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Abstract

We show that low trend inflation strongly affects the dynamics of a standard Neo-Keynesian model where monetary policy is described by a standard Taylor rule. Moreover, trend inflation enlarges the indeterminacy region in the parameter space, substantially altering the so-called Taylor principle. The main results hold for different types of Taylor rules, inertial policy rules and indexation schemes. The key message is that, whatever the set up, the literature on Taylor rules cannot disregard average inflation in both theoretical and empirical analysis.

JEL classification: E31, E52.

Keywords: Sticky Prices, Taylor Rules and Trend Inflation

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1 Introduction

Average inflation in the post-war period in developed countries was moderately different from zero and varied across countries.[1] Nonetheless, much of the vast literature on monetary policy rules worked with models log-linearized around a zero inflation steady state (see e.g., Clarida et al., 1999, Galí, 2003, Woodford, 2003, or the book edited by Taylor, 1999).

To address this inconsistency, we extend the standard small scale Neo-Keynesian model to allow for positive trend inflation.[2] First, we add a Taylor rule to describe the monetary authority behavior and then, examine how the properties of our economy change as the trend inflation varies. We find that moderate levels of trend inflation: (i) modify the determinacy region in the parameters space; (ii) alter the impulse response function of the model economy after a cost-push shock. As a consequence, trend inflation significantly affects also the (unconditional) variances of key variables, such as inflation and output.

With respect to (i), we show that trend inflation substantially changes the well-known Taylor principle for equilibrium determinacy under rational expectations. This result is due to the relative prices distortions that trend inflation causes in the steady state of the model, a surprisingly neglected issue in the literature. The long-run Phillips curve is highly non-linear in the Neo-Keynesian model. It is positively sloped in the inflation-output plane, when steady state inflation is zero. However, because of the strong price-dispersion effect, the slope turns quite rapidly negative for extremely low values of trend inflation. We will show that this feature has significant implications for the celebrated Taylor principle. The results in most of the literature therefore are based on a case (i.e., zero steady state inflation) that is both empirically unrealistic and theoretically special.

Our key result is then generalized and proved to be robust to: (i) different kinds of Taylor type rules proposed in the literature (contemporaneous, backward-looking, forward-looking and hybrid, see e.g., Clarida et al., 2000, Bullard and Mitra, 2002); (ii) inertial Taylor rules for all the cases in (i); (iii) indexation schemes (see, e.g., Yun, 1996 and Christiano et al., 2005); (iv) different parameter values.

[1] For example, Schmitt-Grohe and Uribe (2004a,b) calibrate trend inflation for the U.S. as 4.2%, based on data from 1960-1998. In the same period, Germany, Italy, Spain, and the UK exhibited average inflation rates of 3.22%, 8.12%, 7.1% and 9% respectively (source: OECD).

[2] Here, we abstract for other possible form of frictions, since we want to investigate the relationship between Taylor rules and trend inflation. In what follows, we shall use indifferently trend inflation or long-run inflation to denote the inflation rate in the deterministic steady state.

In summary, this research shows that the literature on monetary policy rules cannot neglect trend inflation, as the specification of the theoretical model (and the results) is very sensitive to low and moderate trend inflation levels, as generally observed empirically in western countries.

The seminal analysis in Clarida et al. (2000), which might be misleading, can be taken as an example. Indeed, Clarida et al. (2000) data set features an average inflation of roughly 4% the US economy, quite different from zero inflation (see Table II, p.157, therein). Their analysis, however, is based on a theoretical model that assumes zero trend inflation. On the one hand, positive trend inflation changes the determinacy region, such that one needs to take trend inflation into account in order to label the equilibrium determinate. On the other hand, once an equilibrium is identified to pass from determinate to indeterminate or vice versa, it is yet to be investigated whether this is due to a change in the monetary policy regime (i.e., a change in the Taylor rule parameters) and/or to a change in the trend inflation level.

A further contribution of the paper is to provide a "neat" presentation of the standard log-linear Neo-Keynesian model approximated around a general trend inflation level with and without indexation schemes. As such, this article generalizes the model in Ascari and Ropele (2004) by allowing for indexation schemes, and complements a series of other recent papers. Indeed, not many articles in the literature investigate the effects of different levels of trend inflation on the standard Neo-Keynesian model.[3]

King and Wolman (1996) and Ascari (1998) are early papers that look at the effects of trend inflation on the properties of the steady state of such a model. Following these contributions, Graham and Snower (2004a,b) and Karanassou et al. (2005) study the long-run relationship between inflation and output in the Neo-Keynesian framework. Ascari (2004) examines, instead, the effects of trend inflation on the dynamics of the standard model. The analysis in Ascari (2004) is extended by Amano et al. (2005). Ascari and Ropele (2004) analyzes how optimal short-run monetary policy changes with trend inflation. Cogley and Sbordone (2005) estimates the New Keynesian Phillips Curve allowing for trend inflation. The key finding by Cogley and Sbordone (2005) is that once shifts in trend inflation are properly taken into account, the NKPC is structural. That is, a Calvo pricing model with constant parameters fits the data very well with no need for indexation or backward-looking component.

[3]A few papers do allow for non-zero steady state inflation in their analysis, but they do not look at what happens when trend inflation changes. Khan et al. (2003) solve the optimal monetary policy problem and then investigate the dynamics of the economy around the given optimal steady state inflation level. Schmitt-Grohe and Uribe (2004a,b) simulates the model under different Taylor type rules calibrating average inflation on US data.

Finally, Kiley (2004) and Hornstein and Wolman (2005) are the two most related paper to ours. Kiley (2004) investigates the effect of trend inflation of a model in which prices are staggered a là Taylor (1979) and monetary policy is described by Taylor rules. Hornstein and Wolman (2005) looks at a model similar to Kiley (2004), but extended to allow for firm-specific capital. Our paper complements these very recent papers by assuming the more popular Calvo (1983) staggered pricing framework, and by generalizing the results to different Taylor type rules and indexation schemes.

2 The Model

In this section, we extend the basic New Keynesian framework of Clarida et al. (1999), Galí (2003) and Woodford (2003) to allow for positive trend inflation and price indexation.

Households

Households live forever and their expected lifetime utility is:

$$E_0 \sum_{t=0}^{\infty} \beta^t \left[\log C_t + \chi_m \frac{(M_t/P_t)^{1-\sigma_m} - 1}{1-\sigma_m} - \chi_n \frac{N_t^{1+\sigma_n}}{1+\sigma_n} \right], \tag{1}$$

where $\beta \in (0,1)$ is the subjective rate of time preference and E_0 is the expectation operator conditional on time $t = 0$ information. The instantaneous utility function is increasing in the consumption of a final good (C_t) and real money balances (M_t/P_t) and decreasing in labor (N_t). The positive parameters σ_m and σ_n represent inverse intertemporal elasticity of substitution in real money balances and labor respectively; χ_m and χ_n are positive constants. At a given period t, the representative household faces the following nominal flow budget constraint

$$P_t C_t + M_t + B_t \leq P_t w_t N_t + M_{t-1} + (1 + i_{t-1}) B_{t-1} + F_t + T_t \tag{2}$$

where P_t is the price of the final good, B_t represents holding of bonds offering a one-period nominal return i_t, w_t is the real wage, and F_t are firms' profits that are returned to households. In addition, each period the government makes lump-sum nominal transfers to households of T_t. The household's problem is to maximize (1) subject to the sequence of budget constraints (2), yielding the following first order conditions:

$$\begin{aligned}
\textit{labor supply} &: \quad \chi_n N_t^{\sigma_n} C_t = w_t, && (3)\\
\textit{money demand} &: \quad \chi_m (M_t/P_t)^{-\sigma_m} C_t = i_t / (1 + i_t), && (4)\\
\textit{consumption Euler eq.} &: \quad 1/C_t = \beta E_t [1/C_{t+1} (1 + i_t) P_t/P_{t+1}]. && (5)
\end{aligned}$$

Equations (3), (4), (5) have the usual economic interpretation.

Final Good Producers

In each period t, a final good Y_t is produced by perfectly competitive firms, using a continuum of intermediate inputs $Y_{i,t}$ and a standard CES production function $Y_t = \left[\int_0^1 Y_{i,t}^{(\theta-1)/\theta} di\right]^{\theta/(\theta-1)}$, with $\theta > 1$. Taking prices as given, the final good producer chooses intermediate good quantities $Y_{i,t}$ to maximize profits, resulting in the usual demand schedule: $Y_{i,t} = (P_{i,t}/P_t)^{-\theta} Y_t$. The zero profit condition of final good producers leads the aggregate price index $P_t = \left[\int_0^1 P_{i,t}^{1-\theta} di\right]^{1/(1-\theta)}$.

