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Abstract:
The dates of U.S. business cycle are reported by NBER with a considerable delay, so an early notion of turning points is of particular interest. This paper proposes a novel sequential approach designed for timely signaling these turning points. A directional cumulated sum decision rule is adapted for the purpose of on-line monitoring of transitions between subsequent phases of economic activity. The introduced procedure shows a sound detection ability for business cycle peaks and troughs compared to the established dynamic factor Markov switching methodology. It exhibits a range of theoretical optimality properties for early signaling, moreover, it is transparent and easy to implement.

Keywords: Business cycle; CUSUM control chart; Dynamic Factor Markov switching models; Early signaling; NBER dating

JEL classification: C44, C50, E32

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1 Introduction

Identification and early detection of business cycle phases remains in the focus of macroeconomic research and policy advise. At least since Burns and Mitchell (1946) economic activity is perceived to be separated in two distinct states: expansions, where broad economic growth prevails, and recessions, where economic activity is contracting. Following this perception, the National Bureau of Economic Research (NBER) provides a dating of U.S. turning points which mark transitions from an expansion to a subsequent recession (peak) and vice versa (trough). These turning points are commonly regarded and widely accepted as the dates of the U.S. business cycle phases. The approach of NBER relies primarily on five time series of economic variables, namely GDP, industrial production, manufacturing and trade sales, personal income less transfers and employment. A downturn hitting all these variables simultaneously can be seen as a recession.

Since 1978 the NBER committee tries to maintain the dating close to the current edge. However, the NBER announcements still come with a considerable time delay, so there is an interest to have early signals about NBER business cycle turning points. Several statistical models or algorithms have been proposed to mimic the NBER dating. These methods exploit mostly only the latter four time series, available at monthly frequency, whereas GDP is reported quarterly and has a disadvantage of a relatively large publication lag. An established statistical approach to replicate the dates of NBER turning points is a dynamic factor Markov switching (DFMS) model proposed by Chauvet (1998). Recently, Chauvet and Piger (2008) show that DFMS is well suited for dating the U.S. business cycle in real time. Another dating approach is developed by Harding and Pagan (2006). They apply the nonparametric algorithm of Bry and Boschan (1971), which identifies turning points in each of four considered monthly time series, and suggest a way to synchronize these points. A further extension of this nonparametric approach is proposed by Leamer (2008). However, the algorithm of Bry and Boschan relies on moving averages, so a sufficient number of (future) observations beyond a turning point itself are needed to provide a dating. For this reason their method is less suitable for the purpose of an early detection of business cycle phase changes.

This paper elaborates statistical methods for on-line *signaling* decisions about the current phase of the U.S. business cycle. The DFMS approach can be applied both for dating and early warning purposes. DFMS signals about business cycle turning points would rely on filtered state probabilities at the current edge. Next, we apply two conditional DFMS decision rules for signaling turning points. These rules are based on the symmetric $p^* = 0.5$ and asymmetric $p^* = 0.8$ probability borders. Given the current phase to be an expansion, a recession probability estimate should exceed the border p^* for the first time in order to signal the start of a recession.

As an alternative to DFMS signaling, we develop a novel approach which provides sequential on-line decisions about business cycle turning points. The problem of early signaling is reformulated to the setup common in statistical process control (SPC). Control charts are SPC decision rules for sequential monitoring of changes in process parameters. We suggest to assess unknown turning points by a mean control chart based on monthly innovations in the NBER series of interest. A chart should provide a quick on-line detection of changes in the innovation mean vectors, which are believed to reflect business cycle phases. A direction of a coming change, e.g. currently an expansion, expected to be followed by a recession, is assumed to be known. Since the distribution parameters of innovations, characterizing both expansion and recession states, could be estimated from the data, the multivariate directional cumulated sum (CUSUM) control chart of Healy (1987) is appropriate for such early signaling task. This chart is transparent and has a simple design. Moreover, it shows a set of optimal detection properties for the common performance criteria, see Moustakides (1986) and Ritov (1990).

A practical application of the suggested control chart requires assumptions concerning the stochastic process to monitor. In SPC a monitored process should consist of serially independent (or at least weakly dependent) increments with a known distribution function. Since the observable time series used by NBER for the dating purpose are strongly autocorrelated, the vector autoregressive (VAR) representation is applied for extracting serially uncorrelated increments. The mean vectors of these VAR increments are assumed to represent the business cycle. This is a different to DFMS, where the business cycle is assessed by a latent state variable. Note that DFMS specifies autoregressive dynamics both for the latent factor as well as for the vector of model residuals. Thus the assumptions concerning process dynamics in our sequential procedure are comparable to those of DFMS.

Validation and comparison of the signaling methods is done for the NBER monthly data from 1969 to 2008. The dataset taken in the study corresponds to the vintage in December 2008. The number of periods (months), identified differently from the NBER dating, serves as a goodness measure. Moreover, we differentiate between false signals and delays in detection of turning points. The full sample information is used for the initial estimation of DFMS and VAR model parameters. Further, expanding conditional information sets are exploited for an early signaling purpose. Decisions at the current edge are based on fully revised conditional data, providing “pseudo” real time analysis (Layton, 1996).

Full sample symmetric DFMS proves to be the best dating approach which misclassifies only 16 months over the whole sample of 499 months. It serves as a benchmark for the performance evaluation of early signaling algorithms. Symmetric conditional DFMS with probability border $p^* = 0.5$ provides 35 falsely identified months, while asymmetric conditional DFMS with $p^* = 0.8$ misclassifies 45 months. However, the number of false signals for asymmetric DFMS is about twice smaller than for symmetric one. Our novel sequential approach with appropriately chosen parameter values yields less than 40 mistakes. The control chart with the best choice of critical values misclassifies only 33 months. Moreover, this chart provides just 11 mistakes starting from “the great moderation” period in the mid 80s (Stock and Watson, 2002). Conditional DFMS misclassifies for the same period 14 months for the symmetric and 21 months for the asymmetric methods. This difference arises primarily due to the fact that the control chart detects the current recession, dated by NBER to start in December 2007, with only 1 month delay, compared to 7 and 8 month delays of the conditional DFMS methods.

Note that full sample DFMS has also difficulties with dating this turning point. The results from the sequential procedure stay sound for the VAR parameter estimates based on the data subsamples as well as for the reasonable choices of the control chart parameters. The numbers of delay and false signals for the control charts are comparable to those of the symmetric conditional DFMS method. Since DFMS and sequential approach rely on different assumptions concerning the data generating process, we see them not as competitors but as complementary tools for early signaling of NBER business cycle turning points. However, our sequential methodology has advantages to be more stable and easier to implement compared to DFMS.

The rest of the paper is structured as follows. Section 2 presents the approaches for early warnings of business cycle turning points. In particular, we adapt the DFMS approach for signaling at the current edge and suggest the sequential methodology for this purpose. Section 3 introduces the control chart which is a statistical decision rule for the task of sequential monitoring. Section 4 describes the dataset used in the study and presents estimation results. Section 5 compares competing approaches for their signaling ability at the current edge, while Section 6 concludes.

2 Business Cycle Dating Methods

2.1 Dynamic factor Markov switching approach

The DFMS approach is an established method for dating turning points (peaks and troughs) between phases of the U.S. business cycle. DFMS relies on the idea that a business cycle can be divided in two distinct phases of economic activity: expansions and recessions. It suggests to extract business cycle phases from time series of coincident economic indicators. DFMS has been proposed by Chauvet (1998) as a generalization of the Markov switching model of Hamilton (1989) for business cycle modeling. Next, we briefly present the DFMS methodology in line with the study of Chauvet.

