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by

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Keywords: interbank market, network models, financial crisis

JEL classification: G21, G01, E42

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Network Analysis of the e-MID Overnight Money Market: The Informational Value of Different Aggregation Levels for Intrinsic Dynamic Processes[†]

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Abstract

In this paper, we analyze the network properties of the Italian e-MID data based on overnight loans during the period 1999-2010. We show that the networks appear to be random at the daily level, but contain significant non-random structure for longer aggregation periods. In this sense, the daily networks cannot be considered as being representative for the underlying 'latent' network. Rather, the development of various network statistics under time aggregation points toward strong non-random determinants of link formation. We also identify the global financial crisis as a significant structural break for many network measures.

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1 Introduction and Existing Literature

Interbank markets are crucial for the functioning of the economy. However, as painfully illustrated by the global financial crisis (GFC) in 2007/08, creating links at the micro-level may generate systemic risk at the macro-level. Thus, the structure of the interbank network, with the banks having connections in terms of credit relationships, is important for its stability. The economy depends on stable interbank markets, since short-term money market rates affect those of longer maturities and thus the real economy. From this viewpoint, it appears quite surprising that the economic profession has not been concerned much with the functioning of interbank markets until recently. The usual focus is on the overnight segment of the interbank deposit market, since it tends to be the largest spot segment of money markets. In this paper, we analyze the network properties of the Italian e-MID (electronic market for interbank deposits) data based on overnight loans during the period 1999-2010.

Most existing studies on the structure of real interbank markets have been conducted by physicists trying to get an idea of the topology of different interbank markets. Examples include Boss *et al.* (2004) for the Austrian interbank market, Inaoka *et al.* (2004) for the Japanese BOJ-Net, Soramäki *et al.* (2006) for the US Fedwire network, Bech and Atalay (2010) for the US Federal funds market, and De Masi *et al.* (2006) and Iori *et al.* (2008) for the Italian e-MID (electronic market for interbank deposits). The most important findings reported in this literature are: (1) most interbank networks are quite large (e.g. more than 5000 banks in the Fedwire network), (2) interbank networks are sparse, meaning that only a minority of all possible links do actually exist, (3) degree distributions appear to be scale-free (with coefficients between 2-3), (4) transaction volumes appear to follow scale-free distributions as well, (5) clustering coefficients are usually quite small, (6) interbank networks are small worlds and (7) the networks show disassortative mixing with respect to the bank size, so small banks tend to trade with large banks and vice versa.

Most relevant for our study are the two papers on the e-MID. We should stress here that the e-MID data are the only interbank data which can be purchased freely without any restrictions. In contrast, getting access to similar datasets for other markets is usually far more complicated. The authors analyze daily networks from 1999-2002 and find intradaily and intramonthly seasonalities. The authors conclude that the networks appear to be random at the daily level. This finding is in stark contrast with the findings of preferential lending relationships in the Portuguese interbank market by Cocco *et al.* (2009). In this paper, we are mostly concerned with matching these seemingly incompatible findings, by showing that the aggregation period has an effect on the informational value of the underlying networks. The main finding is that daily networks indeed feature a substantial amount of randomness and cannot be considered as being representative for the underlying 'latent' network. This is illustrated on the basis of a number of network statistics which are compared to those of random networks. Furthermore, we find a substantial amount of asymmetry in the network. Last but not least, we find that the GFC can be identified as a significant structural break for many network measures.[1]

The remainder of this paper is structured as follows: Section 2 gives a brief introduction into (interbank) networks, Section 3 introduces the Italian e-MID trading system and gives an overview of the data set we have access to. Section 4 describes our findings and Section 5 concludes and discusses the relevance of these findings for future research.

2 Networks

A network consists of a set of N nodes that are connected by M edges (links). Taking each bank as a node and the interbank positions between them as links, the interbank network can be represented as a square matrix of dimension $N \times N$ (data matrix, denoted $\mathbf{D}$).[2] An element d_{ij} of this matrix represents a gross interbank claim, the total value of credit extended by bank i to bank j within a certain period. The size of d_{ij} can thus be seen as a measure of link intensity. Row (column) i shows bank i's interbank claims (liabilities) towards all other banks. The diagonal elements d_{ii} are zero, since a bank will not trade with itself.[3] Off-diagonal elements are positive in the presence of a link and zero otherwise.