Intermediate Goods Producers

Intermediate inputs $Y_{i,t}$ are produced by a continuum of firms $i \in [0,1]$ with technology $Y_{i,t} = N_{i,t}$. Prices are sticky, with intermediate goods producers in monopolistic competition setting prices according to a standard discrete-time version of the Calvo (1983) mechanism. In each period there is a fixed probability $(1-\alpha)$ that a firm can re-optimize its nominal price, i.e., $P_{i,t}^*$. With probability α, instead, the firm must: *either* keep its nominal price unchanged; *or* index its nominal price to steady state inflation (e.g., Yun (1996)); *or* index its nominal price to past inflation rate (e.g., Christiano et al. (2005)). In general, the problem of a price-resetting firm can be formulated as

$$\max_{p_{i,t}^*} \quad E_t \sum_{j=0}^{\infty} \alpha^j \mathcal{D}_{t,t+j} \left[\frac{P_{i,t}^* \left(\overline{\pi}^{\varepsilon j}\right)^{1-\Im} \left(\Pi_{t,t+j-1}^{\varepsilon}\right)^{\Im}}{P_{t+j}} Y_{i,t+j} - \Gamma_{i,t+j} \right],$$

$$\text{s.t. } Y_{i,t+j} = \left[\frac{P_{i,t}^* \left(\overline{\pi}^{\varepsilon j}\right)^{1-\Im} \left(\Pi_{t,t+j-1}^{\varepsilon}\right)^{\Im}}{P_{t+j}} \right]^{-\theta} Y_{t+j} \text{ and} \tag{6}$$

$$\Pi_{t,t+j-1} = \begin{cases} \left(\frac{P_t}{P_{t-1}}\right)\left(\frac{P_{t+1}}{P_t}\right) \times \cdots \times \left(\frac{P_{t+j-1}}{P_{t+j-2}}\right) & \text{for for } j = 1, 2, \cdots \\ 1 & \text{for } j = 0. \end{cases} \tag{7}$$

where $\Gamma_{i,t}$ is the real total cost function, $\mathcal{D}_{t,t+j}$ is the stochastic discount factor, $\overline{\pi}$ is the level of trend inflation (introduced below), $\Pi_{t,t+j-1}$ represents the *cumulative gross inflation rates* (CGIR, hereafter), and $\varepsilon \in [0,1]$ captures the degree of price indexation. $\Im$ is an indicator function that takes a value of either zero or one. $\Im = 1$ and $\varepsilon \in [0,1]$ yield price indexation to past inflation; $\Im = 0$ and $\varepsilon \in [0,1]$ yield price indexation to trend inflation; finally, $\varepsilon = 0$ yields the case of no price indexation. The solution is a formula for the optimal reset price:[4]

$$\frac{P_{i,t}^*}{P_t} = \frac{\theta}{\theta-1} \frac{E_t \sum_{j=0}^{\infty} \alpha^j \mathcal{D}_{t,t+j} \left\{ \Pi_{t+1,t+j}^{\theta} Y_{t+j} \Gamma_{t+j}' \left(\overline{\pi}^{-\theta\varepsilon j} \right)^{1-\Im} \left(\Pi_{t,t+j-1}^{-\theta\varepsilon} \right)^{\Im} \right\}}{E_t \sum_{j=0}^{\infty} \alpha^j \mathcal{D}_{t,t+j} \left\{ \Pi_{t+1,t+j}^{\theta-1} Y_{t+j} \left[\overline{\pi}^{(1-\theta)\varepsilon j} \right]^{1-\Im} \left[\Pi_{t,t+j-1}^{(1-\theta)\varepsilon} \right]^{\Im} \right\}}, \tag{8}$$

where $\Gamma_t' \equiv \partial \Gamma_t (Y_{i,t}) / \partial Y_{i,t}$ denotes the real marginal costs function, that is simply equal to the real wage, given the linear technology.

To fully understand the effects of trend inflation on the optimal resetted price, it is insightful to look at the case of no indexation (i.e., $\varepsilon = 0$), for which the equation (8) becomes

$$\frac{P_{i,t}^*}{P_t} = \frac{\theta}{\theta-1} \frac{E_t \sum_{j=0}^{\infty} \alpha^j \mathcal{D}_{t,t+j} \left\{ \Pi_{t+1,t+j}^{\theta} Y_{t+j} \Gamma_{t+j}' \right\}}{E_t \sum_{j=0}^{\infty} \alpha^j \mathcal{D}_{t,t+j} \left\{ \Pi_{t+1,t+j}^{\theta-1} Y_{t+j} \right\}}, \tag{9}$$

and then focus on the steady-state behavior of (9). In the standard case of zero trend inflation, $\overline{\pi} = 1$ and the CGIRs attached to future expected terms are equal to one at all times. Future expected terms are discounted by $\alpha\beta$. With positive trend inflation, $\overline{\pi} > 1$ and two effects come into play. First, CGIRs at different time horizons shift upwards, changing the effective discount factors $\alpha\beta\overline{\pi}^{\theta}$ and $\alpha\beta\overline{\pi}^{\theta-1}$ in the numerator and denominator respectively. Accordingly, when intermediate firms are free to adjust, they will set higher prices to try to offset the erosion of relative prices and profits that trend inflation automatically creates. Second, future terms in (9) are progressively multiplied by larger CGIRs. This means that optimal price-setting under trend inflation reflects future economic conditions more than short-run cyclical variations. Price-setting firms become more "forward-looking", as does inflation. These are the main effects of trend inflation on the dynamics of the model. Extending the same reasoning to (8), it is easy to see that indexation mitigates the two effects just described (in steady state the two cases $\Im = 0$ and $\Im = 1$ are equivalent).

Relative price dispersion and real marginal costs

At the level of intermediate firms, it holds true that $(P_{i,t}/P_t)^{-\theta} Y_t = N_{i,t}$. Integrating this expression over i yields $Y_t s_t = \int_0^1 N_{i,t} di = N_t$, where the variable $s_t \equiv \int_0^1 (P_{i,t}/P_t)^{-\theta} di$. In other words, the variable s_t measures the relative price dispersion

[4]In a deterministic steady state equation (8) converges if and only if $\alpha\beta\overline{\pi}^{\theta(1-\varepsilon)} < 1$. Given α, β and ε, this condition constrains the maximum level of trend inflation. Throughout the analysis, we will therefore look at levels of trend inflation that meet this restriction. The case of full price indexation to past inflation is discussed in details in Ropele (2007). See also Sahuc (2005) for the partial indexation case.

across intermediate firms and can be shown to evolve as

$$s_t = (1-\alpha)\left(\frac{P_{i,t}^*}{P_t}\right)^{-\theta} + \alpha\left[\frac{(P_t/P_{t-1})}{(\overline{\pi}^{\varepsilon})^{\Im}(P_{t-1}/P_{t-2})^{\varepsilon(1-\Im)}}\right]^{\theta} s_{t-1}. \tag{10}$$

Schmitt-Grohé and Uribe (2004b) shows that s_t is bounded below at one. s_t represents the resource costs (or inefficiency loss) due to relative price dispersion under the Calvo mechanism: the higher s_t, the more labour is needed to produce a given level of output. The variable s_t directly affects the real marginal costs via the the labor supply equation (3): $\Gamma_t' = w_t = \chi_n Y_t^{\sigma_n} s_t^{\sigma_n} C_t$.

Government

The government injects money into the economy through nominal transfers, so $T_t = M_t^s - M_{t-1}^s$where M^s is the aggregate nominal money supply. Most importantly, we assume that steady state money supply evolves according to the following fixed rule: $M_t^s = \overline{\pi} M_{t-1}^s$, where $\overline{\pi}$ is the (gross) steady-state growth rate of the nominal money supply.

Market clearing conditions

The market clearing conditions in the goods, money and labour markets are: $Y_t = C_t$; $Y_{i,t}^s = Y_{i,t}^D = [P_{i,t}/P_t]^{-\theta} Y_t$, $\forall i$; $M_t = M_t^s$; and $N_t = \int_0^1 N_{i,t} di$.

3 A generalized New Keynesian Phillips Curve

Log-linearizing (3) and (5), and using the market clearing condition $\hat{Y}_t = \hat{C}_t$, we obtain:

$$\sigma_n \widehat{N}_t + \widehat{Y}_t = \widehat{w}_t, \tag{11}$$

$$\widehat{Y}_t = E_t \widehat{Y}_{t+1} - \left(\widehat{i}_t - E_t \widehat{\pi}_{t+1}\right), \tag{12}$$

where hatted variables denote percentage deviations from deterministic steady state.

The log-linearization of equations (8) and (10) leads to a system of three first-order difference equations that characterize the generalized NKPC under trend inflation (and price indexation):

$$\begin{cases} \boldsymbol{\Delta}_t = \beta \overline{\pi}^{1-\varepsilon} E_t \boldsymbol{\Delta}_{t+1} + \kappa \widehat{Y}_t + \kappa \frac{\sigma_n}{1+\sigma_n} \widehat{s}_t + \eta E_t \left[(\theta-1)\boldsymbol{\Delta}_{t+1} + \widehat{\phi}_{t+1}\right] \\ \\ \widehat{\phi}_t = \alpha\beta \overline{\pi}^{(\theta-1)(1-\varepsilon)} E_t \left[(\theta-1)\boldsymbol{\Delta}_{t+1} + \widehat{\phi}_{t+1}\right] \\ \\ \widehat{s}_t = \xi \boldsymbol{\Delta}_t + \alpha \overline{\pi}^{\theta(1-\varepsilon)} \widehat{s}_{t-1} \end{cases} \tag{13}$$

where $\mathbf{\Delta}_t \equiv \widehat{\pi}_t - \Im\left(\varepsilon\widehat{\pi}_{t-1}\right)$, $\widehat{\phi}_t$ is an auxiliary variable and the coefficients κ, η and ξ are convolutions of parameters, *inter alia*, trend inflation, and price indexation (see the Appendix). Our generalization encompasses the standard NKPC. Indeed, when $\overline{\pi} = 1$, then $\eta = \xi = 0$, and both the auxiliary variable and the price dispersion measure are irrelevant for inflation dynamics (up to first order). The system (13) therefore reduces to $\mathbf{\Delta}_t = \beta E_t \mathbf{\Delta}_{t+1} + \kappa \hat{Y}_t$.

