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Abstract:

As growing attention is paid to climate change adaptation as an actual policy issue, the significant meaning of climate variability in adaptation decisions is beginning to be recognized. By using a real option framework, we shed light on how climate change and climate variability affect individuals' (farmers') investment decisions with regard to adaptation. As a plausible case in which the delay carries policy implications, we investigate farmers' choices when adaptation involves the use of an open-access resource (water). The results show that uncoordinated farmers with a high risk aversion may under-adapt while farmers with a low risk aversion would over-adapt under the same conditions. Private adaptation should be supported or discouraged accordingly if farmers are not convinced about the possibilities of collective resource management in the long run.

Keywords: Adaptation to climate change, climate variability, risk and uncertainty, real option, water, open-access resources

JEL classification: D81, Q20, Q54

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1 Introduction

The global average surface temperature has already significantly risen over the past century (IPCC (2007)), and studies suggest that even with the best of our mitigation efforts, negative effects of global climate change on human lives will become noticeable within decades, especially in the developing world (IPCC (2007); World Bank (2010b)). Accordingly, interests in climate change adaptation are growing in policy debates.

While adaptation to climate change is not a problem of global public good as climate change mitigation is, it still involves some important economic questions. A relatively well-explored issue is the efficient allocation of resources between mitigation and adaptation (e.g., Tol (2005); de Bruin et al. (2009); Onuma and Arino (2011)). Also, some of the adaptation measures, such as construction of seawalls, are public or collective in nature, and this means that their implementation constitutes a classic economic problem of public good provision (Mendelsohn (2000)).

Meanwhile, there are a large number of adaptation measures that are private in nature, such as the switching of crops in farming. Simple reasoning tells us that such private adaptation just takes place spontaneously as the benefits of adaptation are brought to the individuals who take efforts – in other words, private adaptation does not generate the needs for policy intervention. In line with this logic, previous economic studies have paid relatively little attention to this aspect of climate change adaptation as a potential policy problem.

In this paper, we argue that this simple reasoning does not accurately capture the entire picture of private adaptation to climate change. A key feature here is that humans do not directly perceive a gradual change of climate but the baseline change of climate is hidden behind inter-annual fluctuations of weathers. Two well-known determinants of those inter-annual weather fluctuations are El Niño and La Niña, but there are many other factors, and their combined effects make precise predictions of weather patterns difficult on a multi-year time scale (Rosenzweig and Hillel (2008)). Many of the private adaptation measures are thus taken primarily as a response to those weather fluctuations that are influenced by climate change, not directly to the baseline change of climate. While weather fluctuations themselves do not create a market failure, we show that they add a great deal of complexity to people's adaptation decisions, and in the presence of a market failure due to some other mechanism – in this paper we examine the use of an open-access resource – they lead to remarkably nuanced policy implications.

We analyze this problem by employing a real option framework. The key idea is that under uncertainty, individuals have incentive to delay fixed investments for adaptation, a switch from a weather-sensitive production alternative, whose prime example is the farming of a crop to another production alternative that is either weather-sensitive or weather-resistant. Indeed, benefits of such delaying in adaptation have already been pointed out by some non-theoretical analyses of climate change adaptation. For example, a recent World Bank report (World Bank (2010a), p.92) states that "countries should want to *delay* adaptation decisions as much as possible...while awaiting greater certainty about climate and socioeconomic scenarios" (emphasis in the original). By delaying, farmers might be able to capture relative bumper harvests in the near future even from a crop with an overall declining yield that is better to be abandoned eventually. We show that this factor is significant, as is for a variety of cases analyzed by the real option framework (e.g., Dixit and Pindyck (1994)). Coupled with a market failure in the form of uncoordinated use of an open-access resource (water), this option feature of adaptation decisions becomes a significant determinant for policy actions to improve the social welfare. For example, our results show that uncoordinated farmers with a high risk aversion may under-adapt although farmers with a low risk aversion would over-adapt under the same conditions. If farmers remain suspicious about viability of long-term coordination on resource use, public interventions to support or discourage adaptation could improve the social welfare.

The paper is organized as follows. In the next section, we present a brief summary of a relevant literature. Section 3 is the core part of the paper, which discusses the model for cases without and with the use of an open-access resource. Section 4 concludes.

2 Relevant Literature

The role of risk and uncertainty in adaptation decisions regarding climate change is qualitatively widely recognized (e.g., World Bank (2010b); Howden et al. (2007)), but few economic studies have shed light on its economic implications yet. As discussed in the Introduction, a number of economic studies have investigated climate change adaptation, but they mainly address different issues such as the resource allocation between climate change mitigation and adaptation.

The focus of this paper is the role of risk from persistent climate variability as a delaying factor for substantial adaptation measures in the face of a long-run climate change. Studies that examine potential adaptation methods to climate change in agriculture indicate that while some of adaptation measures are incremental measures such as shifts in planting seasons, there are also larger but potentially effective measures that involve significant initial investments, such as the adoption of irrigation, a switch to an alternative farming crop (for example, from the heat-sensitive wheat farming to the heat-resistant fruit farming), and a shift from crop farming to livestock farming (which tends to be less water intensive) (e.g., Howden et al. (2007); Mendelsohn (2000); Seo and Mendelsohn (2008)). The latter type of adaptation measures entails high monetary and non-monetary initial costs to farmers and thus is not easily reversible. The farmers would need a particular consideration for the irreversible nature of decisions under uncertainty of climatic conditions in the future.

The real option approach is an effective analytical method to study this aspect of irreversible investment. Conceptually based on similar ideas to those of the seminal studies of the quasi-option value (Arrow and Fisher (1974); Henry (1974)), it is an established methodology and is applied to a variety of economic problems (Dixit and Pindyck (1994)). There is also a fair amount of literature employing the real option approach to investigate the policy of climate change mitigation (reviewed by Golub et al. (2011)). But applications of this approach to the context of climate change adaptation are still few, at least in terms of the investigation of generic theoretical aspects (rather than project evaluations in specific contexts, such as Linquiti and Vonortas (2012)). As related groups of academic studies, a substantial number of empirical studies examine the role of weather shocks on rural households (e.g., Rosenzweig and Binswanger (1993) and related studies), and a theoretical literature exists on agricultural commodities and price shocks (e.g., Gouel and Jean (2012) and references therein). But they do not address the investment aspect of adaptation to climate change.

3 Model

3.1 Basic Framework

We describe the basic framework of our adaptation model. The model is based on the framework of real option analysis (e.g., Dixit and Pindyck 1994), which highlights the significance of the option value for investment decisions under uncertainty.

Here, the farmers with identical characteristics in the economy consider switching their modes of production in the face of long-run change of climate, which comes as an exogenous process to them.

By paying a fixed cost, farmers who engage in a vulnerable farming mode may permanently shift to an alternative production that can better stave off the long-run damage of climate change, such as irrigated farming, the cultivation of a heat-resistant crop, livestock farming, and non-farm employment. The fixed cost can take various forms, such as an investment in irrigation facilities, an alteration of farmland layout and topsoil to accommodate a new crop, a purchase of livestock, and training of farmers themselves.

For simplicity, we consider that the fixed cost is significant enough to prevent a two-way switching (for individual farmers, a shift of production is permanent once it is made) and also a simultaneous engagement in the two production modes. The latter assumption is consistent with general observations that many actual farmers have a main crop of cultivation although they may diversify livelihoods to some extent in order to mitigate the risk of income shocks.

Farmers make their decisions of switching by weighing the fixed cost and the long-term or time-discounted expected gains from the switching. The future net gains from the switching, however, are not known to the farmers because the outputs from the two production modes *always* fluctuate.

Climate variability plays a critical role in the model. Under risk of future productions, the farmers may temporarily refrain from making new investments for the alternative production mode, *even if the other production mode, which is better suited in a changed climate, brings them a net increase of expected outputs*. Note that these preferences of farmers to delay new investments are not necessarily the same as risk aversion. They can also come from the farmers' profit-maximizing behavior through waiting for a favorable timing of switching productions. This also means that a delay might be caused not only by the output variability in the alternative production mode (to which they switch) but also from the output variability in the current production mode (in which they are currently engaging).

The economy produces two climate-sensitive goods (agricultural products), 1 and 2. In this paper, we consider two cases, one without the use of an open-access resource and the other with it. The open-access resource could be interpreted as water, as the adoption of irrigation is considered a major method for climate change adaptation (e.g., (Mendelsohn and Dinar, 2003)).

