflation rates, inflation actually exerts a contractionary effect on output, and inflation actually has an upper bound above which producers prefer to reset their new prices to infinity, which probably says more about the staggered price-setting model than about

reality. Previous papers on the effects of trend inflation have already come to this result. Ascari (2004), Sahuc (2006), Bakhshi, Khan, Burriel-Llombart, and Rudolf (2007), Cogley and Sbordone (2008), and Ascari and Merkl (2009), among others, discuss how the equilibrium of the New Keynesian model changes when trend inflation changes—basically, high steady-state inflation exerts a contractionary effect on output because it introduces distortions into relative prices. Models which are linearized around a zero-inflation steady state predict that positive trend inflation is very mildly expansionary. When extended to discuss deviations in trend inflation, such models miss out on the nonlinear nature of the wedge that price dispersion places between labor input and output. At 1970s rates of trend inflation (about 1.5% per quarter), the model predicts a slight stagflationary effect of higher inflation. Stagflation should be associated with lower real commodity prices, not higher real commodity prices, since low demand for output should result in low demand for commodities.

In the steady state, the wedge (14) introduced into the sticky-price sector by gross trend inflation Π takes the form:

$$Y^S = \left(\omega \Pi^{\theta} + (1-\omega) \left(\frac{1 - \omega \Pi^{\theta-1}}{1 - \omega} \right)^{\frac{\theta}{\theta-1}} \right)^{-1} z N^S .$$ (19)

Taking a first derivative of the wedge reveals that it is maximized at a gross inflation rate of one, and the wedge has a negative second derivative everywhere. Put another way, the distortions from inflation are small around zero inflation and are highly nonlinear. For a moderately high rate of trend inflation, the price system breaks down altogether and output goes to zero. The distortions from price dispersion above low rates of trend inflation overwhelm any small gain in output which comes from the traditional New Keynesian aggregate supply channel. Near price stability, the wedge does not matter, and the positive equilibrium effects of stickiness on aggregate supply dominate.

Even looking in the neighborhood of price stability, one can see that trend inflation does not have much of an effect on trend output or commodity prices. While the current model is a little bit more complicated than that, a standard sticky-price New Keynesian model linearized around a zero-inflation steady state can offer good intuition. The linearized aggregate supply equation (as explained in great detail by Woodford (2003)) can be written in the form:

$$y_t = k(\pi_t - \beta E_t \pi_{t+1}).$$ (20)

For steady-state net inflation near zero, one could write the approximate steady-state version of the linearized model as:

$$y = k(1 - \beta)\pi.$$ (21)

For an effective discount factor β near one and for any reasonable value of k, the effect of a marginal change in trend inflation on output must be small. When adjusting for growth, β equals a number very close to one (0.998 in the calibration which we use), so trend inflation must have a very small effect on real variables. Basically, what happens is that when trend inflation rises, reset prices become very high in anticipation of future inflation and wipe out any stimulative effect from current inflation.

5.3 A historical decomposition

The theoretical model predicts that trend inflation should have little positive effect on output or real commodity prices (or even a contractionary effect), while a temporary burst of inflation should be stimulative to both. At first glance, this finding would rule out the idea that the rise in trend inflation is responsible for the 1970s commodity price shocks. This would be true only if long-run inflation and short-run inflation were orthogonal. The estimated impact matrix from the baseline VAR says otherwise. A 1% increase in quarterly trend inflation results in a mean additional rise in nominal spending of 5.17%, with a positive effect occurring in 99.5% of MCMC draws. Figure 8 shows what happens after a 1% increase in quarterly trend inflation accompanied by a 5.17% short-run inflation shock. Under the inelastic calibration, a 1% permanent increase in quarterly inflation should raise commodity prices by 19% on impact with the effect dying out over time and becoming slightly negative. A 19% rise is substantial, though it is somewhat less than the 35% rise estimated by the VAR.

The top panel of Figure 9 shows the effects of inflation shocks throughout the sample as estimated from the baseline VAR. The bottom panel of Figure 9 shows the combined effects of both sets of inflation shocks throughout the sample as estimated from the model using the calibration for the inelastic economy. Both sets of estimates are produced the same way.

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Based on the residuals from the VAR (or the VAR representation of the theoretical model), it is possible to recover the underlying structural shocks, since there are three reduced-form disturbances and three structural shocks. We then take the estimated shocks from the first quarter of 1971 onward and then feed them into the VAR or the theoretical model. Based on these shocks, we can trace out the conditional behavior of real commodity prices.

Both sets of estimates tell a similar story regarding the 1973-74 commodity price spike. Based on the behavior of inflation at the time, the VAR predicts a 30% price spike and the theoretical model predicts a 20% price spike from the first quarter of 1973 through the third quarter of 1974. The short-run inflation which accompanied the unanchoring of monetary policy and the end of price controls seems to have led to higher commodity prices on top of the large shocks to the supply of commodities. The VAR and model disagree as to the nature of the 1979 price spike. The 1979 episode did not come with a spike in core PCE inflation, unlike the 1973-74 episode. Therefore the 1979 episode was not stimulative to the economy and should not have resulted in substantially higher real commodity prices; the model attributes that episode almost entirely to supply shocks. Both sets of estimates (more so the VAR) also indicate that the disinflation of the early and mid 1980s should have resulted in a fall in real commodity prices.

Neither the VAR nor the model attributes much of the behavior of commodity prices after the mid-1980s to shocks to inflation, save a downward blip in 2008. Even though shocks to inflation have a large estimated effect, the shocks themselves are small, so they do not contribute much to overall commodity price dynamics. Most commodity price dynamics are driven by our estimated net commodity supply shocks, so most movements in commodity prices reflect real driving forces rather than pressure on U.S. core inflation. The belief expressed by Calvo (2008) and Taylor (2009) that large fluctuations in commodity prices are attributable to inflationary shocks originating in U.S. monetary policy is not warranted by the data. Instead, those spikes appear to be driven by changes in the net supply of commodities available to U.S. producers and consumers.

6. Conclusion

Our VAR with economically plausible long-run restrictions has strengthened the vague consensus that inflation affects commodity prices. Our results indicate that inflation affects

real commodity prices in a robust and positive way. The estimated effect of short-run inflation on commodity prices appears consistent with macroeconomic theory—our estimates center on the three to four percent range for a one percent permanent nominal spending shock, which is what a reasonably calibrated theoretical model would predict. We also find that we can explain some of the runup in commodity prices during the mid-1970s by noting the behavior of monetary policy at that time. Policymakers during the 1973-74 episode let inflation overshoot its long run target, causing long-run inflation, short-run inflation, and real commodity prices to move together.