As stressed by Ascari and Ropele (2004), trend inflation dramatically alters the inflation dynamics compared to the usual Calvo model with $\overline{\pi} = 1$. First, trend inflation enriches the dynamic structure adding two new variables: the variable $\widehat{\phi}_t$, which is a forward-looking variable, and the variable $\widehat{s}_t$, which is instead, a predetermined variable. Second, trend inflation directly affects the NKPC coefficients. Since price-setting becomes more "forward-looking", trend inflation leads to a smaller coefficient on current output gap and a larger coefficient on future expected inflation for standard calibration values. In the plane $(\widehat{Y}_t, \widehat{\pi}_t)$, consequently, the short-run NKPC flattens. In other words, the contemporaneous relation between $\widehat{\pi}_t$ and $\widehat{Y}_t$ progressively weakens, and the inflation rate becomes less sensitive to variations in the output gap and more forward looking. Third, trend inflation increases the autoregressive parameter in the price dispersion equation yielding, *ceteris paribus,* a more persistent inflation adjustment path. Fourth, the effects of trend inflation are counterbalanced by price indexation: the larger the degree of price indexation, the smaller the effect of trend inflation. In the limiting case of full indexation, the effects of trend inflation are completely neutralized.

4 Determinacy and Taylor rule

In this section we analyze how trend inflation affects the rational expectations equilibrium (REE, henceforth) determinacy properties.[5]

4.1 No indexation

To begin with, we assume no indexation (i.e., $\varepsilon = 0$), and that monetary authority sets the short run nominal interest rate according to the classic contemporaneous Taylor rule, i.e., $\hat{\imath}_t = \phi_\pi \hat{\pi}_t + \phi_Y \hat{Y}_t$. Figure 1 illustrates determinacy regions for different levels of annualized trend inflation, i.e., 0%, 2%, 4%, 6% and 8%, in the discretized plane

[5] Note that: (i) as usual, indeterminacy refers to a situation in which the number of explosive eigenvalues is lower than the number of forward-looking variables; (ii) since, as common, we are studying linear approximations of equilibria, all our statements relates to local properties of the equilibria and to small deviations from steady states.

(ϕ_π, ϕ_Y). Furthermore, we set $\alpha = 0.75$, $\beta = 0.99$, $\theta = 11$, and $\sigma_n = 1$.

Result 1. REE determinacy. *In the no indexation case, trend inflation unambiguously affects the REE determinacy properties: as $\bar{\pi}$ increases, the determinacy region rapidly gets smaller, increasing the possibility of sunspots fluctuations.*

As shown in Figure 1, raising trend inflation from zero to 2% visibly modifies the determinacy region. The determinacy frontier closes like scissors. The number of implementable interest rate rules drop by a remarkable -53.5%. Moving to a higher trend inflation, say 4% or 6%, the contraction is even more evident. Contemporaneously, the determinacy region slightly shifts rightward. Finally, at 8% trend inflation, the determinacy region shrinks by an impressive 99% compared to the case of zero trend inflation. Only 1% of initial policy rules still ensure REE uniqueness. These rules, in particular, are characterized by a strong reaction to inflation and unresponsiveness to output gap.

Result 2. Break-Down of the "Taylor principle". *With trend inflation, the Taylor principle and the "generalized Taylor principle", i.e., $\phi_\pi + \phi_Y \frac{\partial \hat{Y}}{\partial \hat{\pi}}|_{LR} > 1$, only hold as necessary conditions for REE determinacy.*

In the recent monetary policy literature, it has been shown that the contemporaneous Taylor rule ensures REE determinacy if and only if

$$\phi_\pi + \phi_Y \frac{\partial \hat{Y}}{\partial \hat{\pi}}|_{LR} > 1, \tag{14}$$

for ϕ_π and ϕ_Y non-negative and at least one strictly positive (where LR stands for long run). As stressed by Bullard and Mitra (2002) and Woodford (2001, 2003) among others, condition (14) generalizes the original Taylor principle, i.e., $\phi_\pi > 1$: the short-run nominal interest rate should rise by more than the increase of inflation *in the long run.*

In the case of zero inflation, $\frac{\partial \hat{Y}}{\partial \hat{\pi}}|_{LR}$ is given by $\frac{1-\beta}{\kappa}$, and (14) has thus three main implications. First, (14) is a necessary and sufficient condition in the positive orthant of the space (ϕ_π, ϕ_Y). Second, (14) implies a trade off between ϕ_π and ϕ_Y: values of ϕ_π smaller than one still preserve the REE determinacy provided the central bank responds more aggressively to output deviations. Third, in reality this trade-off is very weak. Indeed, the role of ϕ_Y has been largely neglected, because β is calibrated to be very close to one and $\phi_\pi > 1$ suffices for condition (14) to be satisfied (see e.g., Clarida et al., 2000).

The "generalized" Taylor principle continues to be a crucial condition for determinacy in a model with trend inflation. In our model economy with trend inflation$\frac{\partial \hat{Y}}{\partial \hat{\pi}}|_{LR}$

is a heavily complicated expression (see equation (26) in the Appendix 8.2). Plotting (14) for different values of trend inflation then, we obtain exactly the left-lateral frontier in Figure 1. For standard calibration, however, as trend inflation rises, the derivative $\frac{\partial \hat{Y}}{\partial \hat{\pi}}|_{LR}$: (i) switches sign very quickly, from positive to negative, and (ii) increases in absolute value.

As a consequence, trend inflation rapidly and steadily overturns all the implications of (14) discussed above for the zero inflation case. First, condition (14) ceases to be a sufficient in the positive orthant of the space (ϕ_π, ϕ_Y). Indeed, (14) only partially draws the upper determinacy frontier in Figure 1. The lower determinacy frontier progressively shifts upwards and eventually crosses the line defined by condition (14) in the positive orthant. Note then that the classic Taylor principle (i.e., $\phi_\pi > 1$) does not suffice to ensure REE determinacy any longer, because the smallest admissible value of ϕ_π positively co-moves with $\overline{\pi}$. In the case of 6% inflation, for example, ϕ_π needs to be roughly higher than two. Second, even for trend inflation levels close to zero, the trade off between ϕ_π and ϕ_Y disappears, because the slope of the upper frontier switches sign. Thus, a central bank that wants to lower ϕ_π, must at the same time respond less aggressively to the output gap to avoid indeterminacy. Equivalently, a central bank much concerned with output variations has to be even tighter on inflation. Moreover, the higher trend inflation the flatter the upper determinacy frontier and the larger the increase in ϕ_π per unit of ϕ_Y. Third, the coefficient on the output gap now plays a key role, even for moderate levels of trend inflation. As an example, in Figure 1 we highlight with a cross the classical Taylor rule, i.e., $\phi_\pi = 1.5$ and $\phi_Y = 0.5$. As soon as trend inflation is larger than 2%, the Taylor rule yields REE indeterminacy. Hence, in real world applications, the value of ϕ_Y cannot be neglected, and it should be generally very low for realistic values of trend inflation.

To understand these results, it is important to consider the steady state relationship between inflation and output, a surprisingly neglected issue in the Neo-Keynesian literature. The long-run NKPC is extremely non-linear around $\overline{\pi} = 1$: it is positively sloped for $\overline{\pi} = 1$ (because of a discounting effect), but then quite rapidly slopes negatively, because of the strong relative price dispersion effect. It follows that $\frac{\partial \hat{Y}}{\partial \hat{\pi}}|_{LR}$ is positive if the model is log-linearized around a zero inflation steady state, while it turns negative for very low levels of positive trend inflation (see Appendix 8.2). As we discussed above, these effects have radical implications on the celebrated Taylor principle. The results in most of the literature are therefore based on a particular case, i.e., $\bar{\pi} = 1$, which is theoretically special as well as empirically unrealistic.

In summary, Figure 1 shows that as trend inflation rises implementable monetary rules call for increasingly large and positive coefficients on inflation and small coefficients on output gap. Eventually, for large enough values of trend inflation the central bank has no choice but being an inflation targeter. These results agree with the policy prescription of Schmitt-Grohe and Uribe (2004a,b) and of Bullard and Mitra (2002). Even though dealing with two rather different problems, these articles robustly suggest monetary policy rule characterized by a high coefficient on ϕ_π and a close to zero coefficient on ϕ_Y. Allowing for trend inflation casts doubt on the *leaning against the wind* policy prescription in Clarida et al. (1999). As trend inflation increases, the central bank cannot take the risk of responding to the output gap, but must just focus only inflation. Ascari and Ropele (2004) also shows that this is true for the optimal monetary policy and provides basic intuition of why this happens.

4.2 Price Indexation

In this section non-adjusting intermediate firms index their prices *either* to past inflation *or* to trend inflation. In both cases, we set $\varepsilon = 0.5$.

