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Abstract: Many labor market models use both idiosyncratic productivity and a vacancy free entry condition. This paper shows that these two features combined generate an equilibrium comovement between matches on the one hand and unemployment and vacancies on the other hand, which is observationally equivalent to a constant returns Cobb-Douglas function commonly used to model match formation. We use German administrative labor market data to show that the matching function correlation solely based on idiosyncratic productivity and free entry is very close to the empirical matching function. Consequently, we argue that standard matching function estimations are seriously biased if idiosyncratic productivity plays a role for match formation. In this case, they are not suitable for the calibration of labor market models.

Keywords: matching function, idiosyncratic productivity, job creation, vacancies
JEL codes: E24, E32, J63, J64

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1 Introduction

"The matching function is a modeling device that occupies the same place in the macroeconomist's tool kit as other aggregate function, such as the production function (. . .). Like the other aggregate functions its usefulness depends on its empirical viability and on how successful it is in capturing the key implications of heterogeneities and frictions in macro models."
(Petrongolo and Pissarides, 2001, 391-392)

It is conventional practice to model job creation by assuming a Cobb-Douglas contact/matching function with constant returns to scale (CRS).[1] The parametrization of the contact function is usually guided by estimations based on data for matches, vacancies and unemployment. Many papers also assume that the creation of jobs is influenced by match-specific idiosyncratic productivity, i.e. only worker-firm pairs above a certain productivity threshold are formed.[2] In addition, the free entry of vacancies or firms is a standard assumption in modern labor market models such as Mortensen and Pissarides (1994) or Pissarides (2000).

This paper revisits the matching function from two perspectives. First, we show that in a wide class of models the combination of idiosyncratic productivity and free entry of vacancies generates an equilibrium comovement between matches on the one hand and unemployment and vacancies on the other hand, which is observationally equivalent to the empirically observed matching function correlation. Thus, many papers would be able to replicate one of the most important stylized fact of the labor market, even without using a traditional contact function.

Second, we show that the combination of idiosyncratic shocks and a traditional Cobb-Douglas (CRS) contact function increases the weight on vacancies relative to the assumed contact function. Why is this important? From an empirical perspective our paper sounds a cautionary note on using matching function estimations in order to parameterize contact functions in theoretical models. If idiosyncratic productivities play a role for job creation, matching function estimations lead to biased results for the elasticity of the actual contact function with respect to vacancies and unemployment. In addition, our paper has important theoretical implications. From a normative perspective it is well known that the Hosios (1990) rule internalizes search externalities and thus provides constrained efficiency. However, if the estimated elasticities of the matching function are misspecified, Hosios rule does not provide guidance

[1] In what follows, "contact function" refers to the theoretical function that establishes contacts between workers and firms. Due to idiosyncratic shocks, not all of the contacts may become matches. "Matching function" refers to the empirical connection between matches on the one hand and vacancies and unemployment on the other hand.

[2] For a seminal contribution with idiosyncratic productivity see Jovanovic (1979). For a recent proposition see Brown et al. (forthcoming). Traditional search models (e.g. McCall, 1970; Mortensen, 1987) rely on exogenous wage distributions. If they are interpreted as the result of some underlying idiosyncratic productivity heterogeneity, they fall into the same category of models. In addition, there are many papers that combine a contact function, a vacancy free entry condition and idiosyncratic productivity (e.g. Krause and Lubik, 2007).

whether unemployment is inefficiently high or low. In addition, it may be difficult to judge how certain policy interventions affect welfare. From a positive perspective, policies or technological changes that act via the vacancy posting margin – for instance a vacancy posting subsidy or new technologies that reduce vacancy posting costs – may have a much smaller effect.

We are the first to show that the combination of free entry of vacancies and idiosyncratic productivity generates a linear comovement between the job-finding rate and market tightness, which is observationally equivalent to the conventional Cobb-Douglas CRS matching function (see Petrongolo and Pissarides, 2001, for a review). In different words, in a model with some kind of idiosyncratic productivity and free entry of vacancies there will be a conventional matching function correlation pattern on top of the correlation that is already captured by the contact function in place. We show that this pattern emerges even in an extreme scenario with a degenerate contact function where the probability of a worker to make a contact does not depend on the aggregate number of vacancies. What is the underlying mechanism? Under a positive aggregate productivity shock, firms have an incentive to hire workers with lower idiosyncratic productivity. Thus, the job-finding rate rises. In addition, a positive productivity shock increases the returns from posting a vacancy. Thus, firms compete for the larger pie of profits, more of them enter the market and thus increase the market tightness in the economy. These two effects combined lead to a positive equilibrium movement between the job-finding rate and market tightness.

We use German administrative labor market data in order to assess whether this mechanism is quantitatively meaningful. We calibrate the idiosyncratic productivity distribution in our model from individual wage data which allows us to infer an elasticity of matches with respect to vacancies. This elasticity is very close to the corresponding coefficient in a standard matching function estimation. Our results suggest that the interplay of idiosyncratic productivity and free entry of vacancies is an important driver of the matching function correlation usually found in the data. In addition, we show that if idiosyncratic productivity matters for match formation, the contact function in the model and the empirical matching function obtained from standard regressions (see e.g. Blanchard and Diamond, 1990) will diverge.

The rest of the paper proceeds as follows. Section 2 derives a simple model with idiosyncratic productivity and free entry of vacancies. In this version, idiosyncratic shocks hit only in the first period of employment. However, all our results extend to more general cases such as models with endogenous separations and permanent idiosyncratic productivity differences. Section 3 derives an analytical expression for the equilibrium comovement of the job-finding rate and the market tightness in this framework. We start with a degenerate contact function with no role for vacancies in contact creation as this allows us to isolate the effect of idiosyncratic productivity. Our analysis reveals a connection between the shape of the idiosyncratic productivity distribution and the corresponding matching function correlation. This allows us in Section 4 to use an empirical wage distribution to check quantitatively how important idiosyncratic shocks are for the aggregate matching function correlation. We compare our equilibrium matching function correlation to a conventional Cobb-Douglas CRS matching function estimation. Interestingly, the two are very close. Thus, idiosyncratic productivity and free entry of vacancies may drive a large part of the comovement between the job-finding rate and the market tightness in the data. Finally, we combine a non-degenerate contact function with our calibration for the idiosyncratic productivity. We show that the bias in the estimated matching function due to idiosyncratic productivity is substantial. Therefore, it is important to understand the underlying mechanism.

2 A Simple Model

2.1 Model Environment

Our economy is populated with a continuum of workers who can either be employed or unemployed. Employed workers are separated with an exogenous probability ϕ. Unemployed workers search for a job. We assume that they get in contact with a firm with probability $p_t \leq 1$. The contact probability may either be driven by a standard contact function as in Mortensen and Pissarides (1994) and Pissarides (2000) or it may be degenerate, as standard in search models (e.g. McCall, 1970; Mortensen, 1987) or as assumed in selection models (Brown et al., forthcoming; Lechthaler et al., 2010).

When unemployed workers get in contact with a firm, they draw an idiosyncratic productivity realization ε_{it}, i.e. some workers are more productive than others. This nests the case of search and matching models where endogenous separations hit before production takes place (e.g. Krause and Lubik, 2007) or the stochastic job matching model (Pissarides, 2000, chapter 6). Firms will only hire workers when the productivity realization, ε_{it}, is at least as large as the cutoff productivity $\tilde{\varepsilon}_t$, that makes a firm indifferent between hiring and not hiring. For illustration purposes, we start with a model where idiosyncratic productivity shocks are only drawn in the first period of employment. However, this assumption is without loss of generality. We show analytically in the Appendix that we obtain the same results for two additional polar cases. First, when we assume that the idiosyncratic shock, ε, is redrawn every period (both for new contacts and for existing matches) and *iid* across workers and time. Second, when we assume that ε is only drawn when a new contact is made, but it remains the same for the entire period of employment.

Firms have to post vacancies to obtain a share of the economy wide applicants (namely, the firm's vacancy divided by the overall number of vacancies, which is determined by a free-entry condition). With a traditional contact function, more vacancies lead to more contacts. By contrast, with a degenerate contact function, more vacancies do not lead to more contacts. To illustrate our point, we will start with a degenerate contact function and show afterwards how a traditional contact function affects our results.

