The Solow Model in the Empirics of Growth and Trade*

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ABSTRACT

Translated to a cross-country context, the Solow model (Solow, 1956) predicts that international differences in steady state output per person are due to international differences in technology for a constant capital output ratio. However, most of the empirical growth literature that refers to the Solow model has employed a specification where steady state differences in output per person are due to international differences in the capital output ratio for a constant level of technology. My empirical results show that the former specification can summarize the data quite well by using a measure of institutional technology and treating the capital output ratio as part of the regression constant. This reinterpretation of the cross-country Solow model provides an implication for empirical studies of international trade. Harrod-neutral technology differences as presumed by the Solow model can explain why countries have different factor intensities and may end up in different cones of specialization.

I. INTRODUCTION

When I first heard about the Solow model (Solow, 1956) about 25 years ago, I learned that that this growth model was written as a response to the Harrod-Domar model and as such was mainly concerned with the existence, stability, and adjustment to a steady state. Steady state growth was shown to be the result of exogenous technological change. The empirical relevance of the Solow model for understanding long-run economic growth or cross-country differences in the level of development was certainly not an issue, as documented in the textbooks of the time, e.g., Burmeister and Dobell (1970), Jones (1975), and Hacche (1979).

Today, the Solow model is presented in a very different way. Recent textbooks rarely mention the Harrod-Domar model, and the Solow model is now mainly used to explain observed cross-country differences in the level of income, e.g., Jones (2002) and Weil (2004). Probably it is the loss of historical context in combination with what has become a standard empirical approach that explains why parts of the applied growth literature appear to perceive the Solow model as emphasizing factor accumulation as the major determinant of cross-country income differences. For instance, Easterly and Levine (2001) find in a survey of empirical research that factor accumulation is not a major determinant of growth and development, and conclude that this stylized fact speaks against growth models in the tradition of the Solow model.

The seminal paper by Mankiw et al. (1992) (henceforth MRW) is arguably the reason for the fundamental change in the textbook presentation of the Solow model. MRW use regression analysis to demonstrate that their specification of a human capital augmented Solow model provides an excellent description of cross-country data. The MRW paper has generated a large body of subsequent empirical research[1] that discusses the robustness of this result and, implicitly, the empirical relevance of the Solow model. The major counter-evidence comes from a paper by Klenow and Rodriguez-Clare (1997) (henceforth KRC), who employ development accounting methods to show that differences in (residual) technology rather than differences in the capital output ratio are the major determinants of cross-country income differences. The actual textbook treatment of this apparent contradiction is to present, without much discussion, differences in factor accumulation and differences in "productivity" (technology) as independent determinants of development that are emphasized by different growth models. What has been neglected in the empirics of growth, and in recent textbook presentations, is that the Solow model suggests otherwise.

In my view, the fundamental insight provided by the Solow model is not adequately captured by the empirical specification employed by MRW, which has led to some confusion in the applied literature just because the MRW specification is generally presumed as an estimate of the Solow model. The prevailing confusion becomes immediately evident once the historical context is reconsidered. The Harrod-Domar model has emphasized exogenous factor accumulation as a determinant of knife-edge growth. As a response to the Harrod-Domar model, the Solow model has shown that steady state growth is driven by technological change, while the adjustment to stable steady state growth is achieved by endogenous changes in factor accumulation. That is, the Solow model does not emphasize factor accumulation as a determinant of long-run growth.

This is not to suggest that the MRW specification of the Solow model is falsely based on factor accumulation as the decisive explanatory variable. What has largely gone unnoticed in

[1] For detailed references to this literature, see, http://www.bris.ac.uk/Depts/Economics/Growth/refs/augsol.htm, and http://www.bris.ac.uk/Depts/Economics/Growth/refs/levels.htm.

the applied literature is that the MRW specification is neither based on factor accumulation nor on technology as explanatory variables, but on a variable that is presumed to be a constant in the Solow model, namely the capital output ratio. At this point one may ask with some justification whether it is reasonable to apply the Solow model to a cross-country context at all. My answer to this question is borrowed from a well known paper on the empirical analysis of growth, which was written in 1957. Either this kind of empirical analysis appeals or it doesn't; if it does, I think one can draw some crude but useful conclusions from the results, but not necessarily in the way suggested by MRW.

I develop my argument in three steps. The next section applies the textbook Solow diagram to a cross-country context and highlights the implication that differences in technology explain steady state differences in income *and* in factor intensity. Section 3 presents a brief reinterpretation of the applied literature in light of the standard Solow diagram and shows that an empirical specification of the Solow model that allows for international technology differences and presumes a constant capital output ratio provides an excellent description of the cross-country data. Section 4 uses this stylized fact to reconsider the neoclassical model of trade and growth (Findlay and Grubert, 1959). Given the Solow model, Harrod-neutral differences in technology cannot be separated from steady state differences in capital intensity, which implies that the factor endowment point in the Lerner diagram should not be treated as independent from the level of (Harrod-neutral) technology. Up to now, this implication appears to have been neglected in the empirics of international trade.

II. THE SOLOW DIAGRAM AND CROSS-COUNTRY GROWTH EMPIRICS

Figure 1 reproduces the textbook diagram for the steady state of the Solow model. The steady state capital intensity (capital per worker), which is given by the intersection of the investment function and the depreciation function, determines the steady state income (output per worker), which is shown as point *A*. The subsequent textbook exercise is to ask what happens with the steady state income if there is an exogenous increase in the investment rate. As it turns out, a higher share of investment in GDP means a higher steady state income on the same production function, shown as point *B*. So the diagram seems to imply that the Solow model can be estimated by regressing output per worker on the share of investment in GDP conditional on a constant level of technology, which is the approach chosen by MRW.

The question is whether the MRW translation of the textbook Solow diagram into a cross-country context really captures the central insight of the Solow model, or is simply motivated by the availability of cross-country measures of output and investment in the large international data set that has become available since the 1980s (Summers and Heston, 1988) and is now referred to as the Penn World Tables (PWT).[2] Standard textbook exercises apart, steady state growth in the Solow model implies a shift of the production function rather than a movement along a production function, and more precisely a shift of the production function along a constant capital output ratio, as presented in Figure 2. Translating Figure 2 into a cross-country context appears to imply either a regression of output per worker on the level of technology conditional on a constant capital output ratio, or a growth accounting exercise to estimate a residual measure of the level of technology, which is the approach chosen by KRC.

The KRC approach is obviously closer to a cross-country estimate of the Solow model than the MRW approach. But in the applied literature and in recent textbooks, the opposite assessment prevails, which has not least been fueled by the KRC paper itself. In my view it is

[2] This dataset is now available as the Penn World Tables at http://pwt.econ.upenn.edu/.

not without irony that the empirical results by KRC, which are perfectly in line with a cross-country adoption of the Solow model, have been used to criticize the "neoclassical revival" in the empirics of growth, which is supposed to have been initiated by MRW and other studies along the same lines. My Figures 1 and 2 suggest that it is actually the other way round. The MRW findings do not necessarily support the idea that the Solow model can be usefully applied in a cross-country context. By contrast, the KRC findings do show that the Solow model can be usefully applied in a cross-country context by highlighting the empirical relevance of technology differences for understanding observed differences in output per person. But the KRC paper claims to show otherwise because the Solow model is associated with factor accumulation as the major determinant of the level of development. Hence it is probably fair to say that the cross-country growth literature as pioneered by MRW is based on a certain degree of confusion about how theoretical models of growth, and especially the Solow model, can be translated into empirical specifications.

Before going into the details of alternative empirical specifications, it may be useful to see whether some of the stylized facts about long-run growth that have been identified in time series data (Kaldor, 1961) also show up in cross-country data. The fact to be explained in cross-country studies is the large variation of output per person. Figure 3 ranks output per person for 162 countries in 2000 on the vertical axis, and each country's population size (in percent of the world population) on the horizontal axis. As a crude approximation, the figure suggests that about 15 percent of the world's population lives in countries with high average per capita incomes, and about 80 percent of the world's population lives in countries with comparatively low average per capita incomes. Hence with some justification, it makes sense to think about the world economy as consisting of two groups of countries. In terms of Figure 2, the poor countries may be represented by point *A* and the rich countries may be represented by point *B*.