Interbank data usually give rise to directed, sparse and valued networks.[4] However, much of the extant network research ignores the last aspect by focusing on binary adjacency matrices only. An adjacency matrix $\mathbf{A}$ contains elements a_{ij} equal to 1, if there is a directed link from bank i to j and 0

[1] In a companion paper, we focus explicitly on fitting the degree distribution, see Fricke *et al.* (2012). The main findings are (1) The degree distributions are unlikely to be scale-free, and (2) the in- and out-degrees do not follow the same distribution.

[2] In the following, matrices will be written in bold, capital letters. Vectors and scalars will be written as lower-case letters.

[3] This is true when we think of individual banks as consolidated entities.

[4] Directed means that $d_{i,j} \neq d_{j,i}$ in general. Sparse means that at any point in time the number of links is only a small fraction of the $N(N-1)$ possible links. Valued means that interbank claims are reported in monetary values as opposed to 1 or 0 in the presence or absence of a claim, respectively.

otherwise. Since the network is directed, both **A** and **D** are asymmetric in general. In this paper, we also take into account valued information by using both the raw data matrix as well as a matrix containing the number of trades between banks, denoted as **T**. In some cases it is also useful to work with the undirected version of the adjacency matrices, $\mathbf{A}^u$, where $a^u_{ij} = \max(a_{ij}, a_{ji})$.

As usual, some data aggregation is necessary to represent the system as a network. In the following, we use quarterly networks.

3 The Italian interbank market e-MID

The Italian electronic market for interbank deposits (e-MID) is a screen-based platform for trading of unsecured money-market deposits in Euros, US-Dollars, Pound Sterling, and Zloty operating in Milan through e-MID SpA.[5] The market is fully centralized and very liquid; in 2006 e-MID accounted for 17% of total turnover in the unsecured money market in the Euro area. Average daily trading volumes were 24.2 bn Euro in 2006, 22.4 bn Euro in 2007 and only 14 bn Euro in 2008.

Available maturities range from overnight up to one year. Most of the transactions are overnight. While the fraction was roughly 80% of all trades in 1999, this figure has been continuously increasing over time with a value of more than 90% in 2010.[6] As of August 2011, e-MID had 192 members from EU countries and the US. Members were 29 central banks acting as market observers, 1 ministry of finance, 101 domestic banks and 61 international banks. We will see below that the composition of the active market participants has been changing substantially over time. Trades are bilateral and are executed within the limits of the credit lines agreed upon directly between participants. Contracts are automatically settled through the TARGET2 system.

The trading mechanism follows a quote-driven market and is similar to a limit-order-book in a stock market, but without consolidation. The market is transparent in the sense that the quoting banks' IDs are visible to all other banks. Quotes contain the market side (buy or sell money), the volume, the interest rate and the maturity. Trades are registered when a bank (aggressor) actively chooses a quoted order. The platform allows for credit line checking before a transaction will be carried out, so trades have to be confirmed by

[5]The vast majority of trades (roughly 95%) is conducted in Euro.

[6]This development is driven by the fact that the market is unsecured. The recent financial crisis made unsecured loans in general less attractive, with stronger impact for longer maturities. See below. It should be noted, that there is also a market for secured loans called e-MIDER.

both counterparties. The market also allows direct bilateral trades between counterparties.

The minimum quote size is 1.5 million Euros, whereas the minimum trade size is only 50,000 Euros. Thus, aggressors do not have to trade the entire amount quoted.[7] Additional participant requirements, for example a certain amount of total assets, may pose an upward bias on the size of the participating banks. In any case, e-MID covers essentially the entire domestic overnight deposit market in Italy.[8]

We have access to all registered trades in Euro in the period from January 1999 to December 2010. For each trade we know the two banks' ID numbers (not the names), their relative position (aggressor and quoter), the maturity and the transaction type (buy or sell). As mentioned above, the majority of trades is conducted overnight and due to the global financial crisis (GFC) markets for longer maturities essentially dried up. We will focus on all overnight trades conducted on the platform, leaving a total number of 1,317,679 trades. The large sample size of 12 years allows us to analyze the network evolution over time. Here we focus on the quarterly aggregates, leaving us with 48 snapshots of the network.