Result 3. *Price indexation counteracts the effects of trend inflation on REE determinacy properties described in the previous Section.*

Figure 2 clearly confirms the qualitative results induced by trend inflation shown above. Partial indexation, however, mitigates these effects. The closure of determinacy regions is now less critical. Importantly, the lowest admissible value of ϕ_π becomes less sensitive to trend inflation. Again, as trend inflation increases, the central bank has a smaller set of implementable policies. Thus, the monetary authority should respond more to inflation and less to output gap.

Result 4. *For a given level of trend inflation, price indexation to past inflation always yields a set of implementable interest rate rules greater than under long-run indexation.*

It is worth observing that this result arises because with price indexation to past inflation the lower frontier tilts downwards, while the upper frontier exhibits a very similar behavior in both indexation cases. This means that most of the extra policy options, available for the monetary authority in the past inflation indexation case, regard the peculiar possibility of more pro-cyclical monetary policy (i.e., more negative values of ϕ_Y). In other words, the central bank can still ensure determinacy of equilibrium, if it remains looser on inflation deviations but, oddly enough, it responds more pro-cyclically to $\hat{Y}_t$. Finally, full past inflation indexation restores the pivotal role of the original Taylor principle, because $\phi_\pi > 1$ becomes then a necessary and sufficient condition for determinacy (see Ropele, 2007).

4.3 Inertial interest rate rule

Empirical works on Taylor rules show that central banks tend to adjust the nominal interest rate in response to changes in economic conditions only gradually (see, e.g., Rudebusch, 1995, Judd and Rudebusch,1998 or Clarida et al., 2000). Moreover, the recent monetary literature has emphasized the importance of inertia in the conduct of monetary policy with a forward-looking private sector (e.g., Woodford, 2003). Thus, in this section we explore the effects of trend inflation on the REE determinacy properties when our contemporaneous Taylor rule allows for inertia, that is: $\hat{\imath}_t = \phi_\pi \hat{\pi}_t + \phi_Y \hat{Y}_t + \phi_i \hat{\imath}_{t-1}$.

It is well known that in the standard model with zero steady state inflation, interest rate inertia makes indeterminacy less likely. Figure 3 reports our results for $\phi_i = 0.5, 1, 2$ and 5, showing that the somewhat counterintuitive feature that explosive rules enlarge the determinacy region survives in the trend inflation case. As discussed in Rotemberg and Woodford (1999), in a similar model but with zero inflation steady state, it is exactly the possibility of explosiveness of the nominal interest rate that keeps the model on track.[6] Indeed, in a zero trend inflation model, condition (14) becomes $\phi_\pi + \phi_Y (1-\beta)/\kappa > 1 - \phi_i$, such that $\phi_i \geq 1$ is a sufficient condition for a determinate equilibrium in the positive orthant. Moreover, a sufficient condition $\phi_\pi > 1 - \phi_i$ can be easily checked from any Taylor rule estimate. Note that this latter implies no role for ϕ_Y.

Result 5. *With trend inflation, interest rate inertia makes the Taylor principle, i.e., $\phi_\pi > 1$, plainly insignificant. It is the value of ϕ_Y that actually matters for REE determinacy.*

Again, trend inflation radically changes the implications of the model. Looking at panel B, it is evident that there is no more a sufficient condition on ϕ_π (provided that is positive) or on ϕ_i. On the contrary, for sufficiently high levels of trend inflation, we can eventually state a sufficient condition on ϕ_Y. As stressed in Section 4.1, this is due to the switch in the sign of $\frac{\partial \hat{Y}}{\partial \hat{\pi}}|_{LR}$. Moreover, $\frac{\partial \hat{Y}}{\partial \hat{\pi}}|_{LR}$ is increasing with trend inflation in absolute value. For values as high as 6% trend inflation, $\frac{\partial \hat{Y}}{\partial \hat{\pi}}|_{LR}$ is so high

[6] The case of no feedback from inflation and output gap on the nominal interest rate (i.e., $\phi_\pi = \phi_Y = 0$) is of course indeterminate, for values of ϕ_i bigger than 1.

(in absolute value), that ϕ_Y becomes the crucial monetary policy parameter for (14) to be satisfied. Graphically the frontier that corresponds to condition (14) flattens in Figure 3. It follows that what matters for determinacy is that monetary policy should not respond to the output gap, when monetary policy is characterized by an inertial (or superinertial) Taylor rule and moderate trend inflation (6% to 8%).

5 Dynamic Analysis

We now assess how trend inflation impinges on the impulse response functions (IRFs, henceforth) and output/inflation efficient policy frontier. To this purpose, as in Galí (2003), we append to the first equation in (13) a cost-push shock u_t, whose law of motion is $u_t = 0.8u_{t-1} + \eta_t$ and η_t ~i.i.d N(0,1), and set $\phi_\pi = 1.5$ and $\phi_Y = 0.5$ (and $\phi_i = 0$), as in the original Taylor specification.

5.1 Impulse response functions

Figure 4 displays the IRFs of the output gap, inflation rate, nominal and real interest rate to a unit cost-push shock in the case of both no indexation and price indexation to past inflation.[7] Each panel reports a family of IRFs associated to different levels of trend inflation for which the REE is determinate. Panel A shows the case of zero inflation steady state and no price indexation. In response to a unit cost-push shock, the monetary rule calls for a large nominal interest rate increase, sufficient enough to determine a positive real interest rate. Such a response, in turn, opens up a series of negative output gaps that gradually drives the inflation rate back to equilibrium.

Result 6. *Positive trend inflation shifts outwards IRFs of output and inflation, following a cost-push shock.*

Consider the case of 2% trend inflation and no indexation. Although the qualitative pattern is similar to that under zero inflation steady state, some key differences are worth stressing. First, trend inflation visibly alters the impact effects by producing an outward shift. Second, the outward shift in IRFs remains throughout the whole return path to steady state, thus suggesting a tighter monetary policy and a deeper recession. In short, consistent with the results in Ascari and Ropele (2004), the higher trend

[7] The results of this section do not qualitatively change if other values of $\phi_\pi$and ϕ_Y are chosen. For the standard Taylor rule, we can plot just two IRFs because the REE is not determinate for trend inflation larger than 2% . We do not report the IRFs for price indexation to trend inflation, because such indexation rule only generates a (downward) rescaling with respect to the no indexation case.

inflation, the worse trade-off monetary policy faces: the deeper the recession and the higher the deviation of inflation from steady state. Figure 4 also illustrates the effects of price indexation to past inflation. Parall to the previous case, trend inflation shifts outwards the IRFs. As stressed by Christiano et al. (2005), price indexation to past inflation creates a hump-shape in the IRFs of output and inflation, due to the inclusion of π_{t-1} in the New Keynesian Phillips curve.

5.2 Efficient Policy Frontiers

Next, we analyze the effects of trend inflation on unconditional variances of output and inflation, arguments that typically characterize the central bank's loss function. For different levels of trend inflation, we construct the output-inflation efficient policy frontier by varying, in turn, either ϕ_π or ϕ_Y in the range $[0, 3]$. When varying ϕ_π, we set $\phi_Y = 0.5$; while when varying ϕ_Y, we set $\phi_\pi = 2.5$.[8]

Result 7. *Positive trend inflation moves north-east the efficient policy frontier, yielding worse outcomes for both inflation and output variability.*

This is the main result of this section, and we think a quite important one: it is distinctly shown by the outward shift of the efficient policy frontiers in Figure 5. Combinations of σ_π^2 and σ_Y^2 attainable with zero trend inflation are not anymore so as trend inflation rises: either a higher value of σ_Y^2 is necessary for the same σ_π^2 or vice versa. Moreover, as trend inflation increases, the efficient policy frontier substantially shortens (i.e., it comprises a fewer number of points), in that the REE enters the indeterminacy region. Not surprisingly, Figure 5 also shows that, for a given level of $\overline{\pi}$, price indexation to trend inflation shifts the efficient policy frontier south-west, partially offsetting the effects of trend inflation. The results are strengthened in the case of price indexation to past inflation, as shown in Figure 6.