Figure 4 compares capital per worker and output per worker across 127 countries on the basis of data used by Hall and Jones (1999). This figure reveals an important though probably not overly surprising insight: capital intensity and labor productivity are highly correlated across countries. The implication is that the capital output ratio, which is given by the slope of an imaginary regression line through the observed data points, may be considered as a constant in cross-country data (see also Topel, 1999). This is not to deny that there is a substantial degree of cross-country variation in the capital output ratio, as highlighted by Caselli and Feyrer (2005), but it remains as a stylized fact that the cross-country variation in the capital output ratio appears to be substantially smaller than the cross-country variation in capital per worker or output per worker.

A further stylized is that the shares of capital and labor in factor income have remained fairly constant over time in industrialized countries, as already observed by Kaldor (1961). In cross-country data, factor shares also appear to be fairly constant, at least they do not exhibit a trend with rising levels of income (Gollien, 2002; Bernanke and Gürkaynak, 2001). Once a constant capital output ratio and constant factor shares are accepted as stylized facts of cross-country data, it follows immediately as a third stylized fact that the rate of return to capital is constant across countries as well. In addition, it follows that cross-country differences in the real wage are proportional to cross-country differences in output per person, and differences in output per person are proportional to differences in capital per person. The Solow model reproduces these implications, which hints that it may indeed by used for a description of cross-country data, different from the way suggested by MRW.

Figure 5 summarizes the stylized facts just discussed. Points *A* and *B* are assumed to represent a poor and a rich country, which may be seen as approximations of the two broad groups of countries discussed in Figure 3. A constant capital output ratio, discussed in Figure 4, shows up as the inverse of the slope of the straight line through the origin. A constant rate of return to capital in rich and poor countries shows up as the slope of the straight lines through *A* and *B*, which is identical in each point. Given a constant capital output ratio and given a constant return to capital, which together imply constant factor shares, *A* and *B* cannot be located on a single production function. So the parallel straight lines through *A* and *B* may represent tangents to two production functions in the *y*-*k* space. The intersection of each tangent with the *y*-axis identifies the (real) wage *w* as a function of the capital intensity. As Figure 5 is drawn, the relation between the wage and labor productivity is held constant, which reflects the stylized fact of a constant labor share.[3]

In light of the Solow model, one would interpret points *A* and *B* as being generated by a production function with diminishing returns at different levels of technology. In a time series context, differences in steady state income would result from technological change. Applying the Solow model to a cross-country context, one would conclude that the two points represent combinations of output per person and capital per person across countries that employ different technologies. The empirical question then is how much of the observed international income difference (the difference between *y(A)* and *y(B)*) can be explained in terms of differences along a given production function, which represent differences in capital intensity for a given level of technology, and how much can be explained in terms of differences in the level of the production function, which represent differences in technology.

Technologies as represented by a production function may of course differ in many alternative ways, but once the stylized facts are taken into account it follows that such differences should be neutral with regard to factor shares. Hence unless the production function is of the Cobb-Douglas type, the cross-country income difference in Figure 5 has to reflect a Harrod-neutral difference in technology to allow for steady states. A Harrod-neutral difference in technology means a constant rate of return to capital for a constant capital output ratio in both rich and poor countries, which are the stylized facts that where used to draw Figure 5 in the first place. Thus there is no role for factor accumulation as a determinant of development in the theoretical border case represented by Figure 5. Here the steady state differences in output per person across countries are completely explained by Harrod-neutral differences in technology, and the Harrod-neutral differences in technology also explain the cross-country differences in factor accumulation, namely the difference in capital intensity between *k*(*A*) and *k*(*B*).

This implication of the Solow model is also borne out by the MRW specification, which does not include the capital intensity but the capital output ratio as an explanatory variable. Allowing for cross-country variation in the capital output ratio, as in the MRW specification, is also reasonable because the data of course do not match the theoretical border case of Figure 5. The problem is that MRW do not allow for cross-country variation in technology, obviously for lack of data. Since technology or differences in technology cannot be directly observed, it is principally impossible to know from data on output per person and capital per person which part of the observed difference in income is due to a difference in technology and which part is due to a difference in factor accumulation. This identification problem has been well known for more than a quarter century at least (Nelson, 1973) and has been

[3] In Figure 5, *w(A)* and *w(B)* are fixed at 70 percent of *y(A)* and *y(B)*, which approximately equals the size of labor's share in factor income.

discussed in great detail in theory (Diamond et al., 1978) but, as Hulten (2000) notes in passing, has generally been ignored in the applied productivity literature.

So MRW end up with a specification that explains international steady state income differences with international differences in the capital output ratio, conditional on the level of technology. But the central point of the Solow model, if applied to a cross-country context, appears to be that steady state income differences reflect differences in (Harrod-neutral) technology, conditional on a constant capital output ratio. A new strand of the applied literature probably offers a way out by pointing to proxy measures of cross-country differences in a broad concept of technology.

Countries obviously differ, among other things, by geographic and climatic conditions, disease ecologies, and institutional frameworks. Most of these factors appear to be fairly persistent over time; some, like climate and geography, do not change at all in economically relevant time. All of these factors can be expected to impact on output per worker, either directly or indirectly through their effects on factor accumulation and population growth. Such a broad concept of technology differences has recently been approximated by measures such as frost frequency (Masters and McMillan, 2001), malaria prevalence (Sachs, 2001 and 2003), and institutional infrastructure (Hall and Jones, 1999; Acemoglu et al., 2001; Easterly and Levine, 2003; Rodrik et al.; 2004), where the chosen proxy variables may be considered as identifying international differences in agricultural technology, health technology, and institutional technology.

Hence a revised cross-country specification of the Solow model may rely on the initial MRW framework but at the same time could allow for cross-country differences in technology as measured by cross-country differences in climate, health, and institutions. Given that the cross-country data reflect only small deviations from a steady state, one would expect to find that the cross-country variation in the measures of technology rather than the cross-country variation in the capital output ratio explains cross-country differences in level of development.

III. ALTERNATIVE EMPIRICAL IMPLEMENTATIONS[4]

There is no question that there are different possibilities to derive empirical specifications from the Solow model. A Cobb-Douglas production function with Harrod-neutral technology is an obvious possibility to begin with. Abstracting from all detail and focusing on the simplest case with just two factors of production, we have

$$Y = K^{\alpha} (AL)^{(1-\alpha)} \quad , \tag{1}$$

where Y is output, K is capital, A is technology, and L is labor. Since K is an endogenous variable, this equation has to be rearranged to allow for an unbiased estimate of the production elasticity. Dividing by L with $Y/L = y$ and $K/L = k$, taking logs, and rearranging terms in a way to have the capital output ratio on the right-hand side gives

$$\ln y = \ln A + \frac{\alpha}{(1-\alpha)} \ln(k/y) + \varepsilon \quad , \tag{2}$$

[4] This section partly draws on Gundlach (2005).

with ε as an i.i.d. error term. A standard textbook exercise based on the expression for the steady state capital intensity shows that the marginal product of capital (*MPK*) equals $MPK = \alpha(n+g+\delta)/s_k$, where *n* is the rate of labor force growth, *g* is the rate of technological change, δ is the depreciation rate, and s_k is the share of saving in GDP. Since the share of saving can be approximated by the share of investment in GDP $(I/GDP))$ and α equals the marginal product of capital times the capital output ratio, it follows that equation (2) can be rewritten as

$$\ln y = \ln A + \frac{\alpha}{(1-\alpha)}\left(\ln(I/GDP) - \ln(n+g+\delta)\right) + \varepsilon \quad , \tag{3}$$

which is the specification employed by MRW to estimate the Solow textbook model in a cross-country context.[5]

The major empirical problem encountered by MRW was the lack of plausible cross-country measures of *A*. To solve this identification problem, they assumed that technology *A* evolves according to $A(t) = A(0)\, e^{g \cdot t}$ and is the same in all countries apart from a random component, so $A(0) = b + \varepsilon$, where *b* is a constant. Furthermore, *g* and δ are also assumed to be constant and identical across countries.[6] Given this set of assumptions,

$$\ln y_i = c + \frac{\alpha}{(1-\alpha)}\left(\ln(I_i / GDP_i) - \ln(n_i + g + \delta)\right) + \varepsilon \tag{4}$$

may be estimated by OLS across a sample of countries $i = 1...n$, with $c = b + gt$ as the regression constant.