4 Results

In this section, we look at the network structures formed by interbank lending over various horizons of time aggregation of the underlying data. We will see that comparing various network measures at different levels of time aggregation reveals interesting features suggestive of underlying behavioral regularities. Given that most studies focus on overnight data, it has become quite standard to focus on networks constructed from daily data. Here we find, that, at least for the Italian interbank network, it may be more sensible to focus on longer aggregation periods, namely monthly or quarterly data. We also discuss in how far the network structure has changed (and, in how far it has remained intact) after the default of Lehman Brothers in September 2008.

[7] The minimum quote size could pose an upward bias for participating banks. It would be interesting to check who are the quoting banks and who are the aggressors. Furthermore it would be interesting to look at quote data, as we only have access to actual trades.

[8] More details can be found on the e-MID website, see http://www.e-mid.it/.

4.1 General Features

In total, 350 banks (255 Italian and 95 foreign) were active at least once during the sample period. However the number of active banks changes substantially over time as can be seen from the left panel of Figure 1.[9] We see a clear downward trend in the number of active Italian banks over time, whereas the additional large drop after the onset of the GFC is mainly due to the exit of foreign banks. The right panel shows that the decline of the number of active Italian banks went along with a relatively constant trading volume in this segment until 2008. This suggests that the decline of active Italian banks was mainly due to mergers and acquisitions within the Italian banking sector. Given the anonymity of the data set, it is impossible to shed more light on this interesting issue. The overall upward trend of trading volumes was due to the increase of active foreign banks until 2008, while their activities in this market virtually faded away after the onset of the crisis. Interestingly, the average volume per trade tends to increase over time, as can be seen from the strong negative trend in the total number of trades in Figure 2, at least for the Italian banks.

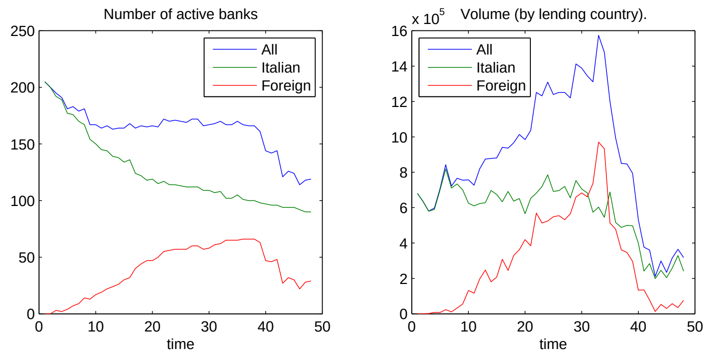

Figure 1: Number of active banks (left) and traded volume (right) over time. We also split the traded volume into money lent by Italian and foreign banks, respectively.

An interesting question in this regard is, who trades with whom. Figure 3 illustrates this for the number of trades (top) and the transacted volume (bottom) by country. For example, the green lines show the total number of trades (traded volumes) of foreign banks lending money to Italian banks, relative to all outgoing trades of foreign banks. Similarly, the blue lines show the total number of trades (traded volumes) of money flowing between

[9] Similar developments are reported by Bech and Atalay (2010) for the federal funds market.

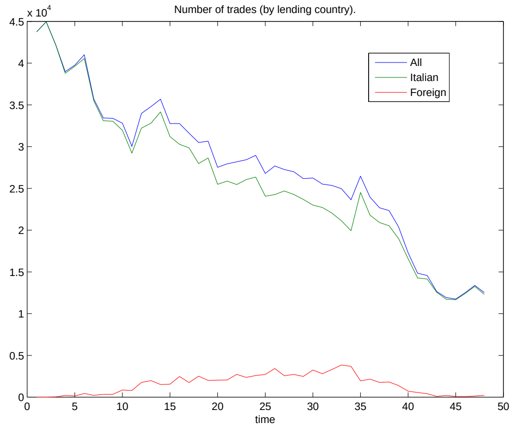

Figure 2: Number of trades per quarter.

