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Abstract

This paper analyzes the cost of disinflation under real wage rigidities in a micro-founded New Keynesian model. Unlike Blanchard and Galí (2007) who carried out a similar analysis in a linearized framework, we take non-linearities into account. We show that the results change dramatically, both qualitatively and quantitatively, for the steady states and for the dynamic adjustment paths. In particular, a disinflation implies a prolonged slump without any need for real wage rigidities.

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1 Introduction

In a very insightful paper Blanchard and Galí (2007) (BG henceforth) advocate the introduction of real wage rigidities in the standard new Keynesian (NK) model. They show that real wage rigidities would generate both more realistic policy trade-offs, by breaking what BG called the *divine coincidence,* and a more realistic empirical behavior of inflation, by generating inflation inertia.

In order to show an example of these two previous features brought about by the introduction of real wage rigidities, in Section 4, BG look at the cost of a classical monetary policy experiment: a disinflation (from 4% to zero).

In this paper, we show that, like others in the literature, the analysis of the real effects of a disinflation in BG is flawed because it abstracts from non-linearities, being based on the log-linear formulation of the standard NK model. Such a procedure is clearly not suited for analyzing the response of the model after a disinflation, because the standard NK model is non-linear, giving rise to non-superneutrality of money. A disinflation experiment is therefore a movement from one steady state to a different one and cannot be analyzed by log-linearizing the model around one of the two steady states.

It may be argued that a log-linear analysis is valid in an approximated sense if the model is "almost" linear. This paper demonstrates that this is not the case. Indeed, we show that the results in Section 4 in BG are inaccurate both qualitatively and quantitatively.

2 The Model

The model is as in BG, that is, a standard NK model where:

(i) Firms produce a differentiated product using the following production function[1]

$$Y_t = F^\alpha N_t^{1-\alpha} \tag{1}$$

where Y is output, and F and N are non-produced[2] and labor inputs, respectively;

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[1] Throughout the paper, capital letters refer to levels, whereas small letters denote the logarithm of a variable.

[2] We deviate slightly from the notation by BG who use the letter M for the non-produced good, which we reserved for money.

(ii) Firms' pricing is described by the usual Calvo mechanism, where θ is the fraction of firms not adjusting their price in any given period;

(iii) Households have the following instantaneous and separable utility function

$$U\left(C_t, \frac{M_t}{P_t}, N_t\right) = \frac{C_t^{1-\sigma}}{1-\sigma} + d_m \frac{\left(\frac{M_t}{P_t}\right)^{1-\nu}}{1-\nu} - d_n \frac{N_t^{1+\varphi}}{1+\varphi} \tag{2}$$

where C is composite consumption (with elasticity of substitution between different types of goods equal to ε).

(iv) BG assume the following partial adjustment model for the real wage: $w_t/p_t = \gamma(w_{t-1}/p_{t-1}) + (1-\gamma)mrs_t$, where mrs_t is the marginal rate of substitution between consumption and labor supply in logarithms and w_t/p_t is the real wage in logarithms. Accordingly, we add the same real wage rigidities to the model, but in a non-linear fashion, that is

$$\frac{W_t}{P_t} = \left(\frac{W_{t-1}}{P_{t-1}}\right)^\gamma (MRS_t)^{1-\gamma}. \tag{3}$$

The Technical Appendix describes all model equations in detail. Note that we add real money balances in the utility function because a disinflation describes a path for the money supply and therefore we do need money demand.

3 Calibration

We calibrate the money demand in the same way as in Chari et al. (2000, CKM henceforth, see pp. 1160 f.). While they have a non-separable utility function, we used a separable form as in BG.

Given the money demand

$$d_m C_t^\sigma (1+i_t) = i_t m_t^\nu \tag{4}$$

and taking the logarithm

$$\ln m_t = \frac{\ln d_m}{\nu} + \frac{\sigma}{\nu} \ln C_t - \frac{1}{\nu} \ln\left(\frac{i_t}{1+i_t}\right) \tag{5}$$

we obtain the same analytical form as CKM (p. 1161, see equation (25)):

$$\ln \frac{M\left(s^t\right)}{\bar{P}\left(s^t\right)} = -\eta \ln \frac{\omega}{1-\omega} + \ln c\left(s^t\right) - \eta \ln\left(\frac{R\left(s^t\right)-1}{R\left(s^t\right)}\right) \tag{6}$$

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To obtain the same interest rate elasticity of money demand, we set $\nu = 2.5641$ (CKM: $\eta = 0.39$). To obtain the same output elasticity, we set $\sigma = 2.5641$ as well (CKM: $\omega = 0.94$). Furthermore, d_m is set to 0.063832.

As in CKM, d_n is calibrated in such a way that people devote one third of their time to work (under zero steady state inflation). The elasticity of substitution between different product types (ε) is set to 10.

Furthermore, we use a standard quarterly discount rate of one percent ($\beta = 0.99$) and a quadratic disutility of labor ($\varphi = 1$), see e.g. Galí (2003). The quarterly probability of not re-setting the prices (θ) is either set to 50 percent (see Bils and Klenow, 2004) or to 75 percent, as in most of the calibrations in the literature. The degree of decreasing returns to labor ($1 - \alpha$) is either 0.975 (BG write that the share of oil in production is roughly 2.5 percent) or 0.67 (as in CKM) in our calibration.