2.2 Contacts

Contacts are assumed to follow a Cobb-Douglas function with CRS

$$c_t = \mu v_t^{\gamma} u_t^{1-\gamma}, \tag{1}$$

where c_t denotes the overall number of contacts, μ is the contact efficiency, and u_t and v_t are beginning of period unemployment and vacancies respectively. The contact probability for a worker is thus $p_t = \mu\theta_t^{\gamma}$ and the contact probability for a firm $q_t = \mu\theta_t^{\gamma-1} = p_t/\theta_t$, where $\theta_t = v_t/u_t$ denotes market tightness. With $\gamma = 0$ the contact function is degenerate in the sense that more vacancies do not lead to more contacts and jobs in the aggregate. This simplifying assumption will be the starting point for our analysis in Section 3.

2.3 The Selection Decision

Once a contact between a searching worker and the firm has been established, firms decide whether to hire/select a particular worker or not. There is a random worker-firm pair specific idiosyncratic productivity shock, ε_{it}, which is *iid* across workers and time[3], with density function $f(\varepsilon_t)$ and the cumulative distribution $F(\varepsilon_t)$. ε_t is observed by the worker and the firm. Thus, the expected discounted profit, $\pi_t^E(\varepsilon_t)$, of hiring an unemployed worker is equal to the current aggregate productivity minus the current wage (which may be a function of ε), $w_t(\varepsilon_t)$, plus the idiosyncratic productivity shock, ε_t, plus the expected discounted future profits:

$$\pi_t^E(\varepsilon_t) = a_t + \varepsilon_t - w_t(\varepsilon_t) + \delta(1-\phi) E_t(\pi_{t+1}), \tag{2}$$

with

$$\pi_t = a_t - w_t + \delta(1-\phi) E_t(\pi_{t+1}), \tag{3}$$

where δ is the discount factor and ϕ is the exogenous separation probability. In the baseline scenario, incumbent worker-firm pairs are not subject to idiosyncratic productivity shocks, i.e. there is no ε_t and the wage for existing worker-firm pairs is not dependent on any idiosyncratic shock realization.

The firm selects an unemployed worker whenever there is an expected positive surplus:

$$\tilde{\varepsilon}_t = w_t(\varepsilon_t) - a_t - \delta(1-\phi) E_t(\pi_{t+1}). \tag{4}$$

Thus, the selection rate is given by:

$$\eta_t = \int_{\tilde{\varepsilon}_t}^{\infty} f(\varepsilon)\, d\varepsilon. \tag{5}$$

[3]Due to the *iid* assumption, we abstract from the worker-firm pair specific index i from here onward.

2.4 Vacancies

As in Pissarides (2000, chapter 1), we assume that each vacancy corresponds to one firm. For entering the market, firms have to pay a fixed vacancy posting cost κ. The value of a vacancy Ψ is

$$\Psi_t = -\kappa + q_t \eta_t E_t \left[\pi_t^E | \varepsilon_t \geq \tilde{\varepsilon}_t \right] + (1 - q_t \eta_t) \Psi_t, \tag{6}$$

where $q_t = c_t / v_t$ is the probability that a vacancy, v_t, leads to a contact, c_t (i.e. overall contacts divided by overall vacancies). Thus:

$$\Psi_t = -\kappa + q_t \eta_t \left(a_t + \frac{\int_{\tilde{\varepsilon}_t}^{\infty} (\varepsilon_t - w(\varepsilon_t)) f(\varepsilon_t) d\varepsilon_t}{\eta_t} + \delta (1 - \phi) E_t (\pi_{t+1}) \right) + (1 - q_t \eta_t) \Psi_t, \tag{7}$$

Firms will post vacancies up to the point where the value is driven to zero (free entry condition), i.e.

$$\frac{\kappa}{q_t \eta_t} = a_t + \frac{\int_{\tilde{\varepsilon}_t}^{\infty} (\varepsilon_t - w(\varepsilon_t)) f(\varepsilon_t) d\varepsilon_t}{\eta_t} + \delta (1 - \phi) E_t (\pi_{t+1}). \tag{8}$$

It is straightforward to see that the model nests the standard matching model where all workers are selected (i.e. with no role for idiosyncratic shocks), by setting $\eta_t = 1$ and $\varepsilon_t = 0$. In this case, the right hand side is $a_t - w_t + \delta (1 - \phi) E_t (\pi_{t+1}) = a_t - w_t + \delta (1 - \phi) E_t \frac{\kappa}{q_{t+1}}$.

Note that even in the case of a degenerate contact function, it is perfectly rational for firms to enter the market. Under a positive aggregate productivity shock, the expected returns of hiring a worker increase. Thus, more firms will enter the market to compete for these profits until the free-entry condition holds again. This makes vacancies procyclical.

2.5 Wages

We assume that a larger idiosyncratic productivity shock leads to a proportionally larger wage:

$$w(\varepsilon_t) = w_t + \alpha \varepsilon_t, \tag{9}$$

where α is the proportional component. w_t is the wage net of contemporaneous ε_t realization (i.e. the wage that holds in future periods if there are no future idiosyncratic shocks). w_t may be a function of current and future variables such as aggregate productivity, market tightness, future expected cutoff points or unemployment benefits, but not the current idiosyncratic productivity realization. Thus, our wage equation is very general and also nests standard Nash bargaining, i.e. a privately efficient wage formation.

2.6 Employment

We assume an economy with a fixed labor force L, which is normalized to 1. Thus, the employment stock is equal to the employment rate, n. Thus, the employment dynamics in this economy is determined by

$$n_{t+1} = (1 - \phi - p_t \eta_t)\, n_t + p_t \eta_t. \tag{10}$$

The number of searching workers is thus equal to the number of unemployed workers at the beginning of period t, i.e.

$$u_t = 1 - n_t. \tag{11}$$

2.7 Labor Market Equilibrium

The labor market equilibrium consists of the equations for firms' profits (3), the productivity cutoff point (4), the selection rate (5), the vacancy free entry condition (8), the contact function (1), the wage equation (9), the employment dynamics equation (10) and the definition of unemployment (11).

3 Analytics

This section shows analytically that our simple model with idiosyncratic productivity shocks and free entry of vacancies generates an equilibrium matching function relationship. We prove for a degenerate contact function that the estimated weight on vacancies in this matching function correlation is described by the first derivative of the expected idiosyncratic productivity shock. We illustrate the implications for different distributions and cutoff points. In a next step, we show how our results differ for a non-degenerate contact function. Finally, we check for the robustness of our results. To obtain analytical results, all derivations in this section are based on a steady state version of our model, i.e. we assume that there is no aggregate uncertainty and we analyze the reaction of the job-finding rate and vacancies with repect to permanent changes in aggregate productivity.

3.1 Degenerate Contact Function

For illustration purposes, we start with a degenerate contact function ($\gamma = 0$). The matching function correlation (i.e. the connection between the job-finding rate and market tightness) can be described by three equations, namely the hiring cutoff point $\tilde{\varepsilon}$, the job-finding rate η, and the market tightness, defined as $\theta = v/u$:

$$\tilde{\varepsilon} = \frac{w - a}{(1 - \delta\,(1 - \phi))\,(1 - \alpha)}, \tag{12}$$

$$\eta = \int_{\tilde{\varepsilon}}^{\infty} f(\varepsilon)\, d\varepsilon, \tag{13}$$

$$\theta = \frac{p\eta}{\kappa} \left(\frac{a - w}{(1 - \delta(1 - \phi))} + \frac{(1 - \alpha) \int_{\tilde{\varepsilon}}^{\infty} \varepsilon f(\varepsilon)\, d\varepsilon}{\eta} \right). \tag{14}$$

In standard empirical matching function estimations, the job-finding rate (jfr) is regressed on the market tightness, where β_1 shows how strongly the job-finding rate and the market tightness comove in percentage terms, namely:

$$\ln jfr_t = \ln p_t \eta_t = \beta_o + \beta_1 \ln \theta_t + \epsilon_t, \tag{15}$$

where β_1 is the elasticity of matches with respect to vacancies in a Cobb-Douglas CRS specification.