Without the use of an open-access resource, the output levels of two production processes, Y_1 and Y_2, are given by

$$Y_1 = X_1 \left(\bar{L} - L\right) \tag{1}$$
$$Y_2 = X_2 L \tag{2}$$

where L is the number of farmers engaging in production 2 and $\bar{L}$ is the total number of farmers in the economy. The stochastic productivity parameters X_1 and X_2 represent exogenous shocks in productivity due to weather patterns. With normalizing the price of good 1 to one and defining the relative price of good 2 as P, the earnings of farmers producing good 1 and good 2 are Y_1 and PY_2, respectively. Throughout this paper, P is set to be constant. A constant P is a plausible representation for a small open economy subject to idiosyncratic local climatic patterns, as the agricultural products are sold at the given world prices whose fluctuations do not match with those of local outputs. Note, however, that P can in principle be endogenized as well, and in so doing, this framework could also be used for a general-equilibrium modeling.[1]

The productivity parameters X_1 and X_2 follow geometric Brownian motions

$$dX_1 = \mu_1 \, X_1 \, dt + \sigma_1 \, X_1 \, dz_1 \tag{3}$$
$$dX_2 = \mu_2 \, X_2 \, dt + \sigma_2 \, X_2 \, dz_2 \tag{4}$$

where $\sigma_1, \sigma_2 \geq 0$. Without loss of generality, we can limit our discussions to the cases where $\mu_1 < \mu_2$, i.e., production of 1 becomes comparatively unfavorable in the long run (2 is relatively favorable under a warm climate). Note that μ_1 can be both positive and negative. Setting $\sigma_i = 0$, $i = 1, 2$, the above formulations can also represent a weather-resistant production mode such as manufacturing.

[1]The relative price can be changed by a shift in the total composition of productions in an economy, in other words, the process of adaptation itself can alter P. Incorporating this property into the model produces remarkably diverse cases, many of which entail socially sub-optimal adaptation as a result of a missing risk market (Narita and Quaas, in progress)

The stochastic processes of X_1 and X_2 might be determined by a combination of variables. For example, X_1 could be a linear function of normalized stochastic temperature $\tilde{T}$, i.e.,

$$X_1 = J\tilde{T}$$

or, it could be formulated with one additional stochastic variable (such as "R" as in "rainfall"), for example,

$$X_1 = \hat{J}\tilde{T}^{\chi}R^{\omega}$$

Note that X_1 still follows a geometric Brownian motion if both $\tilde{T}$ and R follow geometric Brownian motions.[2]

X_2 could also be formulated in a similar way, but with a different parametrization. This implies that X_1 and X_2 do not have to be perfectly correlated, i.e. the correlation θ between X_1 and X_2 is between minus one and one, $-1 \leq \theta \leq 1$.

Next we turn to the case where the production process 2 uses an open-access resource, while the production process of 1 remains the same as in the above. The production process 2 takes an alternative formulation to reflect resource use. We use a simple-as-possible formulation for that purpose: L farmers are using process 2, which has as inputs a stochastic input X_{21} (temperature) and an amount q of the resource (water), which is used under conditions of open access. The resource does not deplete, but the marginal extraction cost increases with the total amount of flow extraction from the common pool (i.e., the extraction cost function takes a convex shape). Note that goods 1 and 2 do not have to be distinguishable, in other words, the model could be interpreted also as a description of switching from rainfed to irrigated farming of the same crop.

An individual farmer's output from process 2 is then given by

$$y_2 = q^{\alpha}\, X_{21}^{1-\alpha} - c\,(q, \tilde{q}, L)\; X_{22} \tag{5}$$

The term $q^{\alpha}\, X_{21}^{1-\alpha}$, where α satisfies $0 \leq \alpha \leq 1$, represents the revenue for the farmer, and the cost is composed of the deterministic cost function $c\,(q, \tilde{q}, L)$ and a stochastic parameter X_{22}. Costs depend on the rate of extraction by the respective farmer, q, the number of farmers using that resource L, and the extraction rate by other (identical) farmers, $\tilde{q}$.

The parameter α of the revenue function is particularly important: The larger the value of α, the more important is the resource for production 2. One can thus expect that the market failure associated with overusing the resource increases with α. If $\alpha = 0$ and X_{21} is redefined as X_2, the resource is irrelevant for the production of 2, and this case is reduced to the above case without an open-access resource.

When the collective extraction costs of resource take a convex function with the exponent $\eta > 1$ and the farmers bear the costs of extraction proportional to their

[2]Note that in a similar fashion, X_1 can also incorporate fluctuations due to economic factors, e.g., exogenous demand shocks.

share of extraction, c is given by the following function on a common-pool resource (the generalized case of an open-access resource):

$$c\left(q,\tilde{q},L\right)=c_0\left[q+\left(L-1\right)\tilde{q}\right]^{\eta}\cdot\frac{q}{q+\left(L-1\right)\tilde{q}} \tag{6}$$

With or without the open-access resource, farmers can switch their production mode from 1 to 2 by paying a fixed cost $I>0$. They make their switching decisions according to their time-discounted stream of utility given by

$$\int_0^{\infty} U\, e^{-\rho t}\, dt$$

where ρ is the pure time preference, and U is a function of flow consumption h

$$U\left(h\right)=G\left(h\right) \tag{7}$$

where

$$G\left(h\right)=\frac{h^{1-\epsilon}}{1-\epsilon} \tag{8}$$

and ϵ is the relative risk aversion parameter ($\epsilon>0$ and $\epsilon\neq 1$) .

3.2 Adaptation without an Open-Access Resource

We first consider adaptation without the use of an open-access resource. This case does not exhibit any market failure and thus does not involve policy mechanisms by itself, but it still provides some useful insights and serves as a benchmark for later discussions with a market failure.

The farmers spend all their earnings for consumption. The instantaneous utilities for the farmers producing 1 and 2, U_1 and U_2 , are thus given by

$$U_1\left(X_1\right)=G\left(X_1\right) \tag{9}$$

$$U_2\left(X_2\right)=G\left(PX_2\right) \tag{10}$$

The instantaneous private net gain for a farmer from switching from the production of 1 to 2 is

$$\Delta U\left(X_1,X_2\right)=U_2\left(X_2\right)-U_1\left(X_1\right)=\frac{\left(PX_2\right)^{1-\epsilon}-\left(X_1\right)^{1-\epsilon}}{1-\epsilon} \tag{11}$$

The time-discounted private net gain for switching from 2 to 1 is thus given by

$$v\left(X_1,X_2\right)=E\int_0^{\infty}\Delta U\left(X_1,X_2\right)\, e^{-\rho t}\, dt \tag{12}$$

Solving the integral and taking expectation leads to

$$v\left(X_1,X_2\right)=\frac{1}{1-\epsilon}\left[\frac{\left(PX_2\right)^{1-\epsilon}}{\rho-\left(1-\epsilon\right)\mu_2+\frac{1}{2}\epsilon\left(1-\epsilon\right)\sigma_2^2}-\frac{\left(X_1\right)^{1-\epsilon}}{\rho-\left(1-\epsilon\right)\mu_1+\frac{1}{2}\epsilon\left(1-\epsilon\right)\sigma_1^2}\right] \tag{13}$$

Farmers switch production if those time-discounted gains exceed the sum of the initial fixed cost and the option value. The option value of moving to production 2, f, is a function of X_1 and X_2 and satisfies

$$\rho f\left(X_1,X_2\right)dt=E\left[df\left(X_1,X_2\right)\right]$$

By using Ito's Lemma, we obtain the following differential equation for f

$$\frac{1}{2}\sigma_1^2X_1^2f_{X_1X_1}+\frac{1}{2}\sigma_2^2\left(PX_2\right)^2f_{X_2X_2}+\theta\,\sigma_1\sigma_2X_1X_2\,f_{X_1X_2}+\mu_1X_1f_{X_1}+\mu_2X_2f_{X_2}-\rho f=0 \tag{14}$$

Boundary conditions (the value-matching and smooth-pasting conditions) give closed-form solutions of the above. To obtain solutions, it is necessary to treat the level of one of the stochastic variables (X_1 or X_2) as given. Here, we treat X_2 as given and derive a threshold X_1^* above which $f>v-I$, that is, it is favorable not to switch production.

The value matching condition is

$$f\mid_{X_1=X_1^*}=v\mid_{X_1=X_1^*}-I \tag{15}$$

The smooth pasting conditions are

$$f_{X1}\mid_{X_1=X_1^*}=v_{X1}\mid_{X_1=X_1^*} \tag{16}$$
$$f_{X2}\mid_{X_1=X_1^*}=v_{X2}\mid_{X_1=X_1^*} \tag{17}$$

With those equation and conditions, the solution of f should take the following form

$$f=A\,X_1^{\beta_1}\left(PX_2\right)^{\beta_2} \tag{18}$$

A, β_1, β_2 and X_1^* are the solutions of the following set of equations:

$$\beta_2=\left(\beta_1-1+\epsilon\right)\frac{\left(PX_2\right)^{1-\epsilon}}{\left(1-\epsilon\right)D_2I-\left(PX_2\right)^{1-\epsilon}} \tag{19}$$

$$\frac{1}{2}\sigma_1^2\beta_1\left(\beta_1-1\right)+\frac{1}{2}\sigma_2^2\beta_2\left(\beta_2-1\right)+\theta\sigma_1\sigma_2\beta_1\beta_2+\mu_1\beta_1+\mu_2\beta_2-\rho=0 \tag{20}$$

$$A=\left(\frac{-\beta_2D_2}{\beta_1D_1}\right)^{\frac{\beta_1}{1-\epsilon}}\frac{1}{D_2\beta_2}\left(PX_2\right)^{1-\epsilon-\beta_1-\beta_2} \tag{21}$$

$$X_1^* = \left(\frac{-\beta_2 D_2}{\beta_1 D_1} \right)^{\frac{1}{-1+\epsilon}} P X_2 \tag{22}$$

where D_1 and D_2 are

$$D_1 = \rho - (1-\epsilon)\,\mu_1 + \frac{1}{2}\epsilon\,(1-\epsilon)\,\sigma_1^2 \tag{23}$$

$$D_2 = \rho - (1-\epsilon)\,\mu_2 + \frac{1}{2}\epsilon\,(1-\epsilon)\,\sigma_2^2 \tag{24}$$

As for β_1, two values, one positive and the other negative, can satisfy the above conditions (obtained as solutions of a quadratic equation). However, only the negative solution of β_1 has an economic meaning since the option value f should become negligible as X_1 becomes large (if X_1 is very large, production switching will never be necessary, and the option to switch to production 2 is valueless). If the corresponding A and X_1^* for the negative β_1 are positive, this means that there is a range of X_1 in which $f > v - I$, in other words, it is beneficial not to exercise switching (hold the option) even if the expected net return from switching is positive. This leads to the following proposition.