The first log differences are applied to a four-dimensional vector of the observable monthly NBER macroeconomic variables (industrial production, retail sales, employment and personal income) in order to provide the process stationarity. The transformed coincident vector is denoted by $\mathbf{z}_t$ at the time point $t \in \mathbb{N}$. Assume that the observable vector $\mathbf{z}_t$ is driven by a latent factor f_t as

$$\mathbf{z}_t = \boldsymbol{\lambda} f_t + \mathbf{u}_t, \qquad t = 1, ..., T, \tag{1}$$

where $\boldsymbol{\lambda}$ is a vector of factor loadings, $\mathbf{u}_t$ represents Gaussian model residuals. The factor f_t is believed to correspond to the NBER business cycle. Its dynamics is assessed by the second order autoregressive model with i.i.d. residuals ϵ_t:

$$f_t = \mu_t + \gamma_1(f_{t-1} - \mu_{t-1}) + \gamma_2(f_{t-2} - \mu_{t-2}) + \epsilon_t, \qquad \epsilon_t \sim \mathcal{N}(0, 1). \tag{2}$$

The expectation of the factor $E(f_t) = \mu_t$ is assumed to be defined via a latent Markov switching process with two regimes:

$$\mu_t \; = \; S_t\,\mu_0 + (1 - S_t)\,\mu_1. \tag{3}$$

The regime at month t is characterized by a binary 0/1 state variable S_t. The economy is considered at t to be in an expansion for $S_t = 1$ and in a recession for $S_t = 0$. The underlying Markov process for the latent variable S_t is described via the transition probabilities

$$P(S_t = 1|S_{t-1} = 1) = p_{11} \quad \text{and} \quad P(S_t = 0|S_{t-1} = 0) = p_{00},$$

where p_{11} denotes the probability to stay in a phase of expansion, while p_{00} corresponds to the probability to stay in a phase of recession. Accordingly, p_{01} denotes the probability to get into a recession after an expansion and p_{10} is the probability to leave a recession. The transition probabilities are restricted by setting $p_{11} + p_{01} \equiv 1$ as well as $p_{00} + p_{10} \equiv 1$.

The Gaussian model residuals $\mathbf{u}_t$ are assumed to have a diagonal covariance matrix and to follow a second order autoregressive process for all four vector components:

$$u_{i,t} = \rho_{i,1} u_{i,t-1} + \rho_{i,2} u_{i,t-2} + \varepsilon_{i,t} \quad \text{with} \quad \varepsilon_{i,t} \sim \mathcal{N}(0, \sigma_i^2), \quad i = 1, ..., 4. \tag{4}$$

The DFMS dating relies on inferences from the full sample information set $\mathcal{I}_T$. The smoothed probabilities $P(S_t = 1|\mathcal{I}_T)$ and $P(S_t = 0|\mathcal{I}_T)$ of the state variable S_t are estimated given the set $\mathcal{I}_T$. Probabilities $P(S_t|\mathcal{I}_T)$ need to be transferred into decisions concerning turning points in the business cycle. For this purpose Chauvet and Piger (2008) suggest to date a peak (begin of a recession) if three consecutive months have a probability for being in a recession $P(S_t = 0|\mathcal{I}_T)$ higher than the predetermined border value $p^* = 0.8$. Then the begin of a recession is dated with month τ, *prior* to these three consecutive months, where the probability $P(S_\tau = 0|\mathcal{I}_T)$ gets over the mark 0.5 for the first time. Similarly, three consecutive months with a probability lower than $1 - p^*$ are needed for dating a trough.

This paper focuses on the task of early on-line *signaling* of NBER business cycle turning points. Thus information available at the current edge t is only a conditional set $\mathcal{I}_t$, $t \leq T$, but not the full set $\mathcal{I}_T$. The DFMS algorithm of Chauvet and Piger suggests *dating* decisions *in real time*, i.e. with some delay, because future inferences (as described above) are required in their procedure for current decisions. For this reason DFMS should be adapted for early signaling of turning points, such that its decisions would rely solely on the filtered probabilities $P(S_t|\mathcal{I}_t)$. We analyze two conditional DFMS rules to link filtered probabilities to binary decisions about a current state of the business cycle. Given a current expansion, the first approach signals a recession at t if $P(S_t = 0|\mathcal{I}_t) > 0.5$. Equivalently, the same probability border is used for signaling an expansion after a recession: an expansion is assumed to start at t if $P(S_t = 0|\mathcal{I}_t) \leq 0.5$ while $P(S_{t-1} = 0|\mathcal{I}_{t-1}) > 0.5$. This approach is further referred as symmetric DFMS with $p^* = 0.5$.

The second conditional DFMS approach uses asymmetric probability borders. Given that the algorithm assumes an expansion at $t - 1$, a peak and thereby the start of a recession is signaled at

t if $P(S_t = 0|\mathcal{I}_t) > p^* = 0.8$. Thus all recession probabilities less or equal $p^* = 0.8$ are interpreted such that the current expansion continuous. Accordingly, given a recession at $t-1$, a change to an expansion at t is signaled if the recession probability $P(S_t = 0|\mathcal{I}_t)$ falls under the border $1-p^* = 0.2$. Such signaling mechanism should reduce a number of false warnings compared to symmetric DFMS because a higher probability is required for a turning point decision. However, this might come at the cost of longer delays in signaling a new business cycle phase. This method is further referred as asymmetric conditional DFMS with $p^* = 0.8$.

The comparison of DFMS decisions about business cycle turning points with the reported NBER dates serves as a goodness measure of signaling procedures. Note that information of the NBER dates is not directly used in the DFMS methodology. The full sample DFMS algorithm is able to reproduce NBER dates of the U.S. business cycle with a high precision. However, the practical implementation of DFMS is complicated by the technical issues arising from estimation of the model parameters. First, Chauvet's approach relies only on the approximate maximum likelihood estimator of Kim (1994), whereas the exact procedure is hardly possible due to the a dimension of the integration problem. Then, since DFMS presumes a mixture of normal distributions, the issue of multimodality constitutes another serious problem. Consequently, it cannot be guaranteed that a numerical maximization would lead to a global maximum and not just to a local one. Unfortunately, the curse of multimodality applies to Bayesian estimation as well (Chauvet and Piger, 2008). In fact, the Bayesian estimation approach is highly prior sensitive in the considered framework. These problems complicate the implementation of DFMS and make the corresponding dating and/or signaling decisions hardly transparent.

2.2 Sequential methodology for early signaling of turning points

Sequential methods are designed for timely detection of changes in parameters of a process of interest. This paper aims to detect changes in phases of the U.S. business cycle as soon as they occur. Andersson et al. (2005) propose several sequential decision rules for monitoring turning points in a univariate business cycle leading index. This paper introduces a novel signaling approach which exploits innovations in the multivariate observable time series $\mathbf{z}_t$. Two distinct mean vectors of innovations in the process $\{\mathbf{z}_t\}_{t=1}^T$ are believed to represent expansion and recession phases of the business cycle. Our methodology relies on sequential monitoring of changes in the process means for an early detection of an unknown (next) turning point after the last announced NBER date.