4 Disinflation

4.1 Steady State Effects

The obvious starting point to analyze a disinflation experiment is to look at the steady state, since the standard NK model is non-linear and non-superneutrality arises. In this respect BG (p. 47) write: "As is well known, the standard NKPC implies the presence of a long-run trade-off, however small, between inflation and the output gap. (...) in the standard NK model, disinflation implies a permanently lower-level output."

BG use the standard linearized Phillips curve to make their point:

$$\pi_t = \beta \pi_{t+1} + \kappa y_t \tag{7}$$

Dropping the time indices implies a positive long-run trade-off between inflation and output: $y = \frac{1-\beta}{\kappa}\pi$. This conclusion is an artifact of the model being linearized around zero inflation, as shown in Figure 1.[3] Indeed, while it is true that the tangent of the curve in the graph at zero inflation exhibits a positive slope equal to $\frac{1-\beta}{\kappa}$, the relationship between steady state output and inflation is non-linear. The effects of non-linearities are

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[3] In Figure 1, steady state output at zero inflation was normalized to one, and quarterly inflation rates are annualized.
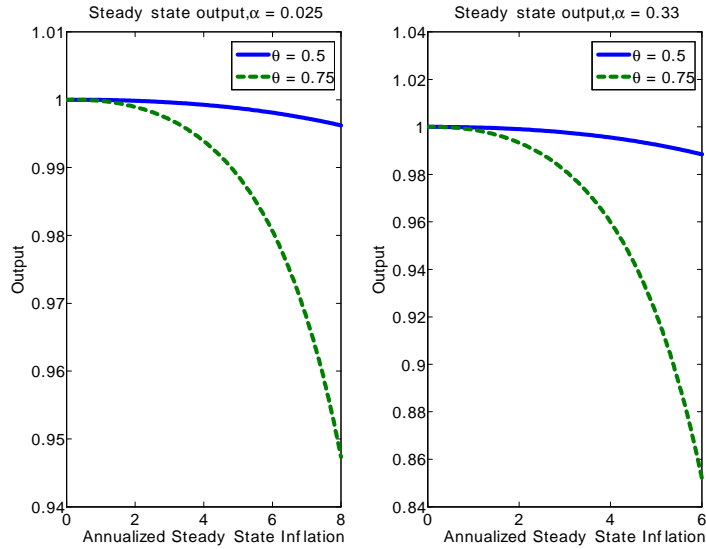
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Figure 1: Steady state relationship between output and (annualized) inflation

quite powerful and turn up very quickly, inverting the relationship from positive to negative (see Ascari and Rankin, 2002, Ascari, 2004 and Yun, 2005).[4]

Quite obviously the strength of the steady state effects due to the non-linearities depends on the parameters governing them, and in particular α, θ and φ. In this respect, we show the graphs for the two values of α (the degree of decreasing returns to labor) used by BG, and for $\theta = 0.5$ (probability of not re-setting the prices), as in BG and Bils and Klenow (2004), as well as $\theta = 0.75$, by far the value most commonly used in the literature, see e.g. Galí (2003).

Our simulations show that non-linearities make the steady state relationship between inflation and output more complex than described by BG. Indeed, it is positive only for very small level of inflation. It follows that the long-run effects of the BG disinflation experiment, i.e., from 4% to zero, are ambiguous and can be sizeable, depending on the calibration chosen. Finally, it is worth noting that the long-run effects depend very much on the particular starting point. Steady state output changes are very different when disinflating from 8% to 4%, rather than from 4% to zero.

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[4]In the language of Graham and Snower (2004), BG only take the "time discounting effect" into account, whereas they ignore the "employment cycling" (product cycling for sticky prices) and "labor supply smoothing" (production smoothing for sticky prices) effects.

4

Some authors may argue that at least in analyzing the steady state properties of the standard NK model one should allow for indexation. Partial indexation would flatten and move the output peak somewhat to the right. However, only full indexation to steady state inflation would be reconcilable with BG's linearized equations (see Ascari, 2004).[5] But complete indexation to steady state inflation would lead to an entirely vertical (flat in our Figure 1) long-run Phillips curve, thus wiping out any trade-off.

4.2 Disinflation Dynamics

4.2.1 Standard NK Model

BG (p. 47) make the following qualitative statement: "Hence, at the time of disinflation (period 0) output declines by $dy(0) = -\left((1-\beta)/\kappa\right)\pi^*$, remaining at the lower level thereafter, with no additional transitional dynamics coming into play." Quantitatively: "In the standard NK model, the real effects of disinflations mentioned above tend to be small, at least for plausible parameter values."

BG's assessment of the effects of a disinflation in a standard NK model is based on a specific log-linearized version of the model. Figure 2 shows the output dynamics (in percentage deviation) in the fully non-linear standard NK model, following a sudden decrease in the rate of growth of money from 4% to zero, as in BG.[6] From a qualitative point of view, it is evident that transitional dynamics come into play, and they do not necessarily seem to be at odds with empirical observations: output drops on impact and then sluggishly returns to its new steady state value after roughly two and a half years. From a quantitative point of view, the effects are quite big: the slump on impact is about 3.5% of the starting output level, and output remains below the steady state

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[5]Indeed, in what follows, we also consider the case where non-resetting firms automatically index their prices to the steady state inflation rate. This is motivated by the fact that full indexation is the only way to obtain the standard New Keynesian Phillips curve (i.e., $\pi_t = \beta E_t \pi_{t+1} + \kappa y_t$, as used by BG) by log-linearizing the model around the steady state, independently of the steady state inflation rate. See the Appendix.