Proposition 1. *If a combination of a negative β_1, a positive A and an X_1^* satisfying the above* (18) – (22) *exists, the farmers engaging in 1 do not switch to 2 until X_1 becomes as low as X_1^*, even if a switch to 2 brings a positive expected gain $v - I > 0$.*

The above solutions represent only the individual decisions by farmers and not the social optimum. The social optimum is found by replacing the time-discounted private gain v with the time-discounted social gain from a farmer's switching.

The time-discounted social gain is obtained as follows. First, by using U_1 and U_2, the instantaneous social welfare is expressed as:

$$B\,(L, X_1, X_2) = \left(\bar{L} - L\right)\, U_1\,(X_1) + L\, U_2\,(X_2) \tag{25}$$

The partial derivative of B with regard to L (B_L) gives the instantaneous marginal social benefit of switching by a farmer to production 2.

$$B_L = U_2\,(X_2) - U_1\,(X_1) + \left\{ L\,\frac{\partial U_2}{\partial L} + \left(\bar{L} - L\right)\,\frac{\partial U_1}{\partial L} \right\} \tag{26}$$

The time-discounted social gain is obtained by replacing B_L for ΔU in (12). The social optimum is calculated by the same procedure as in the case of private solutions. In other words, socially optimal switching of a farmer takes place only when $E \int_0^\infty B_L\, e^{-\rho t}\, dt - I$ is greater than or equal to the option value.[3]

[3]Note that this general form of B_L implies that the return to a switch generally depends on L, and in

In the present case, as U_1 and U_2 are independent of L, i.e., $\frac{\partial U_2}{\partial L} = \frac{\partial U_1}{\partial L} = 0$. Hence, B_L is simply given by:

$$B_L = U_2(X_2) - U_1(X_1) \tag{27}$$

Thus $B_L = \Delta U$. This is paraphrased as in the following Proposition.

Proposition 2. *The solutions for private decisions represented by* (18) – (22) *are identical to the social optimum.*

As the effects of parameters are mostly ambiguous in sign, numerical examples are useful for illustrating the model solutions. Figure 1 shows the relationship between f, $v-I$ and X_1^*. Farmers gain a net positive expected return from switching when $v-I > 0$. In other words, when X_1 reaches as low as X_1^0 on the graph, a switching from 1 to 2 is already beneficial for farmers. However, the option value (the value of waiting) f is greater than $v - I$ at $X_1 = X_1^0$, and it means that a decision to switch later brings a larger expected return than a switching at $X_1 = X_1^0$. Accordingly, it is favorable for the farmers to switch only when X_1 reaches X_1^* on the graph, whose level is determined by equation (22).

Figure 2 shows a numerical example of threshold productivity of 1, X_1^*, for two different levels of risk aversion, $\epsilon = 0.5$ and $\epsilon = 2$. The other parameters are set as follows: $\rho = 0.1$; $P = 1$; $\mu_1 = -0.01$; $\sigma_1 = 0.15$; $\mu_2 = \sigma_2 = 0$; $I = 10$. Note that these settings imply that the productivity of 2 is deterministic and has no baseline increase or decrease, i.e., 2 is fully "climate-proof." The dashed curves describe the levels of X_1 below which the net expected benefit of switching to 2 exceeds that of remaining in 1 (i.e., $v - I = 0$ is satisfied on the curves), whereas the thick curves represent the loci of X_1^*, which is the threshold where the farmers actually have an incentive to switch even in taking account of the option value f (the value of switching not immediately but in the future). The solid curve for $\epsilon = 0.5$ signifies the $v - I = 0$ locus when $\sigma_1 = 0$ (i.e., adaptation is fully deterministic). The switching is justified in the areas below the thick curves, in accordance with the intuition that a lower productivity of 1 promotes the switching away from 1. The figure exhibits general increasing patterns of the curves with X_2, which conforms to intuition (a high productivity of 2 makes a switching attractive). Furthermore, significant differences between the dashed and thick curves. This means that under fluctuations of climatic patterns, the farmers might abstain from adaptation investments even well after the time point when the production of 1 becomes unfavorable

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this sense, whether a switch today is socially optimal depends on how L evolves in the future. However, it is possible that the farmers today or the social planner today may not know how the farmers in the future decide to switch (i.e., whether they are rational or not), and this feature could add another dimension to this problem. While this issue highlights an intriguing question of strategic interactions among the farmers both at present and in the future regarding adaptation decisions, in this paper we limit our scope to the cases where the farmers or the social planner evaluate a switching decision by a farmer as if all the others remain in the same production mode throughout the entire future. See Dixit and Rob (1994) for a similar discussion of switching decisions between two economic sectors.

enough to bring the net expected benefit for switching from 1 to 2. This characteristic essentially originates from the fact that by delaying adaptation, the farmers might be able to capture relatively good future harvests of 1 in case that future climatic conditions (despite the general trend of unfavorable climate change) turn out to be favorable for production of 1. Put differently, farmers under uncertainty adapt later (start switching with a lower X_1) than under deterministic climate, as seen in the difference between the solid and thick curves for $\epsilon = 0.5$.

An interesting pattern observed in Figure 2 is the effect of risk aversion. Overall, risk aversion significantly alters the locations of the thick and dashed curves. For strongly risk-averse farmers, a high productivity of 2 adds only a relatively weak incentive to switch, as the utility increase due to an income gain from switching is relatively small for those farmers. Hence in the range of high X_2's, the curves for $\epsilon = 0.5$ are located above those for $\epsilon = 2$. Also, corresponding to the characteristic that the switching takes place relatively swiftly with a low risk aversion, the divergence between the dashed and thick curves is relatively wide with a low risk aversion, i.e., the option value has a great meaning with a low risk aversion. Meanwhile, the same figure also shows that strong risk aversion favors the switching in the area of low X_2's, reflecting the fact that volatility of production is lower for 2 than for 1 in this example, i.e., production of 1 is riskier than that of 2 and thus deters risk-averse farmers.

The example in Figure 2 includes stochasticity only in production 1, but the model can describe the effects of stochasticity of both production modes and also of the correlation between the two. Figure 3 shows an example of patterns of X_1^* as a function of the correlation of two stochastic parameters, θ. The graph shows results for $\epsilon = 0.5$ and $\epsilon = 2$, the dashed, thick, and solid curves represent $\sigma_2 = 0$, 0.05, 0.15, respectively. The other parameters are set as follows: $\rho = 0.1$; $P = 1$; $\mu_1 = -0.01$; $\sigma_1 = 0.15$; $\mu_2 = 0$; $I = 10$. The graph shows that X_1^* generally increases with θ, and a greater value of stochasticity σ_2 makes the curve steeper. As the $v - I = 0$ locus does not change with θ (note that (13) does not include θ in the formulation), a high correlation between productivity levels of the two production modes thus weakens the significance of the option value, in other words, two types of stochasticity cancel off each other when they are correlated.

3.3 Adaptation with an Open-Access Resource

The above sub-section 3.2 shows that climate variability could slow down adaptation to the long-run climate change. But in a perfectly competitive economy, the delay is still socially optimal. With market imperfections, however, the private decisions and the social optimum diverge, and this divergence has policy implications. In this section, we examine the important case that adaptation involves the use of an open-access resource (water). This case is consistent with the prevalent characteristic of agriculture that the irrigated farming is better able to insulate itself from weather shocks than the rainfed farming is (e.g., (Mendelsohn and Dinar, 2003; Schlenker et al., 2005)) – in other words, the adoption of irrigation, which is to use local water resources, is a possible adaptation measure to climate change. Without coordination, the use of an open-access resource leads to socially suboptimal outcomes (overuse). That is why private adaptation and socially optimal adaptation diverge and therefore the necessity of public interventions – which would take a form of standard instruments to regulate a common resource, such as taxes or quantity restrictions – arises. Here we study the implications of open-access resource use in one production process on individually and socially optimal adaptation.