This specification deserves second thoughts, because it explains the cross-country variation in output per person with the cross-country variation in the capital output ratio, conditional on a constant level of technology. By contrast, the textbook Solow model would explain the cross-country variation in output per person with the cross-country variation in technology, conditional on a constant capital output ratio. Of course one could argue that this is a moot point to debate simply because the Solow model was not meant to be applied to a cross-country context, where the capital output ratio may vary more substantially than in a time series context. But if one does apply the Solow model to a cross country context, like MRW and a large subsequent literature, then it appears to be more reasonable to use an alternative specification that first of all allows for variation in the measure of technology rather than in the capital output ratio.

[5] MRW augment the Solow textbook model by a third factor of production, namely human capital. Their empirical results are meant to support the augmented Solow model, not the textbook Solow model, but this does not make a difference for the point to be discussed here. As an aside, human capital *per output*, which equals the conditional share of human capital investment in GDP, should be used as an explanatory variable in the augmented Solow model (as in MRW). Using the level of human capital *per worker* as an explanatory variable, as in some other empirical studies, cannot be motivated with reference to the Solow model.

[6] MRW assume average values of 1 percent for *n* and 3 percent for δ. An alternative parameterization that is sometimes used in the literature assumes a value of 5 percent for δ.

As already pointed out, one possible reinterpretation of equation (2) can be motivated by recent empirical studies of the role of institutions as a fundamental determinant of development (Hall and Jones, 1999; Acemoglu et al.; 2001; Easterly and Levine, 2003; Rodrik et al., 2004). The institutional framework of a country may be considered as a "technology" that changes very slowly over time but differs substantially across countries. Given that the quality of institutions can be measured across countries, the variable A in equation (2) may be redefined to allow in principle for various country-specific "technology" variables, as informally suggested by Solow (2001). That is, A can be assumed to grow for each individual country i with the same constant rate g over time t (as in MRW), but at different levels which are determined by various factors X_i such that

$$A_i(t) = A(0)e^{gt}\ e^{\phi_k X_{ik}} \quad , \qquad (5)$$

where $A(0)$ stands for the initial level of a narrow concept of technical knowledge that is the same for all countries, and X_k may capture factors $k=1,...,l$ such as institutions and other potential determinants of development that differ across countries but remain fairly stable over time. Equation (5) suggests that persistent differences in X across countries would explain persistent differences across country-specific production functions, which in turn would shift over time due to the common constant rate g.

With this modification of the technology term A and by imposing the alternative restriction that the capital output ratio is part of the regression constant, equation (2) can be rewritten as

$$\ln y_i = \ln A(0) + g\,t + \frac{\alpha}{(1-\alpha)}\ln(k/y) + \phi_k\ X_{ik} + \varepsilon_i \quad , \qquad (6)$$

which reproduces the basic structure of the regression equations used by Hall and Jones (1999), Acemoglu et al. (2001), Easterly and Levine (2003), and Rodrik et al. (2004). By implicitly imposing a cross-country restriction on the capital output ratio rather than on the technology term, these studies reproduce the non-parametric accounting result of KRC with a parametric methodology, namely that international differences in a broad concept of technology account for international differences in output per worker. Notwithstanding all sorts of empirical estimation problems, it appears that this result is much closer to the basic message of the Solow model than the results presented by MRW.

Along these lines, KRC criticize the MRW approach and claim that the cross-country data do not support the Solow growth model. Again abstracting from all detail, KRC use a variant of equation (2) as a non-parametric accounting equation in levels and estimate the technology term as a residual by assuming a stylized share of capital in factor income:

$$\ln A(t)_i = \ln y_i - \frac{\alpha}{(1-\alpha)}\ln(k/y)_i \quad . \qquad (7)$$

Since the international variation in the capital output ratio is small relative to the international variation in output per worker, they find, somehow by default and perfectly in line with the Solow diagram, a large contribution of the residual (cross-country differences in Harrod-neutral technology) in explaining cross-country differences in output per worker. KRC interpret their accounting result as providing strong empirical evidence against the restriction imposed in the MRW approach, namely that technology does not systematically differ across countries. Therefore, KRC conclude that research needs to be re-focused on models that emphasize differences in technology rather than differences in factor accumulation. But they fail to see that their own results are not necessarily in conflict with a cross-country application of the Solow model.

Table 1 reports the results for estimating alternative cross-country specifications of the Solow model on the basis of the original MRW data. Column (1) reproduces the MRW point estimates[7] for a human capital augmented Solow model on the basis of equation (4), where the level of technology is presumed to be part of the regression constant. In the empirical specification presented in Table 1, *school* measures the proportion of the working-age population that is in secondary education, which is taken as a proxy variable for the share of investment in human capital. The (reproduced) MRW results suggest that the international variation in output per worker can be explained by the international variation in the two capital output ratios, conditional on a constant level of technology (see also equations (2) and (3) above).

Column (2) reports results based on equation (6), which resembles a specification used by Acemoglu et al (2001). Here, the capital output ratio rather than the level of technology is presumed to be part of the regression constant. A measure of the *risk of expropriation* is used as a proxy variable for international differences in the quality of institutional technologies, and a measure of settler mortality in the former colonies in the early 19th century is used as an instrument variable to account for the potential endogeneity of the quality of institutions. Acemoglu et al. (2001) point out that early settler mortality is likely to have determined whether the institutions that were adopted would favor the exploitation or the accumulation of existing resources. They also show that institutions appear to be fairly persistent over time, which implies that settler mortality can be considered as a valid instrument variable for present institutions. My empirical result, which is based on the MRW data on output per working-age person in 1985 as the dependent variable, points to a large positive effect of a variation in institutional quality on the level of development, as in the study by Acemoglu et al. (2001).[8] My point estimate of 0.71 implies that a one unit increase in the measure of institutional quality is associated with a 0.71 percent difference in output per worker. What this means in quantitative terms can be seen by comparing two countries, say Chile and Nigeria, where the difference in the measure of institutional quality is 2.3. So my point estimate predicts a 1.6 log-difference between the log GDPs (per working-age person) of the two countries, or approximately a 4-fold difference ($e^{1.6} - 1$) in output per working-age person. In the data used, output per working-age person actually differs by a factor of 4.7 between Chile and Nigeria. This result suggests that the international variation in output per worker can be explained by international differences in a broad concept of technology conditional on a constant capital output ratio.

The next step, presented in column (3) of Table 1, is to estimate a specification where both the level of technology and the capital output ratio are allowed to vary across countries. For this purpose, I simply match the two previous specifications.[9] The results should certainly not be overemphasized and call for a number of robustness tests. But so far my point estimates

[7] See Mankiw et al. (1992), their Table II, column (1).
[8] See Acemoglu et al. (2001), their Table 4, column (1); their dependent variable is output per capita in 1995.
[9] A similar specification has been used by Masters and McMillan (2001), who consider frost frequency as a proxy variable for the potential impact of geography (or agricultural "technology") on development.

suggest that international differences in the two capital output ratios are not important for explaining international differences in output per person once a measure of international technology differences is taken into account. In addition, the estimated coefficient on the measure of institutional quality hardly changes as compared with column (2). In my view, this empirical result is perfectly in line with a cross-country application of the Solow model, but it appears to be in conflict with the MRW cross-country translation of the Solow model.