[6]In Figure 2 , we thus set $\gamma = 0$ and use the benchmark calibration (with $\alpha = 0.025$ and $\theta = 0.5$) as described in section 3. Moreover, we plot two paths for two different cases: no indexation ($\chi = 0$) and full indexation ($\chi = 1$) to the steady state rate growth rate of money. The paths displayed in Figure 2 and onwards are obtained using the software DYNARE developed by Juillard (1996) and others at CEPREMAP.
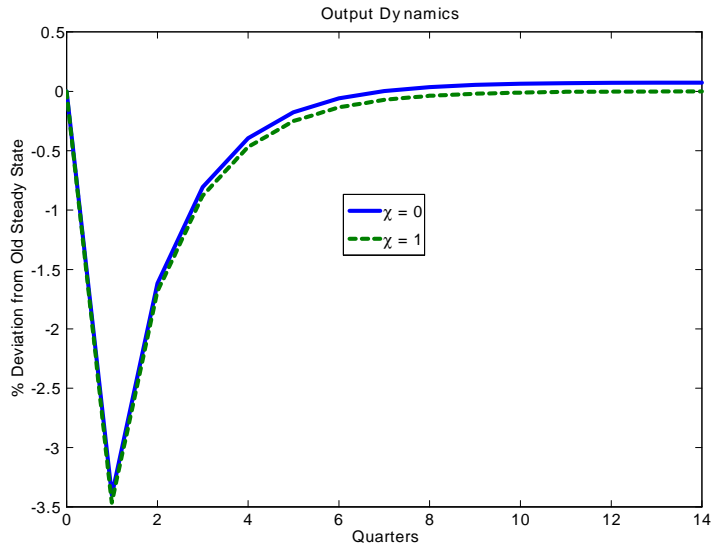
Figure 2: Output response after a disinflation from 4% to 0

value all along the adjustment path. It is worth stressing that this path is obtained for the standard microfounded NK model and standard calibration values.[7]

Two channels induces the slump in output. First, a microfounded money demand implies a higher level of real balances in the new steady state after the disinflation. Thus, the price level has to slow down more than the money supply during the adjustment phase, and this requires output to fall (see, e.g., Blanchard and Fischer, 1989, chp. 10, Ascari and Rankin, 2002 and references therein). Second, a state variable emerges when considering the full non-linear model: the price dispersion term, as shown by Schmitt-Grohé and Uribe (2007). Price dispersion, indeed, has a backward-looking dynamics, hence delivering adjustment dynamics following a disinflation (see equations (51) and (49) in the Appendix).

To sum up, BG write that the standard NK model cannot capture the empirical evidence of the negative effects of a disinflation. Again, we instead claim that the linearization is responsible for this feature, and thus not the NK model *per se*.

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[7]For a thorough analysis of the effects of disinflations in various versions of the NK model see Ascari and Ropele (2006).
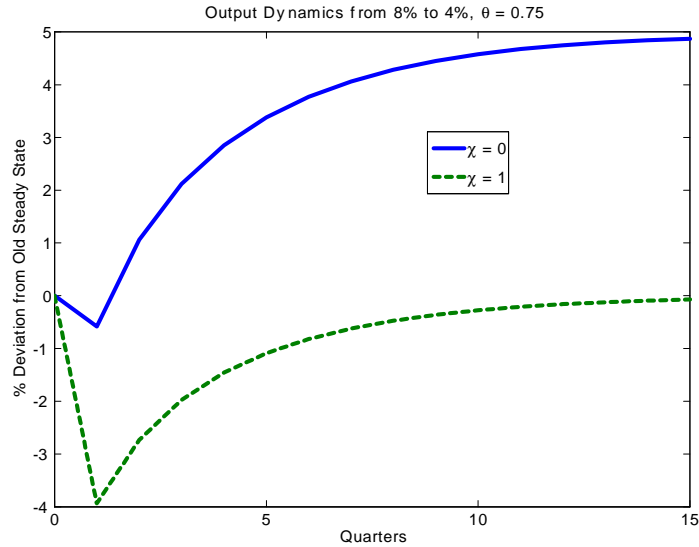
Figure 3: Output response after a disinflation from 7% to 3%

Remark: In Figure 2 we plot two paths for two different cases: no indexation ($\chi = 0$) and full indexation ($\chi = 1$) to the steady state rate growth rate of money. For the benchmark calibration, the two paths are almost identical, showing that our result does not depend on the degree of indexation. The reader should in any case keep in mind that indexation would matter more, whenever effects arising from non-linearities are stronger. Indeed, given the benchmark calibration in Figure 2, the old and new steady states are very close. However, whenever the long-run effects are instead sizeable (because of different starting inflation values and/or different calibration), indexation would obviously also matter for the dynamic adjustment path. This is an important point, exemplifying how long-run effects and short-run dynamics interrelate with each other in a full non-linear model. Just as an illustration, Figure 3 shows the output dynamics following a disinflation from 8% to 4% when $\theta = 0.75$, under the two cases of no and full indexation (see Ascari and Ropele, 2006).