6 Robustness

We explored whether the results of the previous sections are robust to simple variants of the Taylor rule commonly used in the literature (i.e., forward-looking interest rate rule, backward-looking interest rate rule, and various kinds of hybrid interest rate rules) and to changes in the structural parameters of the model. The general conclusion is that the

[8] The value for ϕ_π is different from the one used in the previous section, for convenience of presentation. The efficient policy frontiers would exhibit otherwise very few points as trend inflation increases, because the REE would quickly become indeterminate.

key results found in the previous analysis persist. Moderate trend inflation substantially changes the determinacy region in the parameter space and the dynamic properties of the model economy in all cases. In this section, we briefly report a few results worthy of note.[9]

Backward looking interest rate rule

When the monetary authority sets the nominal interest rate as a function of lagged inflation and lagged output gap, i.e., $\hat{\imath}_t = \phi_\pi \hat{\pi}_{t-1} + \phi_Y \hat{Y}_{t-1}$, positive levels of long run inflation surprisingly increase the set of determinate policy rules, relative to the case of zero inflation steady state. Panel A of Figure 7 illustrates the standard case of zero inflation steady state in this instance. The panel is divided into four parts by two lines: one is almost horizontal at $\phi_Y = 2$, the other one corresponds to the equivalent of condition (14). Note that in the parameters space above the almost horizontal line at $\phi_Y = 2$, the determinacy region now lies *on the left of condition (14) and not on its right*, where the explosive region lies. The other panels of Figure 7 shows the effect of increasing trend inflation. Graphically it is still the same, as the line corresponding to (14) again visibly rotates clockwise.[10] However, due to the fact that now the determinacy region lies partly on the left and partly on the right of this line, the effect of trend inflation is less clear-cut. Roughly speaking, dividing the parameters space in two regions, as trend inflation increases: (i) above the almost horizontal line at $\phi_Y = 2$, the instability region progressively shrinks and gives way to new determinate combinations; (ii) below the almost horizontal line at $\phi_Y = 2$, the indeterminacy region enlarges and reduces the number of implementable (i.e., determinate) rules. Note that while (ii) is the usual effect analyzed in the previous sections, (i) is the peculiarity of the lagged interest rate rule. Given our calibration, (i) is stronger, thus positive trend inflation always delivers a larger determinacy region with respect to the case of zero inflation steady state. As trend inflation takes on higher values, then, a central bank following a lagged interest rate rule is progressively left with two options to guarantee determinacy. On the one hand, it might respond more to inflation deviations, and be more cautious towards the output gap, in line with previous analysis. On the other hand, the central bank can instead respond aggressively to output gap, i.e. $\phi_Y > 2$, regardless to the value of ϕ_π. Again, trend inflation makes the Taylor principle useless and the value of ϕ_Y more important. Introducing inertia in a lagged interest rate rule shifts upward the almost horizontal line in Figure 7. As a result, the effect described in (i) becomes progressively less important

[9] The interested reader can download the extended working paper version from the authors' webpage.

[10] The other almost horizontal line is in contrast only slightly sensitive to changes in trend inflation for our calibration values.

and disappears from the parameters space for superinertial policies. The properties of the model economy, at last, become very similar as with the other monetary policy rules.

Sensitivity analysis

We also checked the robustness of our findings to changes in the structural parametrization. Figure 8 reports the REE determinacy regions, when one of the following parameters values: $\theta = 4$, $\alpha = 0.5$ and $\sigma_n = 5$, are changed in turn in the case of contemporaneous interest rate rule and no indexation.[11] As expected (see Ascari, 2004), a lower value of the elasticity of substitution across goods, or a lower value of the Calvo parameter, make the determinacy frontier to close less rapidly compared to the baseline calibration (see panels A and B). This leaves room for a relatively larger set of implementable policies for a given trend inflation. Considering higher values of the inverse of the intertemporal elasticity of labour supply does not change qualitatively the results presented above (see panels C).

7 Conclusions

Despite that average inflation in the post-war period in developed countries was moderately different from zero, much of the vast literature on monetary policy rules worked with models log-linearized around zero inflation. In this paper, we generalize a standard Neo-Keynesian model with Calvo staggered prices by taking a linear approximation around a general trend inflation level. We then look at how the properties of our model economy change as the trend inflation level varies when monetary policy follows a Taylor rule.

The results show that trend inflation greatly affects the previous results in the literature. In particular, moderate levels of trend inflation modify the determinacy region in the parameters space, substantially changing the *Taylor principle*. Moreover, trend inflation alters the impulse response functions of the model economy after a cost-push shock. In line with Ascari (2004) and Ascari and Ropele (2004), this paper therefore shows that the Neo-Keynesian framework is quite sensitive to variations in the trend inflation level, in the sense that higher trend inflation makes monetary policy much less effective in controlling the dynamics of the economy. Our key results are then generalized and proved to be robust to: (i) different kinds of Taylor type rules; (ii) inertial Taylor rules for all the cases in (i); (iii) indexation schemes; (iv) different parameter values.

[11] The qualitative effects of changes in the values of these parameters are in accordance with intuition, and robust across different type of rules, indexation and inertia.

In summary, the literature on monetary policy rules is based on a case (i.e., zero steady state inflation) that is both empirically unrealistic and theoretically special. The specification of the theoretical model, and consequently all the results, are very sensitive to low and moderate trend inflation levels, as empirically observed in western countries. Our analysis therefore shows that the literature cannot neglect trend inflation in either empirical or theoretical analysis. As non-superneutrality is basic feature of the standard model, future work should aim to integrate the long-run properties and the short-run dynamics into a full non-linear analysis.

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8 Appendix

8.1 NKPC coefficients

$$\kappa \equiv \frac{\left[1-\alpha\pi^{(\theta-1)(1-\varepsilon)}\right]\left[1-\alpha\beta\pi^{\theta(1-\varepsilon)}\right]}{\alpha\pi^{(\theta-1)(1-\varepsilon)}}\left(1+\sigma_n\right)$$

$$\eta \equiv \left(\pi^{1-\varepsilon}-1\right)\beta\left[1-\alpha\pi^{(\theta-1)(1-\varepsilon)}\right]$$

$$\xi \equiv \frac{\theta\alpha\bar{\pi}^{(\theta-1)(1-\varepsilon)}\left(\bar{\pi}^{1-\varepsilon}-1\right)}{1-\alpha\bar{\pi}^{(\theta-1)(1-\varepsilon)}}$$

For standard calibration values, one can show that $\frac{\partial\kappa}{\partial\bar{\pi}}<0$ and $\frac{\partial\xi}{\partial\bar{\pi}}<0$.

8.2 Generalizing the Taylor principle to trend inflation

Here we generalize the Taylor principle as discussed in Woodford (2003, chp. 4) to the case of non-zero steady state inflation.

In the standard Neo-Keynesian model, with zero inflation steady state, a contemporaneous interest rate rule, i.e. $\widehat{i}_t=\phi_\pi\widehat{\pi}_t+\phi_Y\widehat{Y}_t$, with both ϕ_π and ϕ_Y greater than zero, the original Taylor principle, which called for $\phi_\pi>1$, has been generalized to

$$\phi_\pi+\left(\frac{1-\beta}{\kappa}\right)\phi_Y>1. \tag{15}$$

As stressed by Woodford (2003, chp. 4), the logic is that the **long run multiplier** of $\hat{\pi}$ on $\hat{\imath}$ must exceed one:

$$\frac{\partial\hat{\imath}}{\partial\hat{\pi}}=\phi_\pi+\phi_Y\frac{\partial\hat{Y}}{\partial\hat{\pi}}=\phi_\pi+\left(\frac{1-\beta}{\kappa}\right)\phi_Y>1 \tag{16}$$

since given the standard NKPC, i.e. $\hat{\pi}_t=\beta E_t\hat{\pi}_{t+1}+\kappa\hat{Y}_t$, the partial derivative of output with respect to inflation is indeed equal $(1-\beta)/\kappa$. Notice that in the plane $(\phi_\pi;\phi_Y)$ the condition can be rewritten as

$$\phi_Y>\frac{\kappa}{1-\beta}\left(1-\phi_\pi\right),$$

which defines the area in Figure 1 in the paper, corresponding to the case of zero trend inflation.

In this section, we show that in our model with no indexation, i.e. $\varepsilon=0$:

(i) the derivative $\partial\hat{Y}/\partial\hat{\pi}$ depends on trend inflation;

(ii) for standard calibration values, the derivative $\partial \hat{Y}/\partial \hat{\pi}$ turns negative very soon as trend inflation is positive;

(iii) for standard calibration values, the derivative $\partial \hat{Y}/\partial \hat{\pi}$ increases in absolute value as trend inflation increases.

As in the standard case, to derive the partial derivative $\partial \hat{Y}/\partial \hat{\pi}$ we make use of the generalized NKPC, which in the case of no indexation is given by:

$$\begin{aligned}
\widehat{\pi}_t &= \beta\overline{\pi}E_t\widehat{\pi}_{t+1} + \kappa\widehat{Y}_t + \kappa\frac{\sigma_n}{1+\sigma_n}\widehat{s}_t + \eta E_t\left[(\theta-1)\,\widehat{\pi}_{t+1} + \widehat{\phi}_{t+1}\right], && (17)\\
\widehat{\phi}_t &= \alpha\beta\overline{\pi}^{\theta-1}E_t\left[(\theta-1)\,\widehat{\pi}_{t+1} + \widehat{\phi}_{t+1}\right], && (18)\\
\widehat{s}_t &= \xi\widehat{\pi}_t + \alpha\overline{\pi}^{\theta}\widehat{s}_{t-1}. && (19)
\end{aligned}$$

Suppressing in all the equations the time subscript and rearranging yields:

$$\begin{aligned}
\widehat{\pi}\left(1-\beta\overline{\pi}\right) &= \kappa\widehat{Y} + \kappa\frac{\sigma_n}{1+\sigma_n}\widehat{s} + \eta\left[(\theta-1)\,\widehat{\pi} + \widehat{\phi}\right], && (20)\\
\widehat{\phi}\left(1-\alpha\beta\overline{\pi}^{\theta-1}\right) &= \alpha\beta\overline{\pi}^{\theta-1}(\theta-1)\,\widehat{\pi}, && (21)\\
\widehat{s}\left(1-\alpha\overline{\pi}^{\theta}\right) &= \xi\widehat{\pi} && (22)
\end{aligned}$$