The job-finding rate and market tightness are both functions of productivity. By deriving the elasticity of the job-finding rate with respect to productivity and by deriving the elasticity of market tightness with respect to productivity[4], we obtain an analytical expression for the empirical elasticity of the job-finding rate with respect to market tightness[5], namely:

$$\frac{\partial \ln(p\eta)}{\partial \ln a} = \frac{-a \frac{\partial \tilde{\varepsilon}}{\partial a} f(\tilde{\varepsilon})}{\eta}, \tag{16}$$

$$\frac{\partial \ln \theta}{\partial \ln a} = \frac{-\frac{\partial \tilde{\varepsilon}}{\partial a} a (1 - \alpha)}{\left(\frac{a - w}{(1 - \delta(1 - \phi))} + \frac{(1 - \alpha) \int_{\tilde{\varepsilon}}^{\infty} \varepsilon f(\varepsilon) d\varepsilon}{\eta} \right)}. \tag{17}$$

Thus:

$$\frac{\partial \ln(p\eta)}{\partial \ln \theta} = \frac{f(\tilde{\varepsilon})}{\eta} \left(\frac{\int_{\tilde{\varepsilon}}^{\infty} \varepsilon f(\varepsilon)\, d\varepsilon}{\eta} - \tilde{\varepsilon} \right). \tag{18}$$

It is important to emphasize that the comovement of the job-finding rate and market tightness in equation (18) is not a causal relationship. More vacancies do not generate more contacts and jobs in aggregate. However, the model with idiosyncratic shocks and free entry of vacancies generates a positive equilibrium comovement between the job-finding rate and market tightness.

What is the underlying economic mechanism and intuition? When aggregate productivity rises, firms have an incentive to hire workers with lower idiosyncratic productivity. Thus, the job-finding rate is clearly procyclical. When productivity rises, this also increases the returns from posting a vacancy. Thus, firms compete for the larger pie of profits, more of them enter the market and

[4] See Technical Appendix for details.

[5] Note that Merkl and van Rens (2012) show that the job-finding rate and its dynamics are isomorphic in a model with idiosyncratic training costs (under a Pareto distribution) and in the search and matching model. However, their model does not contain any vacancies and is thus silent on the shape of the matching function.

thus increase the market tightness in the economy. These two mechanisms combined lead to a positive equilibrium comovement between the job-finding rate and the market tightness.

Interestingly, the matching function correlation in equation (18) corresponds to the first derivative of the conditional expectation of idiosyncratic productivity:

$$\frac{\partial \frac{\int_{\tilde{\varepsilon}}^{\infty} \varepsilon f(\varepsilon) d\varepsilon}{\eta}}{\partial \tilde{\varepsilon}} = \frac{f(\tilde{\varepsilon})}{\eta} \left(\frac{\int_{\tilde{\varepsilon}}^{\infty} \varepsilon f(\varepsilon)\, d\varepsilon}{\eta} - \tilde{\varepsilon} \right) = \frac{\partial \ln (p\eta)}{\partial \ln \theta}. \tag{19}$$

Thus, up to a first order Taylor approximation, the comovement between the job-finding rate and the market tightness is determined by equation (19). The quality of this approximation will be checked numerically in Section 4.

Figure 1 illustrates the prediction of our model for different distributions. The upper panel plots the density functions of ε for normal, logistic and lognormal distributions and the lower panel plots the first derivative of the conditional expectation of ε with respect to different cutoff points, i.e. the implied weight on vacancies. Two observations are worth pointing out. First, for these standard distributions the weight on vacancies is always between 0 and 1. Second, when the cutoff point is at the left hand side of the peak of the density function, the first derivative of the conditional expectation (i.e. the weight on vacancies) is smaller than 0.5, while it is larger than 0.5 on the right hand side. This will be important later on when we compute a model implied matching function correlation with the help of an empirical wage distribution.

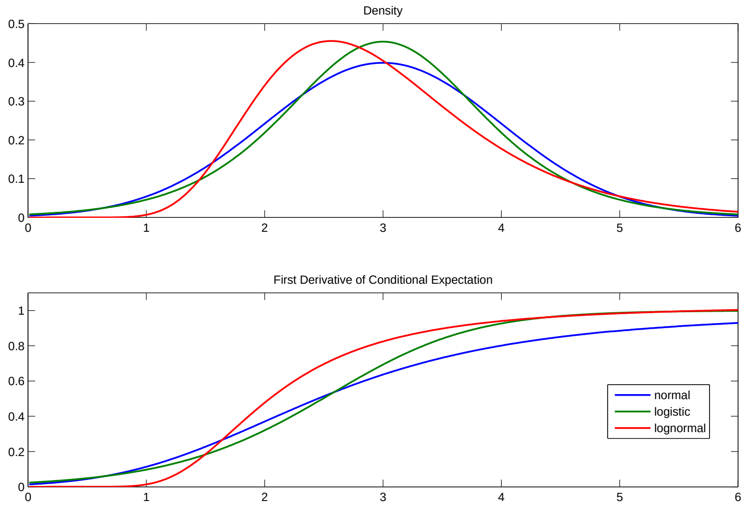
Why does the matching function correlation have a weight larger than 0.5 on the right hand side of the peak of the density function and a weight smaller than 0.5 on the left hand side? The reason is that market tightness is driven by the free entry condition of vacancies (see equation (14)). When aggregate productivity increases, workers with lower idiosyncratic productivity are hired, i.e. the hiring cutoff moves to the left. On the left hand side of the peak of the density function, a small mass of additional workers with low idiosyncratic productivity will be hired. Thus, vacancies move by a lot because the additional hiring activity does not lower the average idiosyncratic productivity by much. Large vacancy movements relative to the job-finding rate lead to a small estimated coefficient in equation (15).

3.2 Traditional Contact Function

Now, let us assume a traditional contact function with $0 < \gamma < 1$. In this case, the probability for a worker to make a contact ($p = c/u$) depends on aggregate productivity. In our real business cycle framework, we thus expect a procyclical movement of the contact rate ($\frac{\partial p}{\partial a} > 0$).

To analyze the implications of this modification, we recalculate the elasticity of the job-finding rate with respect to market tightness:

Figure 1: Predicted matching coefficients for standard distributions



Notes: Density function and first derivative of conditional expectation for different standard distributions (namely, normal, logistic, and lognormal). For comparability reasons, the variance is normalized to 1 and the mean is set to 3 (the lognormal distribution requires a positive mean).

$$\frac{\partial \ln (p\eta)}{\partial \ln a} = -\frac{af(\tilde{\varepsilon})\frac{\partial \tilde{\varepsilon}}{\partial a}}{\eta} + \frac{\partial \ln p}{\partial \ln a}, \quad (20)$$

$$\frac{\partial \ln \theta}{\partial \ln a} = \frac{-\frac{\partial \tilde{\varepsilon}}{\partial a} a (1-\alpha)}{\left(\frac{a-w}{(1-\delta(1-\phi))} + \frac{(1-\alpha)\int_{\tilde{\varepsilon}}^{\infty} \varepsilon f(\varepsilon) d\varepsilon}{\eta}\right)} + \frac{\partial \ln p}{\partial \ln a}. \quad (21)$$

The elasticities of the job-finding rate and market tightness with respect to productivity are the elasticities with a fixed contact rate plus the elasticity of the contact rate with respect to productivity. Defining $\xi_{jfr/\theta} = \frac{\partial \ln jfr}{\partial \ln \theta}$, $\xi_{\eta/a} = -\frac{af(\tilde{\varepsilon})\frac{\partial \tilde{\varepsilon}}{\partial a}}{\eta}$, $\xi_{\theta/a} = \frac{-\frac{\partial \tilde{\varepsilon}}{\partial a} a(1-\alpha)}{\left(\frac{a-w}{(1-\delta(1-\phi))} + \frac{(1-\alpha)\int_{\tilde{\varepsilon}}^{\infty} \varepsilon f(\varepsilon) d\varepsilon}{\eta}\right)}$ and $\xi_{p/a} = \frac{\partial \ln p}{\partial \ln a}$, we can write the elasticity of the job-finding rate with respect to market tightness as:

$$\xi_{jfr/\theta} = \frac{\xi_{\eta/a} + \xi_{p/a}}{\xi_{\theta/a} + \xi_{p/a}}. \quad (22)$$

Taking first derivatives allows us to see how this elasticity changes with a procyclical contact rate:

$$\frac{\partial \xi_{jfr/\theta}}{\partial \xi_{p/a}} = \frac{\xi_{\theta/a} - \xi_{\eta/a}}{\left(\xi_{\theta/a} + \xi_{p/a}\right)^2}. \quad (23)$$

In the previous section, we have shown that for a variety of standard assumptions and cutoff points, the elasticity of the selection rate with respect to market tightness is smaller than 1 ($\frac{\xi_{\eta/a}}{\xi_{\theta/a}} < 1$). Thus, the numerator of (23) is positive, and $\frac{\partial \xi_{jfr/\theta}}{\partial \xi_{p/a}} > 0$, i.e. a stronger procyclicality of the contact rate increases the weight of vacancies in an estimated matching function. In different words: If both a traditional contact function and idiosyncratic shocks are important for match formation, both of them contribute to a positive weight on vacancies in an estimated matching function.