4.2.2 Real Wage Rigidities

BG (p. 48) write: "Hence, a permanent reduction in inflation of 4 percentage points in (annualized) inflation lowers the level of output by roughly 50 basis points in the quarter the policy is implemented, an effect about 10 times larger than in the standard model."

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BG claim that real wage rigidities: (i) are necessary to obtain a dynamic response of output after a disinflation, and (ii) they increase the impact effect on output and thus the overall costs of a disinflation manifold. We already saw that a dynamic path for output is obtained in the standard model without the need for any real rigidities.

Figure 4 shows that BG's assessment of the role that real rigidities play for the dynamic adjustment after a disinflation is qualitatively right.[8] Indeed, real wage rigidities cause stronger and more persistent effects on output. From a quantitative point of view, however, the effects are by no means of the order of magnitude suggested by BG. In the extreme case assumed by BG, i.e., $\gamma = 0.9$, the impact effect is only twice as large as in the standard model. Moreover, during the adjustment, output oscillates and the sudden slump is followed by a prolonged boom that partly compensates the initial output loss, with respect to the case without real wage rigidities, where convergence is instead monotonic. Moreover, Figure 4 does not suggest a "relatively fast convergence to the new steady state."

Finally it is worth noting that only very extreme values of γ tend to have sizeable effects on the output response, since for values smaller than 0.5, the quantitative effects of real rigidities are small (more on that in the next subsection).

BG stress the importance of real rigidities for inflation dynamics. Indeed, in Section 6 of their article, BG show that real wage rigidities are able to generate inflation inertia and give raise to a log-linearized Phillips curve equation which is very similar to the ad hoc specification used in the empirical literature. This point can be visualized by plotting the dynamic response of inflation, as in Figure 5. Inflation indeed displays more inertia for higher values of γ. Moreover, for the calibration chosen by BG, i.e., $\gamma = 0.9$, inflation exhibits a hump-shaped response. However, (i) again only extreme values of γ tend to have significant effects; (ii) the numbers are rather disconcerting. Inflation falls immediately with little inertia whatsoever in any case: the first quarter after the disinflation, inflation is -51% (in annualized terms) if $\gamma = 0$, and -28% if $\gamma = 0.9$.

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[8]In Figure 4 and in the following ones, we assume full indexation, benchmark calibration and again 4% to zero disinflation experiment. Note that the steady state values do not depend on γ. The paths for output would be almost identical if we had assumed no indexation.
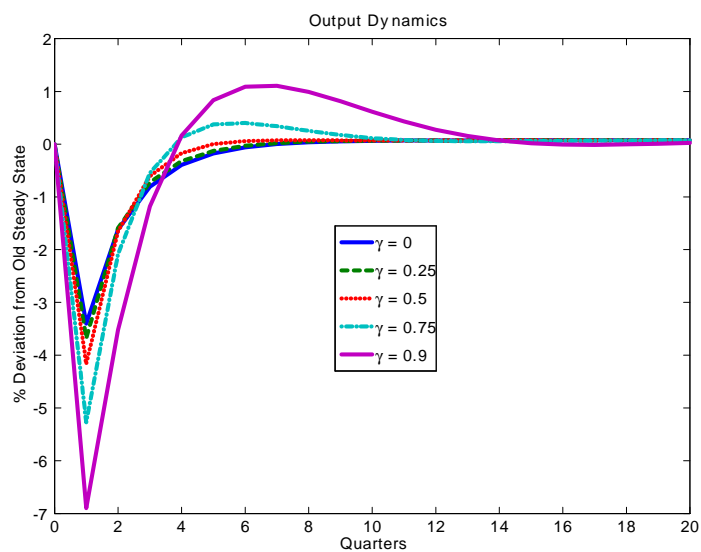
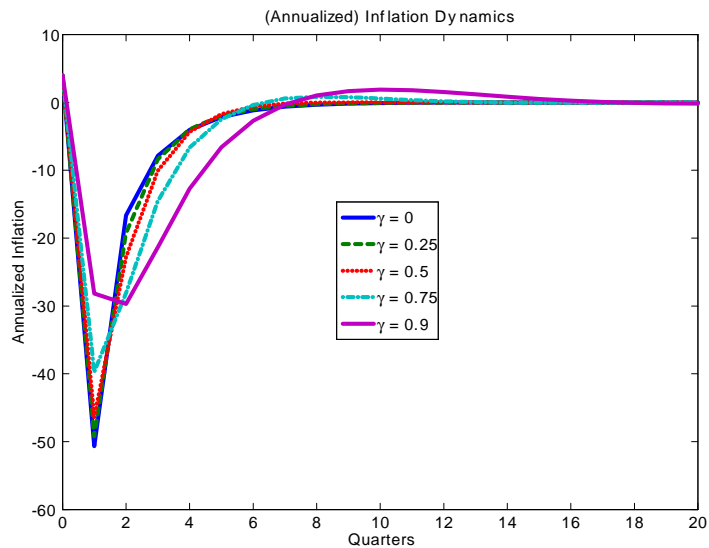Figure 4: The effect of real rigidities on the output response to a disinflation