A serial correlation should be removed from the observable time series $\mathbf{z}_t$ in order to make a signaling mechanism tractable. This is required because the impact of a serial correlation could deteriorate statistical inferences from a sequential decision rule (Woodall, 2000). The autocorrelation in the process $\{\mathbf{z}_t\}_{t=1}^T$ is assessed by applying the following VAR(p) model:

$$\phi(L)\mathbf{z}_t = \mathbf{x}_t, \tag{5}$$

where the lag polynomial $\phi(L)$ captures (vector-) autoregressive dynamics. The mean of the innovation vector $\mathbf{x}_t$ is assumed to depend on NBER business cycle announcements s_t as

$$\mathbf{x}_t = \boldsymbol{\mu}_1 s_t + \boldsymbol{\mu}_0(1 - s_t) + \boldsymbol{\varepsilon}_t, \quad \boldsymbol{\varepsilon}_t \sim \mathcal{N}(\mathbf{0}, \boldsymbol{\Sigma}), \tag{6}$$

whereby Gaussian residuals $\boldsymbol{\varepsilon}_t$ are independent in time. Hereafter we set $\mu_{1,i} > \mu_{0,i}$ for all $i = 1, ..., 4$ without loss of generality. Reformulation of model (5) exhibits the VAR representation

$$\mathbf{z}_t = \sum_{l=1}^{p} \phi_l \mathbf{z}_{t-1} + \boldsymbol{\mu}_1 s_t + \boldsymbol{\mu}_0(1 - s_t) + \boldsymbol{\varepsilon}_t,$$

which can be estimated efficiently via the OLS methodology.

The binary observable 0/1 variable s_t reflects the NBER business cycle decisions and corresponds to the latent state variable S_t in DFMS. Thus our sequential approach considers states of the business cycle as unknown but non-stochastic, whereas DFMS treats them as latent and estimates their probabilities. Due to the model in (5) and (6), the distribution of the innovations $\mathbf{x}_t$ is given by

$$\mathbf{x}_t \sim \begin{cases} \mathcal{N}(\boldsymbol{\mu}_0, \boldsymbol{\Sigma}) & \forall t \text{ with } s_t = 0 \\ \mathcal{N}(\boldsymbol{\mu}_1, \boldsymbol{\Sigma}) & \forall t \text{ with } s_t = 1 \end{cases}, \tag{7}$$

whereas the covariance matrix $\boldsymbol{\Sigma}$ is assumed to be constant and positive definite for both business cycle phases. Thus the state variable s_t is uniquely related to the expectation vector $E(\mathbf{x}_t) = \boldsymbol{\mu}_t$. A case of a recession corresponds to $\boldsymbol{\mu}_t = \boldsymbol{\mu}_0$ and $s_t = 0$, while an expansion is described by $\boldsymbol{\mu}_t = \boldsymbol{\mu}_1$ and $s_t = 1$.

The parsimonious representation in (7) is equivalent to a change point model widely used in SPC (cf. Montgomery, 2005). A business cycle phase is reflected not by the common factor f_t as in DFMS, but by the means of the innovation process $\{\mathbf{x}_t\}$. Intuitively, both VAR and DFMS approaches presume a similar role of innovations but rely on different assumptions concerning the data generating process $\{\mathbf{z}_t\}$. The advantage of the sequential approach is a straightforward estimation which could be conducted via the OLS methodology for the VAR model in (5) and (6). The reported NBER realizations of 0/1-binary process $\{s_t\}$ exhibit compact homogenous time periods of recessions or expansions, which could have very different durations. These differences in phase durations suggest that a DFMS model with two regimes and time homogenous transition probabilities might be not the best choice for a speedy detection of NBER turning points. On the contrary, the sequential approach allows very different durations of business cycle phases in a change point framework.

For a formal presentation of the sequential decision problem assume that the current state of the business cycle at $t = 0$ is a recession, i.e. $s_{t=0} = 0$. The state variable s_t changes to a subsequent expansion only at time τ, i.e. $s_\tau = 1$ and $s_{\tau-1} = 0$ with an unknown change point $\tau \geq 1$. The distribution of $\mathbf{x}_t$ before a change (in-control distribution) with $0 \leq t < \tau$ is $\mathbf{x}_t \sim \mathcal{N}(\boldsymbol{\mu}_0, \boldsymbol{\Sigma})$.

Accordingly, the distribution of $\mathbf{x}_t$ after a change (out-of-control distribution) for $t \geq \tau$ is $\mathbf{x}_t \sim \mathcal{N}(\boldsymbol{\mu}_1, \boldsymbol{\Sigma})$. The task is to detect a change from the in- to the out-of-control distribution as soon as it occurs. SPC suggests control charts as a statistical tool for the detection of such changes. There should be a decision between the null and alternative hypotheses at each point in time t:

$$H_{0,t}: \ E(\mathbf{x}_t) = \boldsymbol{\mu}_0, \qquad H_{a,t}: \ E(\mathbf{x}_t) = \boldsymbol{\mu}_1 \neq \boldsymbol{\mu}_0, \qquad t \geq 1. \tag{8}$$

A control chart allows to differentiate between the hypotheses in (8). A one-sided or single-directional control chart consists of a control statistic Z_t and a critical value $c > 0$. A signal is given at t and hypothesis $H_{0,t}$ is rejected in favor of alternative $H_{a,t}$ if the control statistic exceeds the critical value for the first time, i.e. $Z_t > c$. The monitoring procedure is restarted after every signal. In case of no signal it is believed that hypothesis $H_{0,t}$ still holds. A signal at t is a correct one if $t \geq \tau$, otherwise it is a false alarm.

The detection problem of NBER turning points is definitely a sequential task, because it is unclear how many periods could elapse before a change would happen. This makes a main difference to a usual fixed sample test because a sample size is not fixed but random here (Woodall, 2000). Consequently, the performance of control schemes is evaluated not with the size and power measures as in the conventional test theory but with other criteria (Ghosh and Sen, 1991). A good control chart should not give a (false) signal for a long time if $t < \tau$. However, it should give a correct signal quickly if $t \geq \tau$. Both abilities of control charts to give correct signals quickly and not to give false signals for a long time are measured as a function of a run length L distribution. The run length L is a random variable defined as the time before a control chart provides a signal for the first time:

$$L \ = \ \inf\{t \geq 1, t \in \mathbb{N} \,|Z_t > c\}. \tag{9}$$

The most popular performance measure of control charts is the average run length (ARL), which is defined as the expectation $E(L)$. The recent review of other important performance measures can be found in Frisén (2003). If no change happens, i.e. $\tau = \infty$, the expectation $E_{\tau=\infty}(L)$ is called the in-control ARL. If a change happens immediately at $\tau = 1$, the expectation $E_{\tau=1}(L)$ is denoted as the out-of-control ARL. Thus the in-control ARL is defined as the average number of periods without false signals. It corresponds to the size of a usual test procedure. The out-of-control ARL is the average number of periods required for the detection of a real change, which occurs at $\tau = 1$. It is an analog to the power of a test. Naturally, it is desired to have the in-control ARL large compared to the out-of-control ARL.

3 Design of Control Chart

A control scheme suitable for our task is introduced now. Since the vector $\mathbf{x}_t$ forms the process of interest, a multivariate chart should be applied for the monitoring problem in (8). The current (in-control) state of the economy is assumed to be known at the start of the procedure. This implies the knowledge of the direction of a change, e.g. being in a recession we want to signal a change to an expansion as soon as it occurs. Moreover, since estimates of the mean vectors $\boldsymbol{\mu}_0$ and $\boldsymbol{\mu}_1$ as well as of the matrix $\boldsymbol{\Sigma}$ are taken as if they were equal to the true values, the in- and out-of-control distributions in both change directions are completely specified. All this information should be fully exploited by constructing an appropriate control chart.

