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Phillips curve in staggered wage
setting models

by Andrea Vaona

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Keywords: Phillips curve, inflation, nominal inertia, monetary policy, dynamic general equilibrium, money illusion, Stevens' ratio estimation function.

JEL classification: E3, E20, E40, E50.

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Money illusion and the long-run Phillips curve in staggered wage-setting models

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Abstract

We consider the effect of money illusion - defined referring to Stevens' ratio estimation function - on the long-run Phillips curve in an otherwise standard New Keynesian model of sticky wages. We show that if households under-perceive real economic variables, negative money non-superneutralities will become more severe. On the contrary, if households over-perceive real variables, positive money non-superneutralities will arise. We also provide a welfare analysis of our results and we show that they are robust to the inclusion of varying capital into the model. Firms' (over-)under-perception of the real prices of production inputs (strengthens) weakens negative money non-superneutralities. In an appendix, we investigate how money illusion affects the short-run effects of a monetary shock.

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1 Introduction

Money illusion has recently attracted renewed attention by the economics profession - see for instance the literature quoted in Fehr and Tyran (2001, 2007), which developed an experimental approach to the issue, or Cannon and Cipriani (2006), which assessed its empirical relevance. The aim of this paper is to study how it affects the long-run connection between output and inflation within a DSGE model, where, similarly to the relevant literature, we define the long-run as the steady-state, namely a condition characterized by the absence of temporary shocks².

The way we model money illusion here builds on Stevens (1946, 1951), which has been at the centre of an extensive literature surveyed for instance in Graham (1958), Anderson (1970), Schepard (1981), Luce and Krumhansl (1988) and Michell (1999).

We consider money illusion a biased subjective way economic agents have to evaluate real variables. Suppose an individual receives two pieces of information: his/her nominal wage, W , and the general level of prices, P . S/he will have to estimate her/his real wage as a ratio of the two above on the basis of her/his ratio estimation function $F = f(W, P)$. Stevens conjectured that subjective values are powers of real values, so that Stevens' ratio estimation function $f(W, P)$ can be written as:

²The analysis of the transitional dynamics of the system is tackled in the Appendix.

$$f(W, P) = \left(\frac{W}{P}\right)^\xi$$

Such a distortion could arise for at least two reasons. Economic agents are often found to evaluate economic magnitudes combining real and nominal assessments, given that they have a nominal anchor and, at the same time, they are aware that nominal and real values differ (Shafir et al., 1997). Furthermore, following Pelham et al. (1994), there might be a numerosity effect, whereby people sometimes judge quantity on the basis of the number of units into which a stimulus is divided without fully considering other important variables - on this point see, for instance, Wertenbroch et al. (2007) and the literature quoted therein³.

We nest Stevens' original idea in an otherwise standard New Keynesian model of sticky wages with trend inflation. We do so and we assume that agents do not have other behavioral/informational imperfections because only by observing money illusion in isolation from other factors one can properly assess its implications. For this very reason, it seems advisable to consider households' and firms' money illusion separately.

The effect of trend inflation on output in New-Keynesian models has been the subject of a number of studies by now. Pioneering contributions on this issue were King and Wolman (1996) and Ascari (1998). The former study considers a model with a shopping time technology and it obtains a num-

³Both these mechanisms tend to rule out the possibility for economic agents to *learn* the actual value of real variables.

ber of different results, among which there is that long-run inflation reduces firms' markup, boosting the level of output. Ascari (1998), instead, shows that in wage-staggering models money can have considerable negative non-superneutralities once not considering restrictively simple utility and production functions. Deveraux and Yetman (2002) focused on a menu cost model. An analysis of dynamic general equilibrium models under different contract schemes in presence of trend inflation was offered in Ascari (2004). Graham and Snower (2004), instead, examined the microeconomic mechanisms underlying this class of models. In presence of Taylor wage staggering, in a monopolistically competitive labour market, they highlight three channels through which inflation affects output: employment cycling, labour supply smoothing and time discounting. The first one consists in firms continuously shifting labour demand from one cohort to the other according to their real wage. Given that different labour kinds are imperfect substitutes, this generates inefficiencies and tends to create a negative inflation-output nexus. The second one is that households demand a higher wage in presence of employment cycling given that they would prefer a smoother working time, decreasing labor supply and aggregate output. Finally under time discounting the contract wage depends more on the current (lower) level of prices than on the future (higher) level of prices and, therefore - over the contract period - the real wage will be lower the greater is the inflation rate, spurring labour demand and aggregate output. The time discounting effect dominates at lower inflation rates, while the other two effects at higher inflation rates,

producing a hump-shaped long-run Phillips curve. The aim of this paper is to challenge the concept of the NAIRU and the possibility to identify demand and supply shocks assuming the former ones to be temporary and the latter ones to be permanent.

Graham and Snower (2004) was extended in a number of different directions. Graham and Snower (2008) showed that under hyperbolic time discounting positive money non-superneutralities are more sizeable than under exponential discounting. Vaona and Snower (2007, 2008) showed how the shape of the long-run Phillips curve depends on the shape of the production function. Finally, Vaona (2010) extended the model by Graham and Snower (2004) from the inflation-output domain to the inflation-real growth one.

The present contribution shows that the shape of the long-run Phillips curve changes under different degrees of money illusion. We do so by first assuming firms not to be subjected to money illusion as reminiscence of Friedman (1968). However, given that Shafir et al. (1997) found that even firms' decisions can be affected by money illusion, we deal with this case at a later stage.

The rest of this paper is structured as follows. Section 2 illustrates our baseline model. We then move to its calibration and solution. Section 4 sets out our results and their underlying intuition, also providing a welfare analysis of theirs. Section 5 extends the model by considering varying capital. Section 6 considers the case of firms' money illusion. The last section summarizes our findings and concludes. As mentioned above, in the appen-

dix we tackle the issue of how (either households' or firms') money illusion affects the impact of a temporary monetary shock on the model, given that, even if demand shocks cannot be identified on the basis of their transience, it will be interesting to study how transient demand shocks affect an economy.

2 The baseline model

2.1 Firms' cost minimization problem and the government

The model here presented is inspired to those by Ascari (1998) and Graham and Snower (2004). Firms populating the final perfectly competitive product market produce an homogeneous output and they minimize their total real cost subject to their production function

$$\begin{aligned} \min_{n_t(h)} \int_{h=0}^1 \frac{W_t(h)}{P_t} n_t(h) dh \\ s.t. \ y_t = n_t = \left[\int_0^1 n_t(h)^{\frac{\theta_n-1}{\theta_n}} dh \right]^{\frac{\theta_n}{\theta_n-1}} \end{aligned}$$

where $W_t(h)$ and $n_t(h)$ are respectively the nominal wage and the working time of household h , y_t is output, θ_n is the elasticity of substitution among different labour types and P_t is the general level of prices, which, given that we assume perfect competition in the final product market, equals the price of the homogenous final product.

Taking first order conditions one can obtain the following equations

$$n_t(h) = \left[\frac{W_t(h)/P_t}{\lambda_n} \right]^{-\theta_n} n_t \quad (1)$$

$$\lambda_{n,t} = \left\{ \int_0^1 \left(\frac{W_t(h)}{P_t} \right)^{1-\theta_n} dh \right\}^{\frac{1}{1-\theta_n}} \quad (2)$$

where (1) is the demand for labour services of household h and (2) is an aggregate real wage index, whereby $\lambda_{n,t}$ - the Lagrangian multiplier - can be renamed W_t/P_t .