Figure 5: The effect of real rigidities on the inflation response to a disinflation
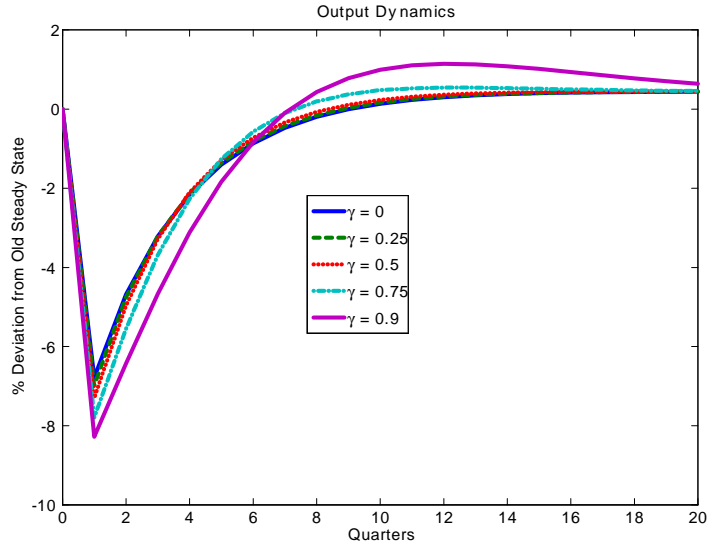
Figure 6: Decreasing returns to scale to labor and the effect of real rigidities

4.2.3 Returns to Scale

BG (p. 49) state: "Finally, it is worth noticing that [...] the quantitative results above change significantly if we assume the presence of decreasing returns. Hence, under our alternative calibration with decreasing returns, the loss of output at the outset of the disinflation is multiplied by a factor of 4 relative to the case with no real rigidities (compared with a factor of 10 in the case of constant returns). The smaller initial impact coexists with a larger persistence."

Figure 6 shows clearly that assuming stronger decreasing returns to scale to labor (DRTS) cause: (i) a higher persistence in the output response; (ii) a downward rescaling of the effect of real rigidities. From a quantitative point of view, however, the effects are not of the size described by BG: actually assuming DRTS makes real rigidities virtually devoid of importance for the output response to a disinflation.

Finally, it is worth visualizing the different paths of the output response for the BG preferred calibration (i.e., $\gamma = 0.9$) under almost constant and DRTS (see Figure 7). With DRTS not only persistence, but also the impact effect is larger. Note that simply by differentiating (25) at p. 48 in BG with respect to α, it is easy to check that BG equations would actually imply the opposite: an increase in α would lower the impact effect of a disinflation.
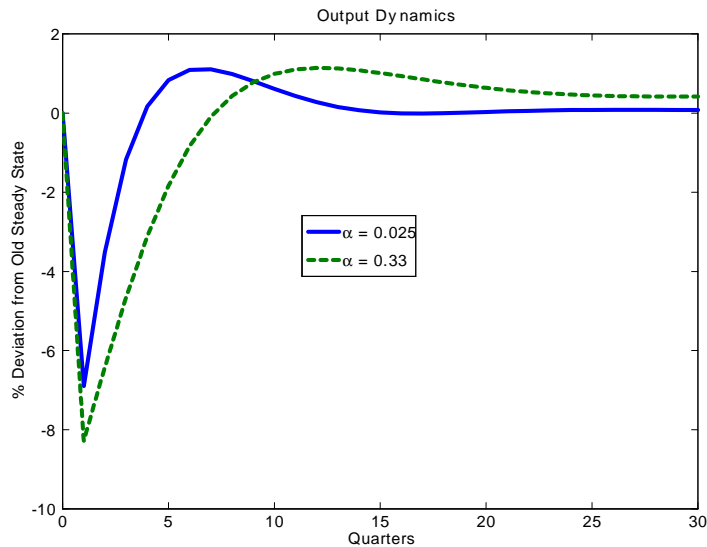
Figure 7: DRTS and ouput response after a disinflation from 4% to 0 ($\gamma = 0.9$)

5 Conclusions

In a stimulating paper BG study the effects of introducing real wage rigidities in a standard NK model. In Section 4, BG look at disinflations. They claim this feature to be crucial for this class of models to explain the cost of disinflation.

In this paper, we show that, like others in the literature, the analysis in BG is flawed because it abstracts from non-linearities, being based on the log-linear formulation of the standard NK model. Indeed, we show that the results in their Section 4 are inaccurate both qualitatively and quantitatively, once the full microfounded and non-linear model is taken into account.

This paper sounds a cautionary note about the log-linearized model as a tool to analyze disinflation experiments theoretically. More generally, we want to advocate the explicit consideration of the effects of non-linearities, whenever necessary and possible.