Below, we discuss sub-optimality of adaptation in focusing on the difference between the private net gain for a farmer from switching the production mode, ΔU, and the social net benefit of switching production B_L. As already discussed in sub-section 3.2, the levels of ΔU and B_L determine the value of switching immediately (v) and the option value (f), and this in turn means that if $\Delta U > B_L$, there is a range of productivity levels where adaptation (switching the production mode) proceeds even if it is not socially optimal. Inefficient adaptation may come in two forms: Individual farmers may adapt to early (over-adaptation) or to late (under-adaptation). Over-adaptation prevails if $\Delta U > B_L$; under-adaptation if $\Delta U < B_L$.

We find that both cases (over- and under-adaptation) are possible, as the order of ΔU and B_L depends on the parameter values. This is because there are two counteracting factors: On the one hand, uncoordinated use of the common-pool resource reduces the farmers' individual benefits of resource use. This factors reduces the individual benefits of adaptation, hence farmers tend to under-adapt. On the other hand, with uncoordinated use of the common-pool resource, farmers do not care about the welfare effects on other farmers when switching production mode and enter the user pool of resource. Hence, farmers tend to over-adapt with uncoordinated resource use. Evidently, if uncoordinated private decisions and the socially-optimal decisions do not match, collective actions such as policy interventions to induce socially-optimal switching may increase the social welfare and thus efficient adaptation. As the analysis in the previous subsection has shown, there is no inherent market failure associated with adaptation itself. Hence, the first-best policy would be to coordinate resource use.

With uncoordinated, open-access extraction of the natural resource, farmers choose extraction levels q^{OA} such that they maximize their individual profits, taking extraction rates of all other farmers as given. The social optimum, by contrast, is found by choosing coordinated extraction levels q^{CE} for all farmers such as to maximize collective output. The respective extraction rates and income levels are specified in the following lemma

Lemma 1. *Extraction rates are*

$$q^{OA} = \left[\frac{\alpha\, X_{21}^{1-\alpha}}{c_0 L^{\eta-1} X_{22}}\right]^{\frac{1}{\eta-\alpha}} \qquad \textit{under open access, and} \tag{28}$$

$$q^{CE} = \eta^{-\frac{1}{\eta-\alpha}} \left[\frac{\alpha\, X_{21}^{1-\alpha}}{c_0 L^{\eta-1} X_{22}}\right]^{\frac{1}{\eta-\alpha}} \qquad \textit{under coordinated extraction.} \tag{29}$$

Income levels are

$$y_2^{OA} = (1-\alpha) \left[\frac{\alpha}{c_0}\right]^{\frac{\alpha}{\eta-\alpha}} L^{-\frac{(\eta-1)\,\alpha}{\eta-\alpha}} X_2 \qquad \textit{under open access, and} \tag{30}$$

$$y_2^{CE} = (\eta-\alpha)\,\eta^{-\frac{\eta}{\eta-\alpha}} \left[\frac{\alpha}{c_0}\right]^{\frac{\alpha}{\eta-\alpha}} L^{-\frac{(\eta-1)\,\alpha}{\eta-\alpha}} X_2 \qquad \textit{under coordinated extraction,} \tag{31}$$

where we define

$$X_2 \equiv X_{21}^{\frac{\eta\,(1-\alpha)}{\eta-\alpha}} X_{22}^{-\frac{\alpha}{\eta-\alpha}}. \tag{32}$$

Proof: see Appendix.

If X_{21} and X_{22} follow geometric Brownian motions, the variable X_2 defined in (32) also follows a geometric Brownian motion, with drift $\mu_2 = \frac{\eta\,(1-\alpha)}{\eta-\alpha}\,\mu_{21} - \frac{\alpha}{\eta-\alpha}\,\mu_{22}$ and volatility $\sigma_2 = \frac{\eta\,(1-\alpha)}{\eta-\alpha}\,\sigma_{21} - \frac{\alpha}{\eta-\alpha}\,\sigma_{22}$.

It is easy to see $\lim\limits_{\alpha\to 1} y_2^{OA} = X_{21}$ (by using $\lim\limits_{x\to +0} x^x = 1$). So with $\alpha \to 0$ (the open-access resource loses its significance for production of 2), the model is reduced to the case studied in the previous subsection.

Next, open-access extraction levels q^{OA} are higher than coordinated extraction levels q^{CE} by a factor $\eta^{1/(\eta-\alpha)} > 1$, which reflects the over-use of the resource under conditions of open access. Because of the over-use, incomes under open access are lower, $y_2^{OA} < y_2^{CE}$, as the first factor in (31) is larger than the corresponding factor in (30):[4]

$$(\eta-\alpha)\,\eta^{-\frac{\eta}{\eta-\alpha}} > 1-\alpha. \tag{33}$$

Note that again, if the resource plays no role for the production mode 2, $\alpha \to 0$, incomes are the same under open access and coordinated extraction.

In the following we shall analyze the question of how the problem of overusing the resource affects the farmer's decision on adaptation to climate change, i.e. the decision which production process to use.

Solutions for private decisions are obtained by following the same procedures as in the previous subsection 3.2. The private instantaneous benefit of adaptation is given by

$$\Delta U\,(L, X_1, X_2) = U_2^{OA}\,(L, X_2) - U_1\,(X_1) \tag{34}$$

[4]This inequality is easily proven as follows: It is straightforward to verify that the left-hand side of the inequality is decreasing and convex in α, while the right-hand side is linear downward sloping in α. For $\alpha = 0$, both sides are equal to one. As for $\alpha = 0$ the slope of the left-hand side is equal to $-(1 + \ln(\eta))/\eta > -1$, and as the left-hand side is convex in α, the strict inequality holds for all $0 < \alpha < 1$.

Here, U_1 is the same as in the model without the open-access resource, and the utility derived from production mode 2, U_2^{OA}, is given by $G\left(y_2^{OA}\right)$.[5]

Meanwhile, the social optimum needs a slightly different formulation from that of the previous case in 3.2. Now, U_2^{CE} is dependent on L, and this means that (26) becomes:

$$B_L\left(L, X_1, X_2\right) = U_2^{CE}\left(L, X_2\right) - U_1\left(X_1\right) + L\,\frac{\partial U_2^{CE}\left(L, X_2\right)}{\partial L} \tag{35}$$

$$= U_2^{CE}\left(L, X_2\right) - U_1\left(X_1\right) - (1-\epsilon)\,\frac{(\eta-1)\,\alpha}{\eta-\alpha}\,U_2^{CE}, \tag{36}$$

where $U_2^{CE} = G(y_2^{CE}) > U_2^{OA}$ (see Lemma 1). Note that these utility levels are positive for $\epsilon < 1$ and negative for $\epsilon > 1$ (cf. equation 8).

The social net benefit of switching production B_L may be lower (or higher) than the private net gain for a farmer from switching the production mode, ΔU. It is lower, i.e. $B_L\left(L, X_1, X_2\right) < \Delta U\left(L, X_1, X_2\right)$, if and only if

$$\Omega\, U_2^{CE} < 0 \qquad \text{with} \tag{37}$$

$$\Omega \equiv 1 - \left(\frac{(1-\alpha)\,\eta^{\frac{\eta}{\eta-\alpha}}}{\eta-\alpha}\right)^{1-\epsilon} - (1-\epsilon)\,\frac{(\eta-1)\,\alpha}{\eta-\alpha} \tag{38}$$

Thus, the question whether social net benefit of switching production is lower or higher than the private net gain can be reduced to the question what is the sign of Ω. For $\epsilon < 1$, $B_L < \Delta U$ if and only if $\Omega < 0$ (as $U_2^{CE} > 0$), while for $\epsilon > 1$, $B_L < \Delta U$ if and only if $\Omega > 0$ (as $U_2^{CE} < 0$).

The parameter cluster Ω captures the above-mentioned two counteracting factors. The first factor, that uncoordinated use of the common-pool resource reduces the farmers' individual benefits of resource use, which causes the tendency to under-adapt, is captured by the first two terms in (38). It follows from (33) that the expression in brackets in the second term is smaller than one, which reflects the result from Lemma 1 that income from production mode 2 is lower under open-access extraction of the resource. Taken together, the first two terms in (38) are positive for $\epsilon < 1$ and negative for $\epsilon > 1$: These two terms reduce the private gain from switching the production process relative to the social gain.

The second factor, that farmers under open access do not care about the welfare effects on other farmers when switching production mode and entering the the user-pool of the resource, which causes the tendency to over-adapt, is captured by the last term in (38). As $\eta > 1$, this term (including the minus-sign in front of it) is negative for $\epsilon < 1$ and positive for $\epsilon > 1$. These two terms increase the private gain from switching the production process relative to the social gain. Which of these two factors dominates

[5] Here, similar to the discussion of B_L in the previous sub-section, we rule out the possibilities that farmers act strategically to increase their payoffs at the expense of others.

depends on the three parameters α, ϵ and η. In the following analysis we will focus on the question how the difference between individual and social instantaneous benefits of switching to production 2 depends on the parameter α of the revenue function for production 2 and on the farmers degree of risk aversion ϵ.