Despite the clear chain of causality which runs from inflation to output to commodity demand and commodity prices, a rise in commodity prices does not necessarily indicate impending inflationary pressure. Shocks to both trend inflation and short-run inflation are small, especially since the 1980s, so while shocks to inflation have a large estimated effect on commodity prices, their overall contribution to commodity prices is dwarfed by shocks to net commodity supply. While a rise in commodity prices may reflect a buildup of inflationary pressure, it most likely reflects global supply and demand factors. As core and long-run inflation have become less variable since the 1970s, their contribution to commodity price dynamics has shrunken accordingly.

Appendix A: The estimation procedure for the Bayesian VAR

We estimate the VAR with long run restrictions using a Markov Chain Monte Carlo in order to get exact confidence intervals for our impulse responses. The feedback coefficients (including the constants c) have an uninformative normal prior distribution, while the variance matrix of the reduced-form residuals has an uninformative inverse Wishart prior distribution. We initialize the VAR using its maximum likelihood (OLS) value and then iterate through 50,001 samples, tossing the first 10,000 samples. Based on the previous sample's draw of the variance-covariance matrix and on the observed data, we draw a set of coefficient values. If the coefficients imply an unstable system (which precludes identification), we redraw. Then based on those coefficients and on the data, we draw a set of variance matrices for the residuals.   On those matrices, we apply the long-run restrictions to recover the contemporaneous impact matrix and take the impulse responses to a one-standard deviation structural shock.

The VAR representation itself follows its usual form, with the stationary observables x forming a k by T matrix, with a full rank of normal residuals which are independent and identically distributed across time and have a full-rank covariance matrix Σ:

$$x_t = c + \sum_{p=1}^{P} A_p x_{t-p} + \varepsilon_t . \tag{A1}$$

The coefficients c and A_p are normal conditional on x and Σ. Letting the stacked OLS estimate of the coefficient matrices equal $\hat{A}$, the stacked coefficients themselves are multivariate normal with a mean of $\hat{A}$ and a variance-covariance matrix of $\Sigma \otimes (xx')^{-1}$. If Σ has an inverse Wishart prior distribution, then its posterior is also inverse Wishart based on the observed covariance of the residuals.

To obtain the implied long-run effects of shocks, one can first represent the reduced form residuals as a function of the long-run residuals using the equation $\varepsilon_t = B\eta_t$. In this case, it is simple to obtain the impact matrix B. To get a unit long-run effect of the long-run residuals, the following equation must hold:

$$I_K = \left(I_K - \sum_{p=1}^{P} A_p \right)^{-1} B,$$

which yields the coefficient matrix B (taking into account the stability restrictions on A):

$$B = I_K - \sum_{p=1}^{P} A_p.$$ (A2)

Then one can recover the long-run residuals by transforming the estimated short-run residuals using the equation $\eta_t = B^{-1}\varepsilon_t$, which is possible since B is in general nonsingular. The variance of the long-run residuals is given by $B^{-1}\Sigma(B')^{-1}$.

The structural shocks ζ_t have a unit variance and are independently and identically distributed across time and from each other. We represent $\eta_t = C\zeta_t$ where C is a lower triangular matrix calculated using the Cholesky decomposition of $B^{-1}\Sigma(B')^{-1}$. The diagonal terms each equal the standard deviation of the long-run shocks, and the off-diagonal terms equal the effects of each of the previous structural shocks ζ_t on the composite residual η_t. Upon obtaining the matrix C, one can then create impulse responses. The impact response equals the matrix product $BC\zeta_t$, and one can iterate through equation (1) to get the impulse responses x^{IRF}.

$$x_t^{IRF} = \sum_{p=1}^{P} A_p x_{t-p}^{IRF} + BC\zeta_t.$$ (A3)

We calculate the impulse response of each variable this way, cumulating the responses of the variables which are taken in first differences. The long-run cumulative response to a given shock can be read from C matrix itself. To get the response of the system to a unit shock (rather than a one standard deviation shock), one need merely divide the impact shock by the appropriate diagonal element of C.

In the baseline case where trend inflation follows an exogenous random walk, the first rows of coefficients are all set to zero, while the Cholesky ordering proceeds as before. In that event, each draw of the VAR coefficients only draws the coefficients in the second through last equations conditional on having already observed the shock to long-run inflation expectations.

The elements of Σ used to calculate the variance-covariance matrix only correspond with the residuals from those equations conditional on the shock to long-run inflation expectations. Both of these things can easily be determined from the first column of the lower triangular decomposition of Σ. The usual OLS coefficient and variance formulas apply after making this transformation to the data.

Appendix B: Steady state of the model

Solving for the steady state of the model involves solving the following system of equations:

$$p^* \sum_{i=0}^{\infty} \left(\frac{\omega \Gamma \Pi^{\theta-1}}{R} \right)^i [(1-\theta)] + E_t \sum_{i=0}^{\infty} \left(\frac{\omega \Gamma \Pi^{\theta}}{R} \right)^i \left[\theta \frac{W}{z} \right] = 0, \tag{B1}$$

or in closed form,

$$p^* \frac{1}{R - \Pi^{\theta-1} \omega \Gamma} (1-\theta) + \frac{1}{R - \Pi^{\theta} \omega \Gamma} \left[\theta \frac{W}{z} \right] = 0 ; \tag{B2}$$

$$(\Pi)^{1-\theta} = (1-\omega) \left(\frac{P^* \Pi}{P^S} \right)^{1-\theta} + \omega ; \tag{B3}$$

$$zN^S = \left(\omega \Pi^{\theta} + (1-\omega) \left(\frac{P^*}{P^S} \right)^{-\theta} \right) Y^S ; \tag{B4}$$

$$xz(N^F)^{\phi} = Y^F ; \tag{B5}$$

$$P^F \phi xz(N^F)^{\phi-1} = W ; \tag{B6}$$

$$\frac{W}{Y} = lN^{\chi} ; \tag{B7}$$

$$N = N^F + N^S ; \tag{B8}$$

$$Y = \left[\alpha^{\frac{1}{\gamma}} (Y^F)^{\frac{\gamma-1}{\gamma}} + (1-\alpha)^{\frac{1}{\gamma}} (Y^S)^{\frac{\gamma-1}{\gamma}} \right]^{\frac{\gamma}{\gamma-1}} ; \tag{B9}$$

$$Y^F = \alpha (P^F)^{-\gamma} Y ; \tag{B10}$$

$$Y^S = (1-\alpha)\left(P^S\right)^{-\gamma} Y ; \tag{B11}$$

and

$$1 = \alpha(P^F)^{1-\gamma} + (1-\alpha)(P^S)^{1-\gamma} . \tag{B12}$$

Real interest rates are stationary and unrelated to inflation, so their long-run values pin down β. Long-run steady states are well behaved as a function of inflation so long as inflation is within the admissible range, so simple numerical search algorithms work well at solving this system.