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Technical Appendix
1. Household

Given the separable utility function

$$U\left(C_t\left(h\right), \frac{M_t\left(h\right)}{P_t}, N_t\left(h\right)\right) = \frac{C_t^{1-\sigma}}{1-\sigma} + d_m \frac{\left(\frac{M_t(h)}{P_t}\right)^{1-\nu}}{1-\nu} - \exp(\xi)d_n \frac{N_t^{1+\varphi}\left(h\right)}{1+\varphi} \tag{8}$$

subject to the budget constraint

$$P_t C_t + \left(1+i_t\right)^{-1} B_t + M_t = W_t N_t - T_t + \Pi_t + B_{t-1} + M_{t-1}, \tag{9}$$

where i_t is the nominal interest rate, B_t are one-period bond holdings, M_t is the nominal money supply, W_t is the nominal wage rate, N_t is the labor input, T_t are lump sum taxes, and Π_t is the profit income. The representative consumer maximizes the intertemporal utility (using the discount factor β)

$$\max_{\{C_t, W_t, B_t, M_t\}} E_0 \sum_{t=0}^{\infty} \beta^t U\left(C_t\left(h\right), \frac{M_t\left(h\right)}{P_t}, N_t\left(h\right)\right), \tag{10}$$

yielding the following first order conditions:

Money demand equation:

$$\frac{U_m}{U_C} = \frac{d_m C_t^{\sigma}}{m_t^{\nu}} = \frac{i_t}{1+i_t} \tag{11}$$

Labor supply equation:

$$\frac{W_t}{P_t} = -\frac{U_N}{U_C} = \frac{\exp(\xi)d_n N_t^{\varphi}}{1/C_t^{\sigma}} = \exp(\xi)d_n N_t^{\varphi} C_t^{\sigma} \tag{12}$$

We introduce real wage rigidities in the same way as BG, that is

$$\frac{W_t}{P_t} = \left(\frac{W_{t-1}}{P_{t-1}}\right)^{\gamma} MRS_t^{1-\gamma} = \left(\frac{W_{t-1}}{P_{t-1}}\right)^{\gamma} \left(-\frac{U_{N_t}}{U_{C_t}}\right)^{1-\gamma} \tag{13}$$

$$\frac{W_t}{P_t} = \left(\frac{W_{t-1}}{P_{t-1}}\right)^{\gamma} \left(\exp(\xi)d_n N_t^{\varphi} C_t^{\sigma}\right)^{1-\gamma} \tag{14}$$

Euler equation:

$$\frac{1}{C_t^{\sigma}} = \beta E_t \left[\left(\frac{P_t}{P_{t+1}}\right)\left(1+i_t\right)\left(\frac{1}{C_{t+1}^{\sigma}}\right)\right] \tag{15}$$

2. Firms' pricing

Final good producers use the following technology

$$Y_t = \left[\int_0^1 Y_{i,t}^{\frac{\varepsilon-1}{\varepsilon}} \, di \right]^{\frac{\varepsilon}{\varepsilon-1}} \tag{16}$$

Their demand for intermediate inputs is therefore equal to

$$Y_{i,t+j} = \left(\frac{P_{i,t}}{P_{t+j}} \right)^{-\varepsilon} Y_{t+j} \tag{17}$$

1. No indexation

The problem of a price-resetting firm can be formulated as

$$\max_{P_{i,t}} \quad E_t \sum_{j=0}^{\infty} \theta^j \Delta_{t,t+j} \left[\frac{P_{i,t}}{P_{t+j}} Y_{i,t+j} - TC_{t+j}^r \left(Y_{i,t+j} \right) \right] \tag{18}$$

$$s.t. \quad Y_{i,t+j} = \left(\frac{P_{i,t}}{P_{t+j}} \right)^{-\varepsilon} Y_{t+j} \tag{19}$$

where $P_{i,t}$ denotes the new optimal price of producer i and $TC_{t+j}^r \left(Y_{i,t+j} \right)$ the real total cost function and $\Delta_{t,t+j}$ is the stochastic discount factor (from period t to period $t+j$). The solution to this problem yields the familiar formula for the standard optimal resetted price in a Calvo setup

$$P_{i,t} = \left(\frac{\varepsilon}{\varepsilon-1} \right) \frac{E_t \sum_{j=0}^{\infty} \theta^j \Delta_{t,t+j} \left[P_{t+j}^{\varepsilon} Y_{t+j} MC_{i,t+j}^r \right]}{E_t \sum_{j=0}^{\infty} \theta^j \Delta_{t,t+j} \left[P_{t+j}^{\varepsilon-1} Y_{t+j} \right]} \tag{20}$$

where $MC_{i,t}^r$ denotes the real marginal costs function.

We express the real marginal costs in terms of the real wage (see section 3 below for the derivation)

$$MC_{i,t}^r = \frac{1}{1-\alpha} w_t \left[(x_t)^{-\varepsilon} Y_t \right]^{\frac{\alpha}{1-\alpha}} \tag{21}$$

in order to be able to re-write the numerator recursively, where $x_t = P_{i,t}/P_t$.

Thus, we obtain:

$$x_t^{1+\frac{\varepsilon\alpha}{1-\alpha}} = \left(\frac{\varepsilon}{\varepsilon-1} \right) \left(\frac{\psi_t}{\phi_t} \right), \tag{22}$$

where

$$\psi_t = E_t \sum_{j=0}^{\infty} (\theta\beta)^j u_c (t+j) \left[\Pi_{t,t+j}^{\frac{\varepsilon}{1-\alpha}} Y_{t+j}^{\frac{1}{1-\alpha}} \frac{1}{1-\alpha} w_{t+j} \right] \qquad (23)$$

$$\phi_t = E_t \sum_{j=0}^{\infty} (\theta\beta)^j u_c (t+j) \left[\Pi_{t,t+j}^{\varepsilon-1} Y_{t+j} \right] \qquad (24)$$

where $\Pi_{t,t+j}^z = \pi_{t+1}^z \pi_{t+2}^z ... \pi_{t+j}^z = \prod_{i=1}^{j} \pi_{t+i}$ for $j \geq 1$ and equal one for $j = 0$, and z is a power.