The government rebates its seigniorage proceeds to households by means of lump-sum transfers, $T_t(h)$. Therefore:

$$\int_0^1 \frac{T_t(h)}{P_t} dh = \int_0^1 \frac{M_t(h)}{P_t} dh - \int_0^1 \frac{M_{t-1}(h)}{P_t} dh$$

where $M_t(h)$ is money holdings of household h at time t .

2.2 Households' maximization problem

Households maximize their discounted expected utility

$$\max_{\{c_{t+i}(h), W_{t+N_i}(h), B_{t+i}(h), M_{t+i}(h)\}} \sum_{j=0}^{\infty} \sum_{i=jN}^{(j+1)N-1} E \left(\beta^i U \left\{ c_{t+i}(h), n_{t+i}(h), \left[\frac{M_{t+i}(h)}{P_{t+i}} \right]^\xi \right\} \right)$$

subject to a series of income constraints perceived as follows

$$\begin{aligned}
c_{t+i}(h) = & \left[\frac{W_{t+i}(h)}{P_{t+i}} \right]^\xi n_{t+i}(h) + \left[\frac{T_{t+i}(h)}{P_{t+i}} \right]^\xi - \left[\frac{M_{t+i}(h)}{P_{t+i}} \right]^\xi + \left[\frac{M_{t+i-1}(h)}{P_{t+i}} \right]^\xi - \\
& - \left[\frac{B_{t+i}(h)}{P_{t+i}} \right]^\xi + \left[\frac{B_{t+i-1}(h)}{P_{t+i}} \right]^\xi \iota_{t+i}
\end{aligned} \tag{3}$$

where β is the discount factor, E is the expectation operator, U is the utility function, c is consumption, B are private bond holdings and ι is the nominal interest rate. The presence of ξ descends from the fact that economic agents are not able to properly assess real variables. The actual real budget constraint can be obtained setting $\xi = 1$.

Further constraints derive from the demand for labour services (1). Note that, in this section, labour demand is not affected by money illusion, notwithstanding that it involves the computation of a ratio of two nominal variables. This is so for two reasons. First, by setting different wages households can, in principle, observe the actual amount of working time that firms demand, so it is not plausible to think that they have a misperception of labour demand. In other words, labour demand is not computed by households, but by firms, which in this section are not subjected to money illusion. Second, this is in accord with the very concept of money illusion. As explained by Shafir et al. (1997) and as briefly mentioned above, money illusion arises because agents have a nominal anchor, for example their colleagues' wages. It is therefore the correct awareness of wage differentials that produces a distorted perception of real variables. For these reasons, it seems appropriate not to consider

any misperception in $\frac{W_{t+i}(h)}{W_{t+i}}$ in the present section.

The first order condition with respect to $W_{t+N_i}(h)$ turns out to be particularly important for the solution of the model

$$\begin{aligned} & \sum_{i=0}^{N-1} E \left\{ \beta^i |U_n(\cdot)| (-1) (-\theta_n) \left[\frac{W_{t+i}(h)}{W_{t+i}} \right]^{-\theta_n-1} \frac{1}{W_{t+i}} y_{t+i} \right\} = \\ & \sum_{i=0}^{N-1} E \left\{ \lambda_{t+i}(h) \beta^i (\theta_n - \xi) \left(\frac{1}{P_{t+i}} \right)^\xi y_{t+i} \left(\frac{1}{W_{t+i}} \right)^{-\theta_n} W_{t+i}(h)^{(\xi-\theta_n-1)} \right\} \quad (4) \end{aligned}$$

where $U_n(\cdot)$ is the first derivative of $U(\cdot)$ with respect to $n_{t+i}(h)$.

Keeping in mind that $W_{t+i}(h)$ is constant through the contract period, one can substitute (2) into (4) to obtain

$$\sum_{i=0}^{N-1} E \left[\beta^i |U_n(\cdot)| (-1) (-\theta_n) n_{t+i}(h) \right] = \sum_{i=0}^{N-1} E \left\{ \lambda_{t+i}(h) \beta^i (\theta_n - \xi) \left[\frac{W_{t+i}(h)}{P_{t+i}} \right]^\xi n_{t+i}(h) \right\} \quad (5)$$

Given that households are symmetrical and have the same first order condition for consumption

$$U_c [c_{t+i}(h), \cdot, \cdot] = \lambda_{t+i}(h) \quad (6)$$

they will all consume the same quantity of the final good. So aggregating one has:

$$\int_0^1 c_t(h) dh = c_t \quad (7)$$

However, in each period, households belonging to different cohorts have different income levels. This implies that, following Ascari (1998), households exchange bonds to keep their consumption constant⁴. Therefore, considering that the total number of households is normalized to 1 and that there exist $\frac{1}{N}$ households in each cohort i , one has that

$$c_t = \sum_{i=0}^{N-1} \frac{W_t(i)}{P_t} \frac{n_t(i)}{N} \quad (8)$$

A first implication of equation (8) is that even though agents do not have a correct perception of their individual budget constraints, these hold anyway. This is thanks to the working of the market. Agents choose their preferred quantity of consumption according to (6) and they buy it spending $P_t c_t$. If they have some savings - because their nominal income exceeds their expenditure - they will use these savings to buy bonds. If, on the other hand, their nominal income is lower than their expenditure, they will sell bonds. All these operations do not involve ratios of nominal variables and so they are not affected by money illusion.

One further possible interpretation of equations (8) and (7) is that, in

⁴Under this respect and regarding equation (3), the present model departs from the one by Graham and Snower (2004), where no assumption is made regarding asset markets (see note 5 at p. 13, where it is also admitted that their model is not suitable for analysing off-steady state dynamics as we do in the Appendix). Consumption is thought, there, to be constant and equal to a fraction of the discounted labour income over the contract period (see equation 12). However, it is left unspecified, there, how, at the beginning of the contract period, households with a low real wage can afford the same level of consumption as households with a high real wage and what happens to the savings of households with a high real wage. Furthermore, the assumption of complete asset markets is customary in the new keyensian literature about nominal rigidities.

the present "right-to-manage" model, households set their wage on the basis of their distorted perceptions and firms set working hours on the basis of the wage set by households. The production process takes then place and, finally, households pool their income before consuming all the same quantity of output. This is why no distortion affects equation (8) and why, upon consuming, the binding budget constraint is (3) with $\xi = 1^5$.

A second implication of equation (8) is that (5) can be rewritten as

$$\sum_{i=0}^{N-1} E \left[\beta^i |U_n(\cdot)| \theta_n n_{t+i}(h) \right] = (\theta_n - \xi) N \sum_{i=0}^{N-1} E \left[\frac{\beta^i \left(\frac{W_{t+i}(h)}{P_{t+i}} \right)^\xi n_{t+i}(h)}{\sum_{i=0}^{N-1} \left(\frac{W_{t+i}(h)}{P_{t+i}} \right)^\xi n_{t+i}(h)} \right] \quad (9)$$

which implies that money illusion affects labour supply and, as explained below, the long-run Phillips curve.

3 Calibration and solution of the baseline model

We specify the utility function as follows

$$U(\cdot) = \log c_{t+i}(h) + \varsigma \frac{[1 - n_t(h)]^{1-\eta}}{1-\eta} + V \left\{ \left[\frac{M_{t+i}(h)}{P_{t+i}} \right]^\xi \right\} \quad (10)$$

⁵The author is indebted to Paolo Bertolotti for suggesting this further interpretation.