One interpretation of my finding is that the observed variation in the capital output ratio largely reflects variation around the steady state and therefore turns out to be economically and statistically insignificant, as indicated by the low point estimates and the large standard errors. Hence a further step would be to estimate a specification of the Solow model that explicitly allows for transitional dynamics around the steady state. Such a specification has also been developed by MRW, who emphasize that the Solow model predicts that countries reach different steady states, conditional on the variation in the determinants of the steady state.

Using the Taylor expansion for approximating around the steady state, MRW derive a regression equation to estimate the speed of convergence conditional on what they hold to be the determinant(s) of the steady state, namely the capital output ratio(s). Again abstracting from all detail, starting from equation (4) and using the same derivation as MRW, one would end up with an equation that describes the growth of output per working-age person as a function of the capital output ratio and the initial level of income.[10] But as before, one could argue that the variation in technology rather than the variation in the capital output ratio should be the conditional factor for the steady state, so a measure of a broad concept of technology should be included in the regression equation that is used to estimate the speed of convergence. In the simplest case with two factors of production, such an equation would read

$$\ln y_i(t) - \ln y(0)_i = (1-e^{-\lambda t})(\ln A(0) + g\,t) \quad + \quad (1-e^{-\lambda t})\frac{\alpha}{(1-\alpha)}\ln\left(k_i / y_i\right) \\ + \quad (1-e^{-\lambda t})\phi X_i \quad - \quad (1-e^{-\lambda t})\ln y(0)_i \quad + \quad \varepsilon_i \tag{8}$$

where the first term represents the regression constant, X is a measure of institutional quality, and λ is the rate of convergence to the steady state, which is predicted to be a function of factor shares and the conditional growth rate of the labor force:

$$\lambda = (1-\alpha)(n + g + \delta) \quad . \tag{9}$$

By estimating a human capital augmented version of equation (8) with the same variables as before, one should expect to find three results. First, there should be no effects of the two capital output ratios, because they are not determining the steady state and because their variation may be considered as random. Second, there should be an economically important effect of the measure of institutional quality, which should be considered as the determinant of the steady state. Third, there should be a rate of convergence in the range of 2 percent, which would be predicted on the basis of a combined share of physical and human capital of about two thirds and a conditional rate of labor force growth of about 6 percent (as assumed in MRW). The results presented in column (4) of Table 1 largely confirm these a priori expectations. There appear to be no economically important effects of the two capital output

[10] As before, I omit the human capital variable to simplify the presentation, which is not essential for the point to be discussed.

ratios. The estimated coefficient on the measure of institutional quality (*expropriation risk*) implies a statistically significant point estimate of the parameter ϕ of 0.89, which is within one standard deviation of the point estimates reported for the equations in levels (see columns (2) and (3)). The rate of convergence is estimated to be 2.6 percent, which is statistically significantly different from zero but not different from 2 percent or 3 percent.[11]

Overall, these empirical results nicely illustrate that the cross-country data do not reject the basic conclusion of Figure 3 discussed in Section 2, namely that steady state differences in factor intensity cannot be separated from steady state differences in technology. Put differently, international differences in technology imply international differences in output per person *and* in capital per person. This insight appears to matter not only for empirical studies of growth and development, but also for empirical studies of trade.

IV. SOLOW MEETS LERNER

The Lerner Diagram (Lerner, 1952) is mainly used as a tool for relating goods prices and factor prices in a two-good, two-factor, Heckscher-Ohlin (HO) trade model, but it can also be used for other purposes (Deardorff, 2002). In a seminal paper, Findlay and Grubert (1959) have utilized the Lerner diagram to study the effects of growth on factor proportions and the pattern of trade. Their paper models growth as exogenous technological change, as in the Solow model. Findlay and Grubert (1959) is still a standard reference for work on trade and growth. What appears to have been neglected in the subsequent literature is that according to the Solow model, steady state differences in factor intensities entirely reflect differences in technology, i.e., differences between production functions and not differences along a production function. In many contributions to the trade literature, factor intensities are treated as being independent from technology differences. Such a modeling strategy implies a Hicks-neutral concept of technology differences, whereas the Solow model implies a Harrod-neutral concept of technology differences.

This benign neglect of steady state implications for the type of technological differences to be used in models of trade and growth also surfaces in recent empirical studies of trade. These studies have emphasized that the well-known failure of the static HO-model to predict observed trade patterns on the basis of factor endowments no longer prevails once the absence of factor price equality even across countries with similar factor endowments can be motivated by Hicks-neutral cross-country differences in technology or a multiple-cone equilibrium.[12] As discussed in the previous sections, the cross-country version of the Solow model would predict that differences in Harrod-neutral technology explain cross-country differences in output per worker *and* capital per worker. Translating this basic insight into the Lerner diagram appears to provide an additional argument why a HO-model without technology differences is likely to fail to predict the pattern of trade among countries with different levels of income.

Figure 6 is a Lerner diagram for two goods and two factors. The curves labeled X_i (i = 1,2) are the unit value isoquants of the two goods that are produced with capital K and labor L, at given commodity prices p_i. The two isoquants represent alternative technologies for producing one euro’s worth of output. The isocost line that is tangent to the two unit value isoquants also represents one euro's worth of output. With $X_i = 1/p_i$, the relative position of an isoquant

[11] Using a value of 5 percent for δ (see footnote 6) and all other things constant one would predict a rate of convergence of 2.7 percent.
[12] See, e.g., Trefler (1995), Davis and Weinstein (2001), Debaere (2003), Schott (2003).

depends on the prices of the goods and on the different techniques that are employed to produce the two goods. With free trade and ubiquitous technology, all countries with factor endowments within the cone of diversification will face the same goods prices, and hence the same factor prices.

As Figure 6 is drawn, the cone of diversification is defined by capital intensities $\tilde{k}_1$ and $\tilde{k}_2$, and there is a free-trade equilibrium with one set of equilibrium factor prices, where *w* is the wage rate and *r* is the profit rate. Countries with capital intensities outside the cone specialize in the production of just one good; countries within the cone chose a least cost combination of available technologies to produce both goods. Production occurs at constant returns to scale, and factors are mobile between the two sectors within countries but immobile across countries. Within cones, factors share their rewards because of trade, but countries outside the cone realize different factor prices.

The question to be addressed is how persistent cross-country differences in technology might affect the equilibrium. Given that differences in the level of development are understood as exogenous differences in technology and given that commodity prices can be treated as fixed, there is a straightforward link between factor prices and the level of development. The former assumption can be motivated by the Solow model, the latter assumption implies that technologies are local in the sense that they are not available for firms in the same sector but located in a different country. In such a setup, there would be goods market equilibrium of a trading world economy, with each country facing the same commodity prices. Thus persistent cross-country differences in technology imply that the corresponding unit value isoquants and the resulting equilibrium factor prices must differ across countries.

In the Lerner diagram, this insight can be introduced by considering a second factor endowment point *B*, which is meant to represent a country that employs a different (institutional) technology than country *A* (Figure 7). Accordingly, country *B* should face different unit value isoquants than country *A*. The relative location of the unit value isoquants for country *B* will thus depend on the specific nature of the presumed differences in technology. The standard approach used in most empirical studies of trade is to model technology differences as being Hicks-neutral. In this case the unit value isoquants for country *B* relative to country *A* would be located inwards on the rays $\tilde{k}_i^A$ subject to a parallel unit value isocost line, because Hicks-neutral technology differences are defined for a constant factor price ratio at a constant factor capital intensity. One reason why the assumption of Hicks-neutral technology differences is so popular in the empirics of trade appears to be that it allows for treating differences in observed capital intensity as being independent from differences in technology. The Solow model would suggest otherwise, since it predicts that steady state differences in capital intensity reflect differences in Harrod-neutral technology. This conceptual difference appears to matter for empirical studies of trade.