To derive the long run multiplier under trend inflation, we compute the following derivatives:

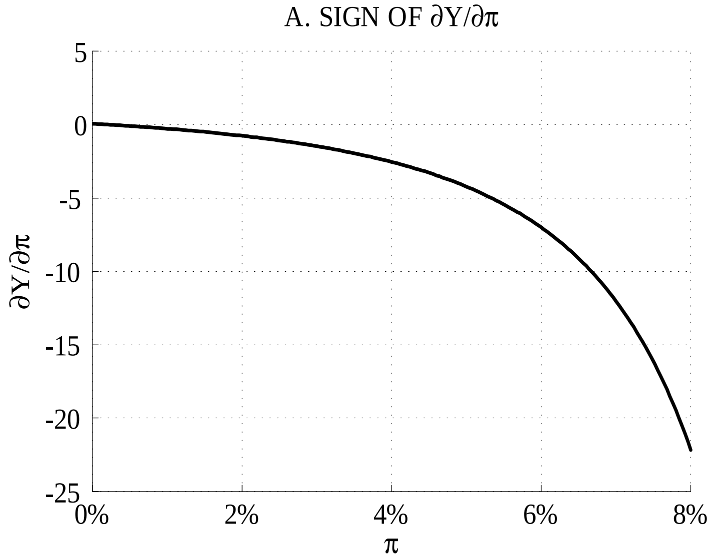
$$\begin{aligned}
\frac{\partial\widehat{Y}}{\partial\widehat{\pi}} &= \frac{1}{\kappa}\left[1-\beta\overline{\pi}-\kappa\frac{\sigma_n}{1+\sigma_n}\frac{\partial\widehat{s}}{\partial\widehat{\pi}} - \eta(\theta-1) - \eta\frac{\partial\widehat{\phi}}{\partial\widehat{\pi}}\right], && (23)\\
\frac{\partial\widehat{\phi}}{\partial\widehat{\pi}} &= \frac{\alpha\beta\overline{\pi}^{\theta-1}(\theta-1)}{1-\alpha\beta\overline{\pi}^{\theta-1}}, && (24)\\
\frac{\partial\widehat{s}}{\partial\widehat{\pi}} &= \frac{\xi}{1-\alpha\overline{\pi}^{\theta}}. && (25)
\end{aligned}$$

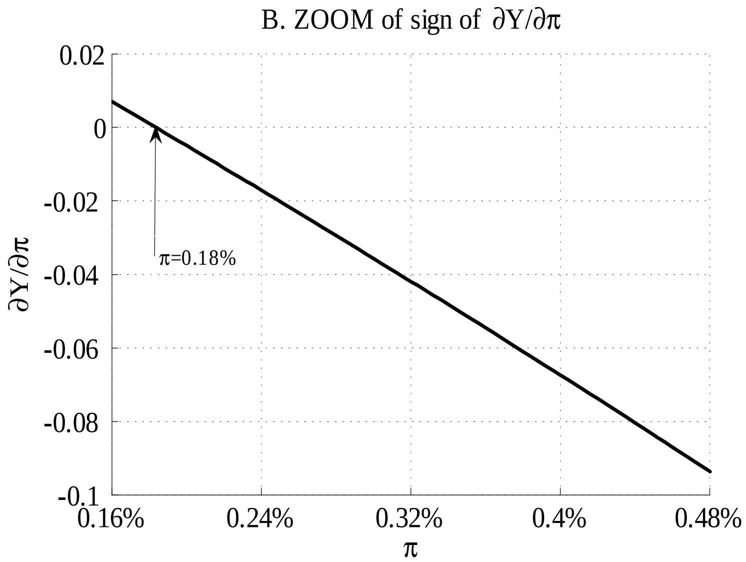
Recalling that $\frac{\partial\hat{\imath}}{\partial\hat{\pi}} = \phi_\pi + \phi_Y\frac{\partial\hat{Y}}{\partial\hat{\pi}} > 1$, it then follows that the generalized condition reads as:

$$\phi_\pi + \phi_Y\underbrace{\frac{1}{\kappa}\left[1-\beta\overline{\pi}-\kappa\frac{\sigma_n}{1+\sigma_n}\frac{\xi}{1-\alpha\overline{\pi}^{\theta}} - \eta(\theta-1) - \eta\frac{\alpha\beta\overline{\pi}^{\theta-1}(\theta-1)}{1-\alpha\beta\overline{\pi}^{\theta-1}}\right]}_{\partial\hat{Y}/\partial\hat{\pi}} > 1. \qquad (26)$$

Clearly, the derivative $\partial\hat{Y}/\partial\hat{\pi}$ depends, *inter alia*, upon trend inflation and nests condition for zero inflation steady state case, because both ξ and η are zero when $\overline{\pi} = 1$.

However, due to the obscure convolution of parameter it is not possible to determine analytically the sign of $\partial\hat{Y}/\partial\hat{\pi}$ as trend inflation varies. To this end, we resort to numerical results. Figure 1 plots $\partial\hat{Y}/\partial\hat{\pi}$ against trend inflation while keeping the remaining parameters at their baseline values ($\beta = 0.99$, $\alpha = 0.75$, $\theta = 11$ and $\sigma_n = 1$). As argued in the main text, this derivative is positive at zero steady state inflation, but it turns quickly to negative (i.e. at 0.18% annual steady state inflation rate, that is $\overline{\pi} = 1.00045$), inverting therefore the slope of the Taylor principle condition in the space $(\phi_\pi; \phi_Y)$ in Figure 1 in the main text.


A. SIGN OF ∂Y/∂π
5
0
-5
-10
-15
-20
-25
∂Y/∂π
0%
2%
4%
6%
8%
π


B. ZOOM of sign of ∂Y/∂π
0.02
0
-0.02
-0.04
-0.06
-0.08
-0.1
∂Y/∂π
π=0.18%
0.16%
0.24%
0.32%
0.4%
0.48%
π

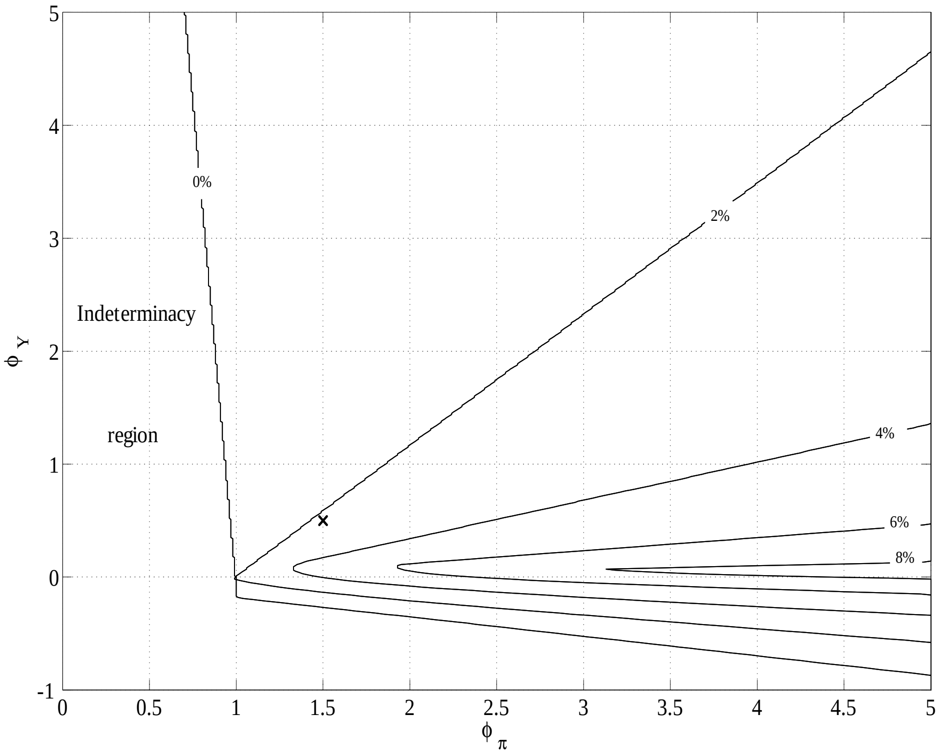


Figure 1: Contemporaneous interest rate rule and the effects of trend inflation. The cross marker identifies the canonical Taylor rule, i.e. $\phi_\pi = 1.5$ and $\phi_Y = 0.5$.