Appendix C: The linearized model

The linearized model is fairly straightforward, with the one complication that we have to look at two sectors at once. The New Keynesian aggregate supply equation no longer has a closed form, making things somewhat more complicated as well. Lower case letters denote linearized objects. Prices are in real terms, using economywide output as the numeraire. We use the code provided by Sims (2002) to solve for the nonexplosive equilibrium of the system, which is locally determinate in the economy which we study.

The asset pricing equation becomes:

$$E_t\left(y_t - y_{t+1} + i_t - \pi_{t+1}\right) = 0. \tag{C1}$$

The economywide production function and sectoral demand equations become:

$$Y^{\frac{\gamma-1}{\gamma}} y_t = \alpha^{\frac{1}{\gamma}} (Y^F)^{\frac{\gamma-1}{\gamma}} y_t^F + (1-\alpha)^{\frac{1}{\gamma}} (Y^S)^{\frac{\gamma-1}{\gamma}} y_t^S; \tag{C2}$$

$$y_t^F = -\gamma p_t^F + y_t; \tag{C3}$$

and

$$y_t^S = -\gamma p_t^S + y_t. \tag{C4}$$

The sticky price-setting equation, as a reminder, is given by

$$\frac{(\theta-1)}{\theta}\frac{P_t^*}{P_t} E_t \sum_{i=0}^{\infty} \omega^i D_{t+i,t}\left[\frac{P_t}{P_{t+i}}\left(\frac{P_t^S}{P_{t+i}^S}\right)^{-\theta} Y_{t+i}^S\right] = E_t \sum_{i=0}^{\infty} \omega^i D_{t+i,t}\left[\frac{W_{t+i}}{z_{t+i}}\left(\frac{P_t^S}{P_{t+i}^S}\right)^{-\theta} Y_{t+i}^S\right],$$

and it can be written as

$$\frac{(\theta-1)}{\theta}\frac{P_t^*}{P_t}(LHS)_t = (RHS)_t,$$

where

$$(LHS)_t = E_t \sum_{i=0}^{\infty} \omega^i D_{t+i,t} \left[\frac{P_t}{P_{t+i}} \left(\frac{P_t^S}{P_{t+i}^S} \right)^{-\theta} Y_{t+i}^S \right],$$

and

$$(RHS)_t = E_t \sum_{i=0}^{\infty} \omega^i D_{t+i,t} \left[\frac{W_{t+i}}{z_{t+i}} \left(\frac{P_t^S}{P_{t+i}^S} \right)^{-\theta} Y_{t+i}^S \right].$$

Around a no-inflation, no-growth steady state, the three equations reduce into the familiar New Keynesian aggregate supply relationship. In the presence of trend inflation, the ability to neatly reduce the size of the model vanishes, so we must carry around the two auxiliary variables *LHS* and *RHS*. The linearization is given by:

$$p_t^* + (lhs)_t = (rhs)_t; \tag{C5}$$

$$(lhs)_t = \frac{Y^S y_t^S}{(LHS)} + \frac{\omega \Gamma \Pi^{\theta-1}}{R} E_t \left(-i_t + \theta(p_{t+1}^S - p_t^S + \pi_{t+1}) + (lhs)_{t+1} \right); \tag{C6}$$

and

$$(rhs)_t = \frac{Y^S W}{z(RHS)} \left(y_t^S + w_t \right) + \frac{\omega \Gamma \Pi^{\theta}}{R} E_t \left((1+\theta)\pi_{t+1} - i_t + \theta(p_{t+1}^S - p_t^S) + (rhs)_{t+1} \right). \tag{C7}$$

The index of sticky prices becomes:

$$(P^S)^{1-\theta} p_t^S = (1-\omega)(P^S)^{1-\theta} p_t^* + \omega \left(\frac{P^S}{\Pi} \right)^{1-\theta} \left(p_{t-1}^S - \pi_t \right). \tag{C8}$$

Sticky-price goods productivity is given by:

$$\frac{N^S z}{Y^S}\left(n_t^S - y_t^S\right) = \omega\Pi^\theta\theta\left(p_t^S - p_t^S + \pi_t\right) + (1-\omega)\left(\frac{P^S}{P^*}\right)^\theta\theta\left(p_t^S - p_t^*\right). \tag{C9}$$

Flexible-goods pricing implies that:

$$p_t^F + z_t + (\phi - 1)n_t^F = w_t, \tag{C10}$$

and flexible-good production is given by:

$$y_t^F = z_t + \phi n_t^F. \tag{C11}$$

Household labor supply is given by:

$$w_t - y_t = \chi n_t, \tag{C12}$$

and market clearing in labor yields the expression:

$$Nn_t = N^F n_t^F + N^S n_t^S. \tag{C13}$$

A one-off change in nominal spending given the inflation target τ would be given by:

$$y_t - y_t + \pi_t = \tau_t + \varepsilon_t^\pi + d_{sr|lr}\varepsilon_t^\tau, \tag{C14}$$

where the last term gives the response of short-run inflation to a shock to long-run inflation. A permanent change in inflation would be given by:

$$\tau_t = \tau_{t-1} + \varepsilon_t^\tau, \tag{C15}$$

and a permanent change in relative flexible-price productivity would be given by:

$$x_t = x_{t-1} + \varepsilon_t^x. \tag{C16}$$

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Table 1: Estimates from VAR: Basic results, alternate specifications and lag lengths