A directional CUSUM procedure of Healy (1987), introduced below, satisfies our requirements for a monitoring scheme. For its presentation assume, as earlier, that the current state of the economy at $t = 0$ is a recession, i.e. $s_{t=0} = 0$. The task is to detect a change point τ such that $\tau = \inf\{t \mid s_t = 1,\ t > 0\}$, which characterizes the start of an expansion. Healy's chart grounds on the quadratic form D, which is defined as

$$D = \left[(\boldsymbol{\mu}_1 - \boldsymbol{\mu}_0)'\boldsymbol{\Sigma}^{-1}(\boldsymbol{\mu}_1 - \boldsymbol{\mu}_0)\right]^{1/2} \geq 0.$$

This non-centrality parameter D is a Mahalanobis distance between two state mean vectors. The statistic T_t of Healy's CUSUM procedure for detecting a change from a recession $\boldsymbol{\mu}_0$ to an expansion $\boldsymbol{\mu}_1$ is given by

$$T_t = (\boldsymbol{\mu}_1 - \boldsymbol{\mu}_0)'\boldsymbol{\Sigma}^{-1}(\mathbf{x}_t - \boldsymbol{\mu}_0)/D. \tag{10}$$

The in- and out-of-control distributions of T_t are derived by Healy (1987, p. 410):

$$T_t \sim \begin{cases} \mathcal{N}(0,1) & \text{in-control, } \mathbf{x}_t \sim \mathcal{N}(\boldsymbol{\mu}_0, \boldsymbol{\Sigma}) \\ \mathcal{N}(D,1) & \text{out-of-control, } \mathbf{x}_t \sim \mathcal{N}(\boldsymbol{\mu}_1, \boldsymbol{\Sigma}) \end{cases}. \tag{11}$$

The control statistic of this chart is defined by a cumulated sum Z_t, which is based on the statistic T_t and distance D:

$$Z_t = \max\{Z_0, Z_{t-1} + T_t - \gamma D\}, \quad t > 0, \quad \gamma \in [0, 1). \tag{12}$$

The proportionality factor γ, the critical value c and the starting value Z_0 are the parameters of this CUSUM chart. A signal is given at time point t if the control statistic exceeds the critical value for the first time, i.e. $Z_t > c > 0$.

The statistic for the symmetric case of a transition from an expansion with $E(\mathbf{x}_t) = \boldsymbol{\mu}_1$ to a recession with $E(\mathbf{x}_t) = \boldsymbol{\mu}_0$ is given as $T_t = -(\boldsymbol{\mu}_1 - \boldsymbol{\mu}_0)'\boldsymbol{\Sigma}^{-1}(\mathbf{x}_t - \boldsymbol{\mu}_1)/D$. The distribution result in (11) remains valid here as well. The control statistic in this case is defined as in (12) by $Z_t = \max\{Z_0, Z_{t-1} + T_t - \gamma D\}$, $t > 0$. A signal occurs at t if $Z_t > c$.

Since statistic T_t is a univariate random variable with distribution given in (11), the chart of Healy is equivalent to the one-sided univariate CUSUM chart of Page (1954). The latter is derived directly from the sequential probability ratio test of Wald. Moustakides (1986, 2008) and Ritov

(1990) prove that a one-sided univariate CUSUM scheme exhibits a range of optimality properties for a known size and a direction of a shift to be detected. In our setting the parameters and the shift direction are assumed to be known, so the chart of Healy suits for the investigated problem under the assumptions met for the process $\{\mathbf{x}_t\}$. Consequently, Healy's chart can be seen as the theoretically optimal procedure for early signaling of turning points in the considered framework. Moreover, it can be easily implemented in practice. An early economic application of a similar monitoring approach is suggested by Theodossiou (1993) for predicting bankruptcy using a vector of firm characteristics.

The conventional values $Z_0 = 0$ and $\gamma = 1/2$ are used for the implementation of Healy's chart in our study. The choice of the critical limit c depends on the in-control distribution of the run length L, defined in (9). After setting the in-control ARL equal to the predetermined value, the limit c is determined by solving the equation:

$$E(L|D, c, T_t \sim \mathcal{N}(0, 1)) \;=\; \xi. \tag{13}$$

The number $\xi > 0$ is the desired in-control ARL, related in our setup to the duration of the current business cycle phase. This equation can be solved exactly, e.g. by means of Monte Carlo simulations. However, the approximative critical value $c \approx f(\xi, D)$ can be obtained from the equation

$$\xi D^2/2 \;\approx\; \exp((1.166 + c)D) - (1.166 + c)D - 1 \tag{14}$$

for a one-sided univariate CUSUM chart with parameters $\gamma = 1/2$ and $Z_0 = 0$. This approximation is suggested by Siegmund (1985, p. 27) and is sufficiently good for our purposes. More discussion concerning the choice of critical value for Healy's chart is provided in the Appendix of the paper.

4 Data Description and Estimation

The monthly dataset used for dating and signaling of the NBER turning points starts at 1967, April and ends at 2008, October, in total 499 months. It consists of four time series of the fully revised data for the vintage in December 2008. The data is taken from the Federal Reserve (industrial production), the Bureau of Labor Statistics (employees on non-farm payrolls) and the Bureau of Economic Analysis (personal income less transfers and manufacturing and trade sales). Figure 1 presents the observed process $\{\mathbf{z}_t\}$, which is built as the first log differences of the NBER series.

[Figure 1 about here.]

The business cycle turning points are defined by NBER announcements, which are available under **http://wwwdev.nber.org/cycles/cyclesmain.html**. NBER dates 13 turning points within the time period between 1967 and 2008, namely seven peaks and six troughs. Thus, our time span contains six enclosed recessions and six enclosed expansions. The average duration of a recession has been about 11 months, while the average duration of an expansion has been about 65 months. However, there are significant differences in the duration of each particular business cycle phase. E.g., the expansion from 1980, August till 1981, July lasts only 12 months. Next we provide the estimation details for the DFMS and VAR models and report the obtained full sample parameter estimates.

4.1 DFMS model estimation

Following Chauvet and Piger, the parameters of the DFMS model are estimated with Bayesian methodology via the Gibbs sampler. The informative priors are based on the approximative maximum likelihood estimates, which are obtained by using the procedure of Kim (1994). The smoothed probabilities for the business cycle variable S_t are calculated by generating draws from the full conditional distribution, as in Kim and Nelson (1999). The full sample Bayesian estimates of the DFMS model parameters are the means of the posterior distributions. These estimates and the corresponding standard deviations of the posterior distributions are summarized in Table 1.

[Table 1 about here.]

The parameter values in Table 1 roughly correspond to their counterparts from the study of Chauvet (1998, Table 2). The mean of the expansionary state is $\mu_1 = 0.63$, while the recessionary state is characterized by the mean $\mu_0 = -3.41$. These values indicate that recessions are rather short events compared to the duration of expansions. This evidence is also supported by the obtained estimates of the transition probabilities. The expected duration of an expansion is round 30 months as the mean of the posterior distribution of p_{11} is 0.9679, see Table 1. The corresponding duration of a recession is roughly 7 months. The filtered state probabilities $P(S_t|\mathcal{I}_t)$, required for decisions at the current edge, are obtained by repeatedly using the Gibbs sampler conditional on the expanding information set $\mathcal{I}_t$ for $t: \ 1 \rightarrow T$.

4.2 VAR model estimation

Since serially uncorrelated observations are required for the purpose of sequential monitoring, we need to estimate the VAR model parameters in order to obtain the process of innovations $\{\mathbf{x}_t\}_{t=1}^T$. The available data is the observable process $\{\mathbf{z}_t\}_{t=1}^T$ as well as the NBER binary state variable s_t, pointing on recessions or expansions.

The VAR model order p is selected in accordance to those of the DFMS models (cf. Chauvet and Piger, 2008), namely $p = 2$. In our setting the parameters of the VAR model ($\phi(L)$, $\boldsymbol{\mu}_0$, $\boldsymbol{\mu}_1$, $\boldsymbol{\Sigma}$) can be estimated with the OLS methodology, see Greene (2003, p. 588ff). The parameter estimates based on the full information set $\mathcal{I}_T$ as well as the standard errors for the mean vectors and VAR coefficients $\phi(L)$ are provided in Tables 2 and 3.