3.3 Robustness Checks

The model we have derived so far is most similar to the selection model by Brown et al. (forthcoming). However, our results hold for a broad set of models that contain idiosyncratic productivity shocks. The Technical Appendix shows that we obtain the same analytical results in a search and matching model with endogenous separations (where iid shocks hit every period) and in a model where idiosyncratic shocks are drawn for the entire span of employment. In all these cases, the first derivative of the expected idiosyncratic productivity shock corresponds to the matching function correlation that we would observe in the absence of a traditional contact function.

4 Theory and Evidence

Our analytical results put us in a position to use wage data as a proxy for the idiosyncratic productivity shocks to calculate the model implied weight on vacancies. We first establish a reference point by estimating an empirical matching function. We then proceed to calibrate the steady state model with individual wage data. For a degenerate contact function we can thus directly calculate the model implied matching function. We also simulate the dynamic model and estimate a matching function from the simulated data. The simulation allows us to test for the quality of our steady state approximation and for the constant returns to scale assumption. In addition, we check how a traditional contact function has to look like in order to obtain the same elasticity of matches with respect to vacancies as in empirical estimations.

For all these exercises, we use administrative labor market data for Germany (see e.g. Dustmann et al., 2009; Schmieder et al., 2012). The German administrative database has several advantages over commonly used U.S. data. First, it provides actual labor market transitions on a daily basis. This means that we do not have to construct labor market flows from unemployment, employment and duration data and we do not face the problem of a time aggregation bias (see, e.g., Shimer, 2005, 2012; Nordmeier, 2012). Second, we can use several control variables that might influence the search and matching process. Third, we can observe wages for new matches. Importantly, these wages are from the same database that we construct our flow data from. Finally, we have real vacancies instead of a job advertising index for longer time series.[6]

4.1 Empirical Matching Function

We estimate a standard Cobb-Douglas CRS matching function for the German labor market. Thus, we regress the job-finding rate jfr on labor market tightness θ, a linear time trend t, and a shift dummy, d_{2005}, which accounts for the redefinition of unemployment in course of the so-called Hartz reforms:[7]

$$\log jfr_t = \beta_0 + \beta_1 \log \theta_t + \beta_2 t + \beta_3 d_{2005} + \psi_t, \tag{24}$$

where the job-finding rate at time t denotes all matches during month t over the beginning-of-month-t unemployment stock and market tightness refers to the beginning-of-month-t vacancy to unemployment ratio. The coefficient β_1 represents the matching elasticity with respect to vacancies and thus is the relevant reference point for our numerical exercises below.

We further include observable control variables to account for the effects of a changing unemployment pool and different search intensities on the aggregate matching probability.[8] Table 1 displays the estimation results of the matching

[6] See Appendix B for a detailed data description.

[7] In 2005, the official unemployment measure in Germany was extended to include recipients of former social assistance.

[8] It is well known that there is duration dependence of individual job-finding rates. Recent research by Hornstein (2012) and Barnichon and Figura (2011) suggests that this may be due

Table 1: Matching function estimations

jfr	(1)	(2)
constant	-2.3498***	-4.3403**
log θ	0.2458***	0.3463***
t	-0.0003***	-0.0054**
d_{2005}	-0.0798***	-0.0766
controls	no	yes
adj. R^2	0.5134	0.6162
DW statistic	1.3664	1.8407
CRS t-statistic	1.0074	1.6141

Note: OLS estimations (1993-2007). ***, ** and * indicate significance at the 1%, 5% and 10% levels. Control variables: *long* , *young*, *old*, *low-skilled*, *high-skilled*, *foreign*, *female*, *married*, *child*, *UB I*.

function specification with and without control variables. Both estimations show a fairly good fit in terms of the adjusted R^2 measure. However, the Durbin-Watson statistic indicates that it is important to control for the composition of the unemployment pool because this specification overcomes the positive autocorrelation in the error term.[9] The point estimate of β_1 in our preferred specification is 0.35 and the 95% confidence interval spans from 0.23 to 0.46. The matching elasticities of vacancies and unemployment are thus roughly one third and two thirds, respectively. These results are in line with the survey of matching function estimations by Petrongolo and Pissarides (2001). Moreover, the constant returns to scale assumption cannot be rejected.

4.2 Model Implied Matching Function

How closely does a model with a degenerate contact function but with free entry of vacancies and idiosyncratic productivity match the estimated matching function? To test for this, we use the wage distribution of the German administrative data for new matches to infer the shape of the actual distribution of idiosyncratic productivity at the cutoff point.[10]

We have assumed that wages are formed according to $w(\varepsilon_t) = w_t + \alpha\varepsilon_t$, where w_t contains aggregate components (e.g. current and future market tightness) and ε_t represents match-specific idiosyncratic productivity. Given that this wage formulation nests Nash bargaining, this is a standard assumption. We will use the proportionality between contemporaneous idiosyncratic productivity and the wage for our empirical analysis.

to composition effects of the unemployment pool. Katz and Meyer (1990) find evidence for an influence of unemployment benefit receipt on workers' job acceptance behavior.

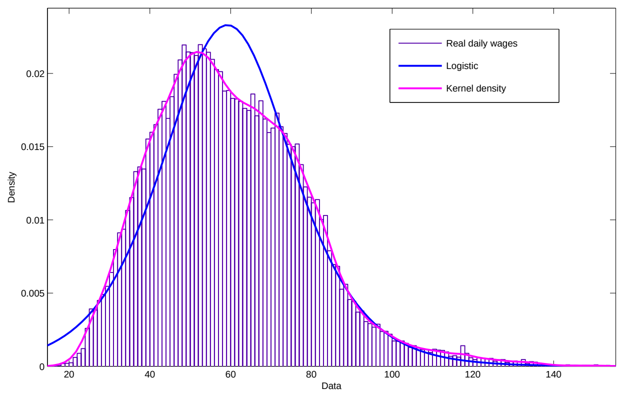
[9] We also performed an IV estimation to account for an endogeneity problem in specification (1), but the coefficients did not change notably. The results of the IV estimation are available on request.

[10] See Appendix B for a description of the wage data.

We focus on wages of a homogeneous reference group as we are not interested in wage differentials that can be explained by observable characteristics such as education, gender or unemployment history. We choose the following baseline group: male, German, not married, no children, age 25-55, medium skilled and short-term unemployed (before being hired). For comparability reasons, we restrict our attention to full-time employment.[11] As a robustness check we report results for various other group compositions in the Appendix.

Our proportionality assumption allows us to infer the shape of the distribution of the idiosyncratic productivity directly from the wage data. In line with our baseline model, we only use wages at the start of an employment spell. The histogram of wages is displayed in Figure 2. When equation (9) holds, idiosyncratic productivity is just a scaled version of this distribution.[12]

Figure 2: Distribution of real daily wages



Notes: Wages are real daily gross wages of new job entrants with the following characteristics: male, German, full-time employed, not married, no children, age 25-55, mediums skilled (Hauptschule or Realschule plus vocational training), short-term unemployed.