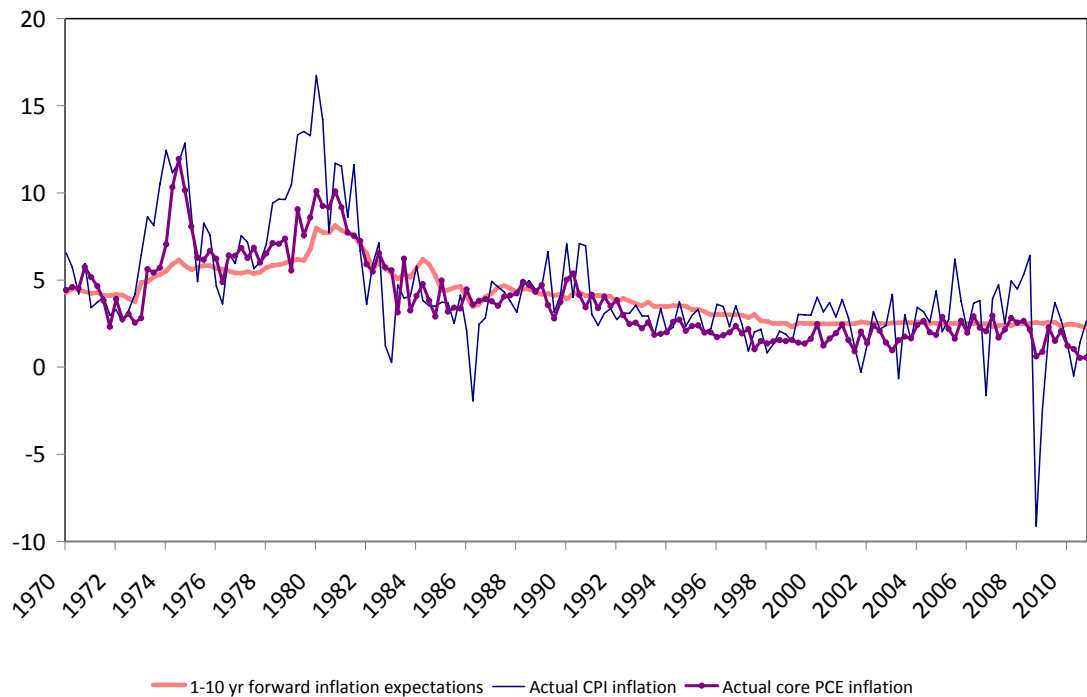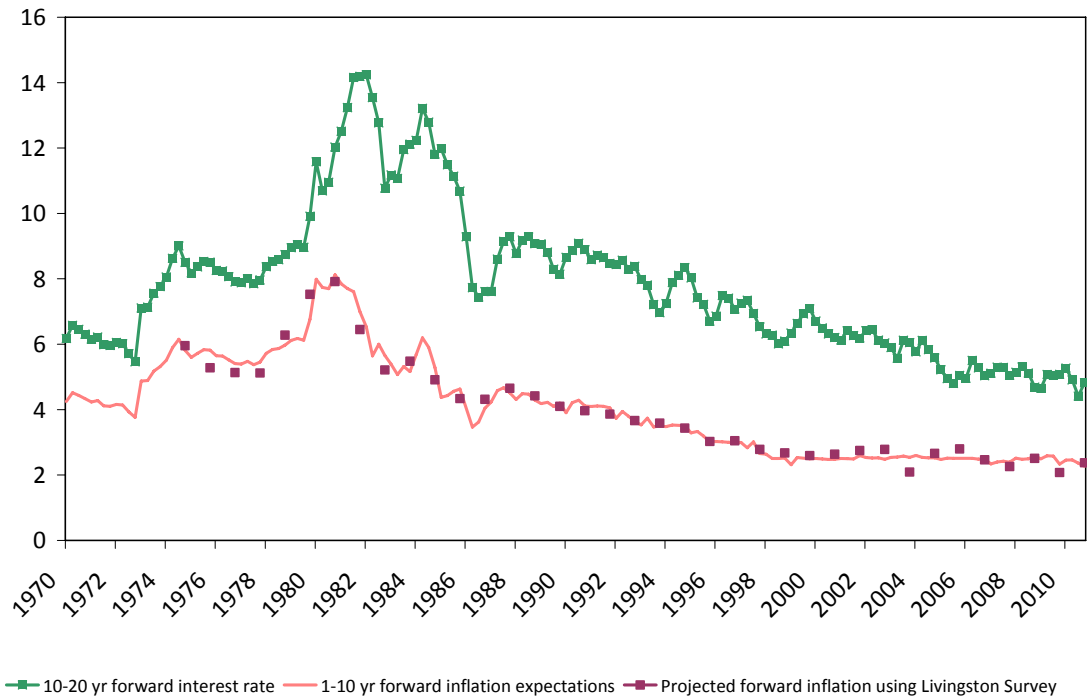|		Baseline	Other sized systems			6 Lags
	Variables in VAR system	3	2	4	5	3
	Lags	4	4	4	4	6
	Beginning of sample	1970	1970	1970	1970	1970
	End of sample	2010	2010	2010	2010	2010
	Trend π^* random walk?	yes	yes	yes	yes	yes
	% of unstable draws	0.01%	0.00%	0.03%	1.12%	0.02%
Effects of shocks to trend π	LR effect on p^F	19.4	28.1	20.7	17.4	25.6
	(p>0)	0.872	0.972	0.898	0.808	0.934
	Max. effect on p^F	35.3	32.7	35.0	29.9	37.5
	(lag)	4	2	4	4	4
	Min. effect on p^F	13.6	14.1	12.8	11.9	13.4
	(lag)	0	0	0	0	0
	Argmax(p>0,1-(p<0))	0.990	0.988	0.987	0.973	0.992
	(lag)	2	2	2	2	2
	LR effect on y	-	-	-2.31	-3.67	-
	(p>0)	-	-	0.265	0.175	-
	Max. effect on y	-	-	2.04	1.78	-
	(lag)	-	-	1	1	-
	Min. effect on y	-	-	-3.02	-4.42	-
	(lag)	-	-	8	10	-
	Argmax(p>0,1-(p<0))	-	-	0.966	0.947	-
	(lag)	-	-	0	0	-
Effects of temp. shocks to π	Max. effect on p^F	3.36	-	2.58	2.17	2.65
	(lag)	1	-	3	3	1
	Min. effect on p^F	0.00	-	0.00	-0.09	-0.54
	(lag)	∞	-	∞	7	5
	Argmax(p>0,1-(p<0))	0.945	-	0.906	0.833	0.827
	(lag)	3	-	3	3	1
	Max. effect on y	-	-	0.295	0.173	-
	(lag)	-	-	1	0	-
	Min. effect on y	-	-	-0.160	-0.203	-
	(lag)	-	-	7	7	-
	Argmax(p>0,1-(p<0))	-	-	0.843	0.243	-
	(lag)	-	-	0	7	-

Source: Authors' calculations as described in text. Level statistics are for posterior median impulse responses after a unit shock to trend inflation or prices. The rows labeled *Argmax(p>0,(1-p)<0)* report the lags at which the strongest inference can be made with respect to the sign of the impulse response, along with the posterior probability that the impulse response is positive at that point.