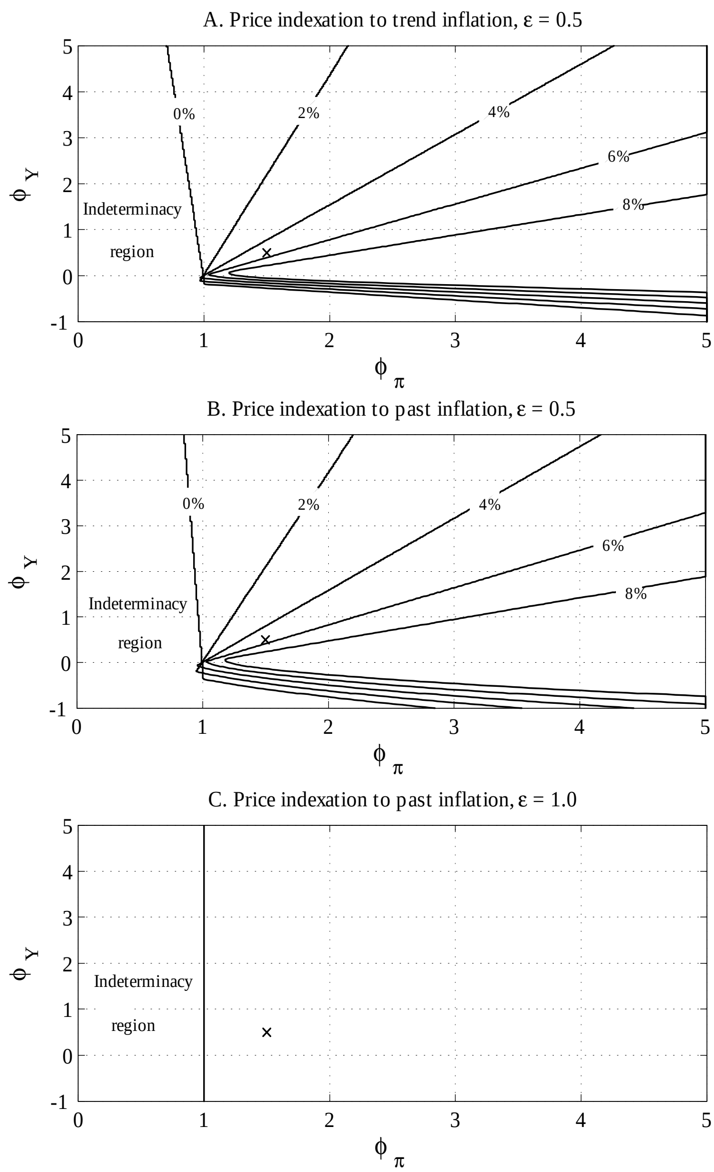


Figure 2: Contemporaneous interest rate rule, price indexation and the effects of trend inflation. The cross marker identifies the canonical Taylor rule, i.e. $\phi_\pi = 1.5$ and $\phi_Y = 0.5$.

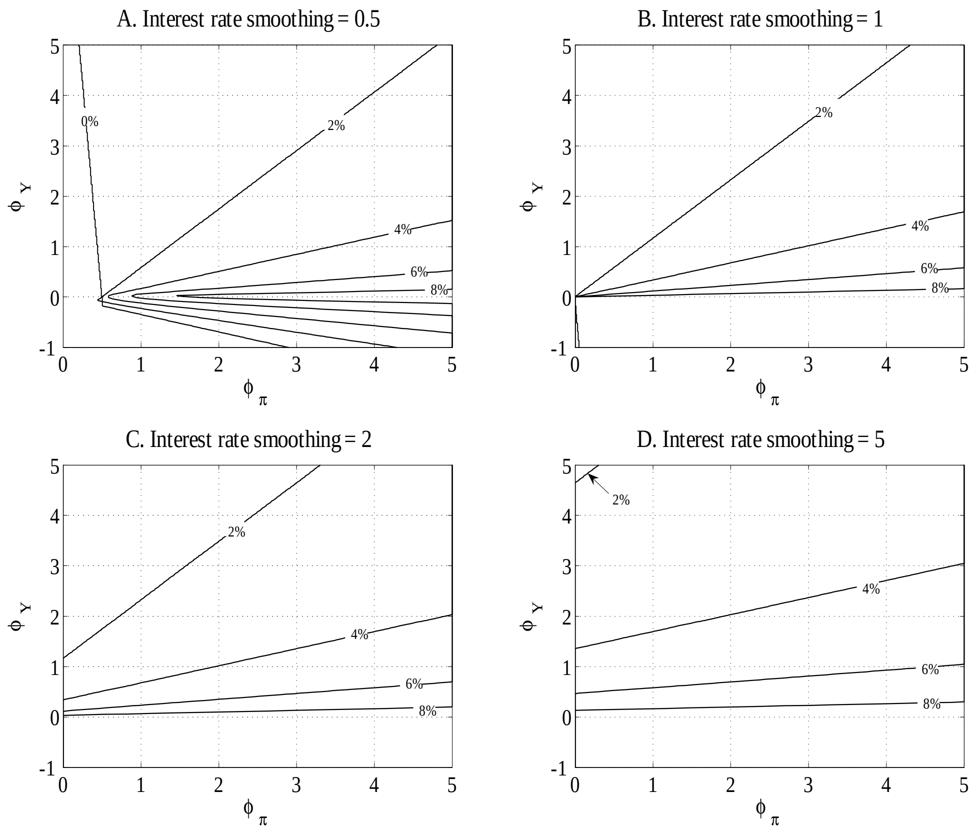


Figure 3: Inertial contemporaneous interest rate rule and the effects of trend inflation.

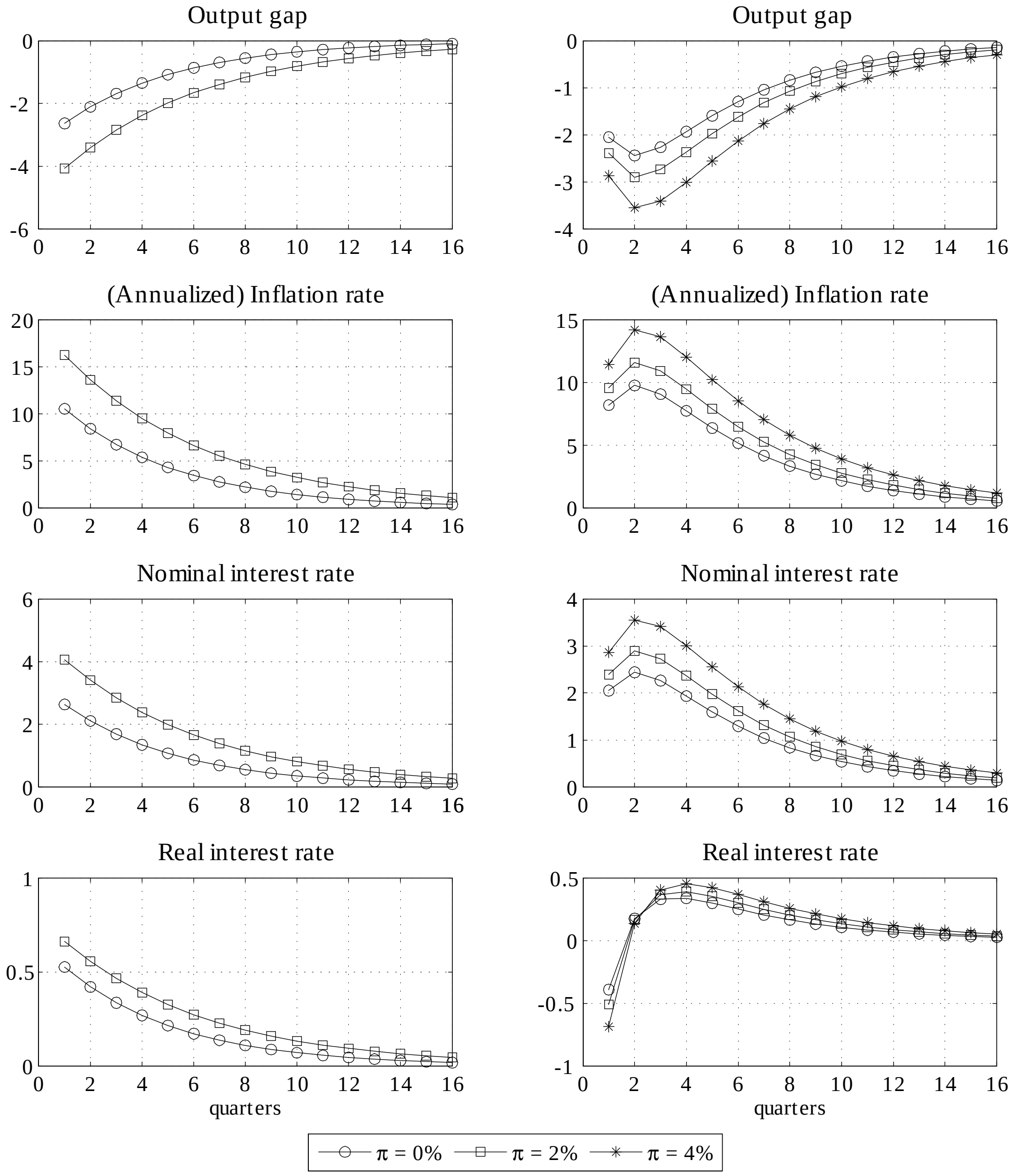


Figure 4: Impulse response functions to a unit cost push shock ($\phi_\pi = 1.5$ and $\phi_Y = 0.5$).