In Figure 7, countries *A* and *B* have the same size of the workforce but differ in capital per worker by the vertical difference between *A* and *B*. Interpreting *A* and *B* as cross-country steady state factor endowments that reflect differences in *Harrod*-neutral technology and by considering the stylized facts as discussed above, it follows that the difference in capital per worker is proportional to the difference in output per worker. Given the stylized facts of constant factor shares, it follows that the cross-country difference in the wage is also proportional to the difference in output per worker.[13]

These implications show up in Figure 7 as follows. For given *L*, the difference in output (or national income) is given by the horizontal difference between the parallel lines through *A* and *B*, so the relative income of country *B* in terms of country *A* is given by Y_B / Y_A. With a constant labor share of 70 percent as before (see Figure 5), it follows that the relative wage of country *B* equals $w_B / w_A = 0.7 \cdot y_B / y_A$. This proportional wage difference is represented by the distance between $1/w_A$ and $1/w_B$. For a constant profit rate *r*, the unit value isocost line for country *B* thus results as $\overline{1/r \;\; 1/w_B}$.

Harrod-neutral differences in technology between countries *A* and *B* are reflected by the horizontal distance between the unit value isoquants, where the distance for each unit value isoquant is confined by the differently sloped unit value isocost lines $\overline{1/r \;\; 1/w_B}$ and $\overline{1/r \;\; 1/w_A}$, which indicate differences in relative factor prices. As it is drawn, Figure 7 suggests that the two countries *A* and *B* are located in different cones of specialization. Whether or not one ends up with different cones of specialization of course depends on the size of the difference in factor endowments. The crucial point is the assumption of Harrod-neutral technology, which allows at least for the possibility of different cones of specialization. By contrast, one would never end up with different cones of specialization by assuming Hicks-neutral technology, independent from the difference in factor endowments. With Hicks-neutral technology, the cross-country differences in the location of the unit value isoquants would be represented as proportional distance along the two rays $\tilde{k}_1^A$ and $\tilde{k}_2^A$ confined by two parallel init value isocost lines, which would indicate identical *relative* factor prices across the two countries (not shown).

Given that cross-country differences in output per person and in capital per person mainly reflect differences in Harrod-neutral technology and are large enough to support different cones of specialization, there is a major implication for the prediction of within-country sectoral factor allocations and the pattern of trade. This is shown in Figure 8, which reproduces the situation depicted in the previous figure but now focuses on sector allocations. For a given factor endowment, the predicted within-country sector allocation can be derived by drawing a parallelogram with the rays representing the borders of the cone and with the origin and the factor endowment point as two opposite corners. As it turns out, the basically untestable a priori technology assumption can make a large difference for the prediction of within-country sector allocations and subsequently for the pattern of trade.

For instance, one may assume that there are no technology differences at all between countries *A* and *B*, so both countries would be located in the cone confined by $\tilde{k}_1^A$ and $\tilde{k}_2^A$. If so, country *B* would be predicted to produce more of good 2 than of good 1, which would be represented by the length of the vectors v_2^A and v_1^A. Thus country B would be said to specialize in the production of the capital intensive good. The same prediction would prevail if one would allow for Hicks neutral differences in technology between countries *A* and *B*, because such differences would not result in different cones of specialization. In such a case,

[13] In a time series context, the steady state condition is that technology, output per worker, capital per worker, and the wage grow with the same rate.

only the *level* of factor prices would differ across countries because there would be two parallel unit value isocost lines, but not the factor price ratio.

But if the cross-country income differences are interpreted as differences in Harrod-neutral technology, then the predicted sectoral factor allocation for country *B* would be different, just because country *B* would end up in a different cone of specialization. In the cone confined by $\tilde{k}_1^B$ and $\tilde{k}_2^B$, country *B* would be locally labor abundant and specialize in the production of the labor intensive good, despite being globally capital abundant relative to country *A*. Hence country *B* would be predicted to be a net exporter of the labor intensive good 1, due to sector allocations represented by the length of the vectors v_1^B and v_2^B. Thus in light of the cross-country interpretation of the Solow diagram, it does not come as a big surprise that an empirical estimate of a Heckscher-Ohlin model based on identical technology across countries with rather different levels of income does not perform better than tossing a coin in predicting international trade flows. The cross-country translation of the Solow model suggests that empirical studies of trade could probably gain by abandoning the Hicks-neutrality assumption in favor of the Harrod neutrality assumption.

V. CONCLUSION

The Solow model concludes that steady state growth is due to technological change. Applied to a cross-country context, one would therefore imply that steady state differences in output per person are due to differences in technology. Under general conditions, only Harrod-neutral differences in technology would be consistent with steady state income differences. Harrod-neutral differences in technology explain steady state differences in factor intensity for a constant capital output ratio and a constant profit rate. In line with this implication, my empirical results suggest that the cross-country data on output per worker can be consistently summarized by a specification that allows for international variation in technology conditional on a constant capital output ratio. This result appears to be more in line with the Solow model than the results presented in the seminal paper on the cross-country empirics of growth by MRW, who find that the international data on output per worker can be consistently summarized by a specification that allows for international variation in the capital output ratio conditional on a constant level of technology.

Leaving aside which empirical specification is actually closer to the Solow model, the idea that international differences in output per person may be explained by international differences in technology appears to provide a useful restriction for empirical studies of trade. Even in a simple neoclassical 2x2 trade model, cross-country differences in Harrod-neutral technology may support different cones of specialization. Since the predicted sectoral factor allocations and the subsequent pattern of trade depend on the cone of specialization that a specific country belongs to, it is hardly surprising that empirical studies of trade that do not allow for international differences in Harrod-neutral technology may fail to predict observed sector allocations and trade patterns. As it seems, the Solow model offers an insight that has not fully been exploited by trade economists.

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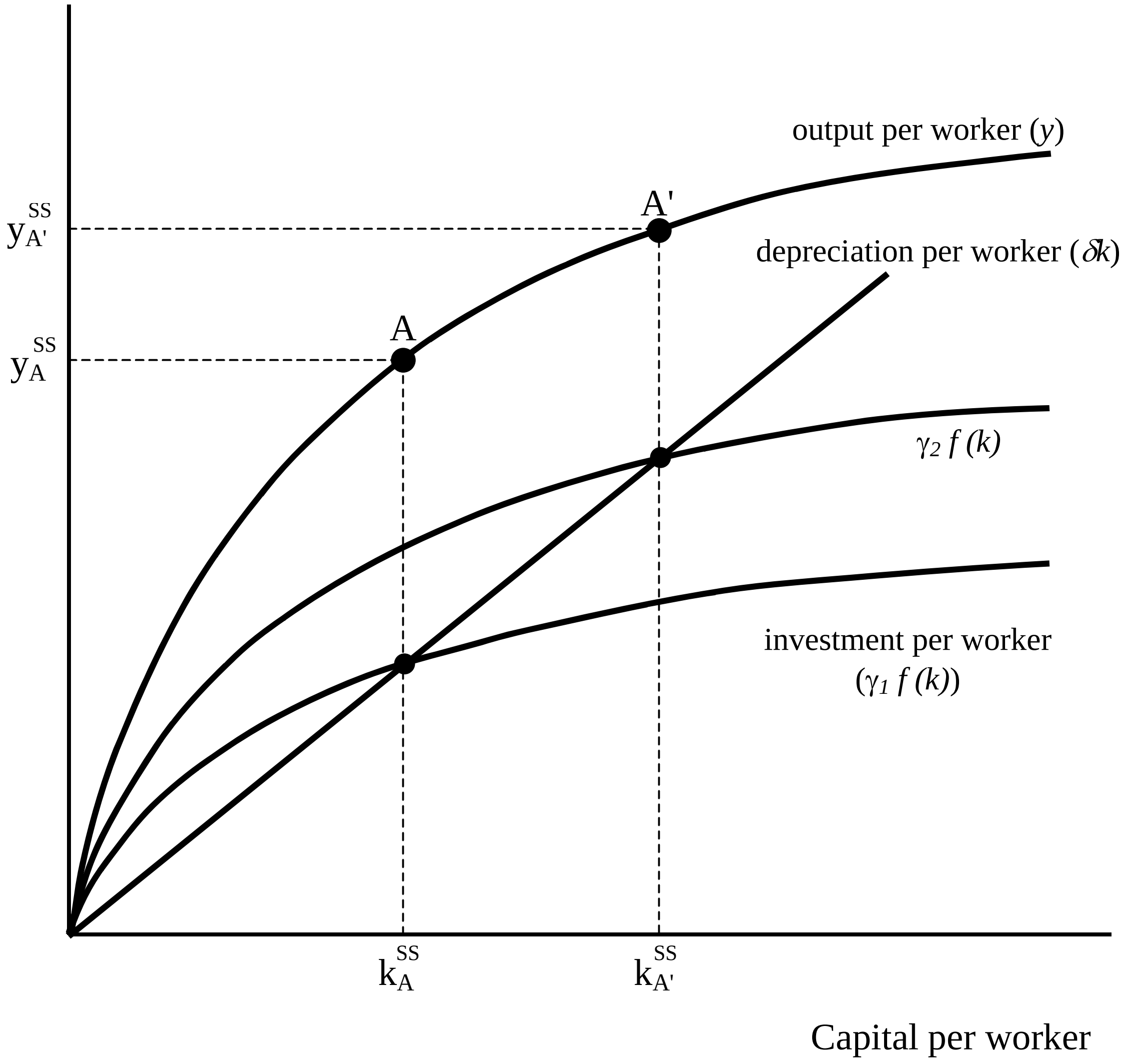
Table 1 - Alternative Cross-Country Estimates of the Solow Model