[Tables 2 and 3 about here.]

The reported VAR estimates confirm that $\mu_{1,i} > \mu_{0,i}$ for all $i = 1, .., 4$. Moreover, we cannot reject the null hypothesis that the covariance matrix $\mathbf{\Sigma}$ is constant for both regimes. The full sample non-centrality parameter estimate $\hat{D} = [(\hat{\boldsymbol{\mu}}_1 - \hat{\boldsymbol{\mu}}_0)'\hat{\mathbf{\Sigma}}^{-1}(\hat{\boldsymbol{\mu}}_1 - \hat{\boldsymbol{\mu}}_0)]^{1/2}$ is equal to 1.4938. The innovation process $\{\mathbf{x}_t\}$ is calculated recursively based on the obtained parameter estimates. Thus the past true dates of the NBER business cycle s_t are not directly required in our setting for constructing the innovations $\mathbf{x}_t$.

5 Empirical Results

5.1 DFMS approach

The dating results of the full sample DFMS models are based on the information set $\mathcal{I}_T$. Smoothed probabilities for the latent variable S_t are transferred into binary decisions about business cycle phases as described in Section 2.1. Figure 2 visualizes the estimated smoothed recession probabilities $P(S_t = 0|\mathcal{I}_T)$ together with the NBER dates. Then DFMS decisions need to be compared to the NBER announcements, whereas we differentiate between false signals and delays in detection of turning points.

[Figure 2 about here.]

Full sample DFMS with asymmetric border $p^* = 0.8$ falsely identifies 19 months. This result corresponds to the dating evidence of Chauvet and Piger, because 9 of these 19 periods are before and 4 are after the time interval considered in their study. The full sample symmetric DFMS rule with border probability $p^* = 0.5$ classifies 16 months wrongly. The misclassified dates for the full sample DFMS approaches are reported in Table 4. Full sample DFMS gives 6 delays and 10 false signals for $p^* = 0.5$, 12 delays and 7 false signals for $p^* = 0.8$ methods. Although DFMS with $p^* = 0.8$ provides more misclassifications, it gives a smaller number of undesired false signals compared to DFMS with $p^* = 0.5$. The majority of misclassifications for full sample DFMS occurs in the surrounding of NBER turning points. Single false signals within recessions 1973/75 and 1981/1982 are the exceptions. They last one month only, while the DFMS dating evidence for the following months confirms the continuation of the recessions. Note that such one month mistake makes no impact on dating from the asymmetric procedure of Chauvet and Piger, because it requires 3 consecutive months with a high expansion probability for a turning point decision.

[Table 4 about here.]

Signaling at the current edge is of importance since a decision maker may be interested not only in a precise dating, but also in early signals about business cycle turning points. Now we consider signaling results for the symmetric and asymmetric conditional DFMS approaches. Since following observations (in $t+1$ etc.) can alter inferences on the latent state variable S_t, earlier decisions of a DFMS algorithm may be changed because of later information. Thus a dating of a full sample DFMS rule does not need to coincide exactly with DFMS signals at the current edge based on conditional information sets $\mathcal{I}_t$, $t \leq T$. Table 4 presents the dates, misclassified by the conditional DFMS rule, both for the symmetric $p^* = 0.5$ and asymmetric $p^* = 0.8$ probability borders.

The precision of the conditional DFMS signaling is inferior compared to the full sample DFMS dating. Symmetric conditional DFMS provides 35 falsely identified periods. There is a considerable number of the false signals (15 times) concerning peaks or troughs. On the contrary, asymmetric conditional DFMS allows only 7 false signals. However, this method provides larger delays in the detection of the turning points, 38 in total. There are some periods with severe misclassifications. E.g., the start of recession in 1973/74 shows up with a delay of 11 months by $p^* = 0.8$ method, while $p^* = 0.5$ method signals wrongly 6 times at the beginning of 1974. Furthermore, the conditional DFMS methods have problems with signaling the start of the current recession, dated by NBER in December 2007. Note that both full sample DFMS methods have problems with precise dating in these periods too. Rather severe mistakes also occur by DFMS signaling the turning points in 1982 and in 1990.

5.2 Sequential monitoring

The sequential methodology for early signaling of changes in the business cycle is implemented as described in Section 3. The VAR parameter estimates from Tables 2 and 3 are used for extracting the innovation process $\{\mathbf{x}_t\}$. Its mean vectors are assumed to correspond to the business cycle phases, as in (7). The charts for detecting an expansion after a recession (a change from $E(\mathbf{x}_t) = \boldsymbol{\mu}_0$ to $E(\mathbf{x}_t) = \boldsymbol{\mu}_1$) with the critical value c_0 and for detecting a recession after an expansion (a change from $E(\mathbf{x}_t) = \boldsymbol{\mu}_1$ to $E(\mathbf{x}_t) = \boldsymbol{\mu}_0$) with the critical value c_1 are started simultaneously at the beginning of the dataset in 1967. Both charts are restarted after every signal with the value $S_0 = 0$. The time evolution of the control statistics for these charts is presented on Figure 3.

[Figure 3 about here.]

Signals occur at dates where the control statistics cross the critical limits c_0 and c_1, denoted as the horizontal lines on Figure 3. As expected, there are much more signals indicating on a current expansion (left) than on a current recession (right). Note that if the current state is assumed to be an expansion, a signal for an expansion due to crossing the limit c_0 would just support this assumption. Since according to NBER the average duration of an expansion is larger than of a recession, it is desired to have $c_1 \geq c_0$, i.e. it should be harder to signal the start of a recession. The best signaling performance is achieved for the critical values around $c_0 = 0.74$ for detecting expansions and $c_1 = 0.94$ for detecting recessions. They correspond to the average duration (the in-control ARL) about 12 months for a recession phase and roughly 17 months for an expansion phase, respectively. Note that the discussed sequential procedure is more sensitive with respect to the choice of c_1, which is responsible for the detection of a recession following a currently assumed expansion, compared to the choice of c_0. Figure 3 (right) illustrates this statement by showing that a further increase of the limit c_1 would make detection of recessions problematic starting from "the great moderation" period in the mid 80s.

The last column in Table 4 provides the dates of delays and false signals for the control charts with the best pair of critical values $c_0 = 0.74, c_1 = 0.94$. The total number of falsely identified months is 33, which consists of 20 delays in the detection and 13 false signals. The largest difficulties arise by signaling recession 1973/1974, where the detection delay lasts 8 months. Moreover, there is a false signal with 3 misclassified periods in 2003. However, the control chart shows a good performance for the recent recession by signaling already in February 2008, while the conditional DFMS approaches provide signals not earlier than in August 2008.

The performance of control charts depends heavily on the choice of the critical values. The best signaling ability of the control charts is found for the critical values $c_0 \in [0.6, 0.9]$, which corresponds to $[9.3, 16.0]$ months for the average duration of a recession, and for $c_1 \in [0.7, 1.0]$ which is about $[11.2, 19.0]$ months for the average duration of an expansion. These values allow to keep the number of falsely identified months below 40, which is comparable to the conditional DFMS results. The signaling ability of the control charts as a percent of correctly dated months is illustrated on Figure 4 depending on the choice of the critical values c_0, c_1.

[Figure 4 about here.]

The overall results for signaling and/or dating performance of the considered approaches are summarized in Table 5. The goodness is evaluated by counting the number of misclassified periods. As earlier, we count separately false signals and detection delays, whereas the former are less desired. The ultimate goal remains, of course, to get no falsely identified periods at all.

[Table 5 about here.]