Suppose $\eta > 0$, (9) will become

$$\sum_{i=0}^{N-1} E \left\{ \beta^i [1 - n_{t+i}(h)]^{-\eta} \theta_n n_{t+i}(h) \varsigma \right\} = (\theta_n - \xi) N \sum_{i=0}^{N-1} E \left[\frac{\beta^i \left(\frac{W_{t+i}(h)}{P_{t+i}} \right)^\xi n_{t+i}(h)}{\sum_{i=0}^{N-1} \left(\frac{W_{t+i}(h)}{P_{t+i}} \right) n_{t+i}(h)} \right] \quad (11)$$

The equilibrium for this model is a sequence $\{c_t, W_t(h), \mu_t, n_t(h), n_t, y_t, \iota_t\}$ satisfying households' utility maximization and firms' profit maximization. However, we show below for the steady state that equations (1), (2) and (11) constitute an autonomous system in $\frac{W^*}{W}, n_0$ and y , where $\frac{W^*}{W}$ and n_0 are the relative real wage of cohort zero and its labour supply respectively.

Consider, first, that the inflation rate (equal to the growth rate of money) is μ ; second, that, being the final product market perfectly competitive, $W_t = P_t$ and, finally, that, after (1), the labour supply of cohorts j and $j + 1$ are connected in the following way:

$$\frac{n_j}{n_{j+1}} = \mu^{-\theta_w}$$

On these grounds, it is possible to solve the sums in (11) and (2) to obtain

$$\sum_{i=0}^{N-1} \beta^i [1 - n_0 \mu^{\theta_n i}]^{-\eta} n_0 \mu^{\theta_n i} = \frac{\theta_n - \xi}{\theta_n \varsigma} N \frac{1 - \beta^N \mu^{(\theta_n - \xi)N}}{1 - \beta \mu^{(\theta_n - \xi)}} \frac{1 - \mu^{(\theta_n - 1)}}{1 - \mu^{(\theta_n - 1)N}} \left(\frac{W^*}{W} \right)^{(\xi - 1)} \quad (12)$$

with

$$\frac{W^*}{W} = \left[\frac{1}{N} \frac{1 - \mu^{N(\theta_n - 1)}}{1 - \mu^{\theta_n - 1}} \right]^{\frac{1}{\theta_n - 1}} \quad (13)$$

(12) can be solved numerically for n_0 . Aggregate output can be easily obtained by substituting n_0 and (13) into (1).

Further notice that the first order condition with respect to bonds implies that in steady state

$$\beta = \left[\frac{P_{t+i}}{P_{t+i+1}} \iota_{t+i} \right]^{-\xi} \approx [R]^{-\xi} \quad (14)$$

where R equals 1 plus the real interest rate.

We calibrate parameters as follows: $N = 2$, $\eta = 2$, $\theta_n = 5$ and the real interest rate to be 4% in steady state, which means that the discount factor is pinned down by (14). Sensitivity analyses regarding these parameters were extensively discussed and performed by Graham and Snower (2004, 2008). It is offered there also a discussion of the reasons why Taylor contracts can be preferable to other kinds of wage contracts and why inserting wage indexation may not be interesting. Furthermore, we stick to Taylor contracts because Ascari (2004) showed that they are less sensitive to trend inflation than Calvo contracts. These issues will not receive further attention here.

Instead we focus on ξ . Notice that $\xi < \theta_n$ not to have a negative mark-up of the reset wage over the ratio between the weighted marginal disutilities from labour and marginal utilities from consumption over the contract period - see equation (39). The two theoretical arguments advanced at the beginning of the present article for introducing ξ in our model cannot offer much guidance regarding its value. Resorting to the literature on pay satisfaction and the subjective value of money will not help either, given that

there are studies - like Giles and Barrett (1971) and Heneman et al. (1997) - that would favour a value of ξ greater than one, but there are also studies - like Brandstätter and Brandstätter (1996) and Worley et al. (1992) - which would favour a value of ξ smaller than one. Therefore, we offer illustrative results for $\xi = 0.7$ and $\xi = 4.9$ - which is very close to θ_n .

4 Results, intuition and welfare analysis of the baseline model

Figure 1 compares the long-run Phillips curves for $\xi = 0.7$ and for $\xi = 1$, while Figure 2 those for $\xi = 4.9$ and for $\xi = 1$. In the first case the long-run Phillips curve in presence of money illusion lies below the one without money illusion, while in the second case the contrary happens. To appreciate the intuition underlying this result the reader should consider equations (14) and (11). For a given real interest rate, $\xi < 1$ rises the discount factor dampening the time discounting effect. Furthermore, it lowers agents' perception of their labour income reducing labour supply. $\xi > 1$ instead produces opposite effects. However, this does not imply that output will grow even in presence of hyperinflation, given that, as it is possible to see in Figure 2, for high inflation rates positive non-superneutralities fade away (turning in the end into negative non-superneutralities).

Building on Woodford (1998) among others, we can specify agents' welfare, \mathcal{W} , assuming the weight of money holdings to be so small to be negli-

gible:

$$\mathcal{W} = \sum_{j=0}^{\infty} \beta^j \left\{ \log c_{t+j} - \varsigma \frac{[1 - n_{t+j}(h)]^{1-\eta}}{1-\eta} \right\} \quad (15)$$

which, after Graham and Snower (2004, p.23), in steady state can be re-written as

$$\mathcal{W} = \frac{1}{1-\beta} \log y - \varsigma \frac{1}{1-\beta^N} \sum_{l=0}^{N-1} \beta^l \frac{[1 - n_0 \mu^{\theta_{nl}}]^{1-\eta}}{1-\eta}$$

Table 1 shows that, varying μ , \mathcal{W} behaves in a similar way to output. Under wage-staggering and $\xi = 4.9$ welfare increases with the money growth rate, levelling off at high inflation rates. For $\xi = 0.7$ welfare decreases as the inflation rate increases.

The next section introduces varying capital in the model.

5 The model with varying capital

5.1 Households' maximization problem

Households maximize their discounted expected utility

$$\max_{\{c_{t+i}(h), W_{t+N_i}(h), B_{t+i}(h), M_{t+i}(h), K_{t+i}(h)\}} \sum_{j=0}^{\infty} \sum_{i=jN}^{(j+1)N-1} \beta^i U \left\{ c_{t+i}(h), n_{t+i}(h), \left[\frac{M_{t+i}(h)}{P_{t+i}} \right]^{\xi} \right\}$$

subject to a series of income constraints perceived as follows

$$\begin{aligned}
c_{t+i}(h) + i_{t+i}(h) = & \left[\frac{W_{t+i}(h)}{P_{t+i}} \right]^\xi n_{t+i}(h) + \left[\frac{T_{t+i}(h)}{P_{t+i}} \right]^\xi - \left[\frac{M_{t+i}(h)}{P_{t+i}} \right]^\xi + \left[\frac{M_{t+i-1}(h)}{P_{t+i}} \right]^\xi - \\
& - \left[\frac{B_{t+i}(h)}{P_{t+i}} \right]^\xi + \left[\frac{B_{t+i-1}(h)}{P_{t+i}} \iota_{t+i} \right]^\xi + \left[\frac{\iota_{t+i}^k(h)}{P_{t+i}} \right]^\xi K_{t+i}(h)
\end{aligned}$$

where K is capital, i investment and ι_{t+i}^k is the nominal rental rate of capital.