Italian banks, as a fraction of all outgoing trades of Italian banks. The general patterns are the same for both Figures: Italian banks lend most of the time to other Italian banks (99.31% on average) and only a negligible amount to foreign banks (0.61% on average). This pattern is remarkably stable over time. In contrast, at the beginning of the sample period, foreign banks mostly used the market in the absence of (many) other foreign counterparties to lend money to Italian banks. This has changed over time and foreign banks mostly later on used the platform to trade with other foreign banks. It is not quite clear why this is the case, the underlying trend seems to point towards structural changes altering the (foreign) banks' behavior. For many research questions, one should therefore only use the subsample of Italian banks. In most of what follows, we stick to this choice.

This leads us to a first glance at the network structure. Figure 4 shows the banking network formed by the 119 active banks (89 Italian) in the last quarter of 2010.[10] The network consists mainly of two components: The very dense part formed by the Italian banks (circles) on the right-hand side and the far less interconnected foreign banks (triangles) on the left-hand side. The higher activity of the Italian banks is not represented in terms of the volume traded. We use total outgoing volume as a proxy for banks size and group the banks into 4 classes according to which percentile (30th, 60th, 90th or above) they belong to. This attribute is shown in the Figure as the size and the brightness of the nodes. We should note that 3 out of 12 banks of group 4 are foreign banks which is in line with their fraction of the total

[10] The Figure was produced using visone, http://www.visone.info/, by Brandes and Wagner (2004).

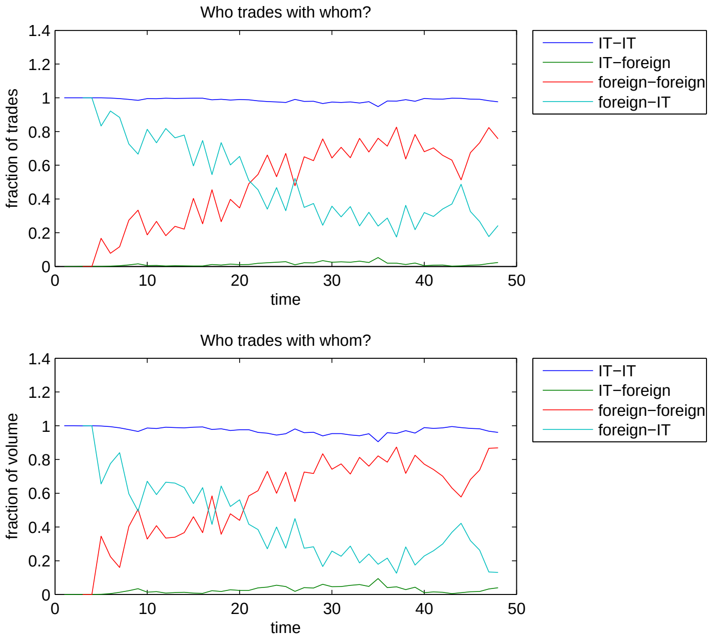

Figure 3: Fraction of trades (top) and traded volume (bottom) between banks from different countries.

banks (30 out of 119). Hence, foreign banks trade less on average (both in terms of volume and number of trades), however, the volume per trade is higher.

It is also interesting to highlight some specific features of the trading behavior of individual banks in this particular quarter, since not all banks use the market in the same way:[11] There are 14 banks with zero in-degree and 29 banks with zero out-degree. Surprisingly these banks are quite heterogeneous and not, as one might expect, just small banks. As an example the highest overall transaction volume of 58.6 bn Euro for a single bank, and therefore roughly 9.3% of the total trading volume, was traded by a German bank borrowing this sum in 90 trades from 8 counterparties. Another interesting case is an Italian bank trading only with one counterparty, lending this other bank 5.02 billion Euro in 76 trades, whereas borrowing just 0.03 billion in 3 trades. Even though these special relationships are quite interesting, the anonymity of the data set makes it impossible for us to say more on the particular relationships that might lead to these interesting outcomes. After this broad overview of the market and the ongoing interactions, we turn to the question of a sensible aggregation period. As should be clear from the

[11] For a detailed analysis of the trading strategies in the e-MID, see Fricke (2012).