Although the conditional DFMS method and control charts exhibit a similar number of misclassified periods, the results for both approaches are much worse than for the full sample DFMS dating. This is not surprising, because full sample DFMS should be more precise due the availability of future information for inferences. Note, however, that there are months when all considered methods fail to mimic the decisions of NBER, such as August 1990 (see Table 4). The performance of the current edge signaling methods could be partially explained by the non-fulfillment of the assumptions for the underlying statistical models. For example, the phenomenon of GDP volatility reduction since the middle 80s, known as “the great moderation”, violates the assumption about the constancy of the covariance matrix both for the DFMS models and control charts. This volatility decline can be well observed in the time series $\mathbf{z}_t$, presented on Figure 1. Moreover, although it holds for the whole period in the study that $\mu_{1,i} > \mu_{0,i}$, $i = 1, ..., 4$, the conditional dynamics for the state means could also be of interest. Much more elaborated econometric models of the NBER business cycle (see, e.g. DeJong et al., 2005, 2009) should be considered in order to take these features into account. These issues are, however, far beyond the scope of the paper and remain for further research.

Our study shows that conditional DFMS and the suggested control chart provide roughly similar goodness of timely signaling using the fully revised data. However, the implementation of the sequential approach is more transparent. The obtained evidence suggests that DFMS and control charts amend and enhance one another. Thus they should be considered simultaneously for the purpose of early warnings of NBER turning points.

6 Summary

This paper elaborates methods for early warnings about the NBER business cycle turning points based on information at the current edge. The established DFMS approach of Chauvet and Piger (2008) for dating in real time is modified to enable signals at t using only the conditional information set $\mathcal{I}_t$. Moreover, we propose a novel approach based on sequential process monitoring for providing timely signals about changes of the business cycle phase. The cumulated sum control chart of Healy (1987) is adapted for on-line monitoring of the innovation mean vector, assumed to be responsible for a business cycle state. A signal from the chart could be interpreted as an evidence for a turning point. Our monitoring methodology has advantages to be stable, transparent and easy to implement.

Four monthly NBER time series (industrial production, manufacturing and trade sales, personal income less transfers and employment) of fully revised data are exploited for empirical evaluation of early signaling algorithms. The empirical results confirm that the developed sequential approach is able to provide quick and precise warnings about business cycle peaks and troughs. The number of periods, misclassified by the control chart over the whole sample is comparable to those from the current edge DFMS methods. Our control charts also show a better signaling ability compared to conditional DFMS for a recent time period from the mid 80s. These findings support the usefulness of the suggested sequential procedure. It should be seen as an amendment to the DFMS approach for a more precise early signaling of U.S. business cycle turning points.

Appendix

An analytical approximation for both in- and out-of-control ARLs of one-sided CUSUM schemes is suggested by Siegmund (1985, p.27ff) for the CUSUM parameters $\gamma = 1/2$ and $Z_0 = 0$. The ARL can be expressed as a function of the non-centrality parameter $D = [(\boldsymbol{\mu}_1 - \boldsymbol{\mu}_0)'\boldsymbol{\Sigma}^{-1}(\boldsymbol{\mu}_1 - \boldsymbol{\mu}_0)]^{1/2}$, the actual shift D^* and the critical value c. The approximation for the ARL is given as

$$ARL(c, \Delta) \approx \frac{\exp(-2\Delta b) + 2\Delta b - 1}{2\Delta^2}, \qquad b = 1.166 + c, \tag{15}$$

where the normalized shift is defined as $\Delta = D^* - D/2$ for a chart aiming to detect a transition from $\boldsymbol{\mu}_0$ to $\boldsymbol{\mu}_1$, $\boldsymbol{\mu}_1 > \boldsymbol{\mu}_0$, and $\Delta = -D^* - D/2$ for a chart detecting a transition from $\boldsymbol{\mu}_1$ to $\boldsymbol{\mu}_0$.

In our situation the actual shift is $D^* = 0$ for the in-control situation. This immediately yields equation (14). The actual out-control shift is $D^* = D$ for detecting expansions and $D^* = -D$ for detecting recessions. Consequently, the normalized shift Δ is given for both expansion and recession charts as

$$\Delta = \begin{cases} -D/2, & \text{in-control,} \\ D/2, & \text{out-of-control.} \end{cases} \tag{16}$$

This means, the charts for detection of expansions and recessions exhibit the same behavior, i.e. they are identical from SPC viewpoint. The approximated in- and out-of-control ARLs are plotted as a function of c on Figure 5 for the distribution parameters given in Table 2.

[Figure 5 about here.]

The detailed discussion of these issues can be found in Montgomery (2005, p. 390ff).

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λ_i	0.3470 (0.0358)	0.4133 (0.0477)	0.1860 (0.0319)	0.1860 (0.0252)
σ_i^2	0.3791 (0.0335)	0.3499 (0.0486)	0.7905 (0.0479)	0.7905 (0.0546)
$\rho_{i,1}$	0.1397 (0.0586)	-0.1512 (0.0659)	-0.3261 (0.0521)	-0.3261 (0.0479)
$\rho_{i,2}$	0.3496 (0.0584)	-0.0749 (0.0612)	-0.0599 (0.0499)	-0.0599 (0.0484)
	$\gamma_1 = 0.6321$ (0.1119)		$\gamma_2 = -0.0132$ (0.0707)	
	$\mu_1 = 0.6305$ (0.2019)		$\mu_0 = -3.4103$ (0.5973)	
	$p_{11} = 0.9679$ (0.0127)		$p_{00} = 0.8541$ (0.0544)	

Table 1: Bayesian posterior full sample parameter estimates for the DFMS model: the expectations with standard deviations of posterior distributions underset in parenthesis.

	$\boldsymbol{\mu}_1$	$\boldsymbol{\mu}_0$	$\boldsymbol{\Sigma}$			
z_1	0.0939 (0.0115)	−0.0903 (0.0184)	0.0215	0.0453	0.0483	0.0127
z_2	0.2020 (0.0491)	−0.5433 (0.0785)	0.0453	0.3924	0.2701	0.0679
z_3	0.3840 (0.0698)	−0.5677 (0.1116)	0.0483	0.2701	0.7942	0.0925
z_4	0.3163 (0.0418)	0.0406 (0.0669)	0.0127	0.0679	0.0925	0.2855

Table 2: Distribution parameters of innovation process $\{\mathbf{x}_t\}$ based on the full sample information. Standard errors are underset in parenthesis. The non-centrality parameter $D = 1.4938$. The variables z_1, z_2, z_3, z_4 denote nonfarm payroll employment, industrial production, manufacturing and trade sales, and personal income less transfers, respectively.

	z_1	z_2	z_3	z_4
$\phi_1^{(1)}$	0.1388 (0.0473)	0.4825 (0.2020)	0.1541 (0.2873)	0.3203 (0.1723)
$\phi_2^{(1)}$	0.0366 (0.0128)	−0.0083 (0.0545)	0.2009 (0.0776)	0.0650 (0.0465)
$\phi_1^{(2)}$	−0.0013 (0.0084)	0.0456 (0.0358)	−0.3493 (0.0510)	0.0121 (0.0305)
$\phi_2^{(2)}$	0.0105 (0.0126)	0.1455 (0.0537)	0.2251 (0.0763)	−0.3400 (0.0458)
$\phi_1^{(3)}$	0.3445 (0.0451)	−0.2541 (0.1925)	−0.4607 (0.2739)	0.2155 (0.1642)
$\phi_2^{(3)}$	−0.0165 (0.0130)	−0.0363 (0.0557)	0.0888 (0.0792)	−0.0031 (0.0475)
$\phi_1^{(4)}$	−0.0042 (0.0083)	0.1205 (0.0355)	−0.0951 (0.0505)	0.0172 (0.0303)
$\phi_2^{(4)}$	0.0161 (0.0127)	0.0638 (0.0540)	0.1044 (0.0769)	−0.0898 (0.0461)

Table 3: VAR(2) model parameter estimates $\phi(L)$ based on the full sample information. Standard errors are underset in parenthesis.