Further constraints derive from the demand for labour services (1). Defining

the capital depreciation rate as δ , the law of motion of capital is

$$K_{t+i}(h) = i_{t+i}(h) + (1 - \delta) K_{t+i-1}(h)$$

which can be substituted into the budget constraint to obtain

$$\begin{aligned}
c_{t+i}(h) + K_{t+i}(h) = & \left[\frac{W_{t+i}(h)}{P_{t+i}} \right]^\xi n_{t+i}(h) + \left[\frac{T_{t+i}(h)}{P_{t+i}} \right]^\xi - \left[\frac{M_{t+i}(h)}{P_{t+i}} \right]^\xi + \left[\frac{M_{t+i-1}(h)}{P_{t+i}} \right]^\xi - \\
& - \left[\frac{B_{t+i}(h)}{P_{t+i}} \right]^\xi + \left[\frac{B_{t+i-1}(h)}{P_{t+i}} \iota_{t+i} \right]^\xi + \left[\frac{\iota_{t+i}^k(h)}{P_{t+i}} \right]^\xi K_{t+i-1}(h) + \\
& + (1 - \delta) K_{t+i-1}(h)
\end{aligned}$$

Taking the first-order derivative with respect to $K_{t+i}(h)$ one has

$$\lambda_{t+i}(h) - E \left[\beta \left(\frac{\iota_{t+i+1}^k}{P_{t+i+1}} \right)^\xi \lambda_{t+i+1}(h) \right] - E [\beta (1 - \delta) \lambda_{t+i+1}(h)] = 0 \quad (16)$$

5.2 Firms' cost minimization problem and the government

We adopt a two-stage budgeting approach after Chambers (1988, pp. 112-113) and Heijdra and Van der Ploeg (2002, pp. 360-363). In the first stage firms choose the amount of each labour kind as in section 2.1. In the second stage, they minimize their total costs by choosing their preferred amounts of capital and of the aggregate labour input as follows⁶

$$\begin{aligned} \min_{K_t, n_t} \frac{l_t^k}{P_t} K_t + \frac{W_t}{P_t} n_t \\ \text{s.t. } y_t = K_t^\alpha n_t^{1-\alpha} \end{aligned} \quad (17)$$

Solving this problem one has that

$$K_t = MC_t \alpha \frac{y_t}{\frac{l_t^k}{P_t}} \quad (18)$$

$$n_t = MC_t (1 - \alpha) \frac{y_t}{\frac{W_t}{P_t}} \quad (19)$$

$$MC_t = \left(\frac{\frac{l_t^k}{P_t}}{\alpha} \right)^\alpha \left(\frac{\frac{W_t}{P_t}}{1 - \alpha} \right)^{1-\alpha} \quad (20)$$

where MC_t is the real marginal cost at time t . Profit maximization further implies the price of the final product to be equal to the firms' nominal

⁶We did not insert an index for firms as they are symmetrical and their number is normalized to one.

marginal cost, which leads us to the following equation

$$P_t = \left(\frac{l_t^k}{\alpha}\right)^\alpha \left(\frac{W_t}{1-\alpha}\right)^{1-\alpha} \quad (21)$$

5.3 Model solution and results

The equilibrium for this model is a sequence $\left\{\mu_t, c_t, n_t(h), n_t, \Pi_t, l_t, K_t, \frac{W_t}{P_t}, \frac{W_t(h)}{P_t}, y_t, \frac{l_t^k}{P_t}\right\}$ - where $\Pi_t = \frac{P_t}{P_{t-1}}$ - satisfying households' utility maximization and firms' profit maximization. For K and c , we could drop the h index because households are symmetrical with respect to their consumption and investment choices.

In steady state (16) implies

$$(R^\xi - 1 + \delta) = \left(\frac{l^k}{P}\right)^\xi \quad (22)$$

In words, agents equate their perception of the real interest rate to their perception of the gross real rental rate of capital. P can be normalized to 1 and (22) together with (21) can be used to find W . Having the ratio between the aggregate wage index and the gross rental rate of capital, (19) and (18) can be used to find K/n , and, by means of the production function (17), y/n . Further consider that in steady state $i = \delta K$, therefore

$$\frac{c}{n} = \left(\frac{K}{n}\right)^\alpha - \delta \left(\frac{K}{n}\right)$$

In the right-hand side of equation (5) divide and multiply by n_{t+i} to obtain

$$\sum_{i=0}^{N-1} E \left[\beta^i |U_n(\cdot)| (-1) (-\theta_n) n_{t+i}(h) \right] = \sum_{i=0}^{N-1} E \left\{ \frac{\beta^i (\theta_n - \xi) \left[\frac{W_{t+i}(h)}{P_{t+i}} \right]^\xi \frac{n_{t+i}(h)}{n_{t+i}}}{\frac{c_{t+i}}{n_{t+i}}} \right\} \quad (23)$$

Specifying the utility function as in (10), one can find that, considering varying capital, (12) turns out to be

$$\sum_{i=0}^{N-1} \beta^i [1 - n_0 \mu^{\theta_n i}]^{-\eta} n_0 \mu^{\theta_n i} = \frac{\theta_n - \xi}{\theta_n \varsigma} \frac{n}{c} \frac{1 - \beta^N \mu^{(\theta_n - \xi)N}}{1 - \beta \mu^{(\theta_n - \xi)}} \left(\frac{W^*}{W} \right)^{-\theta_w} W^{*\xi} \quad (24)$$

which can be solved numerically for n_0 . With n_0 at hand, one can easily recover first n by using (1) and then all the steady state values of the other variables of model.

Figure (3) shows that our results are robust to the inclusion of variable capital. $\xi > 1$ reinforces positive money non-superneutralities, while for $\xi < 1$ the opposite holds true.

6 Money illusion in firms' decisions

Up to this point, we have assumed that money illusion affects households and not firms. In this section, instead, we explore how the long-run Phillips curve is affected by money illusion when it pertains to firms' and not households'

behavior. In so doing we stick to the model with varying capital. Under this assumption $\xi = 0$ and the first stage cost minimization problem of firms turns out to be

$$\begin{aligned} \min_{n_t(h)} \int_{h=0}^1 \left[\frac{W_t(h)}{P_t} \right]^{\xi_f} n_t(h) dh \\ \text{s.t. } n_t = \left[\int_0^1 n_t(h)^{\frac{\theta_n-1}{\theta_n}} dh \right]^{\frac{\theta_n}{\theta_n-1}} \end{aligned}$$

The second stage cost minimization problem instead is

$$\begin{aligned} \min_{K_t, n_t} \left(\frac{t_t^k}{P_t} \right)^{\xi_f} K_t + \left(\frac{W_t}{P_t} \right)^{\xi_f} n_t \\ \text{s.t. } y_t = K_t^\alpha n_t^{1-\alpha} \end{aligned}$$

where ξ_f is the firms' counterpart of ξ .