The first important result is that for low values of α the individual benefits of changing production (under open access conditions) exceed the social benefits (under coordinated resource use), while for high values of α the social benefits are higher.

Proposition 3. *There exists a value $\bar{\alpha}$ for the output elasticity of the resource in production mode 2 such that*

- *for all $\alpha < \bar{\alpha}$ private instantaneous gains from adaptation exceed social gains, $\Delta U(L, X_1, X_2) > B_L(L, X_1, X_2)$.*
- *for all $\alpha > \bar{\alpha}$ private instantaneous gains from adaptation are lower than social gains, $\Delta U(L, X_1, X_2) < B_L(L, X_1, X_2)$.*

Proof: see Appendix.

In the proof we show that for $\alpha < \bar{\alpha}$, the parameter cluster Ω is negative if $\epsilon < 1$ and positive for $\epsilon > 1$.

Thus, if the open-access resource is not very important for generating outputs in production 2 (i.e. $\alpha < \bar{\alpha}$), the individual incentives to switch to production 2 are too high, as farmers do not take into account the negative effect of adaptation on those farmers who are already producing 2. Given the discussion of 3.1, this means that some farmers may switch productions too early (over-adapt) when the social planner would still favor waiting. This conforms to the intuition that we tend to overuse a limited resource if the use of the resource is not essential for our life.[6]

If, by contrast, the open-access resource is important ($\alpha > \bar{\alpha}$), the problem of overuse associated with the resource decreases outputs in production 2 so much that this reduces the individual benefit of adapting below the social benefit. If the resource is important we would thus expect the opposite pattern to the above: Some farmers may delay too long in adaptation (under-adapt).

The critical value of α, where the individual incentives under open access conditions switch from being inefficiently high to being inefficiently low depends on the farmers degree of risk aversion, as stated in the following proposition.

Proposition 4. *The threshold value $\bar{\alpha}$ for the output elasticity of the resource in production mode 2 below which farmers over-adapt to climate change (above which they under-adapt) decreases with risk-aversion ϵ, i.e.*

$$\frac{d\bar{\alpha}}{d\epsilon} < 0.$$

[6] Outside of the context of climate change adaptation, it is easy to find such cases, e.g., the extinction of the Passenger Pigeon in North America due to excessive hunting.

Proof: see Appendix.

Thus, the degree of risk aversion determines whether the the individual incentives are inefficiently high to being inefficiently low.

The results stated formally in Propositions 3 and 4 are illustrated by a numerical example in Figure 4. The threshold values, i.e. the values for α for which $\Omega = 0$, are $\bar{\alpha} = 0.94$ for $\epsilon = 0.2$, $\bar{\alpha} = 0.88$ for $\epsilon = 0.5$, and $\bar{\alpha} = 0.73$ for $\epsilon = 2$.

Considering the dynamic problem, the value of the option to switch the production process is given by (18), with value matching and smooth pasting conditions as above.

The time-discounted private net gain for switching from 1 to 2 is thus given by

$$v^{OA}(X_1, X_2, L) = \frac{1}{1-\epsilon}\left[\left[(1-\alpha)\left[\frac{\alpha}{c_0}\right]^{\frac{\alpha}{\eta-\alpha}} L^{-\frac{(\eta-1)\,\alpha}{\eta-\alpha}}\right]^{1-\epsilon} \frac{X_2^{1-\epsilon}}{D_1} - \frac{X_1^{1-\epsilon}}{D_1}\right] \tag{39}$$

with D_1 and D_2 given by (23) and (24); and the time-discounted social net gain for switching from 1 to 2 is given by

$$v^{CE}(X_1, X_2, L) = \frac{1}{1-\epsilon}\left[\left[(\eta-\alpha)\,\eta^{-\frac{\eta}{\eta-\alpha}}\left[\frac{\alpha}{c_0}\right]^{\frac{\alpha}{\eta-\alpha}} L^{-\frac{(\eta-1)\,\alpha}{\eta-\alpha}}\right]^{1-\epsilon} \times \right.$$
$$\left. \times\left[1-(1-\epsilon)\frac{(\eta-1)\,\alpha}{\eta-\alpha}\right]\frac{X_2^{1-\epsilon}}{D_1} - \frac{X_1^{1-\epsilon}}{D_1}\right]. \tag{40}$$

In the following Proposition, we focus on the case where $\mu_2 = 0$ and $\sigma_2 = 0$, i.e. production 2 is certain and not affected by climate change.

Proposition 5. *If $\mu_2 = 0$ and $\sigma_2 = 0$, the switch under open access conditions is at a lower (higher) level of X_1 than socially optimal if and only if $\alpha > \bar{\alpha}$ ($\alpha < \bar{\alpha}$).*

Proof: see Appendix.

This result is illustrated in Figures 5 and 6. In Figure 5 the threshold levels for X_1 below which it is individually and socially optimal to switch to production mode 2 are shown as functions of labor input in production mode 2 (top panels) and as functions of the productivity parameter X_2 of production mode 2 (bottom panels). We consider two different output elasticities of the resource in production mode 2: $\alpha = 0.5$ in the left panels and $\alpha = 0.95$ in the right panels. For all cases, $\alpha = 0.5$ is below the threshold value $\bar{\alpha}$ identified in Proposition 3, while $\alpha = 0.95$ is above this threshold. Accordingly, in the diagrams for $\alpha = 0.5$, the threshold levels for X_1 below which it is individually optimal to switch to production mode 2 are above those for which it is socially optimal to switch production, reflecting that private gains from adaptation exceed social gains. The converse holds for the panels on the right. The curves $X_1^\star(L)$ shown in the top panels are downward sloping and convex for both levels of risk aversion and both levels of α. The curves $X_1^\star(X_2)$ shown in the bottom panels are upward sloping and convex for the lower degree of risk aversion ($\epsilon = 0.5$) and concave for the higher degree of risk aversion ($\epsilon = 0.95$). In Figure 6 we keep L and X_2 constant and vary the output elasticity of the resource in production mode 2. Again, this illustrates the result of Proposition 5. For both degrees of risk aversion, the switch under open access conditions is already at a higher level of X_1 than socially optimal for low values of α and at a lower level of X_1 for high values of α, i.e. values of α above the threshold level $\bar{\alpha}$.

Those results lead to some nuanced policy implications with regard to public support for private adaptation to climate change. In a way, the above results are not different from those of a standard model of open-access resource use in that the introduction of coordination, in the form taxes or quotas (i.e. limiting resource use to the socially optimal level given by (29)), is a sufficient condition for obtaining the social optimum – in other words, additional public interventions to either promote or discourage adaptation are unnecessary. However, the farmers make an optimal switching only when they believe that coordination is achieved not only at present *but also in the entire future*. Under suspicion of effectiveness of resource management in the future, the farmers may still over- or under-adapt even if coordination is in fact to be provided. This is hardly an unrealistic possibility particularly in some developing countries where the governance system tends to be subject to frequent breakdowns. In such a circumstance, the government should provide explicit incentives to farmers either for more or less adaptation in addition to coordination of resource use.

Meanwhile, if the collective management of the resource is practically infeasible both at present and in the future (because of infeasibility of effective monitoring, etc.), evaluation of adaptation decisions need to be made entirely based on U_2^{OA} (i.e., without U_2^{CE}). In that case, the economy should seek a second-best solution where benefits from uncoordinated resource use are maximized, and this means that adaptation should be always discouraged relative to the level that individual farmers are privately inclined to, e.g., by imposing an entry tax (whose level is set at $(1-\epsilon)\,\frac{(\eta-1)\,\alpha}{\eta-\alpha}\,\int_0^\infty U_2^{CE}\,e^{-\rho t}\,dt$).

4 Concluding Remarks

As humans do not directly perceive a gradual change of climate but experience it primarily as shifting inter-annual fluctuations of weathers, climate variability should have a significant meaning for people's adaptation decisions to climate change. By using a real option framework, we have attempted a description of how climate variability could affect farmers' investment decisions with regard to climate change adaptation. A common characteristic across all cases of the model is that as a reflection of the option value, climate variability delays adaptation – without any market distortion, this delay is in fact socially optimal as well. As a plausible example in which this delay effect carries policy implications, we have discussed a case in which adaptation involves the use of a common pool resource (water). The results show complex dynamics that would justify nuanced potential policy interventions. For example, the model indicates that there exist cases where uncoordinated farmers with a high risk aversion may under-adapt although farmers with a low risk aversion would over-adapt under the same conditions. If farmers remain suspicious about viability of long-term coordination on resource use, public interventions to explicitly support or discourage private adaptation could improve the social welfare.

Climate change adaptation has hardly been seen as it is in this study, and this study's relatively simple model already shows significant complexity of decisions on climate change adaptation combined with continuous risk. Indeed, this paper discusses still only a fraction of the cases that the model is potentially able to examine. This analysis highlights the critical role of climate variability for the designing of climate change adaptation policy, and also, a scope for further research.