This can be re-written recursively:

$$\psi_t = \frac{1}{1-\alpha} u_c (t) Y_t^{\frac{1}{1-\alpha}} w_t + \theta\beta E_t \left(\pi_{t+1}^{\frac{\varepsilon}{1-\alpha}} \psi_{t+1} \right) \qquad (25)$$

and

$$\phi_t = u_c (t) Y_t + \theta\beta E_t \left[\pi_{t+1}^{\varepsilon-1} \phi_{t+1} \right]. \qquad (26)$$

The aggregate price level evolves according to

$$P_t = \left[\int_0^1 P_{i,t}^{1-\varepsilon} di \right]^{\frac{1}{1-\varepsilon}} \implies \qquad (27)$$

$$1 = \left[\theta\pi_t^{\varepsilon-1} + (1-\theta) \left(\frac{P_{i,t}}{P_t} \right)^{1-\varepsilon} \right]^{\frac{1}{1-\varepsilon}}. \qquad (28)$$

2. Partial indexation to long-run inflation (LRI)

Under this assumption, a firm that cannot re-optimize its price updates the price according to this simple rule:

$$P_{i,t} = \bar{\pi}^\chi P_{i,t-1} \qquad (29)$$

where $\bar{\pi}$ is the steady state inflation level and $\chi \in [0,1]$ is a parameter that measures the degree of indexation. If $\chi = 1$, there is full indexation, if $\chi = 0$ there is no indexation and the problem is the same one as in the previous case. The problem of a price-resetting firm then becomes the following

$$\max_{p_t^*(i)} \quad E_t \sum_{j=0}^{\infty} \theta^j \Delta_{t,t+j} \left[\frac{P_{i,t} \bar{\pi}^{\chi j}}{P_{t+j}} Y_{i,t+j} - TC_{t+j}^r (Y_{i,t+j}) \right] \qquad (30)$$

$$s.t. \quad Y_{i,t+j} = \left[\frac{P_{i,t} \bar{\pi}^{\chi j}}{P_{t+j}} \right]^{-\varepsilon} Y_{t+j} \qquad (31)$$

and the first order condition (FOC) is

$$P_{i,t} = \frac{\varepsilon}{\varepsilon - 1} \frac{E_t \sum_{j=0}^{\infty} \theta^j \beta_{t,t+j} \left[P_{t+j}^{\varepsilon} Y_{t+j} MC_{i,t+j}^r \bar{\pi}^{-\varepsilon \chi j} \right]}{E_t \sum_{j=0}^{\infty} \theta^j \beta_{t,t+j} \left[P_{t+j}^{\varepsilon-1} Y_{t+j} \bar{\pi}^{(1-\varepsilon)\chi j} \right]}. \tag{32}$$

Again using (21), we can re-write the numerator recursively, such that

$$x_t^{1+\frac{\varepsilon\alpha}{1-\alpha}} = \left(\frac{\varepsilon}{\varepsilon - 1} \right) \left(\frac{\psi_t}{\phi_t} \right) \tag{33}$$

where

$$\psi_t = E_t \sum_{j=0}^{\infty} (\theta\beta)^j u_c (t+j) \left[\Pi_{t,t+j}^{\frac{\varepsilon}{1-\alpha}} Y_{t+j}^{\frac{1}{1-\alpha}} \frac{1}{1-\alpha} w_{t+j} \bar{\pi}^{-\frac{\varepsilon\chi j}{1-\alpha}} \right] \tag{34}$$

and

$$\phi_t = E_t \sum_{j=0}^{\infty} (\theta\beta)^j u_c (t+j) \left[\Pi_{t,t+j}^{\varepsilon-1} Y_{t+j} \bar{\pi}^{(1-\varepsilon)\chi j} \right]. \tag{35}$$

In recursive form,

$$\psi_t = \frac{1}{1-\alpha} u_c (t) Y_t^{\frac{1}{1-\alpha}} w_t + \theta\beta E_t \left(\pi_{t+1}^{\frac{\varepsilon}{1-\alpha}} \bar{\pi}^{-\frac{\varepsilon}{1-\alpha}} \psi_{t+1} \right) \tag{36}$$

and

$$\phi_t = u_c(t) Y_t + \theta\beta \bar{\pi}^{(1-\varepsilon)\chi} E_t \left(\pi_{t+1}^{\varepsilon-1} \phi_{t+1} \right). \tag{37}$$

The aggregate price level now evolves according to

$$P_t = \left[\int_0^1 P_{i,t}^{1-\varepsilon} di \right]^{\frac{1}{1-\varepsilon}} = \left[\theta\bar{\pi}^{(1-\varepsilon)\chi} P_{t-1}^{1-\varepsilon} + (1-\theta) P_{i,t}^{1-\varepsilon} \right]^{\frac{1}{1-\varepsilon}} \implies \tag{38}$$

$$1 = \left[\theta\bar{\pi}^{(1-\varepsilon)\chi} \pi_t^{\varepsilon-1} + (1-\theta) \left(\frac{P_{i,t}}{P_t} \right)^{1-\varepsilon} \right]^{\frac{1}{1-\varepsilon}}. \tag{39}$$

3. Technology

Production function:

$$Y_{i,t} = F_t^{\alpha} N_{i,t}^{1-\alpha}. \tag{40}$$

For simplicity, we omit F_t^{α} (since we are not explicitly interested in a cost push shock).