The new first order conditions corresponding to equations (1) – (2) and (18)-(20) can be easily computed, once recalling that under firms' money illusion one has that $\lambda_{n,t} = \left(\frac{W_t}{P_t} \right)^\xi$. Specifically the counterpart of equation (1) for the present model is

$$n_t(h) = \left[\frac{W_t(h)/P_t}{\lambda_n} \right]^{-\xi_f \theta_n} n_t \quad (25)$$

Profit maximization still implies that the price level is equal to the nominal marginal cost and therefore that the real marginal cost is equal to one for

$\xi_f \neq 0$. Given that the product market is perfectly competitive, profits will still be zero. The quantities of production inputs will be chosen on the basis of the distorted firms' perceptions. The solution procedure is similar to the one illustrated in the previous section. Equation (24) changes to

$$\sum_{i=0}^{N-1} \beta^i [1 - n_0 \mu^{\xi_f \theta_n i}]^{-\eta} n_0 \mu^{\xi_f \theta_n i} = \frac{\theta_n - 1}{\theta_n \varsigma} \frac{n}{c} \frac{1 - \beta^N \mu^{(\xi_f \theta_n - 1)N}}{1 - \beta \mu^{(\xi_f \theta_n - 1)}} \left(\frac{W^*}{W}\right)^{-\xi_f \theta_w} W^* \quad (26)$$

Figure 4 sets out our results. $\xi_f > 1$ reinforces negative money non-superneutralities, while for $\xi_f < 1$ the opposite holds true. In order to understand the economic intuition, it is possible to inspect equations (25) and (26) and to see that firms' money illusion has a similar effect to an increase in the elasticity of substitution of different labour kinds, which was analysed in Ascari (1998) and Graham and Snower (2004). This increases employment cycling and, also due to the labour supply smoothing effect, leads to the results showed in Figure 4.

7 Conclusions

To conclude this paper shows how the long-run inflation-output nexus behaves in presence of agents mis-perceiving real economic variables. Households' under-perception reduces labour supply and time discounting leading to more negative money non-superneutralities. Households' over-perception, instead, boosts labour supply and time discounting leading to more positive

money non-superneutralities. These results hold true even inserting varying capital in our model. Once considering firms' money illusion, we found that (over-)under-perception (strengthens)weakens negative money non-superneutralities. Also the long-run Phillips curve is more sensitive to firms' rather than to households' money illusion. In the Appendix, we treat how a temporary shock in money growth affects our model for different degrees of money illusion.

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8 Appendix

In this Appendix we consider the impact of a temporary monetary shock on the economy. We stick to the varying capital model. We first analyse the case of households' money illusion and then that of firms' money illusion. Following Edge (2002) money growth is assumed to evolve according to the process

$$\mu_t = (\mu)^{1-\zeta} (\mu_{t-1})^\zeta \exp(\epsilon_t)$$

where ϵ_t is a random shock and ζ an autoregressive parameter, that we set equal to 0.57 after Ascari (2004). We also set the steady state money growth rate, μ , to 1%. Similarly to Edge (2001), the other equations of the system are

$$1 = \left(\frac{l_t^k}{\alpha} \right)^\alpha \left(\frac{W_t}{1-\alpha} \right)^{1-\alpha} \quad (27)$$

$$n_t = \left(\frac{1-\alpha}{\alpha} \right)^\alpha Y_t \left(\frac{W_t}{l_t^k} \right)^{-\alpha} \quad (28)$$

$$K_t = \left(\frac{\alpha}{1-\alpha} \right)^{1-\alpha} Y_t \left(\frac{W_t}{l_t^k} \right)^{1-\alpha} \quad (29)$$

$$n_t(h) = \left[\frac{W_t(h)}{W_t} \right]^{-\theta_n} n_t \text{ for } h \in \left[0, \frac{1}{2} \right] \quad (30)$$

$$n_t(h) = \left[\frac{\frac{W_{t-1}(h)}{P_{t-1}}}{\frac{W_t}{P_t} \Pi_t} \right]^{-\theta_n} n_t \text{ for } h \in \left[0, \frac{1}{2} \right] \quad (31)$$

$$\frac{W_t}{P_t} = \left\{ \frac{1}{2} \left[\frac{W_t(h)}{P_t} \right]^{1-\theta_n} + \frac{1}{2} \left[\frac{W_{t-1}(h)}{P_t \Pi_t} \right]^{1-\theta_n} \right\}^{\frac{1}{1-\theta_n}} \quad (32)$$

$$\frac{c_{t-1}}{c_t} \left(1 - \frac{1}{l_t^\xi}\right) / \left(1 - \frac{1}{l_{t-1}^\xi}\right) = \left(\frac{\mu_t}{\Pi_t}\right)^{-\xi} \quad (33)$$

$$\frac{1}{c_t} = \beta E \left\{ \frac{1}{c_{t+1}} \left[\frac{1}{\Pi_{t+1}} l_t \right]^\xi \right\} \quad (34)$$

$$E(c_{t+1}) = \beta E \left[\left(\frac{l_{t+1}^k}{P_{t+1}} \right)^\xi \right] c_t + \beta(1 - \delta) c_t \quad (35)$$

$$y_t = c_t + K_t - (1 - \delta) K_{t-1} \quad (36)$$

$$[1 - n_t(h)]^{-\eta} n_t(h) + \beta E \{ [1 - n_{t+1}(h)]^{-\eta} n_{t+1}(h) \} = \quad (37)$$

$$= \frac{(\theta_n - \xi)}{\theta_n \varsigma} \left[\frac{W_t(h)}{P_t} \right]^\xi \frac{n_t(h)}{c_t} + \frac{(\theta_n - \xi)}{\theta_n \varsigma} \beta E \left\{ \left[\frac{W_t(h)}{P_t \Pi_{t+1}} \right]^\xi \frac{n_{t+1}(h)}{c_{t+1}} \right\} \quad (38)$$

where (33) comes from the ratio of the first order condition for money holdings at time t and at time $t - 1$.

Figure 5 shows the impulse responses of output to a monetary shock for different values of ξ . Similarly to Edge (2002) we normalized them on the basis of the impact value of output for $\xi = 1$. As it is possible to see, money illusion does not alter output persistence. It only changes the impact value of output as a greater value of ξ reduces it. To appreciate an intuition for this result consider the equation for the nominal reset wage, that can be obtained from the equation (37)-(38)

$$W_t^*(h) = \left\{ \frac{\sum_{i=0}^{N-1} E [\beta^i |U_n(\cdot)| n_{t+i}(h)]}{\sum_{i=0}^{N-1} E \left[U_c(\cdot) \beta^i \frac{n_{t+i}(h)}{P_{t+i}^\xi} \right]} \frac{\theta_n}{(\theta_n - \xi)} \right\}^{\frac{1}{\xi}} \quad (39)$$

As noted by Ascari (2000) among others, the nominal reset wage is a mark-up over the ratio between the weighted marginal disutilities from labour and marginal utilities from consumption over the contract period. Overperception of real variables reduces the impact of a monetary shock because agents believe their real wage is higher than what it is in reality and so they change their nominal wage by a lesser amount. This is the meaning of the exponent of $\frac{1}{\xi}$ in the right hand side of equation (39). The opposite holds true for underperception of real variables, which implies an over-reaction by agents to monetary shocks. Agents' over(under)-reaction produces also over(under)-reaction of output. This intuition is confirmed by the behavior of the real wage of the resetting cohort in our model. Similarly to Huang and Liu (2002), after a monetary shock it declines, however for $\xi = 1.1$ it does so by 45% less than for $\xi = 1$, while for $\xi = 0.9$ by 66% more than in absence of money illusion.