5 Appendix

Proof of Lemma 1

From 6, the marginal production costs with regard to q are given by:

$$\frac{\partial c\left(q,\tilde{q},L\right)}{\partial q}=c_0\left[\left\{q+\left(L-1\right)\tilde{q}\right\}^{\eta-1}+q\left(\eta-1\right)\left\{q+\left(L-1\right)\tilde{q}\right\}^{\eta-2}\right] \tag{41}$$

Since the farmers are identical, $\tilde{q}=q$ at equilibrium. Hence

$$\frac{\partial c}{\partial q}=c_0\left(L+\eta-1\right)L^{\eta-2}q^{\eta-1} \tag{42}$$

Profit-maximizing farmers equate (42) and the marginal revenue, in other words,

$$\alpha\, q^{\alpha-1}\, X_{21}^{1-\alpha}=\ c_0\left(L+\eta-1\right)L^{\eta-2}X_{22}q^{\eta-1}$$

Thus, q under uncoordinated extraction (q^{UE}) is

$$q^{UE}=\left[\frac{\alpha\, X_{21}^{1-\alpha}}{c_0\left(L+\eta-1\right)L^{\eta-2}X_{22}}\right]^{\frac{1}{\eta-\alpha}} \tag{43}$$

This represents q for a common pool resource. q under open access (q^{OA}) is obtained as a special case of the above with $L\gg 1$, which is

$$q^{OA}=\left[\frac{\alpha\, X_{21}^{1-\alpha}}{c_0 L^{\eta-1}X_{22}}\right]^{\frac{1}{\eta-\alpha}} \tag{44}$$

Thus, under open access, each of the identical individual farmer's income becomes

$$y_2^{OA} = (1-\alpha)\left[\frac{\alpha}{c_0}\right]^{\frac{\alpha}{\eta-\alpha}} L^{-\frac{(\eta-1)\,\alpha}{\eta-\alpha}} X_2 \tag{45}$$

Meanwhile, the efficient level of resource extraction is found as if the social planner maximizes the collective output. The social planner would not distinguish the farmers in maximizing collective benefits, i.e., $q = \tilde{q}$. Thus, c is simply described as:

$$c = c_0 L^{\eta-1} q^{\eta} \tag{46}$$

The efficient or coordinated extraction q (q^{CE}) equates the marginal revenue and cost, hence

$$\alpha\left(q^{CE}\right)^{\alpha-1} X_{21}^{1-\alpha} \;=\; \eta c_0 L^{\eta-1} X_{22}\left(q^{CE}\right)^{\eta-1} \tag{47}$$

$$\Leftrightarrow q^{CE} \;=\; \left[\frac{\alpha\, X_{21}^{1-\alpha}}{\eta c_0 L^{\eta-1} X_{22}}\right]^{\frac{1}{\eta-\alpha}} \tag{48}$$

Thus, if the use of resource is coordinated, each of the identical individual farmer's income is

$$y_2^{CE} = \left[\frac{\alpha\, X_{21}^{1-\alpha}}{\eta c_0 L^{\eta-1} X_{22}}\right]^{\frac{\alpha}{\eta-\alpha}} X_{21}^{1-\alpha} - c_0 L^{\eta-1}\left[\frac{\alpha\, X_{21}^{1-\alpha}}{\eta c_0 L^{\eta-1} X_{22}}\right]^{\frac{\eta}{\eta-\alpha}} X_{22} \tag{49}$$

$$= \frac{\eta-\alpha}{\eta}\left[\frac{\alpha}{\eta c_0 L^{\eta-1}}\right]^{\frac{\alpha}{\eta-\alpha}} X_{21}^{\frac{\eta\,(1-\alpha)}{\eta-\alpha}} X_{22}^{-\frac{\alpha}{\eta-\alpha}} \tag{50}$$

Hence, we obtained the formulations for q^{OA} , y_2^{OA} , q^{CE} and y_2^{CE}. QED

Proof of Proposition 3

For $\epsilon < 1$, we have $B_L < \Delta U$ if and only if $\Omega > 0$. For $\epsilon > 1$, we have $B_L < \Delta U$ if and only if $\Omega < 0$. We consider case $\epsilon < 1$ here. The proof for $\epsilon > 1$ is similar and omitted here. For $\alpha = 1$, we then have $\Omega = \epsilon$. For $\alpha = 0$, $\Omega = 0$, but Ω is decreasing with α:

$$\frac{d}{d\alpha}\left(1-\left(\frac{(1-\alpha)\,\eta^{\frac{\eta}{\eta-\alpha}}}{\eta-\alpha}\right)^{1-\epsilon} - (1-\epsilon)\,\frac{(\eta-1)\,\alpha}{\eta-\alpha}\right)\Bigg|_{\alpha=0}$$

$$= -\frac{1-\epsilon}{(\eta-\alpha)^2}\left[\left(\frac{(1-\alpha)\,\eta^{\frac{\eta}{\eta-\alpha}}}{\eta-\alpha}\right)^{1-\epsilon}\left[\frac{(\eta-1)\,(\eta-\alpha)}{1-\alpha} - \eta\,\ln(\eta)\right] - \eta\,(\eta-1)\right]\Bigg|_{\alpha=0}$$

$$= -\frac{(1-\epsilon)\,\ln(\eta)}{\eta} \tag{51}$$

In between, Ω has a minimum at some α', which is determined by

$$\left(\frac{(1-\alpha)\,\eta^{\frac{\eta}{\eta-\alpha}}}{\eta-\alpha}\right)^{1-\epsilon}\left[\frac{(\eta-1)\,(\eta-\alpha)}{1-\alpha}-\eta\,\ln(\eta)\right]=\eta\,(\eta-1) \tag{52}$$

This value α' is unique, as the LHS of this condition is monotonically increasing in α at any value of α solving (52):

$$\frac{d}{d\alpha}\left[\left(\frac{(1-\alpha)\,\eta^{\frac{\eta}{\eta-\alpha}}}{\eta-\alpha}\right)^{1-\epsilon}\left[\frac{(\eta-1)\,(\eta-\alpha)}{1-\alpha}-\eta\,\ln(\eta)\right]\right]\Bigg|_{\alpha=\alpha'}$$
$$=\left(\frac{(1-\alpha)\,\eta^{\frac{\eta}{\eta-\alpha}}}{\eta-\alpha}\right)^{1-\epsilon}\left[-(1-\epsilon)\left[\frac{\eta-1}{1-\alpha}-\frac{\eta\,\ln(\eta)}{\eta-\alpha}\right]^2+\left[\frac{\eta-1}{1-\alpha}\right]^2\right]>0$$

This holds because

$$\frac{\eta-1}{1-\alpha}>\frac{\eta\,\ln(\eta)}{\eta-\alpha}$$

for the value of α' solving (52).

Given these results, there must exist a threshold value $\bar{\alpha}$ such that $\Omega>0$ for all $\alpha>\bar{\alpha}$ and $\Omega<0$ for all $\alpha<\bar{\alpha}$. QED

Proof of Proposition 4

Differentiating Ω with respect to ϵ we obtain

$$\frac{\partial\Omega}{\partial\epsilon}=\ln\left(\frac{(1-\alpha)\,\eta^{\frac{\eta}{\eta-\alpha}}}{\eta-\alpha}\right)\left(\frac{(1-\alpha)\,\eta^{\frac{\eta}{\eta-\alpha}}}{\eta-\alpha}\right)^{1-\epsilon}+\frac{(\eta-1)\,\alpha}{\eta-\alpha}$$
$$\frac{\partial^2\Omega}{\partial\epsilon^2}=-\left(\ln\left(\frac{(1-\alpha)\,\eta^{\frac{\eta}{\eta-\alpha}}}{\eta-\alpha}\right)\right)^2\left(\frac{(1-\alpha)\,\eta^{\frac{\eta}{\eta-\alpha}}}{\eta-\alpha}\right)^{1-\epsilon}<0$$

which holds by condition (33).

Using the condition $\Omega=0$ for $\alpha=\bar{\alpha}$, we have

$$\frac{\partial\Omega}{\partial\epsilon}\bigg|_{\alpha=\bar{\alpha}}=\frac{1}{1-\epsilon}\,\ln\left(1-(1-\epsilon)\,\frac{(\eta-1)\,\alpha}{\eta-\alpha}\right)\left(1-(1-\epsilon)\,\frac{(\eta-1)\,\alpha}{\eta-\alpha}\right)+\frac{(\eta-1)\,\alpha}{\eta-\alpha}$$

which is positive $\epsilon<1$ and negative for $\epsilon>1$, because $(1-x)\,\ln(1-x)>-x$.

Now differentiating the condition $\Omega=0$ for $\bar{\alpha}$ with respect to ϵ, we obtain

$$\frac{\partial\Omega}{\partial\epsilon}+\frac{\partial\Omega}{\partial\alpha}\frac{d\alpha}{d\epsilon}=0$$

As shown in the proof of the previous proposition, $\partial\Omega/\partial\alpha<0$ at $\bar{\alpha}$ for $\epsilon<1$ and $\partial\Omega/\partial\alpha>0$ for $\epsilon>1$. Taken together, we have established the proposed result.