Specification	Variable capital output ratios: MRW (1992) reproduced	Variable technology: AJR (2001) specification	Variable capital output ratios and variable technology	
			Levels	Growth rates
	(1)	(2)	(3)	(4)
$\ln(I/GDP) - \ln(n+g+\delta)$	0.74 (0.12)	-	-0.09(0.35)	0.06 (0.25)
$\ln(school) - \ln(n+g+\delta)$	0.66 (0.07)	-	0.19(0.20)	-0.04 (0.13)
$expropriation\ \ risk$	-	0.71 (0.10)	0.65(0.21)	0.43 (0.17)
$initial\ \ income$	-	-	-	-0.48 (0.13)
Estimated by	OLS	IV	IV	IV
Number of observations	98	57	57	57
Implied θ	-	-	-	0.89 (0.28)
Implied λ	-	-	-	0.026 (0.01)

Notes: The table only reports major findings, detailed regression results are available upon request. Standard error in parenthesis. Dependent variable: output per working-age person in 1985 (in column (4): growth rate of output per working-age person in 1960-85). IV regressions use settler mortality as an instrumental variable. All data taken from Mankiw et al. (1992) except for expropriation risk and settler mortality, which are taken from Acemoglu et al. (2001).

Figure 1 — The Steady State Effect of Different Investment Rates



Source: Adapted from Weil (2005).

Figure 2 — The Steady State Effect of Different Technologies

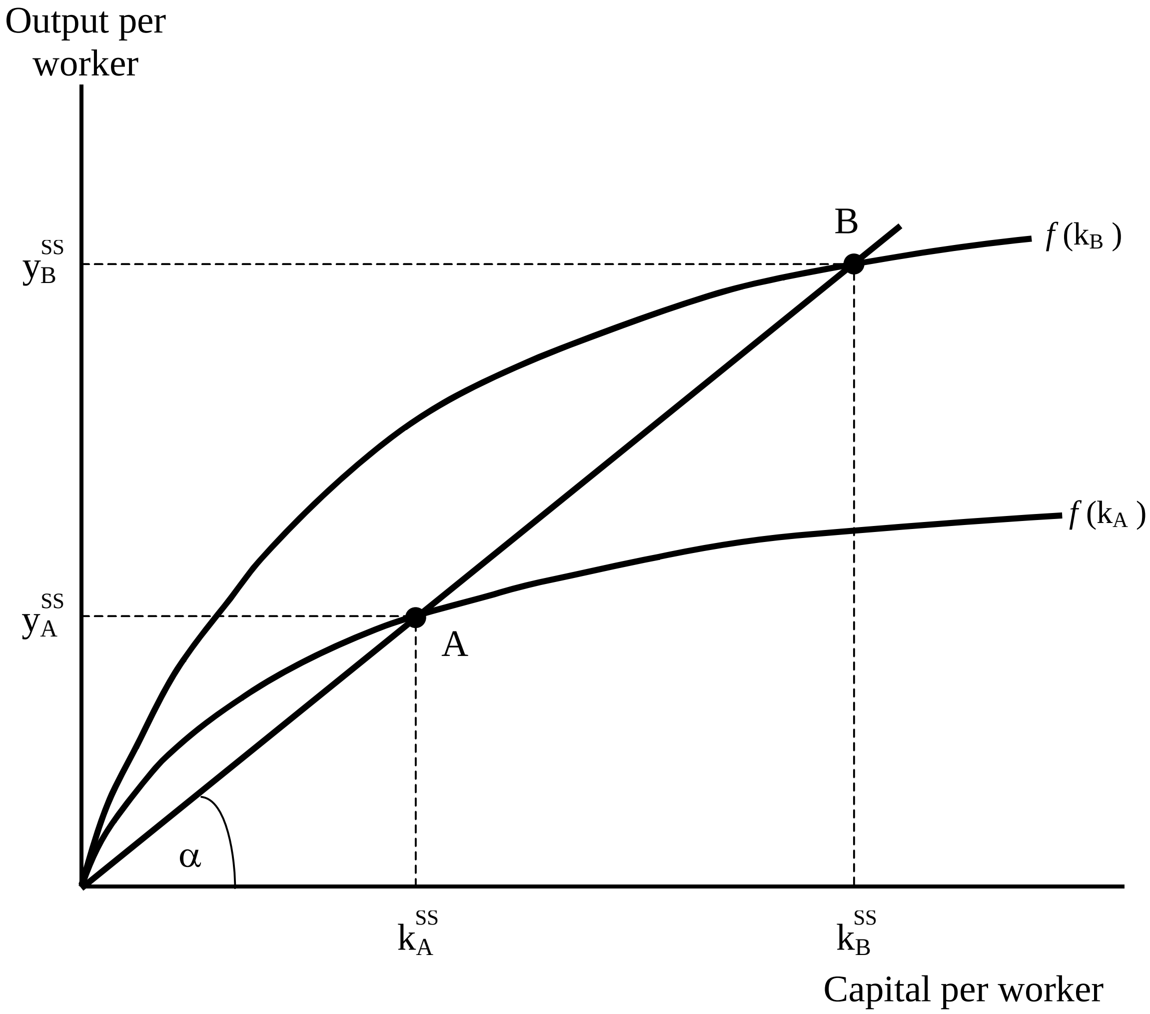
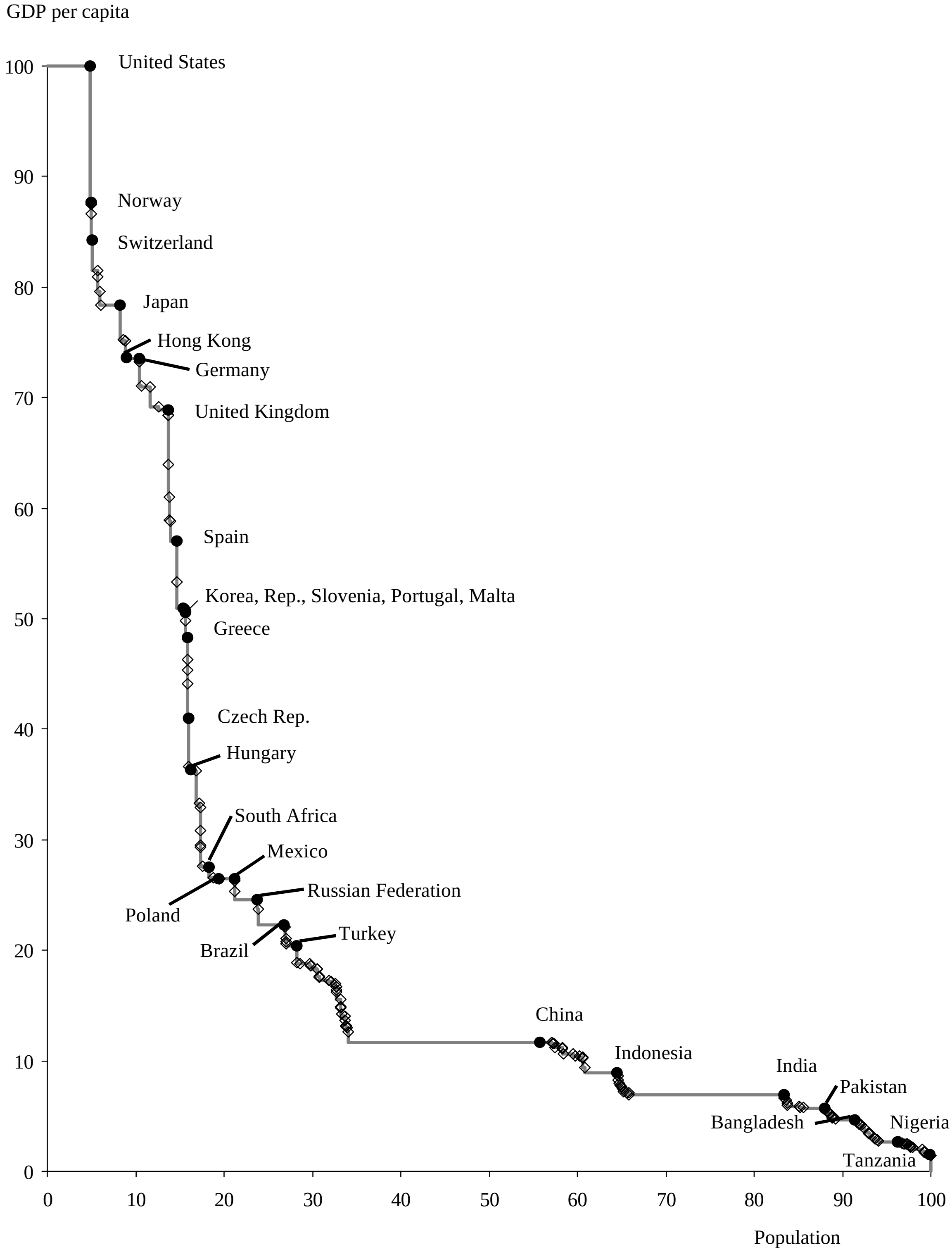