Under firms' money illusion the system of equations is similar to (27) – (38) with the exceptions that $\xi = 1$ and that equations (27) – (32) now are

$$1 = \left[\frac{\left(\frac{l_t^k}{P_t}\right)^{\xi_f}}{\alpha} \right]^{\alpha} \left[\frac{\left(\frac{W_t}{P_t}\right)^{\xi_f}}{1-\alpha} \right]^{1-\alpha} \quad (40)$$

$$n_t = \left(\frac{1-\alpha}{\alpha}\right)^{\alpha} Y_t \left(\frac{W_t}{l_t^k}\right)^{-\alpha\xi_f} \quad (41)$$

$$K_t = \left(\frac{\alpha}{1-\alpha}\right)^{1-\alpha} Y_t \left(\frac{W_t}{l_t^k}\right)^{(1-\alpha)\xi_f} \quad (42)$$

$$n_t(h) = \left[\frac{W_t(h)}{W_t}\right]^{-\xi_f\theta_n} n_t \text{ for } h \in \left[0, \frac{1}{2}\right] \quad (43)$$

$$n_t(h) = \left[\frac{\frac{W_{t-1}(h)}{P_{t-1}}}{\frac{W_t}{P_t}\Pi_t}\right]^{-\xi_f\theta_n} n_t \text{ for } h \in \left[0, \frac{1}{2}\right] \quad (44)$$

$$\frac{W_t}{P_t} = \left\{ \frac{1}{2} \left[\frac{W_t(h)}{P_t}\right]^{(1-\theta_n)\xi_f} + \frac{1}{2} \left[\frac{W_{t-1}(h)}{P_t\Pi_t}\right]^{(1-\theta_n)\xi_f} \right\}^{\frac{1}{(1-\theta_n)\xi_f}} \quad (45)$$

Figure 6 shows the impulse responses of output to a monetary shock for different values of ξ_f . Similarly to above, we normalized them on the basis of the impact value of output for $\xi_f = 1$. Once again money illusion does not alter output persistence. It only changes the impact value of output as a greater value of ξ_f increases it. To appreciate an intuition for this result consider that Huang and Liu (2002, 430) write that, in wage staggering models, wage-setting decisions are dominated by the labour supply smoothing effect. As a consequence, wage setting cohorts, in order to dampen fluctuations in their working hours, rise their wage less than inflation leading to a decline in their real wage. For $\xi_f > 1$ firms overperceive wage differences leading to greater fluctuations in the working hours of the various cohorts. As a con-

sequence the wage setting cohort underreact to a monetary shock leading to a greater decline in their real wage and to a greater output expansion. The contrary happens for $\xi_f < 1$. This intuition is confirmed by the fact that under $\xi_f = 1.1$ the real wage of the resetting cohort declines by 47% more than for $\xi_f = 1$. On the contrary for $\xi_f = 0.9$ the real wage of the resetting cohort declines by 45% less than for $\xi_f = 1$.

Table 1 - Percentage deviation from the welfare level with flexible wages

Money growth rate	$\xi = 4.9$	$\xi = 0.7$
2.5	0.04%	-0.24%
5	0.08%	-0.96%
10	0.12%	-3.60%
15	0.13%	-7.44%

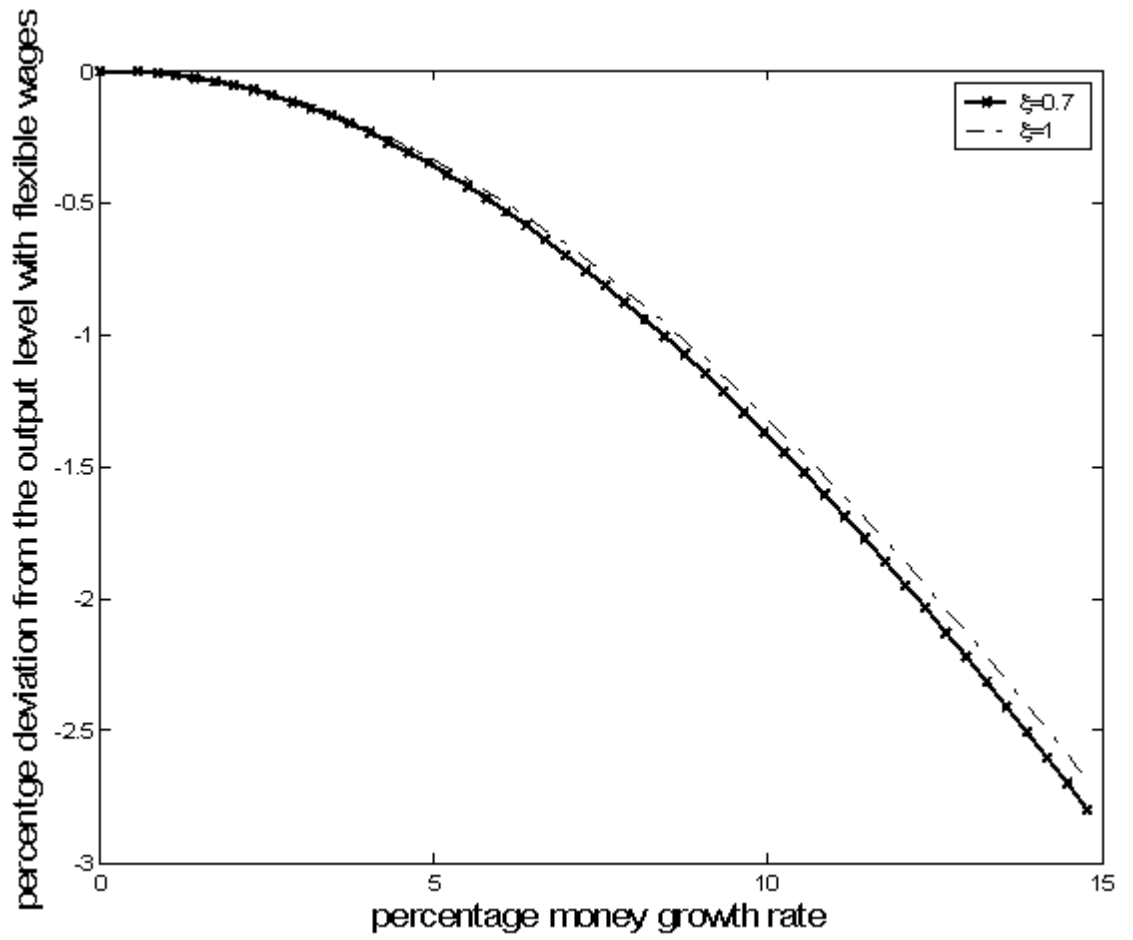


Figure 1: The long-run Phillips curve for $\xi = 0.7$ and $\xi = 1$.