Derivation of thresholds

We define

$$\omega^{OA}(L) = \left((1-\alpha) \left[\frac{\alpha}{c_0} \right]^{\frac{\alpha}{\eta-\alpha}} L^{-\frac{(\eta-1)\,\alpha}{\eta-\alpha}} \right)^{1-\epsilon} \tag{53}$$

fortheopen-accesscaseand

$$\omega^{CE}(L) = \left((\eta-\alpha)\,\eta^{-\frac{\eta}{\eta-\alpha}} \left[\frac{\alpha}{c_0} \right]^{\frac{\alpha}{\eta-\alpha}} L^{-\frac{(\eta-1)\,\alpha}{\eta-\alpha}} \right)^{1-\epsilon} \left(1 - (1-\epsilon)\,\frac{(\eta-1)\,\alpha}{\eta-\alpha} \right) \tag{54}$$

for socially optimal extraction.

The social and individual adaptation problems differ only in whether to use ω^{OA} or ω^{CE}. We shall consider both problems at the same time, using the symbol $\omega \in \{\omega^{OA}, \omega^{CE}\}$.

$$\begin{aligned} A\,X_1^{\beta_1}\,X_2^{\beta_2} &= \frac{1}{1-\epsilon} \left[\frac{\omega\,X_2^{1-\epsilon}}{D_2} - \frac{(X_1)^{1-\epsilon}}{D_1} \right] - I \\ \beta_1\,A\,X_1^{\beta_1}\,X_2^{\beta_2} &= -\frac{X_1^{1-\epsilon}}{D_1} \\ \beta_2\,A\,X_1^{\beta_1}\,X_2^{\beta_2} &= \frac{\omega\,X_2^{1-\epsilon}}{D_2} \end{aligned}$$

$$\begin{aligned} && \frac{\omega\,X_2^{1-\epsilon}}{\beta_2\,D_2} &= \frac{1}{1-\epsilon} \left[\frac{\omega\,X_2^{1-\epsilon}}{D_2} + \frac{\beta_1}{\beta_2}\,\frac{\omega\,X_2^{1-\epsilon}}{D_2} \right] - I \\ \Leftrightarrow && 1-\epsilon &= \beta_2 + \beta_1 - \frac{(1-\epsilon)\,\beta_2\,D_2\,I}{\omega\,X_2^{1-\epsilon}} \\ \Leftrightarrow && \beta_2 &= \frac{1-\epsilon-\beta_1}{1-(1-\epsilon)\,\frac{D_2\,I}{\omega\,X_2^{1-\epsilon}}} \end{aligned}$$

$$\begin{aligned} X_1 &= \left(-\frac{\beta_1\,D_1}{\beta_2\,D_2}\,\omega \right)^{\frac{1}{1-\epsilon}} X_2 \\ A &= \frac{\omega\,X_2^{1-\epsilon}}{\beta_2\,D_2\,X_1^{\beta_1}\,X_2^{\beta_2}} = \frac{1}{\beta_2\,D_2} \left(-\frac{\beta_1\,D_1}{\beta_2\,D_2} \right)^{-\frac{\beta_1}{1-\epsilon}} \omega^{1-\frac{\beta_1}{1-\epsilon}}\,X_2^{1-\epsilon-\beta_1-\beta_2} \end{aligned}$$

Proof of Proposition 5

For $\mu_2 = 0$ and $\sigma_2 = 0$, equation (20) implies

$$\beta_1 = \frac{1}{2} - \frac{\mu_1}{\sigma_1^2} - \sqrt{2\,\frac{\rho}{\sigma_1^2} + \left(\frac{1}{2} - \frac{\mu_1}{\sigma_1^2}\right)^2} < 0$$

Thus, β_1 does not depend on ω.

Note that $\omega^{CE}(L) > \omega^{OA}(L)$ if and only if $\alpha > \bar{\alpha}$.

$$\frac{dX_1}{d\omega} = \frac{1}{(1-\epsilon)\,(1-\epsilon-\beta_1)} \left(-\frac{\beta_1\,D_1}{D_2}\right)^{\frac{1}{1-\epsilon}} \left(\frac{\omega - (1-\epsilon)\,\frac{D_2\,I}{X_2^{1-\epsilon}}}{1-\epsilon-\beta_1}\right)^{\frac{\epsilon}{1-\epsilon}} X_2 > 0$$

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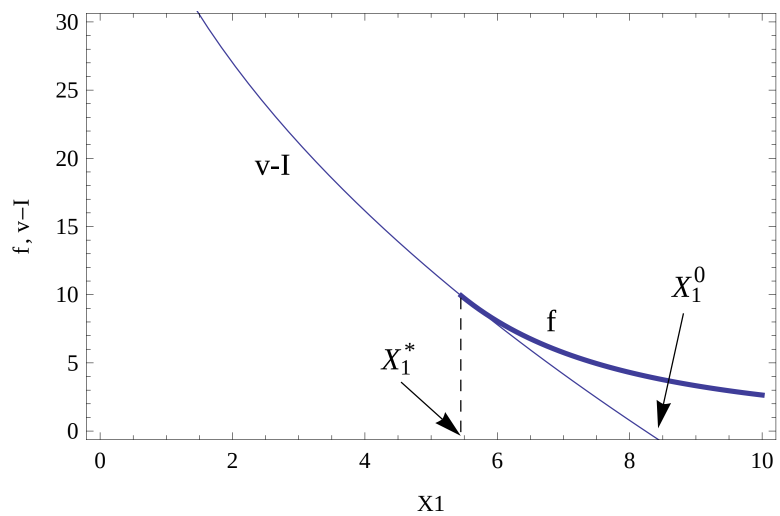

Figure 1: The relationship between f, $v-I$ and X_1^*. Parameter levels are set as follows: $\rho = 0.1,\ P = 1,\ \mu_1 = -0.01,\ \sigma_1 = 0.15,\ \mu_2 = \sigma_2 = 0,\ I = 10,\ X_2 = 10.$

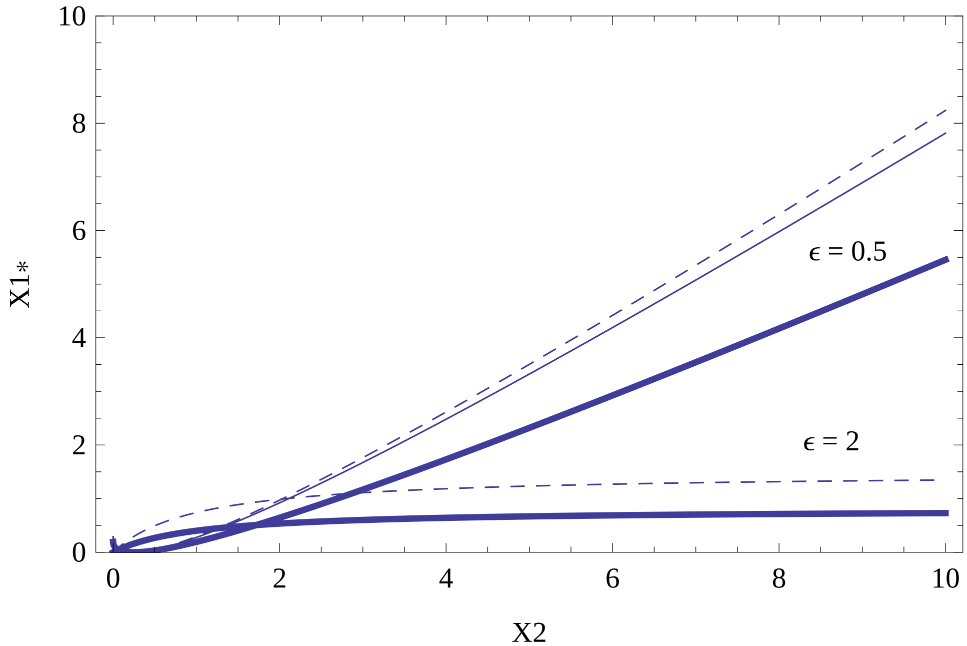

Figure 2: A numerical example of threshold productivity of 1, X_1^*, for two different levels of risk aversion, $\epsilon = 0.5$ and $\epsilon = 2$. The other parameters are set as follows: $\rho = 0.1$; $P = 1$; $\mu_1 = -0.01$; $\sigma_1 = 0.15$; $\mu_2 = \sigma_2 = 0$; $I = 10$. The dashed curves describe the levels of X_1 below which the net expected benefit of switching to 2 exceeds that of remaining in 1 (i.e., $v - I = 0$ is satisfied on the curves), whereas the thick curves represent the loci of X_1^*, which is the threshold where the farmers actually have an incentive to switch even in taking account of the option value f (the value of switching not immediately but in the future). The solid curve for $\epsilon = 0.5$ signifies the $v - I = 0$ locus when $\sigma_1 = 0$ (i.e., adaptation is fully deterministic).