Table 2: Estimates from VAR: Alternate samples, robustness checks

		Baseline	Subsamples		Alt. p^F	Unrestr.
	Variables in VAR system	3	3	3	3	3
	Lags	4	4	4	4	4
	Beginning of sample	1970	1970	1985	1970	1970
	End of sample	2010	1984	2010	2010	2010
	Trend π^* random walk?	yes	yes	yes	yes	no
	% of unstable draws	0.01%	3.38%	3.41%	0.01%	0.06%
Effects of shocks to trend π	LR effect on p^F	19.4	13.7	65.8	16.5	34.0
	(p>0)	0.872	0.759	0.877	0.997	0.950
	Max. effect on p^F	35.3	24.8	65.8	20.1	46.2
	(lag)	4	4	∞	5	3
	Min. effect on p^F	13.6	14.8	14.3	5.2	25.8
	(lag)	0	0	0	0	0
	Argmax(p>0,1-(p<0))	0.990	0.977	0.887	0.999	0.999
	(lag)	2	0	9	7	2
Effects of temp. shocks to π	Max. effect on p^F	3.36	3.92	8.08	0.79	2.98
	(lag)	1	0	3	3	1
	Min. effect on p^F	0.00	-0.04	0.00	0.00	-0.00
	(lag)	∞	10	∞	∞	21
	Argmax(p>0,1-(p<0))	0.945	0.944	0.929	0.914	0.920
	(lag)	3	0	3	4	3

Source: Authors' calculations as described in text. Level statistics are for posterior median impulse responses after a unit shock to trend inflation or prices. The rows labeled *Argmax(p>0,(1-p)<0)* report the lags at which the strongest inference can be made with respect to the sign of the impulse response, along with the posterior probability that the impulse response is positive at that point.

**Figure 1: Measures of inflation expectations and inflation, %, 1970-2010**

Legend (top panel):
- 10-20 yr forward interest rate
- 1-10 yr forward inflation expectations
- Projected forward inflation using Livingston Survey

Legend (bottom panel):
- 1-10 yr forward inflation expectations
- Actual CPI inflation
- Actual core PCE inflation

Sources: St. Louis Fed FRED database (Interest rates), BEA (PCE inflation), BLS (CPI inflation), Philadelphia Fed (Survey of Professional Forecasters, Livingston Survey, and Blue Chip Survey), and authors' calculations. Details given in text. The pre-1979 portion of the inflation expectations series is extrapolated using the 10-20 year forward interest rate and current inflation expectations.

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**Figure 2: Real commodity prices, 1970-2010 (2005 average = 1)**
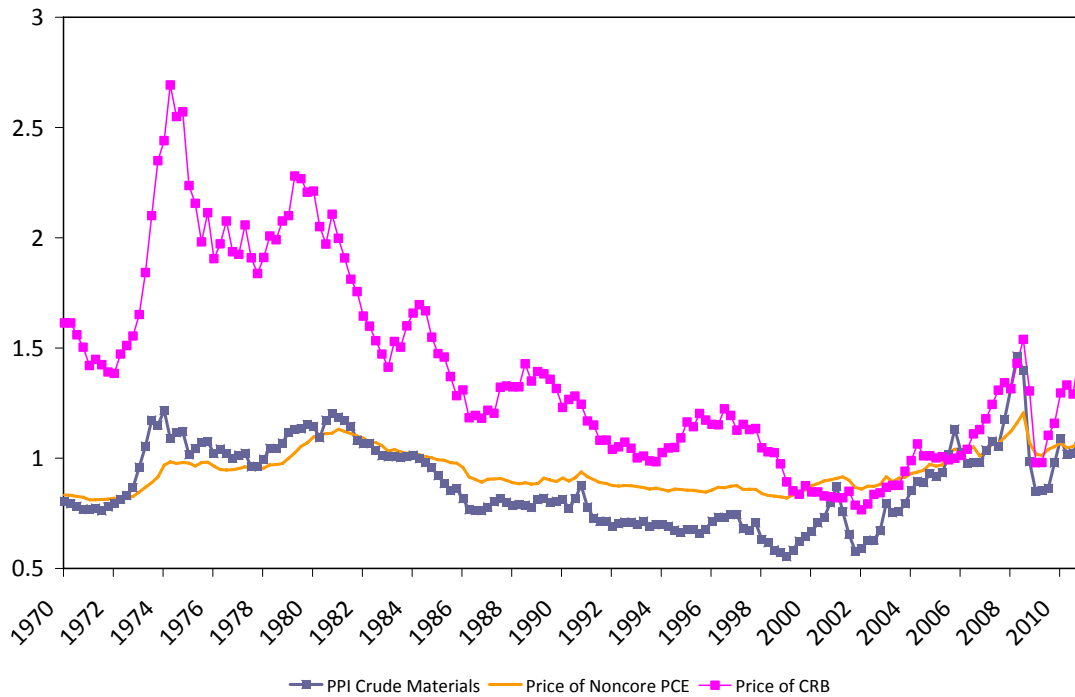
Source: BLS (PPI series), NIPA (PCE series), Datastream (CRB series), and authors' calculations, all normalized so that 2005 average prices=1. The PCE food and energy price index is calculated as a residual from the total PCE and core PCE price indices using the Törnqvist formula. Core PCE prices are the numeraire for all series.