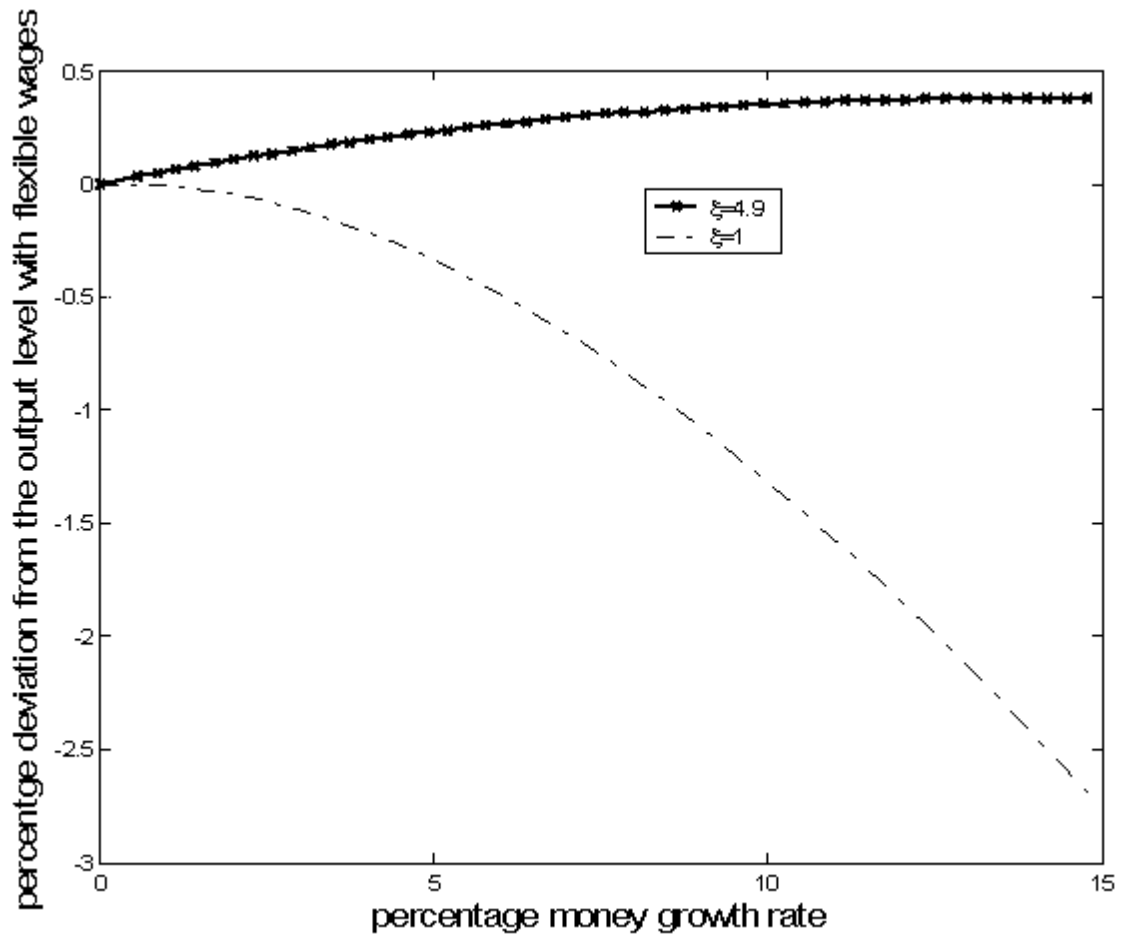


Figure 2: The long-run Phillips curve for $\xi = 4.9$ and $\xi = 1$.

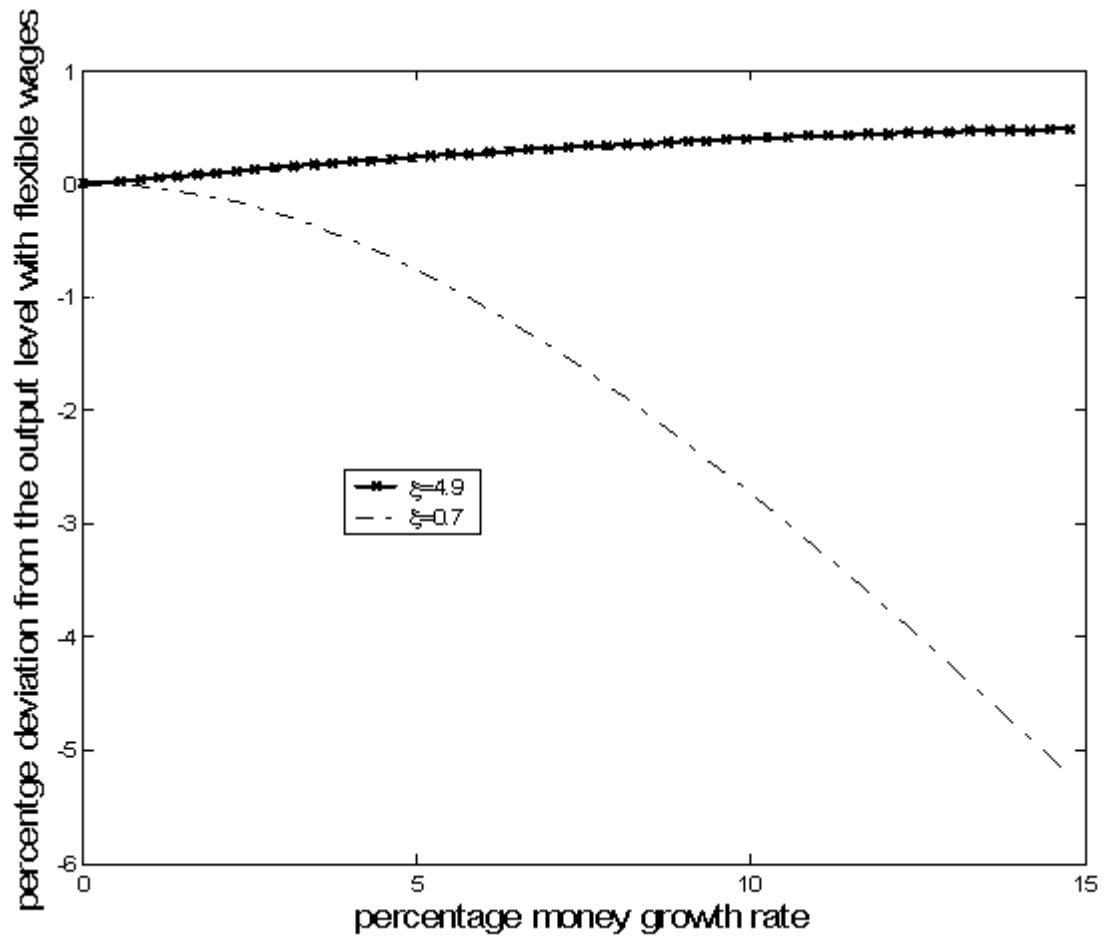


Figure 3: The long-run Phillips curve for different values of ξ and varying capital.