We only observe part of the distribution, namely the realizations of productivity that result in a hire. As our model would predict, the distribution looks truncated on the left side where we expect the cutoff productivity, whereas the distribution of wages flattens out smoothly on the right side for the workers with high idiosyncratic productivity. This truncation is not taken into account when we fit the distribution to the data. Fortunately, all that is relevant for our results is the shape of the distribution at the cutoff point. All that we require is thus some smoothness of the distribution at the hiring cutoff.

[11] Controlling for year fixed effects does not alter our results.

[12] Note that the scaling does not affect the shape of the distribution at the respective cutoff point and does not affect the calculation of equation (19).

According to our model, the relevant cutoff productivity would be determined by the lowest reported wage. In our data this wage is just below 17 Euro per calender day. In order to rule out that our results are driven by outliers we set the cutoff point at the 1st, 5th and 10th percentiles of the wage distribution. These correspond to daily gross wages of 26, 33 and 37 Euro respectively. It is quite standard in the literature to use the 10th percentile as a minimum wage measure. For example, the mean-min ratio, which corresponds to the 50th to 10th percentile ratio, is a conventionally used measure for wage dispersion (see e.g. Hornstein et al., 2011).

We fit the data non-parametrically using a kernel density estimation with a normal kernel (see Figure 2) and numerically calculate the derivative of the conditional expectation using equation (19). This gives us the numbers for the elasticity of matches with respect to vacancies. Based on the wage distribution, it is 0.12, 0.28, and 0.38 for the 1st, 5th, and 10th percentile, respectively (see Table 2). These model based numbers already come remarkably close to our empirical data estimate of 0.35. This is particularly interesting, given that these results are based on a degenerate contact function so far, where vacancies do not affect the aggregate number of contacts.

Table 2: Weight on vacancies, based on steady state approximation

	1st percentile	5th percentile	10th percentile
$\log \theta$	0.12	0.28	0.38

Note: Results are calculated numerically from the non-parametric fit of the distribution using equation (19).

Before we move to the dynamic analysis, it is worthwhile discussing some potential pitfalls of our analysis:

First, wage differentials may be driven by other factors than observables or idiosyncratic productivity, namely luck. This would change our wage equation to $w(\varepsilon_t) = w_t + \alpha\varepsilon_t + \iota_t$, where ι_t is the luck component. But as long as there is no systematic correlation between ε_t and ι_t, the luck component simply adds noise to our analysis, but the results remain valid.

Second, collective bargaining is still the predominant wage formation mechanism in Germany. If collective bargaining prevents that idiosyncratic productivity differentials show up in the wage, our analysis is not valid. However, collective bargaining only defines a lower bound for the wage. If a worker with certain characteristics is particularly productive, firms can easily pay a higher wage. In addition, firms have a certain discretion into which payscale they want to classify a particular worker (i.e. a worker with a lower idiosyncratic productivity can be assigned to a lower payscale). Beyond this, collective bargaining has lost importance over the last decades. However, controlling for year fixed effects in our wage distribution does not alter our results.

Third, we may have chosen our homogeneous reference group inappropriately. In particular, we may have defined it too broadly. Here, we face of course a trade off between the number of observations and a narrower group definition.[13] Therefore, we repeat the dynamic simulation for a set of different reference groups (in particular a finer differentiation along age and education). The results can be found in the Appendix and are fairly similar. We are therefore confident that our results are not driven by the choice of the reference group. In addition, the results in the Appendix show that our preferred reference group represents an intermediate case with respect to the range of estimates.

4.3 Dynamics

So far, the results have been based on our comparative static equation. In order to test for the validity of our results out of steady state, we now simulate the model with shocks to aggregate productivity. This also allows us to check whether non-constant returns to scale are present in the simulated data. Most importantly, we can use the dynamic simulation to quantitatively assess the interplay between a traditional contact function and idiosyncratic productivity.

For the dynamic simulation, we need to parameterize our model. In particular, we need to define a functional form for the idiosyncratic productivity distribution. We therefore fit several standard distributions to the data, i.e. we choose the parameters of the distributions that give the best fit of our data in terms of maximum likelihood. We choose the logistic distribution because it has the best fit. Figure 2 displays the distribution of real daily gross wages and the corresponding logistic distribution. The match is reasonably good especially near the cutoff.[14] We provide all other details on the parametrization in Appendix D. We simulate the model 1000 times with aggregate productivity governed by a first-order autocorrelation process. The simulation is based on a second-order Taylor approximation.[15] Each time we use 180 periods corresponding to the time span used for our empirical matching function estimation.

Again, we estimate a Cobb-Douglas CRS matching function:

$$\log jfr_t = \beta_0 + \beta_1 \log \theta_t + \psi_t. \tag{25}$$

Table 3 compares the estimated coefficient β_1 for different quantiles to the comparative static results when we use equation (19) for the same purpose. The numbers from the simulation exercise and the comparative static exercise are literally the same. This shows that our comparative statics is a very good approximation for the dynamic exercise. Note that the discrepancy between the results in this section and the previous section only stem from the imperfect fit of the logistic distribution compared to the non-parametric fit.

[13] In addition, if we choose a too narrow subgroup, it may be difficult to make an inference for the aggregate matching function.

[14] The fit of course ignores the truncation of wages on the left hand side. In addition, we enforce a specific mass and shape for the part of the wages that we do not observe. This is without loss of generality as the non-observable part of the distribution does not affect our results as long as there is no sharp discontinuity in the vicinity of the cutoff. We are therefore confident that the logistic distribution provides a reasonable approximation for our purposes.

[15] This explicitly allows for some non-linearities not covered by our analytical steady state results.

Table 3: Matching function based on simulation (degenerate contact function)

	1st percentile	5th percentile	10th percentile
	Simulation result		
constant	-2.88	-2.79	-2.69
log θ	0.14	0.22	0.28
	Simulation result (unconstrained)		
constant	-2.89	-2.79	-2.69
log U	0.85	0.78	0.72
log V	0.14	0.22	0.28
	Steady State prediction		
constant	-	-	-
log θ	0.14	0.22	0.28

Note: Steady State results are calculated numerically from the logistic fit of the distribution using equation (19). The dynamic simulation results are OLS estimates from the simulated series using a logistic distribution. Reported coefficients are means over 1000 simulations.

We further analyze whether we have artificially imposed the CRS assumption in our comparative static exercise. We estimate the following unconstrained matching function:

$$\log m_t = \beta_0 + \beta_1 \log v_t + \beta_2 \log u_t + \psi_t, \tag{26}$$

where m_t denotes all matches in period t. Table 3 shows that the sum of estimated coefficients $(\beta_1 + \beta_2)$ is virtually 1. The estimated coefficients are also statistically significant at the 1% level in every single run of the simulation.[16]

Interestingly, when we estimate other functional forms such as CES, the Cobb-Douglas specification is confirmed. As a robustness check we have also performed IV estimations using the lagged value of market tightness as an instrument. This does not alter our results.[17]

4.4 Traditional Contact Function and Idiosyncratic Productivity

Finally, the dynamic simulation puts us in a position to combine idiosyncratic productivity with a traditional contact function. Hence, we assume that the contact probability is defined by $p_t(v_t, u_t) = \mu\theta_t^\gamma$ with $\gamma > 0$, i.e. the job-finding rate is not only driven by the movement of the cutoff point for idiosyncratic productivity, but also by a procyclical contact rate.

We analyze how much of the empirical matching function correlation is due to the contact function and how much is due to idiosyncratic productivity. For this purpose, we again use the logistic distribution for wages (see Table 3) and determine the contact elasticity γ so as to get an overall elasticity of matches

[16] We do not report t-statistics as means over t-values would not have a meaningful interpretation.

[17] Results are available from the authors on request.

with respect to vacancies of 0.35 as found in the empirical matching function. The results are shown in Table 4.

Table 4: Weight on vacancies, dynamic simulations with different contact function specifications.