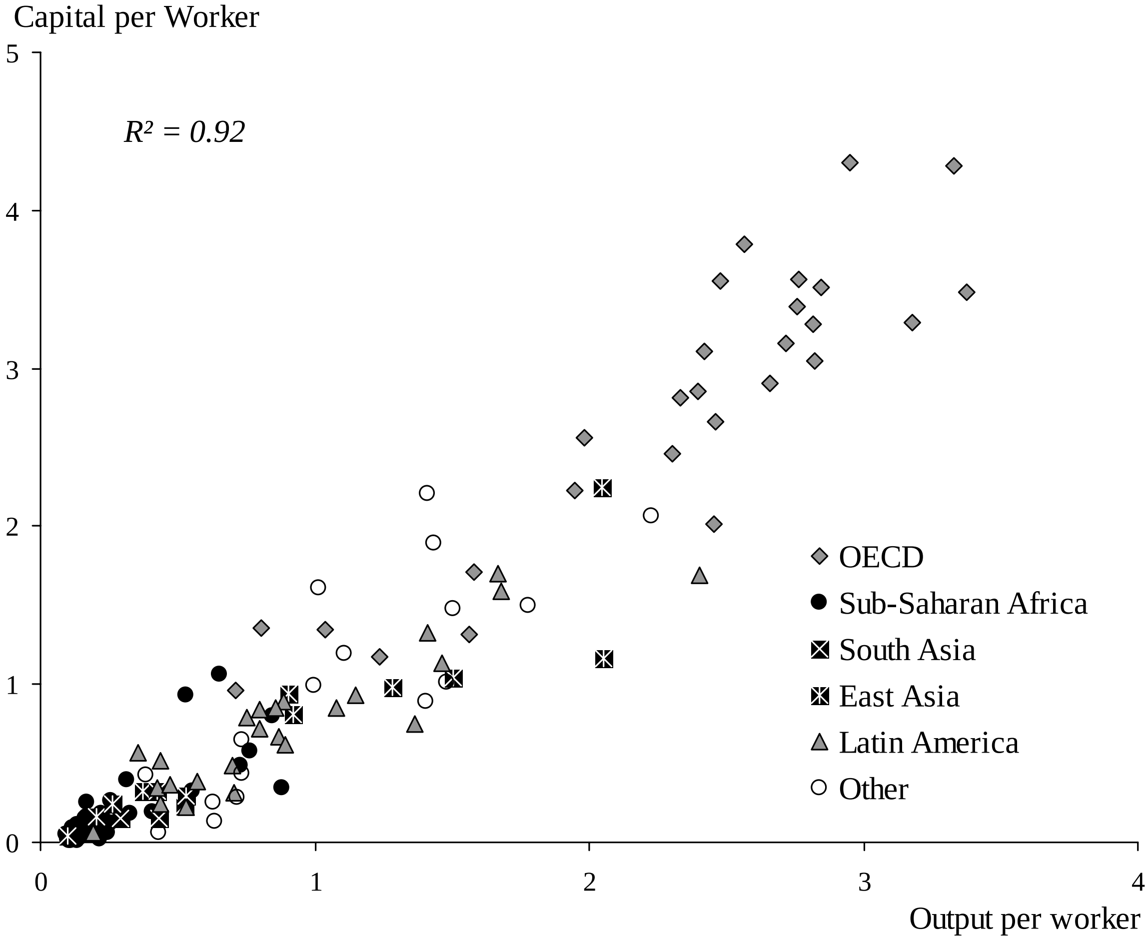
Figure 3 – GDP per Person in the World Economy, 2000



Note: Real GDP per person, current purchasing power parity; 162 countries; United States= 100; total population of 162 countries= 100.

Source: World Bank, World Development Indicators, CDROM.

Figure 4 — The Capital-Output Ratio in International Perspective



Note: Capital per working-age person and output per working-age person, each measured as relative to their mean. 127 countries, ca. 1990. "OECD" excludes Mexico, South Korea, and Turkey, "Other" includes North Africa, Middle East, and former socialist countries.

Source: Data taken from Hall and Jones (1999).