Figure 3: Impulse responses to inflation shocks, 3-variable VAR:
**1% permanent shock to quarterly trend inflation**
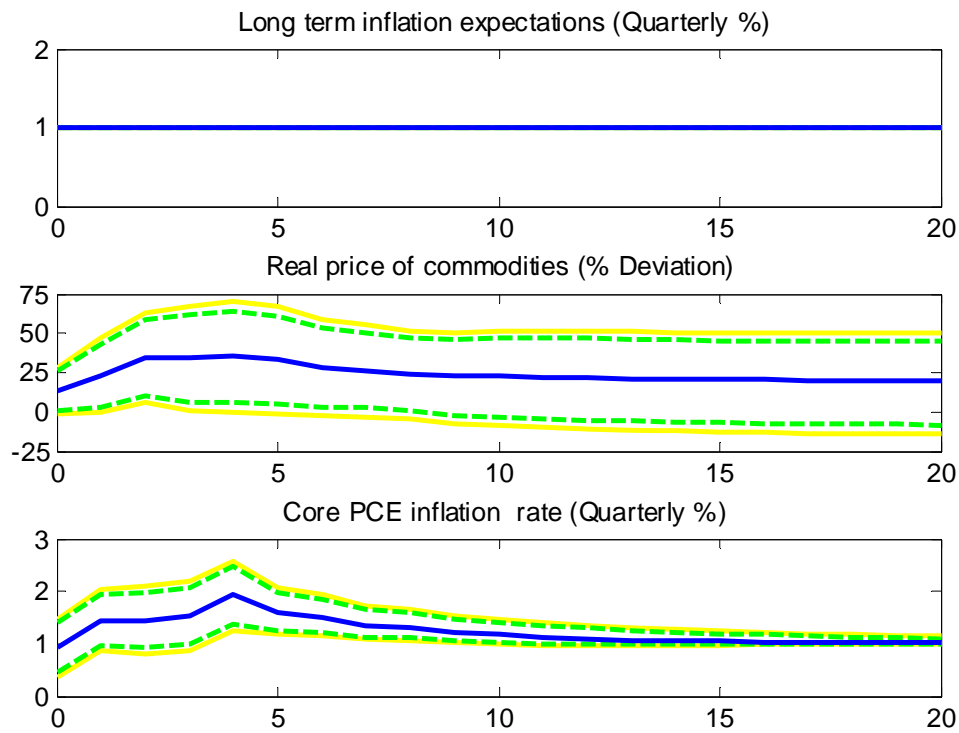
Source: Authors' calculations. Solid dark lines represent the posterior median. Confidence bands cover 95% of the posterior distribution (solid, light yellow) and 90% (dashed, dark green).

**Figure 4: Impulse responses to inflation shocks, 3-variable VAR:
1% permanent shock to price level**
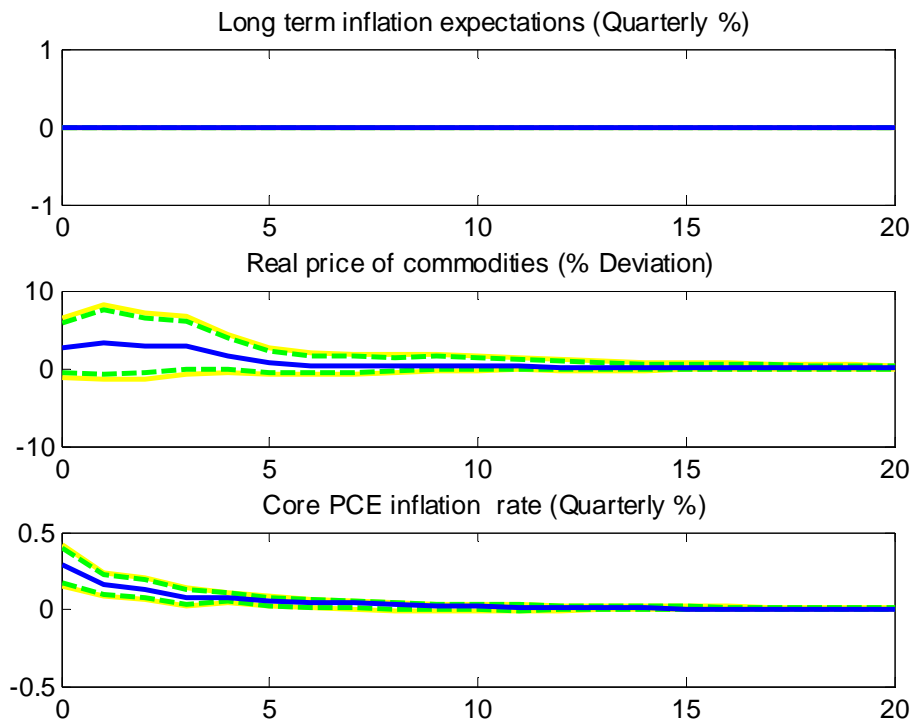
Source: Authors' calculations. Solid dark lines represent the posterior median. Confidence bands cover 95% of the posterior distribution (solid, light yellow) and 90% (dashed, dark green).