NBER dates		(i) full 0.8		(ii) full 0.5		(iii) $p^* = 0.8$		(iv) $p^* = 0.5$		(v) Control Chart	
before	after	signal	fails	signal	fails	signal	fails	signal	fails	signal	fails
		196911	2	*196911*	2			*196911*	1	*196911*	1
196912 (1)	197001 (0)	197001	0	197001	0	197001	0	197001	0	197001	0
								197003	1	*197002*	3
		197011	1	*197011*	1			*197007*	1		
197011 (0)	197012 (1)	197012	0	197012	0	197012	0	197012	0	197012	0
197311 (1)	197312 (0)	197406	6	197312	0	197411	11	197401	1	197408	8
								197402	1		
				197405	1			*197404*	4		
197503 (0)	197504 (1)	197504	0	197504	0	197505	1	197505	1	197505	1
						197801	1	*197801*	1	*197801*	1
						197904	1	*197904*	1	*197904*	1
198001 (1)	198002 (0)	198002	0	198002	0	198004	2	198004	2	198004	2
		198007	1	*198007*	1						
198007 (0)	198008 (1)	198008	0	198008	0	198008	0	198008	0	198009	0
								198102	1		
198107 (1)	198108 (0)	198108	0	198108	0	198111	3	198110	2	198109	1
				198202	1	*198202*	4	*198202*	2	*198202*	1
198211 (0)	198212 (1)	198212	0	198212	0	198301	1	198301	1	198301	1
								198302	1	*198302*	1
										198308	1
199007 (1)	199008 (0)	199009	1	199009	1	199012	4	199011	3	199009	1
199103 (0)	199104 (1)	199104	0	199104	0	199105	1	199105	1	199105	1
		200101	3	*200101*	3						
200103 (1)	200104 (0)	200104	0	200104	0	200107	3	200104	0	200104	0
200111 (0)	200112 (1)	200201	1	200201	1	200204	4	200202	2	200204	4
										200303	3
200712 (1)	200801 (0)	200805	4	200805	4	200809	8	200808	7	200802	1
				200810	1	*200810*	1	*200810*	1	*200810*	1

Table 4: Dates of signals and number of misclassified months for NBER turning points: (i-ii) DFMS full sample with borders $p^* = 0.8$ and $p^* = 0.5$; (iii) conditional DFMS $p^* = 0.8$; (iv) conditional DFMS $p^* = 0.5$; (v) CUSUM chart with critical values $c_0 = 0.74$ and $c_1 = 0.94$; (1) denotes expansion, (0) recession; dates of false signals are in *italic*.

method	$\sum$ delays	$\sum$ false signals	totally
full sample DFMS $p^* = 0.8$	12	7	19
full sample DFMS $p^* = 0.5$	6	10	16
conditional DFMS $p^* = 0.8$	38	7	45
conditional DFMS $p^* = 0.5$	20	15	35
$c_0 = 0.74$, $c_1 = 0.94$	20	13	33
$c_0 \in [0.6, 0.9]$, $c_1 \in [0.7, 1.0]$	$\leq 21^*$	$\leq 22^\dagger$	≤ 40

Table 5: Number of delays and falsely identified months compared to NBER dates for DFMS and CUSUM chart methods; * falsely concluded for recession, $\dagger$ falsely concluded for expansion.

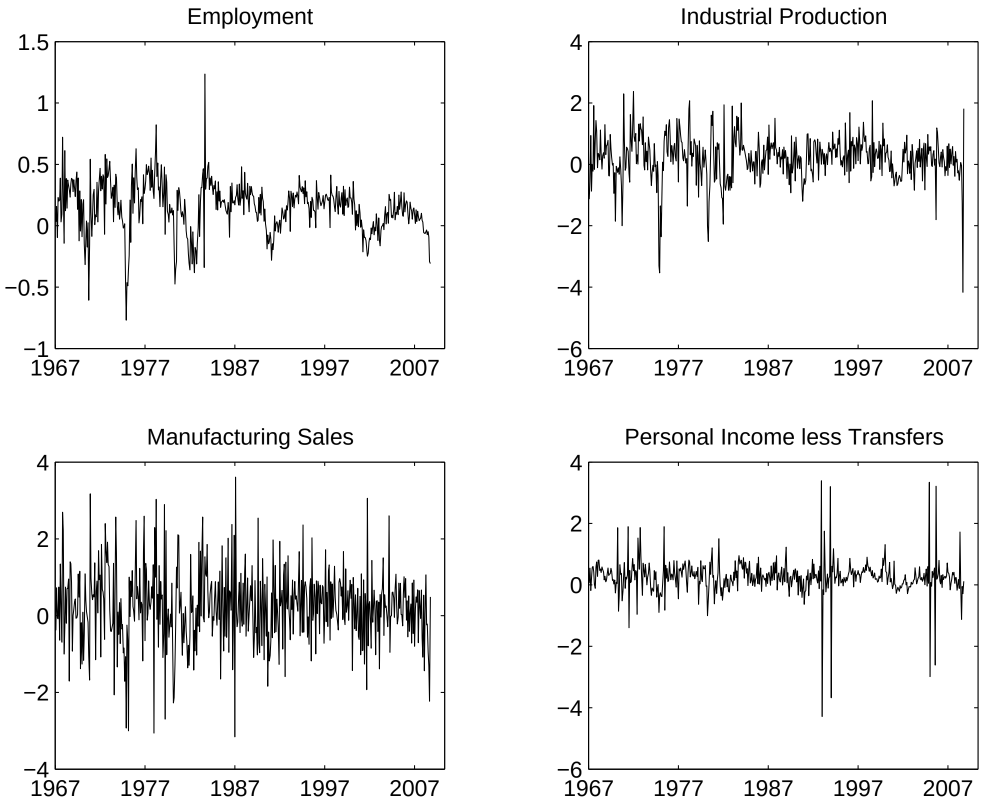

Figure 1: Fully revised time series $\mathbf{z}_t$ for vintage in December 2008, built as the first log differences of industrial production, retail sales, employment and personal income, monthly data for 1967-2008.