	1st percentile	5th percentile	10th percentile
Matching function correlation with $\gamma = 0$			
$\log \theta$	0.14	0.22	0.28
Calibrated elasticity of the contact function (γ)			
$\log \theta$	0.17	0.11	0.06
Combined matching function correlation			
$\log \theta$	0.35	0.35	0.35

Note: The dynamic simulation results are OLS estimates from the simulated series using a logistic distribution. Reported coefficients are means over 1000 simulations.

Our numerical results are in line with our theoretical results from Section 3.2. When a procyclical contact rate and idiosyncratic productivity are combined, this leads to a larger weight on vacancies in an estimated matching function. The first line in Table 4 shows the results for the model simulation with idiosyncratic productivity but with a degenerate matching function. The second line shows the elasticities of a traditional contact function that would correspond to the overall elasticity of matches with respect to vacancies if there was no idiosyncratic productivity. The third line shows the combination of the two mechanisms. Interestingly, the resulting matching function correlation has a weight on vacancies which is roughly equal to the sum of the weight on vacancies in the two cases. More precisely, the sum of the two mechanism is somewhat smaller than the overall weight on vacancies. Thus, there is a small degree of complementarity between idiosyncratic productivity and the traditional contact function.

The results suggest that 50% or more of the observed elasticity of matches with respect to vacancies may actually be driven by idiosyncratic productivity. When we use the 10th percentile of the wage distribution, 80% of the weight on vacancies are driven by idiosyncratic shocks. Thus, our exercise shows that there is a potentially large bias in standard matching function estimations if idiosyncratic productivity plays a role. Hence, in many model applications, the contact functions may be misspecified, assigning too large a role for vacancies in the process of match formation.

Why is this important? Hosios (1990) shows that matching models with a CRS matching function are constrained efficient when firms' bargaining power in Nash bargaining is equal to the weight on vacancies in the contact function. But by using the weight on vacancies from empirical estimations, there is a misspecification. This may lead to a misjudgement of the welfare implications of policy interventions. In addition, the quantitative effects of policy interventions that affect the vacancy margin will be smaller.

5 Conclusion

Our paper shows that a wide class of models with idiosyncratic productivity and a vacancy free entry condition generates a positive equilibrium relationship between matches on the one hand and unemployment and vacancies on the other hand. Although this insight is straighforward, we are the first paper to establish this connection and to work out some interesting implications. We have shown analytically that idiosyncratic productivity and free entry of firms generate a Cobb-Douglas constant returns relationship. This relationship continues to hold for a realistic calibration of the idiosyncratic productivity distribution and for aggregate shocks. Furthermore, we show that even with a degenerate contact function the equilibrium comovement, based on idiosyncratic shocks and free entry of firms, generates a matching elasticity with respect to vacancies which is very close to the data. Our calibration with indiviual wage data suggests that the matching function in many calibrations is seriously misspecified.

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A Theory: Derivations

This Appendix proceeds in three steps. First, we show the intermediate steps for the results in Section 2. This corresponds to the case where the idiosyncratic shocks is only drawn during the first period of employment. Second, we show that the result also holds for a model with an *iid* shock in each period of employment, i.e. a model with endogenous separations (an assumption conventionally used in search and matching models with endogenous separations). Third, we show that the result does not change when workers draw an idiosyncratic shock realization at the beginning of their employment span and this realization does not change over time (an assumption conventionally used for the wage offer distribution in search models).

A.1 Baseline Results

$$\tilde{\varepsilon} = \frac{-(a-w)}{(1-\delta(1-\phi))(1-\alpha)}, \tag{27}$$

$$\eta = \int_{\tilde{\varepsilon}}^{\infty} f(\varepsilon)\, d\varepsilon, \tag{28}$$

$$\theta = \frac{p\eta}{\kappa}\left(\frac{a-w}{1-\delta(1-\phi)} + \frac{(1-\alpha)\int_{\tilde{\varepsilon}}^{\infty} \varepsilon f(\varepsilon)\, d\varepsilon}{\eta}\right). \tag{29}$$

The first derivative of the selection rate with respect to productivity is

$$\frac{\partial \eta}{\partial a} = -\frac{\partial \tilde{\varepsilon}}{\partial a} f(\tilde{\varepsilon}). \tag{30}$$

Thus, the elasticity of the job-finding rate with respect to productivity is

$$\frac{\partial \ln(p\eta)}{\partial \ln a} = \frac{-a\frac{\partial \tilde{\varepsilon}}{\partial a} f(\tilde{\varepsilon})}{\eta}. \tag{31}$$

By applying the implicit functions theorem, we obtain:

$$\frac{d\tilde{\varepsilon}}{da} = \frac{\frac{1-\frac{\partial w}{\partial a}}{(1-\delta(1-\phi))(1-\alpha)}}{\frac{\frac{\partial w}{\partial a}}{(1-\delta(1-\phi))(1-\alpha)}+1}. \tag{32}$$

The first derivative of market tightness with respect to productivity is

$$\frac{\partial \theta}{\partial a} = \frac{pf(\tilde{\varepsilon})}{\kappa}\frac{d\tilde{\varepsilon}}{da}\left(\frac{a-w}{(1-\delta(1-\phi))} + \frac{(1-\alpha)\int_{\tilde{\varepsilon}}^{\infty} \varepsilon f(\varepsilon)\, d\varepsilon}{\eta}\right) + \tag{33}$$

$$\frac{p\eta}{\kappa}\left(-(1-\alpha)\frac{d\tilde{\varepsilon}}{da} + (1-\alpha)\frac{d\tilde{\varepsilon}}{da}\frac{f(\tilde{\varepsilon})}{\eta}\left(-\tilde{\varepsilon} + \frac{\int_{\tilde{\varepsilon}}^{\infty} \varepsilon f(\varepsilon)\, d\varepsilon}{\eta}\right)\right). \tag{34}$$

Thus:

$$\frac{\partial \theta}{\partial a} = -\left(1-\alpha\right)\frac{p\eta}{\kappa}\left(\frac{d\tilde{\varepsilon}}{da}\right). \tag{35}$$

Thus, the elasticity of market tighness with respect to productivity is

$$\frac{\partial \ln \theta}{\partial \ln a} = \frac{-\left(\frac{d\tilde{\varepsilon}}{da}\right) a\left(1-\alpha\right)}{\left(\frac{a-w}{(1-\delta(1-\phi))} + \frac{(1-\alpha)\int_{\tilde{\varepsilon}}^{\infty}\varepsilon f(\varepsilon)d\varepsilon}{\eta}\right)}. \tag{36}$$

Combining equations (31) and (36), we obtain the heterogeneity based matching function:

$$\begin{aligned}
\frac{\partial \ln (p\eta)}{\partial \ln \theta} &= \frac{\left(\frac{a-w}{(1-\delta(1-\phi))} + \frac{(1-\alpha)\int_{\tilde{\varepsilon}}^{\infty}\varepsilon f(\varepsilon)d\varepsilon}{\eta}\right) a\frac{\partial \tilde{\varepsilon}}{\partial a} f\left(\tilde{\varepsilon}\right)}{\left(1-\alpha\right)\frac{d\tilde{\varepsilon}}{da}\eta a} \\
&= \frac{f\left(\tilde{\varepsilon}\right)}{\eta}\left(\frac{\int_{\tilde{\varepsilon}}^{\infty}\varepsilon f\left(\varepsilon\right)d\varepsilon}{\eta} - \tilde{\varepsilon}\right) \\
&= \frac{\partial \frac{\int_{\tilde{\varepsilon}}^{\infty}\varepsilon f(\varepsilon)d\varepsilon}{\eta}}{\partial \tilde{\varepsilon}}.
\end{aligned}$$