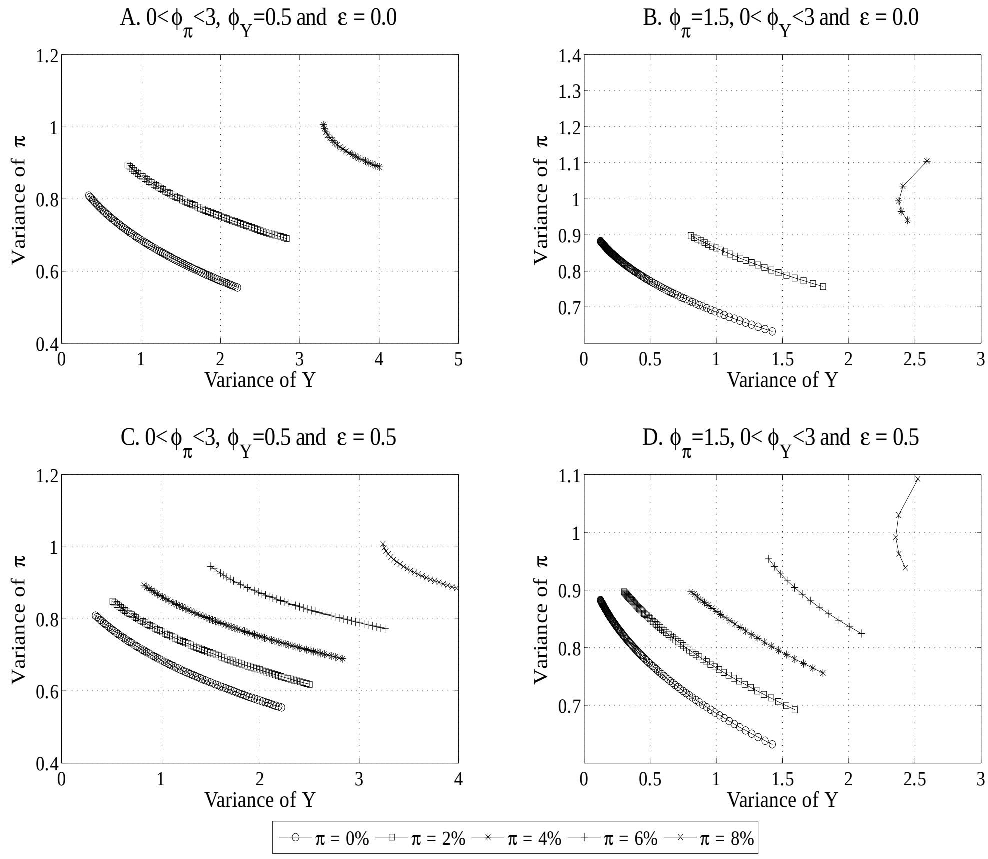


Figure 5: Efficient policy frontiers with contemporaneous interest rate rule and price indexation to trend inflation.

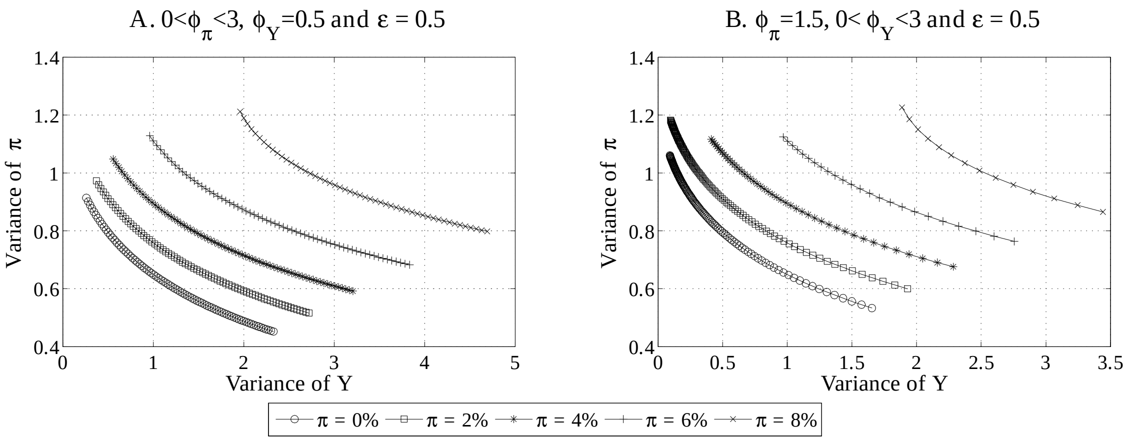


Figure 6: Efficient policy frontiers with contemporaneous interest rate rule and price indexation to past inflation.

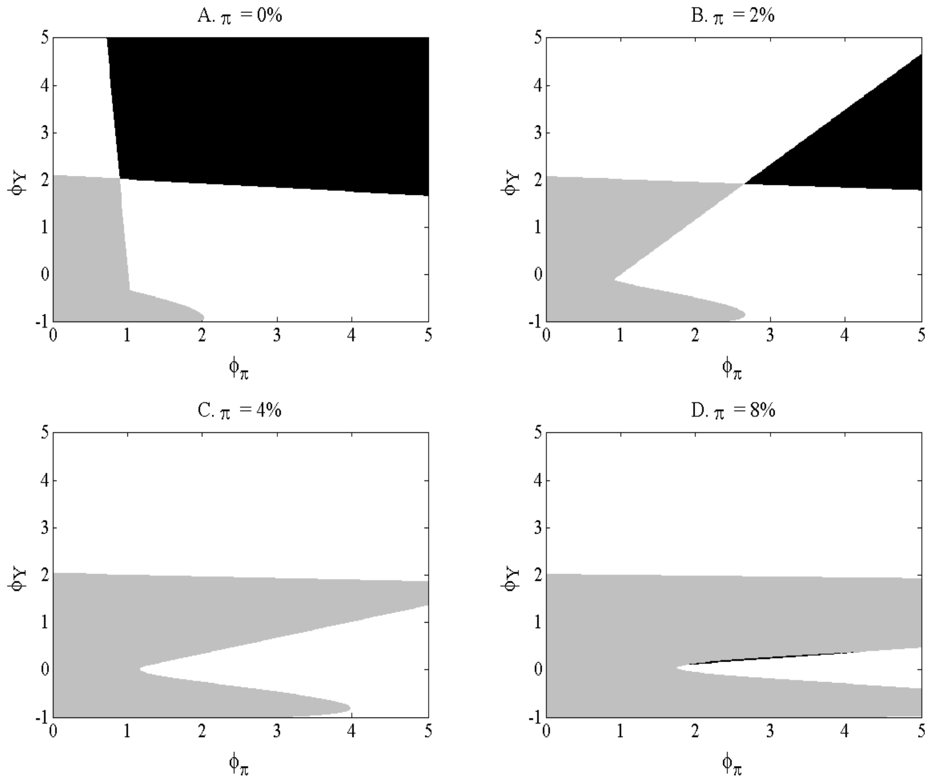


Figure 7: Backward looking interest rate rule and the effects of trend inflation (Black area = REE instability; Grey = REE indeterminacy; White = REE determinacy).

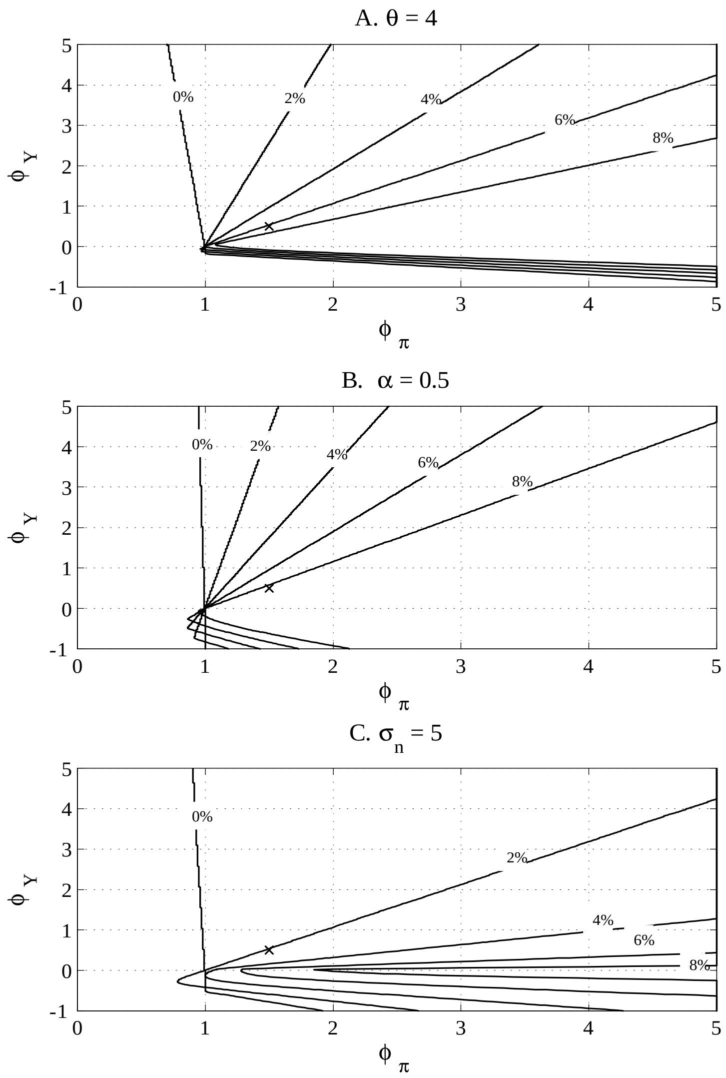


Figure 8: Sensitivity analysis.