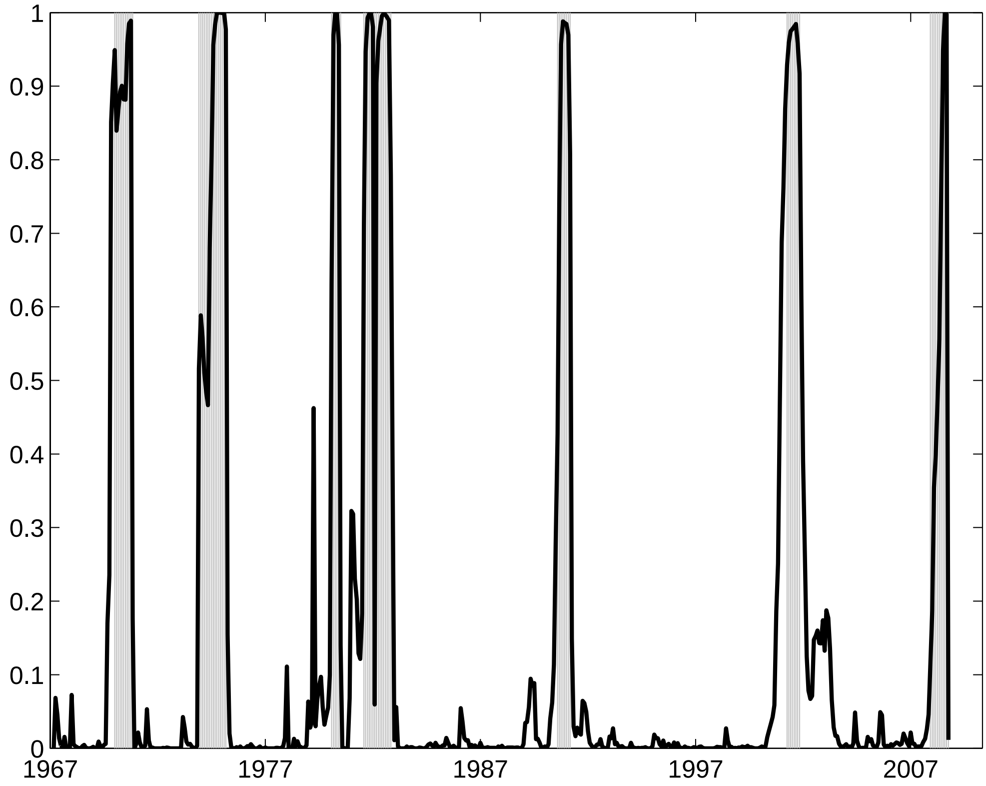

Figure 2: Estimated DFMS full sample probabilities of a recession $P(S_t = 0|\mathcal{I}_T)$ (bold line) and the true NBER recession dates shown in the gray area for the period 1967-2008.

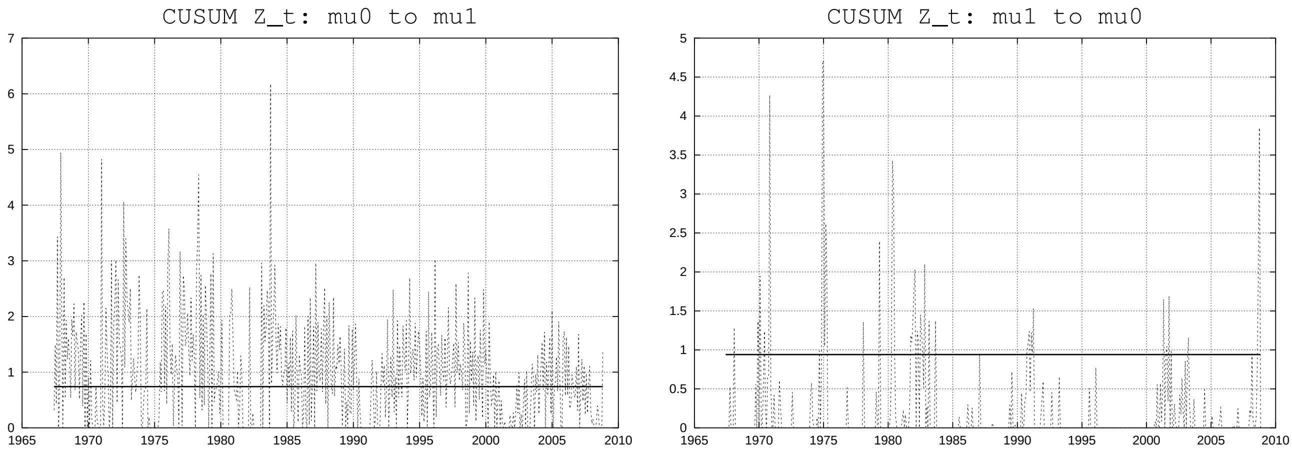

Figure 3: CUSUM Z_t statistics for detecting the shift from $\boldsymbol{\mu}_0$ to $\boldsymbol{\mu}_1$ (recession to expansion, left) and from $\boldsymbol{\mu}_1$ to $\boldsymbol{\mu}_0$ (expansion to recession, right). The horizontal lines denote the critical values $c_0 = 0.74$ (left) and $c_1 = 0.94$ (right).

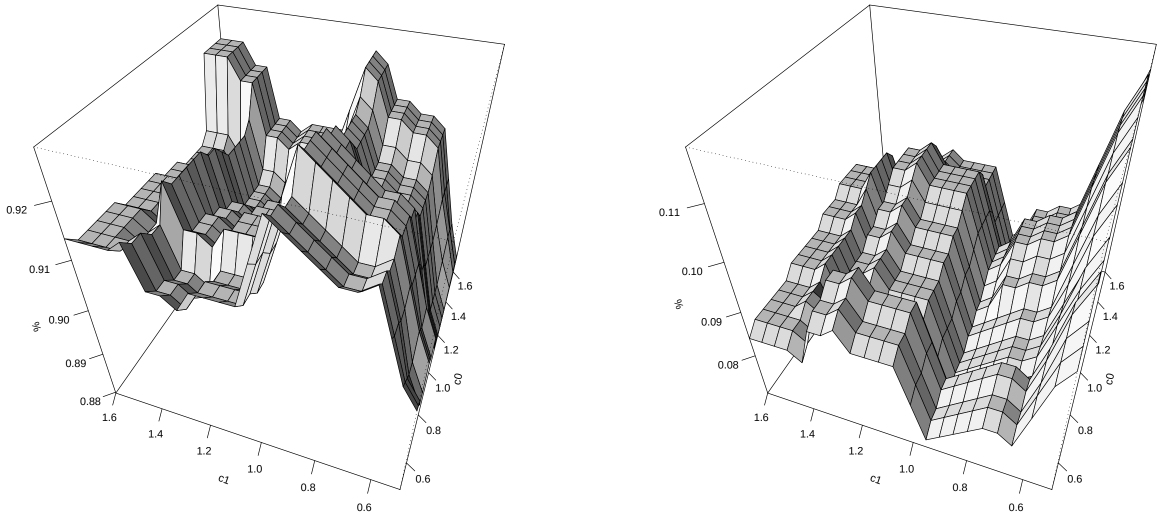

Figure 4: Proportion of correctly (left) and falsely (right) identified months compared to NBER dates for Healy's chart as a function of critical values c_0, c_1.

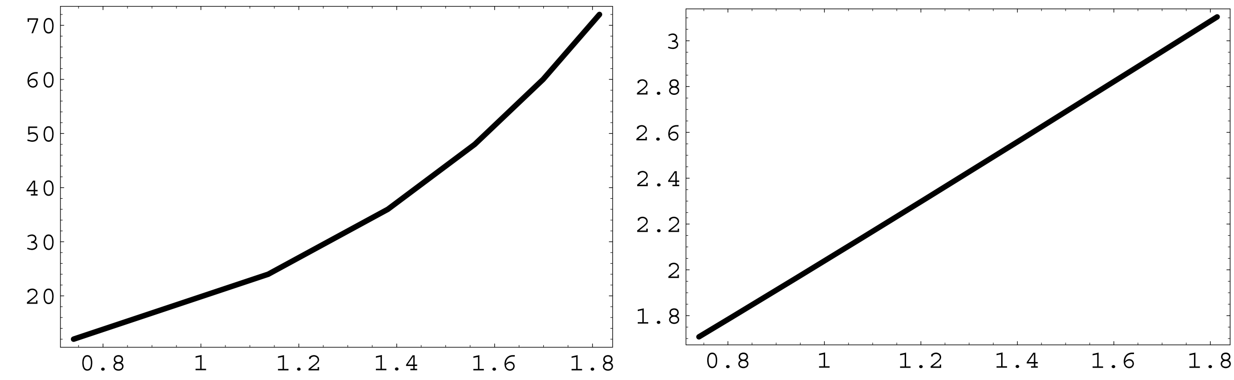

Figure 5: ARL(c): in- (left) and out-of-control ARL (right) as a function of c. The distribution parameters are taken from Table 2.