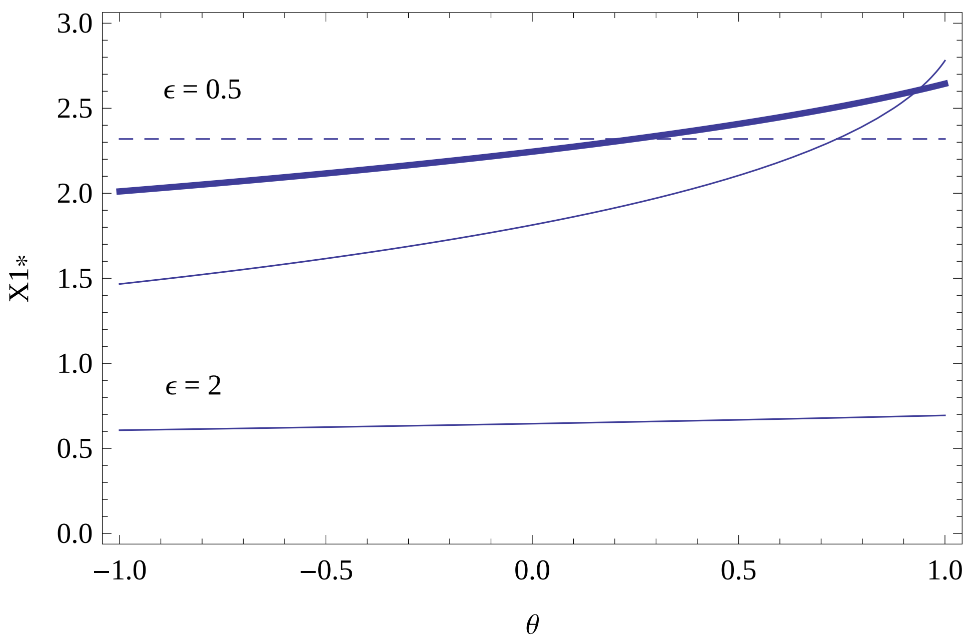

Figure 3: A numerical example of threshold productivity of 1, X_1^* as a function of the correlation of two stochastic parameters, θ . The graphs shows results for $\epsilon = 0.5$ and $\epsilon = 2$, the dashed, thick, and solid curves represent $\sigma_2 = 0$, 0.05, 0.15, respectively. The other parameters are set as follows: $\rho = 0.1$; $P = 1$; $\mu_1 = -0.01$; $\sigma_1 = 0.15$; $\mu_2 = 0$; $I = 10$.

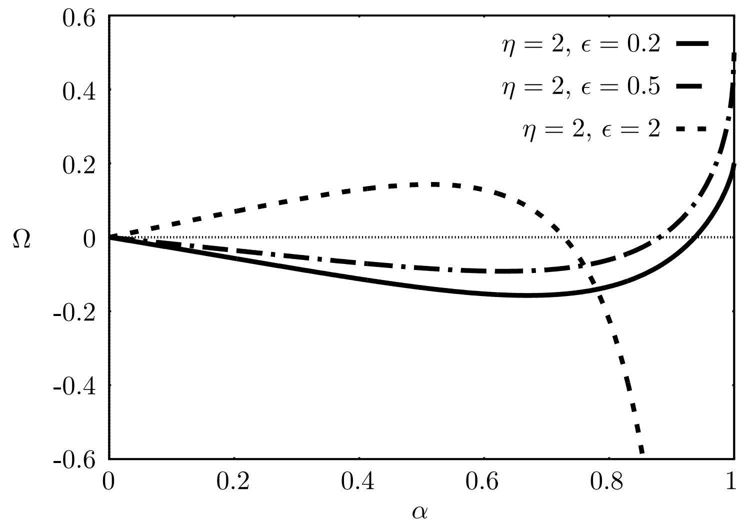

Figure 4: Numerical examples illustrating Propositions 3 and 4.

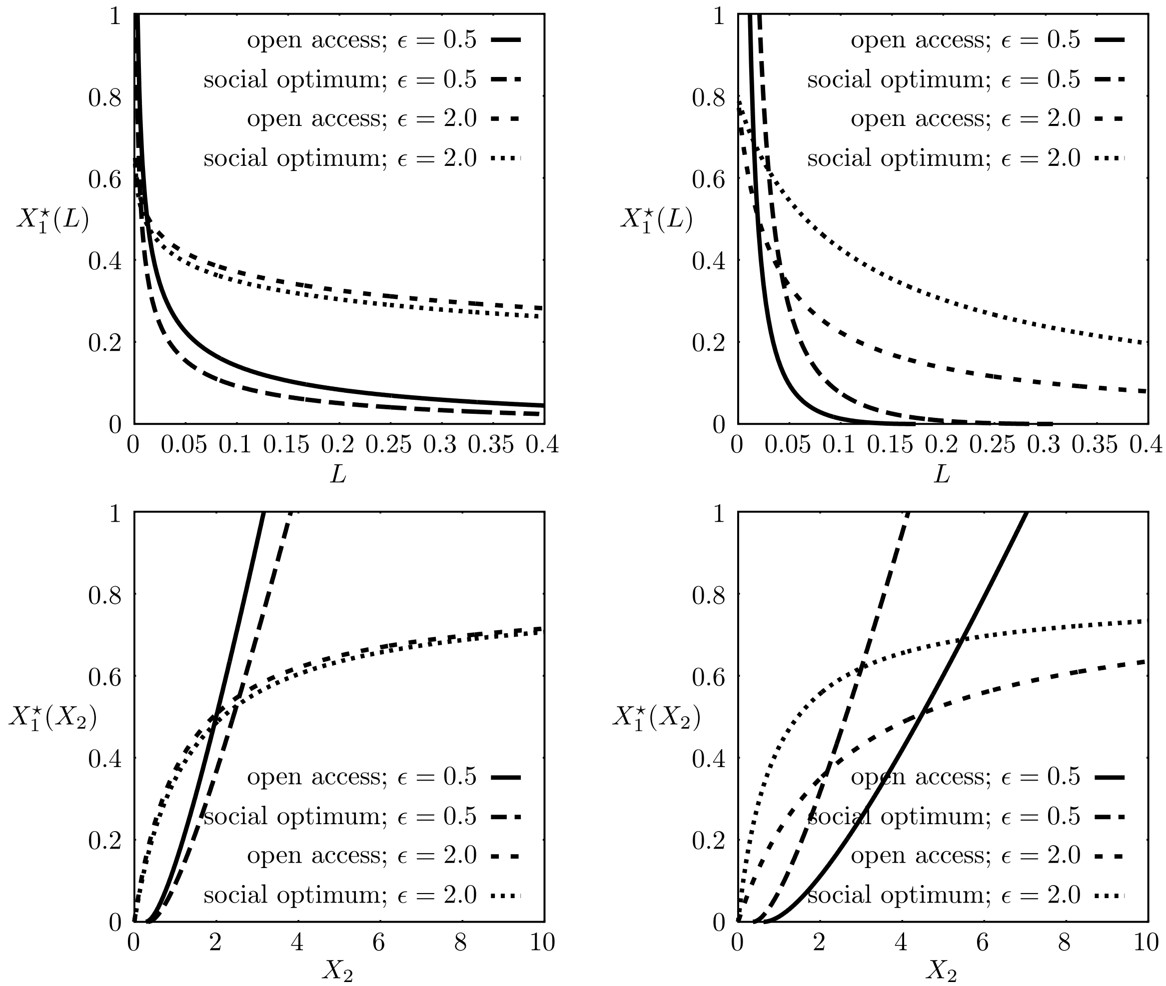

Figure 5: Numerical examples illustrating Proposition 5; using the parameter set as in Figure 1; $c_0 = 1$, $\eta = 2$; $\alpha = 0.5$ in the left panels, $\alpha = 0.95$ in the right panels. For the top panels we assume $X_2 = 1$; for the bottom panels we assume $L = 0.1$.

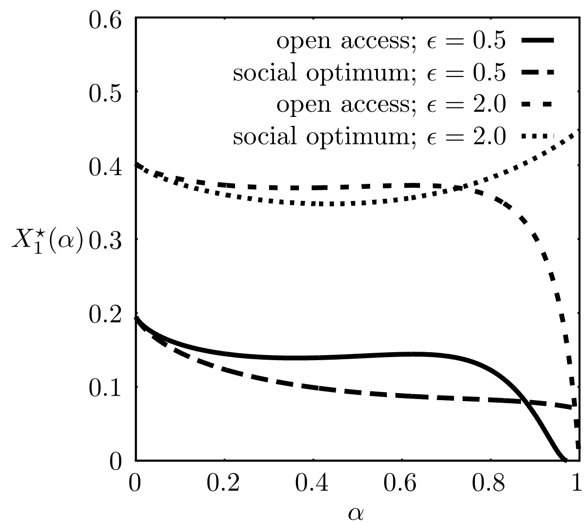

Figure 6: Numerical examples illustrating Proposition 5; using the parameter set as in Figure 1, varying α while keeping fixed $X_2 = 1$ and $L = 0.1$.