Figure 5: Impulse responses to a permanent 1% increase in nominal spending
$\gamma = 0.1$; $\phi = 0.1/1.1$; $\chi = 2$ (Inelastic economy)
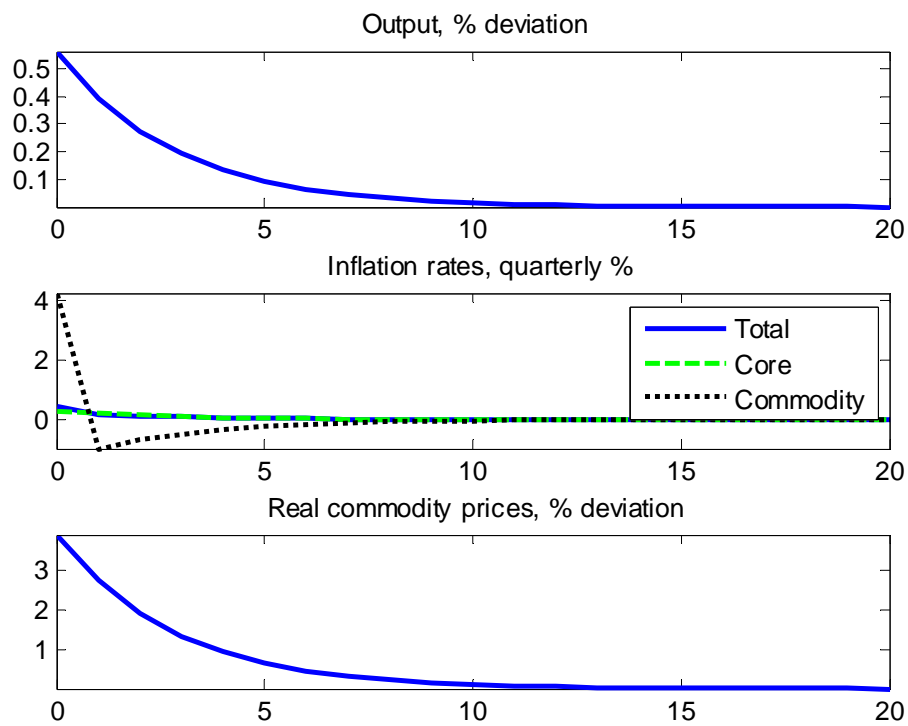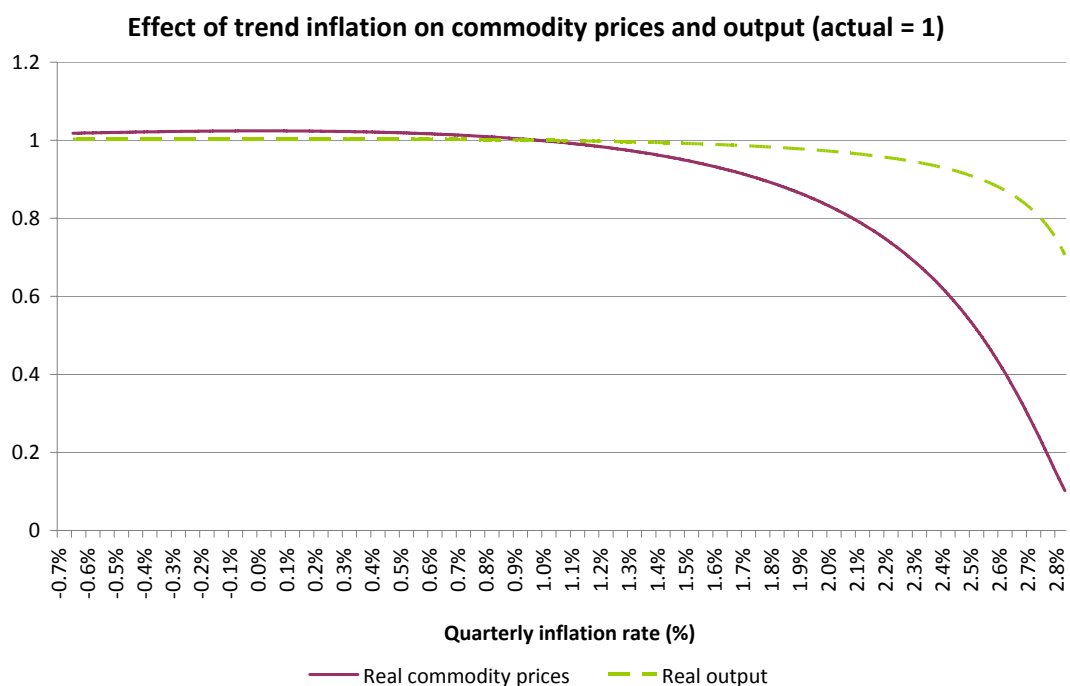

This figure shows the impulse responses to an unanticipated, one-time, one percent permanent increase in nominal spending. The parameterization here is engineered to give a large response in commodity prices out of a set of reasonable parameter values.

Figure 6: Impulse responses to a permanent 1% increase in nominal spending
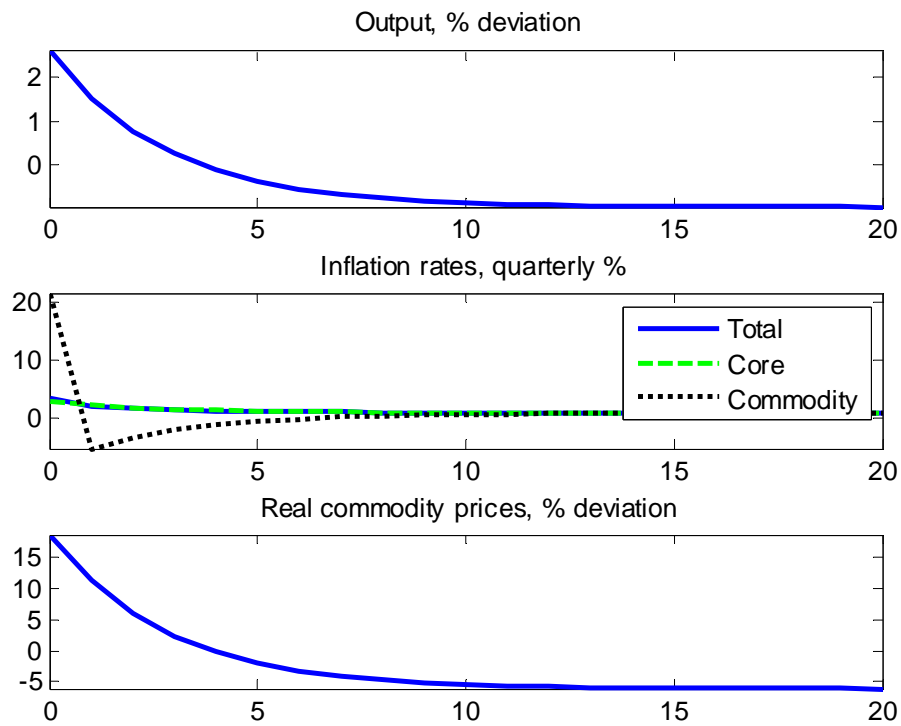$\gamma = 0.5$; $\phi = 0.5/1.5$; $\chi = 0$ (Elastic economy)
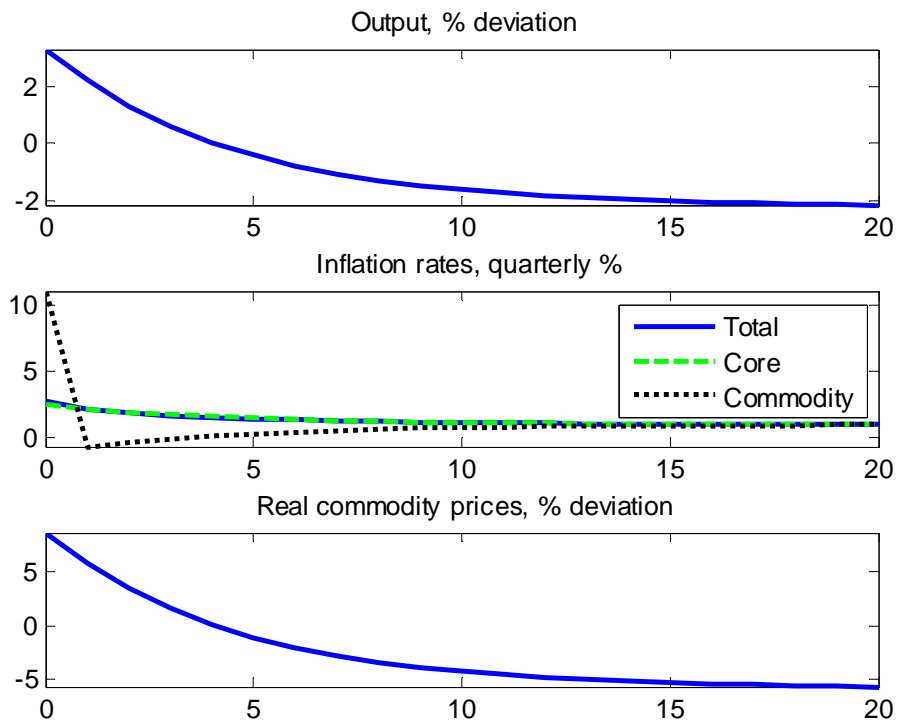
This figure shows the impulse responses to an unanticipated, one-time, one percent permanent increase in nominal spending. The parameterization here is engineered to give a small response in commodity prices out of a set of reasonable parameter values.