Figure 5 — The Solow Model Across Countries: Stylized Facts

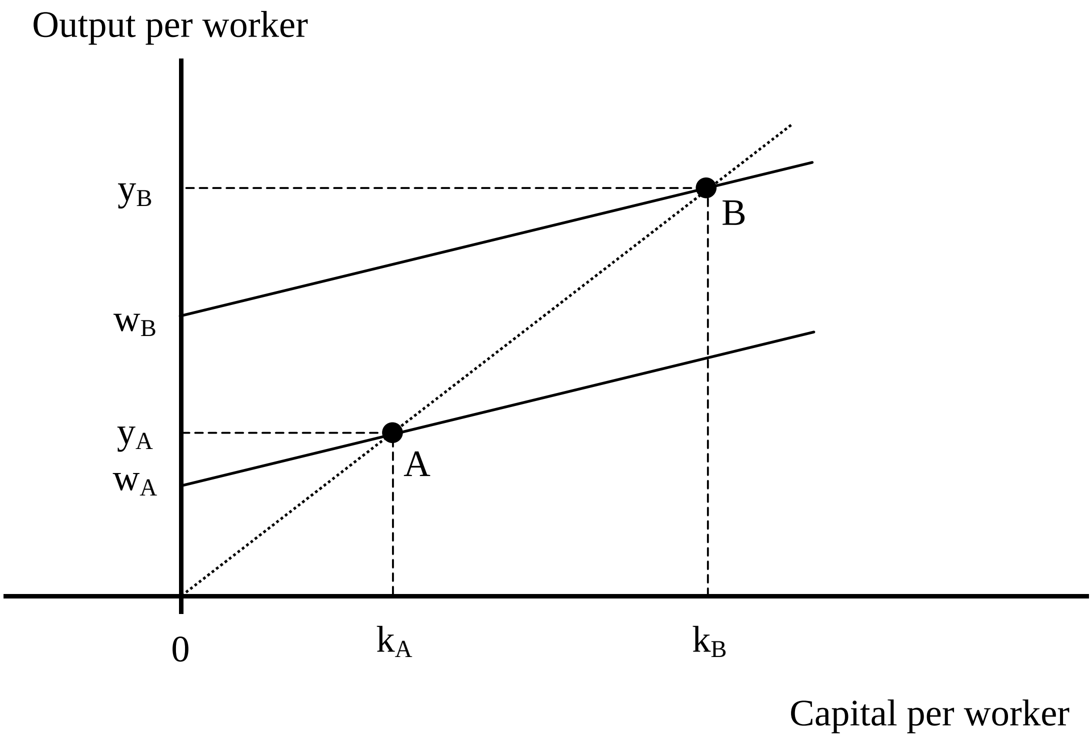


Figure 6 – The Lerner Diagram for Two Goods and Two Factors

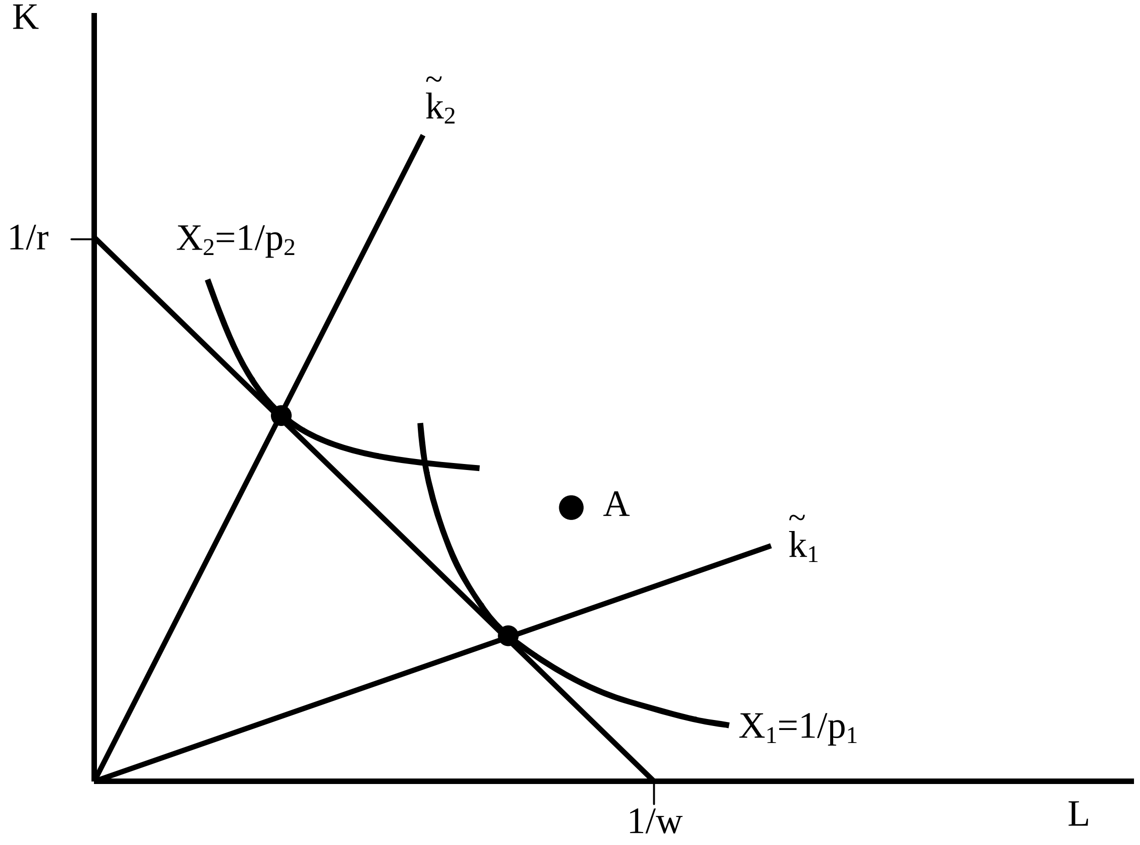


Figure 7 – The Lerner Diagram for Harrod-neutral Technology Differences

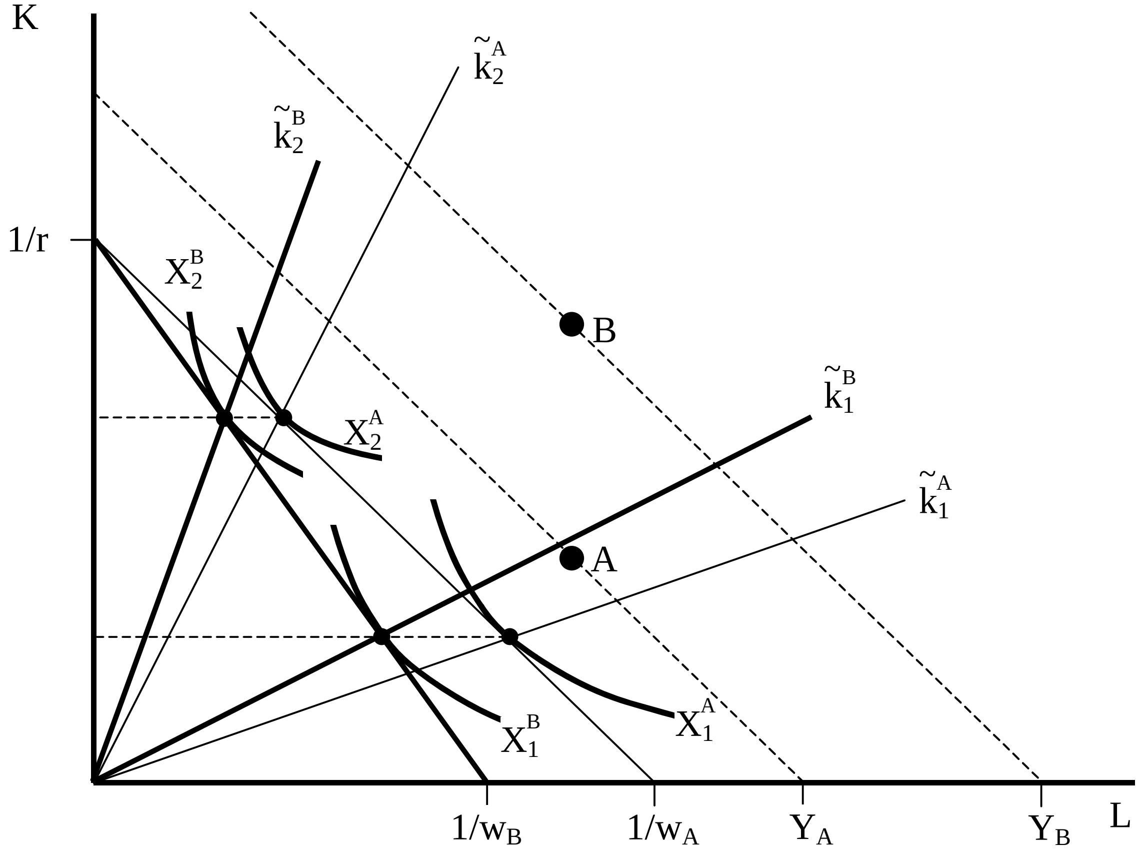


Figure 8 – Sectoral Allocations as a Function of the Cone of Diversification