The labor demand and the real marginal cost of firm i is therefore

$$N_{i,t}^d = [Y_{i,t}]^{\frac{1}{1-\alpha}} \tag{41}$$

$$MC_{i,t}^r = \frac{1}{1-\alpha}\frac{W_t}{P_t}Y_{i,t}^{\frac{\alpha}{1-\alpha}}. \tag{42}$$

Note that now marginal costs depend upon the quantity produced by the single firm, given the decreasing returns to scale. In other words, different firms charging different prices would produce different levels of output and hence have different marginal costs.

Express $MC_{i,t}^r$ as

$$
\begin{aligned}
MC_{i,t}^r &= \frac{1}{1-\alpha}\frac{W_t}{P_t}Y_{i,t}^{\frac{\alpha}{1-\alpha}} \\
&= \frac{1}{1-\alpha}\frac{W_t}{P_t}\left[\left(\frac{P_{i,t}}{P_t}\right)^{-\varepsilon}Y_t\right]^{\frac{\alpha}{1-\alpha}}.
\end{aligned}
\tag{43}
$$

4. Aggregation and price dispersion

The aggregate resource constraint is now simply given by

$$Y_t = C_t \tag{44}$$

and the link between aggregate labour demand and aggregate output is provided by

$$
\begin{aligned}
N_t^d &= \left[\int_0^1 N_{i,t}^d di\right] = \left[\int_0^1 \left[\left(\frac{P_{i,t}}{P_t}\right)^{-\varepsilon}Y_t\right]^{\frac{1}{1-\alpha}}di\right] = \\
&= [Y_t]^{\frac{1}{1-\alpha}}\underbrace{\int_0^1 \left[\left(\frac{P_{i,t}}{P_t}\right)^{-\varepsilon}\right]^{\frac{1}{1-\alpha}}di}_{s_t} = s_t\,[Y_t]^{\frac{1}{1-\alpha}}
\end{aligned}
\tag{45}
$$

where

$$s_t = \int_0^1 \left[\left(\frac{P_{i,t}}{P_t}\right)^{-\varepsilon}\right]^{\frac{1}{1-\alpha}}di \tag{46}$$

is a sort of tax due to price distortions (and the non-linearity of the aggregator). Schmitt-Grohé and Uribe (2007) show that s_t is bounded below at one, so that s_t represents the resource costs due to relative price dispersion under the Calvo mechanism with long-run inflation. Indeed, the higher s_t, the more labor is needed to produce a given level of output. Note that s_t can also be rewritten as a ratio between two different price indexes P_t and $\widetilde{P}_t$

$$s_t = \left(\frac{P_t}{\widetilde{P}_t}\right)^{\varepsilon} \quad \text{where} \quad \widetilde{P}_t = \left[\int_0^1 P_t\,(i)^{-\varepsilon}\,di\right]^{-1/\varepsilon}, \tag{47}$$

as in Ascari (2004). Whenever there is price dispersion these two indexes evolve differently from each other, determining a certain dynamics for s_t, that affects the level of production negatively . s_t would not affect the real variables up to first order whenever there is no trend inflation (i.e., $\bar{\pi} = 1$) or whenever the resetted price is fully indexed to any variable whose steady state level grows at the rate $\bar{\pi}$.

To close the model we just need to solve for the dynamic of s using (46). This would depend on the indexation.

1. **No indexation**

$$s_t = \int_0^1 \left[\left(\frac{P_{i,t}}{P_t} \right)^{-\varepsilon} \right]^{\frac{1}{1-\alpha}} di \tag{48}$$

$$s_t = (1-\theta) \left[\frac{P_{i,t}}{P_t} \right]^{-\frac{\varepsilon}{1-\alpha}} + \theta \pi_t^{\frac{\varepsilon}{1-\alpha}} s_{t-1} \tag{49}$$

2. **Long-run indexation**

$$s_t = \int_0^1 \left[\frac{P_{i,t}}{P_t} \right]^{-\frac{\varepsilon}{1-\alpha}} di \tag{50}$$

$$s_t = (1-\theta) \left[\frac{P_{i,t}}{P_t} \right]^{-\frac{\varepsilon}{1-\alpha}} + \theta \left(\frac{\pi_t}{\pi^\chi} \right)^{\frac{\varepsilon}{1-\alpha}} s_{t-1} \tag{51}$$

5. System of Equations

The following systems of equations are simulated non-linearly:

1. **No Indexation**

Equations (11), (12), (15), (22), (25), (26), (28), (41), and (49).

2. **Indexation**

Equations (11), (12), (15), (22), (36), (37), (39), (41), and (51).

In both cases, the money supply identity equation closes the system

$$m_{t-1} rgm_t = m_t \pi_t, \tag{52}$$

where rgm_t is the rate of growth of money which is reduced under a disinflation.

In the presence of a real wage rigidity, equation (12) is replaced by equation (14).