Figure 7: Steady-state relationship among inflation, commodity prices, and output
$\gamma = 0.1$; $\phi = 0.1/1.1$; $\chi = 2$ **(Inelastic economy)**

Effect of trend inflation on commodity prices and output (actual = 1)
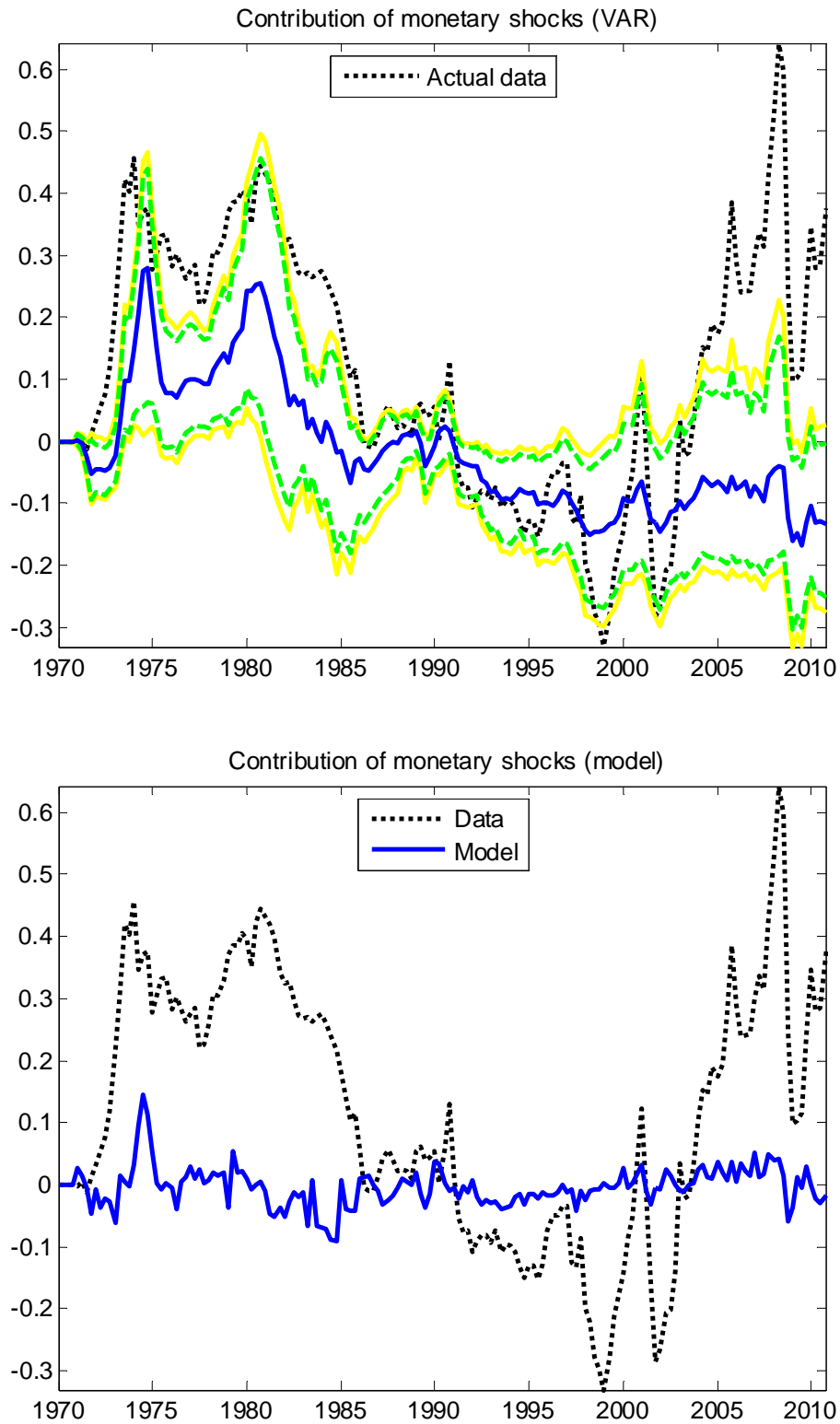

Source: Authors' calculations using the equilibrium model, in the inelastic economy. This figure shows the exact long-run steady values of real commodity prices and output as a function of trend inflation.

Figure 8: Impulse responses to a permanent 1% increase in trend inflation
$\gamma = 0.1; \phi = 0.1/1.1; \chi = 2$ **(Inelastic economy)**

This figure shows the impulse responses to an unanticipated, one-time, one percent permanent increase in nominal spending. The parameterization is chosen because it fits the behavior of the short-run aggregates relatively well.

Figure 9: Simulated contribution of both sets of inflation shocks to commodity prices: 3-variable VAR and model under the inelastic economy

Contribution of monetary shocks (VAR)

Contribution of monetary shocks (model)

Source: Authors' calculations using VAR and inelastic theoretical model. The dotted black line gives real commodity prices renormalized to 1971.I values. Solid dark lines represent the posterior median for the VAR and exact model predictions. Confidence bands cover 95% of the posterior distribution (solid, light yellow) and 90% (dashed, dark green).