A.2 Endogenous Separations

With endogenous separations, the cutoff point is

$$-\tilde{\varepsilon} = a - w - \alpha\tilde{\varepsilon} + \delta\left(1-\phi\left(\tilde{\varepsilon}\right)\right)\left(a - w + \frac{\left(1-\alpha\right)\int_{\tilde{\varepsilon}}^{\infty}\varepsilon f\left(\varepsilon\right)d\varepsilon}{\left(1-\phi\left(\tilde{\varepsilon}\right)\right)}\right) \tag{37}$$

$$+\delta^2\left(1-\phi\right)^2\left(a - w + \frac{\left(1-\alpha\right)\int_{\tilde{\varepsilon}}^{\infty}\varepsilon f\left(\varepsilon\right)d\varepsilon}{\left(1-\phi\left(\tilde{\varepsilon}\right)\right)}\right) + ... \tag{38}$$

$$\tilde{\varepsilon} = \frac{-\left(a - w + \delta\left(1-\phi\left(\tilde{\varepsilon}\right)\right)\left(1-\alpha\right)\frac{\int_{-\infty}^{\tilde{\varepsilon}}\varepsilon f(\varepsilon)d\varepsilon}{1-\phi(\tilde{\varepsilon})}\right)}{\left(1-\delta\left(1-\phi\left(\tilde{\varepsilon}\right)\right)\right)\left(1-\alpha\right)}. \tag{39}$$

As usual, the selection rate is

$$\eta = \int_{\tilde{\varepsilon}}^{\infty} f\left(\varepsilon\right)d\varepsilon. \tag{40}$$

The elasticity of the job-finding rate with respect to productivity is

$$\frac{\partial \ln (p\eta)}{\partial \ln a} = \frac{-a\frac{\partial \tilde{\varepsilon}}{\partial a} f\left(\tilde{\varepsilon}\right)}{\eta}. \tag{41}$$

With endogenous separations, market tightness is:

$$\theta = \frac{p\eta}{\kappa}\left(\frac{a-w+\frac{(1-\alpha)\int_{\tilde{\varepsilon}}^{\infty}\varepsilon f(\varepsilon)d\varepsilon}{(1-\phi(\tilde{\varepsilon}))}}{1-\delta\left(1-\phi\left(\tilde{\varepsilon}\right)\right)}\right) \tag{42}$$

$$= \frac{p\eta}{\kappa}\left(-\left(1-\alpha\right)\tilde{\varepsilon}+\frac{\left(1-\alpha\right)\int_{\tilde{\varepsilon}}^{\infty}\varepsilon f\left(\varepsilon\right)d\varepsilon}{\left(1-\phi\left(\tilde{\varepsilon}\right)\right)}\right), \tag{43}$$

$$\frac{\partial\theta}{\partial a} = \frac{pf\left(\tilde{\varepsilon}\right)}{\kappa}\frac{d\tilde{\varepsilon}}{da}\left(\frac{a-w+\frac{(1-\alpha)\int_{\tilde{\varepsilon}}^{\infty}\varepsilon f(\varepsilon)d\varepsilon}{(1-\phi(\tilde{\varepsilon}))}}{1-\delta\left(1-\phi\left(\tilde{\varepsilon}\right)\right)}\right)+ \tag{44}$$

$$\frac{p\eta}{\kappa}\left(-\left(1-\alpha\right)\frac{d\tilde{\varepsilon}}{da}+\left(1-\alpha\right)\frac{d\tilde{\varepsilon}}{da}\frac{f\left(\tilde{\varepsilon}\right)}{\eta}\left(\tilde{\varepsilon}-\frac{\int_{\tilde{\varepsilon}}^{\infty}\varepsilon f\left(\varepsilon\right)d\varepsilon}{\eta}\right)\right). \tag{45}$$

Taking into account that $\eta = \phi$ in this setting and after some algebra, we obtain:

$$\frac{\partial\theta}{\partial a} = -\left(1-\alpha\right)\frac{p\eta}{\kappa}\left(\frac{d\tilde{\varepsilon}}{da}\right). \tag{46}$$

Thus, the elasticity is

$$\frac{\partial\ln\theta}{\partial\ln a} = \frac{-\left(1-\alpha\right)\frac{d\tilde{\varepsilon}}{da}a}{\left(\frac{a-w+\frac{(1-\alpha)\int_{\tilde{\varepsilon}}^{\infty}\varepsilon f(\varepsilon)d\varepsilon}{\eta}}{(1-\delta(1-\phi(\tilde{\varepsilon})))}\right)}. \tag{47}$$

Combining equations (41) and (47), we obtain the heterogeneity based matching function:

$$\frac{\partial\ln\left(p\eta\right)}{\partial\ln\theta} = \frac{\left(\frac{a-w+\frac{(1-\alpha)\int_{\tilde{\varepsilon}}^{\infty}\varepsilon f(\varepsilon)d\varepsilon}{\eta}}{(1-\delta(1-\phi(\tilde{\varepsilon})))(1-\alpha)}\right)a\frac{\partial\tilde{\varepsilon}}{\partial a}f\left(\tilde{\varepsilon}\right)}{\left(\frac{d\tilde{\varepsilon}}{da}\right)\int_{\tilde{\varepsilon}}^{\infty}f\left(\varepsilon\right)d\varepsilon a} \tag{48}$$

$$= \frac{f\left(\tilde{\varepsilon}\right)}{\eta}\left(\frac{\int_{\tilde{\varepsilon}}^{\infty}\varepsilon f\left(\varepsilon\right)d\varepsilon}{\eta}-\tilde{\varepsilon}\right)$$

$$= \frac{\partial\frac{\int_{\tilde{\varepsilon}}^{\infty}\varepsilon f(\varepsilon)d\varepsilon}{\eta}}{\partial\tilde{\varepsilon}}. \tag{49}$$

A.3 Same Idiosyncratic Shock for the Entire Employment Span

$$-\tilde{\varepsilon} = a-w-\alpha\tilde{\varepsilon}+\delta\left(1-\phi\right)\left(a-w+\left(1-\alpha\right)\tilde{\varepsilon}\right)+\delta^{2}\left(1-\phi\right)^{2}\left(a-w+\left(1-\alpha\right)\tilde{\varepsilon}\right)+... \tag{50}$$

$$\tilde{\varepsilon} = -\frac{a-w}{1-\alpha}. \tag{51}$$

Selection Rate:

$$\eta = \int_{\tilde{\varepsilon}}^{\infty} f(\varepsilon)\, d\varepsilon. \tag{52}$$

The elasticity of the job-finding rate with respect to unemployment is

$$\frac{\partial \ln (p\eta)}{\partial \ln a} = \frac{-a\frac{\partial \tilde{\varepsilon}}{\partial a} f(\tilde{\varepsilon})}{\int_{\tilde{\varepsilon}}^{\infty} f(\varepsilon)\, d\varepsilon}. \tag{53}$$

Market tightness:

$$\theta = \frac{p\eta}{\kappa} \left(\frac{a - w + \frac{(1-\alpha)\int_{\tilde{\varepsilon}}^{\infty} \varepsilon f(\varepsilon) d\varepsilon}{(1-\phi)}}{(1-\delta(1-\phi))} \right). \tag{54}$$

Thus:

$$\frac{\partial \theta}{\partial a} = \frac{pf(\tilde{\varepsilon})}{\kappa} \frac{d\tilde{\varepsilon}}{da} \left(\frac{a - w + \frac{(1-\alpha)\int_{\tilde{\varepsilon}}^{\infty} \varepsilon f(\varepsilon) d\varepsilon}{(1-\phi)}}{(1-\delta(1-\phi))} \right) + \tag{55}$$

$$\frac{p\eta(1-\alpha)}{\kappa(1-\delta(1-\phi))} \left(-\frac{d\tilde{\varepsilon}}{da} + \frac{d\tilde{\varepsilon}}{da} \frac{f(\tilde{\varepsilon})}{\eta} \left(\tilde{\varepsilon} - \frac{\int_{\tilde{\varepsilon}}^{\infty} \varepsilon f(\varepsilon)\, d\varepsilon}{\eta} \right) \right). \tag{56}$$

Taking into account that $\eta = \phi$in this setting and after some algebra, we obtain:

$$\frac{\partial \theta}{\partial a} = -(1-\alpha) \frac{1}{(1-\delta(1-\phi))} \frac{p\eta}{\kappa} \left(\frac{d\tilde{\varepsilon}}{da} \right). \tag{57}$$

Thus, the elasticity is

$$\frac{\partial \ln \theta}{\partial \ln a} = \frac{-\left(\frac{d\tilde{\varepsilon}}{da}\right)}{\left(\frac{\int_{\tilde{\varepsilon}}^{\infty} \varepsilon f(\varepsilon) d\varepsilon}{(1-\phi)} - \tilde{\varepsilon} \right)}. \tag{58}$$