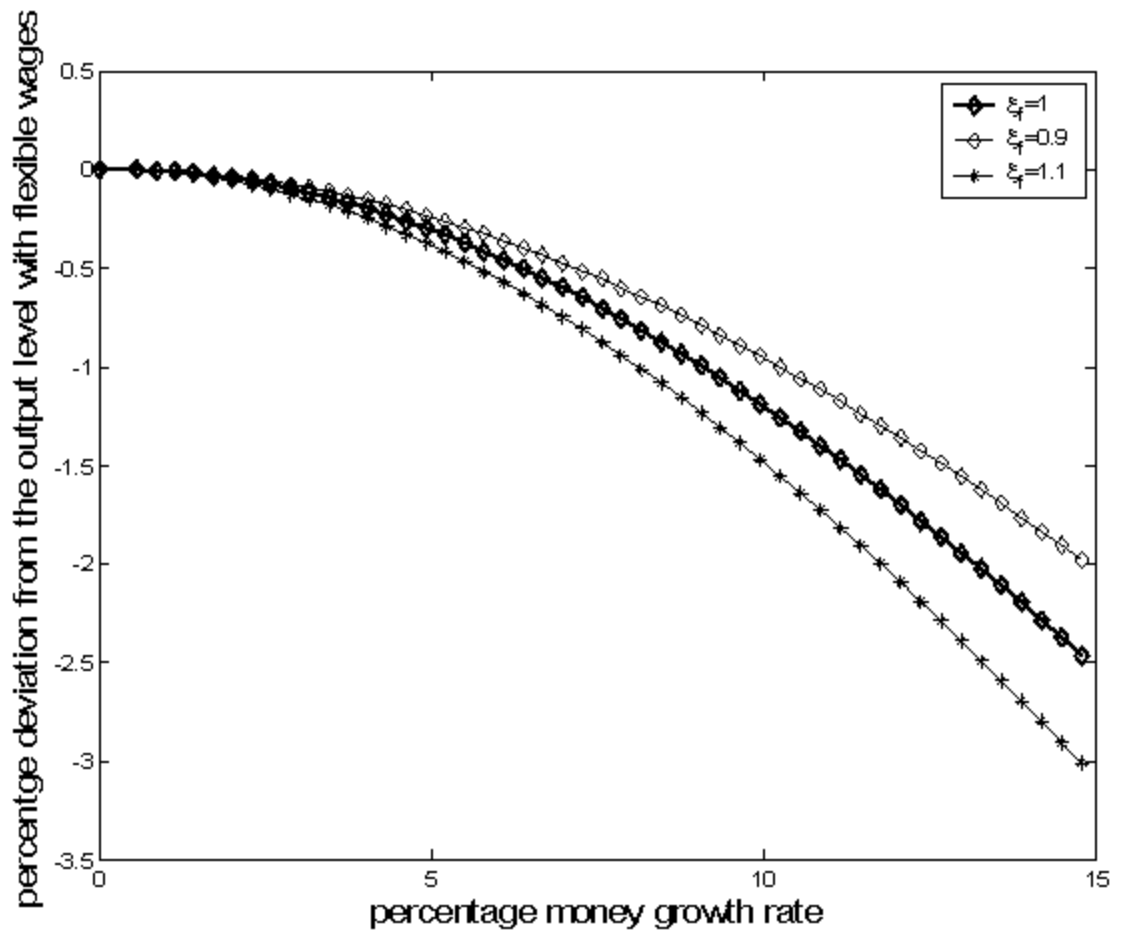


Figure 4: The long-run Phillips curve for different values of ξ_f and varying capital.

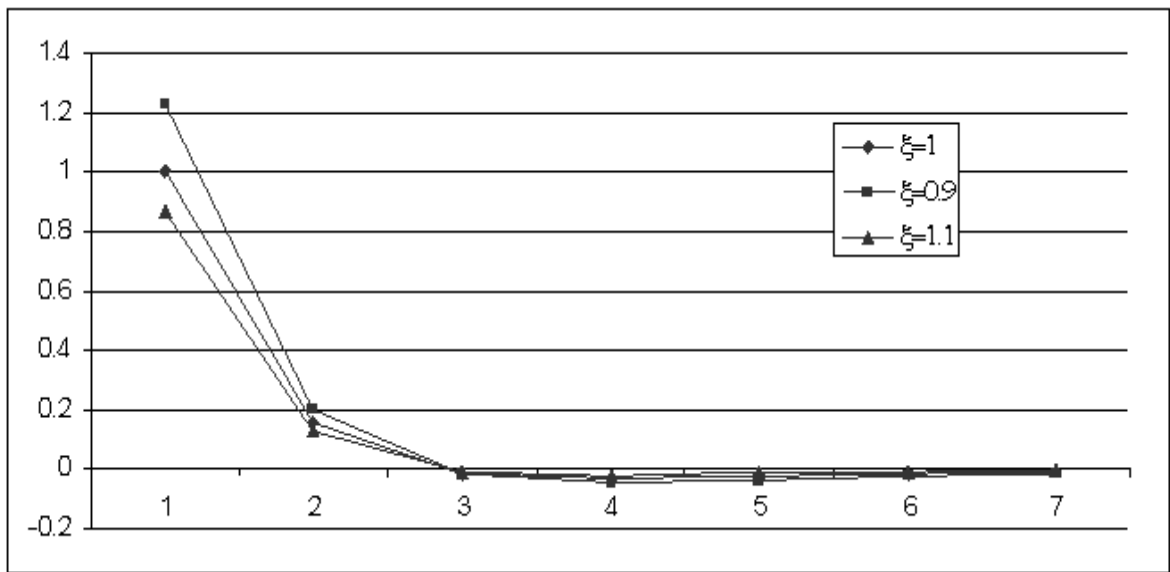


Figure 5: Normalized impulse response functions of output after a monetary shock for different values of ξ . (The normalization base is the impact value for $\xi = 1$).

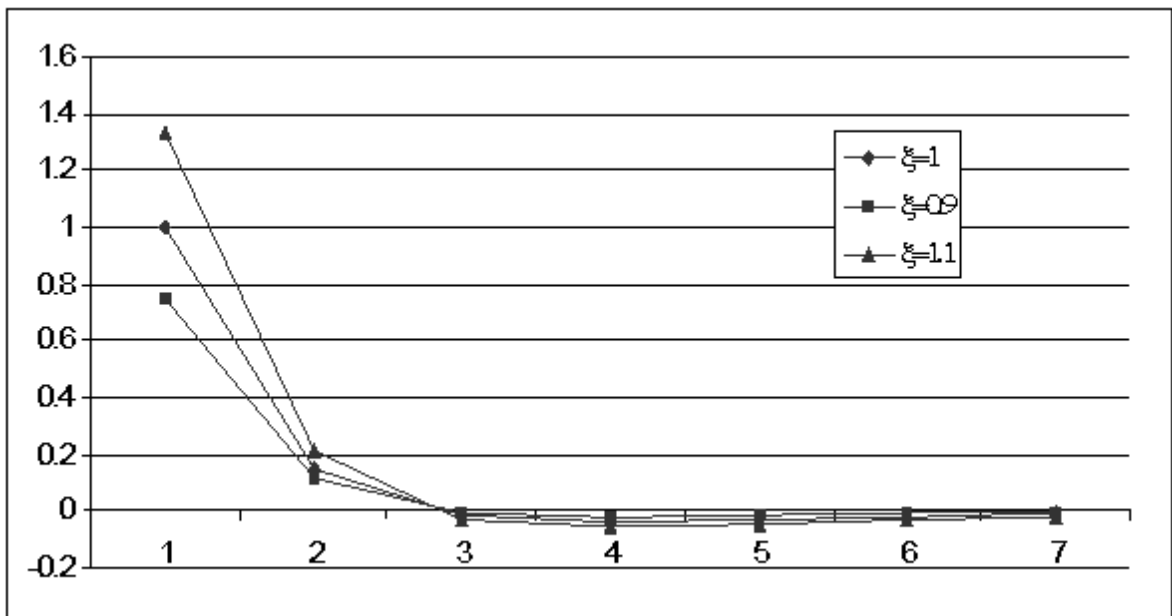


Figure 6: Normalized impulse response functions of output after a monetary shock for different values of ξ_f . (The normalization base is the impact value for $\xi_f = 1$).