Combining equations (53) and (58), we obtain the heterogeneity based matching function:

$$\frac{\partial \ln (p\eta)}{\partial \ln \theta} = \frac{\left(\frac{\int_{\tilde{\varepsilon}}^{\infty} \varepsilon f(\varepsilon) d\varepsilon}{(1-\phi(\tilde{\varepsilon}))} - \tilde{\varepsilon}\right) a \frac{\partial \tilde{\varepsilon}}{\partial a} f(\tilde{\varepsilon})}{\frac{d\tilde{\varepsilon}}{da} \int_{\tilde{\varepsilon}}^{\infty} f(\varepsilon) d\varepsilon a} \tag{59}$$

$$= \frac{f(\tilde{\varepsilon})}{\eta} \left(\frac{\int_{\tilde{\varepsilon}}^{\infty} \varepsilon f(\varepsilon) d\varepsilon}{\eta} - \tilde{\varepsilon} \right)$$

$$= \frac{\partial \frac{\int_{\tilde{\varepsilon}}^{\infty} \varepsilon f(\varepsilon) d\varepsilon}{\eta}}{\partial \tilde{\varepsilon}}. \tag{60}$$

B Data Description

The German administrative database provides coherent definitions of the matching function variables. We use monthly data over the time period from 1993 to 2007. Matches and unemployment are obtained from the Sample of Integrated Labor Market Biographies (SIAB). The SIAB is a 2% random sample of all German residents who are registered by the Federal Employment Agency because of paying social security contributions or receiving unemployment benefits (see Dorner et al., 2010). Unemployment benefits may cover contribution-based benefits, means-tested benefits and income maintenance during training. We use an adjusted measure of unemployment benefit receipt according to Fitzenberger and Wilke (2010) to determine the unemployment stock. Matches are defined as transitions from unemployment to employment subject to social security. Even though marginal employment has become subject to social security since 1999, we do not consider this kind of employment as it is often ascribed to a stepping stone into regular jobs. The number of matches is calculated continuously, i.e. we take into account every daily transition. Hence, we do not neglect any job findings that are reversed within a month. See Nordmeier (2012) for more details on the time series.

Vacancies are taken from the official statistics and cover open positions that are reported to the Federal Employment Agency. The reported vacancies account for about 30-40% of overall vacancies in Germany. However, an adjustment of the reported vacancies by using the reporting rate of the IAB Job Vacancy Survey would not affect our estimation results because the reporting rate does not show a cyclical pattern in our observation period.

For our calibration exercise, we exploit the wage information included in the SIAB. Wages are shown as the employee's gross daily wage in Euros, which was calculated from the fixed-period earnings reported by the employer and the duration of the employment period in calendar days. Because we focus on new full-time jobs, we only consider wages above the marginal part-time income threshold. We use the consumer price index (CPI) from the National Accounts to obtain real daily wages.

Table B.1: Description of control variables

Variables	Extracted series	Definition
Unemployment duration	*long*	Share of long-term unemployed, i.e. unemployment duration $\geq$ 1 year
Age	*young* *old*	Share of unemployed with age $\leq$ 25 years Share of unemployed with age $\geq$ 55 years
Education	*low-skilled* *high-skilled*	Share of unemployed without vocational training (acc. to Fitzenberger et al., 2005) Share of unemployed with university degree (acc. to Fitzenberger et al., 2005)
Nationality	*foreign*	Share of unemployed with immigration background (see Wichert and Wilke, 2012)
Gender	*female*	Share of female unemployed
Family status	*married* *child*	Share of married unemployed Share of unemployed with at least one child
Benefit receipt	*UB I*	Share of contribution-based unemployment benefits recipients (unemployment benefits I)

Data source: SIAB.

C Different Wage Groups

The results in Section 4.3 are based on the distribution of entry wages of a homogenous reference group. We repeat this exercise several times each time changing certain characteristics of the reference group. We consider the following group compositions: The reference group with...

- ... low-skilled (no vocational training) instead of medium-skilled.
- ... women instead of males.
- ... long-term instead of short-term unemployed.
- ... age further differentiated (ten year age spans).
- ... education further differentiated (no degree, vocational training degree, high school, high school and vocational training, technical college, university).

Table C.2 reports for each percentile the highest and the lowest estimates of all groups along with the baseline. Our conclusions from Section 4.3 are robust to the different group compositions. For the 5th percentile, for instance, we get a minimum of 0.13 for the coefficint on vacancies and a maximum of 0.26.

Table C.2: Weights on vacancies and unemployment: Robustness

	1st percentile	5th percentile	10th percentile
		minimum	
constant	-2.98	-2.90	-2.79
log U	0.93	0.87	0.79
log V	0.06	0.13	0.21
		base	
constant	-2.89	-2.79	-2.69
log U	0.85	0.78	0.72
log V	0.14	0.22	0.28
		maximum	
constant	-2.83	-2.73	-2.64
log U	0.81	0.74	0.69
log V	0.19	0.26	0.31

Note: The reported coefficients are means over 1000 simulations. The matching function was estimated unconstrained.

D Parametrization of the Model

We parameterize the model on a monthly basis. For an overview of targets and parameters see Table D.3. We assume Nash bargaining, which ensures private match efficiency. Nash bargaining is a special case of our general wage rule (9). We set the bargaining power of workers to 0.5. Note that our results are completely robust to variations in this parameter. Aggregate productivity is normalized to 1. The discount factor is $0.99^{\frac{1}{3}}$ and the vacancy posting cost is 0.1. The latter only affects the level of market tightness and is otherwise inconsequential. In line with the empirical data for Germany, we set the separation rate to 0.01. The value of non-work is set to 0.8. Unemployment benefits for short-term unemployed in Germany are 60 or 67% of the last net wage. Our value takes into account that there is a value of home production. We simulate the model with an AR(1) process for productivity. The correlation coefficient in the AR(1) process is set to 0.95 and the standard deviation of the shock is 0.44%. We have estimated these values from productivity data from the German National Accounts.[18]

A nice feature of the logistic distribution is that the derivative of the conditional expectation is uniquely determined by the cumulative density to the right of the cutoff point (i.e. the selection rate) for any combination of mean and variance. Thus, we can set one of the two parameters of the distribution freely. In this exercise, we set the standard deviation of the logistic distribution to 1 and let the mean of the distribution be determined endogenously. The 1st, 5th, and 10th percentile of our wages correspond to selection rates of 0.956,

[18]Productivity: Output per hours worked from the Federal Statistical Office (*Statistisches Bundesamt*), 1991Q1 to 2013Q1.

0.921, and 0.885 in the fitted distribution. Note that these do not necessarily correspond to the real selection rate as we do not know the number of workers to the left of the distribution. However, for the dynamics of our model it is irrelevant whether we have a low selection rate with a high contact rate or vice versa. What matters is the shape of the idiosyncratic shock distribution to the right of the cutoff point which we calibrate with wage data. The contact rate is set to match the empirical job-finding rate of 5% per month is steady state.

Note that the mean of idiosyncratic productivity seems unrealistically low in our parametrization. Two comments are in order. First, our baseline model is very simple and does not contain any training cost or fixed costs of production. In addition, the influence of unions may lead to larger average wages. Including these features would potentially lead to a larger average calibrated idiosyncratic productivity in the first period of employment. Second, our results for the elasticity of the matching function with respect to vacancies are independent of our parametrization strategy. The particular combination of mean and variance for a given selection rate does not matter for our key results.

Table D.3: Parameters and Targets

Parameter	Value	Source and/or target
Discount factor	$0.99^{\frac{1}{3}}$	set
Bargaining power	0.5	set
Value of leisure	0.8	set
Separation rate	0.01	SIAB data
Vacancy posting cost	0.1	set
Aggr. productivity	1	normalization
AR-coef. productivity	0.95	National Accounts data
SD productivity	0.0044	National Accounts data
Selection rate	0.956, 0.921, 0.885	wage data
Contact rate	0.052, 0.054, 0.056	jfr: 0.05 (SIAB data)
SD of log dist.	1	set
Mean of log dist.	-9.956, -10.827, -11.378	selection rate