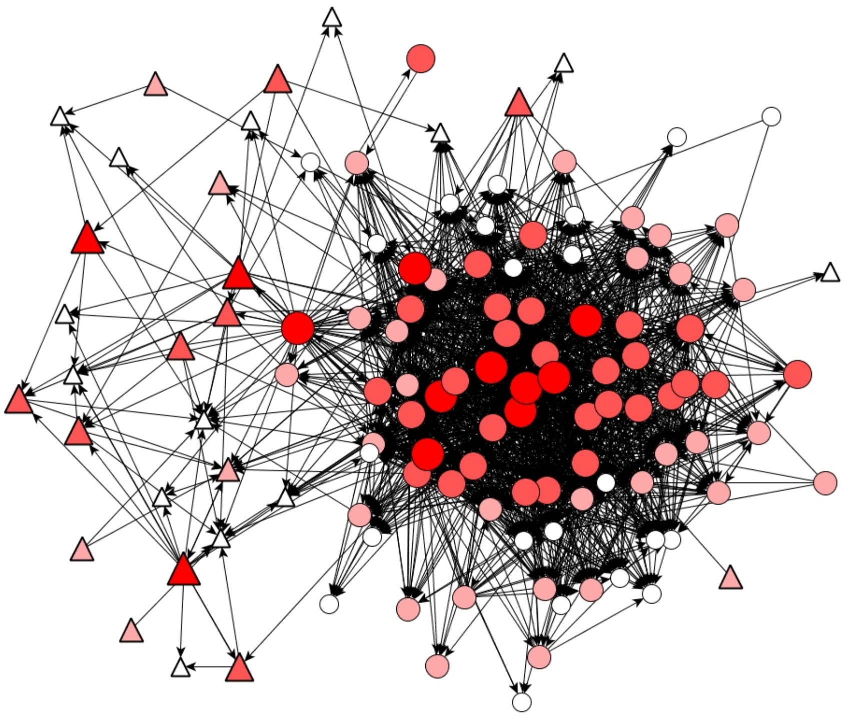

Figure 4: The banking network in the 4th of quarter 2010: triangles are foreign banks. The size of the node as well as the brightness of the red color indicate the size in terms of volume lent.

discussion above, we will mostly focus on the (sub)network formed by Italian banks only.

4.2 Density

The density ρ of a network is defined as the number of existing links (M) relative to the maximum possible number of links. It can be calculated as

$$\rho = \frac{M}{N(N-1)}. \tag{1}$$

Figure 5 illustrates the evolution of the density for four different aggregation periods (day, month, quarter, year). Except for the daily networks the density is quite stable over time and slightly increases until the GFC, which was a significant structural break for the monthly and quarterly network. We should note that the breakpoint (quarter 39), coincides with the quarter during which Lehman Brothers collapsed. The daily density fluctuates much more strongly, but overall increases throughout the sample.[12]

Compared to the findings for other interbank networks, the density of the Italian interbank network is quite high. For example, Bech and Atalay (2010) and Soramäki *et al.* (2006) report an average density of below 1% in daily interbank networks, compared to an average density of roughly 3.1% in our case. The main reasons for the higher density are most likely the relatively small number of participating banks in the market and the transparent market structure which easily allows each bank to trade with any other bank in the market. For comparison, the Fedwire network investigated by Soramäki *et al.* (2006) contains 5,086 institutions.[13] The means of 20.8% for quarterly aggregated networks (13.4% monthly) reveal much higher figures. Obviously the network density is positively related to the aggregation period, but to our knowledge the structure of this relation has not been investigated for interbank networks so far.

For this reason, we compare the aggregation properties of the empirical networks with those of random networks. Here we use Erdös-Renyi networks, i.e. completely random networks, and random scale-free networks, where the out-degrees follow a power-law distribution with scaling parameter 2.3.[14] The

[12] The density for the total network, including the foreign banks, seems to steadily decline over the sample period. This illustrates the fact that the increasing fraction of foreign banks are less interconnected with the (smaller) Italian banks.

[13] Additionally, the electronic nature of the trading platform might make links between any two institutions more likely.

[14] The power-law distribution with tail exponent 2.3 is a common finding in many interbank markets, see e.g. Boss *et al.* (2004). The resulting sequences of the out-degrees are

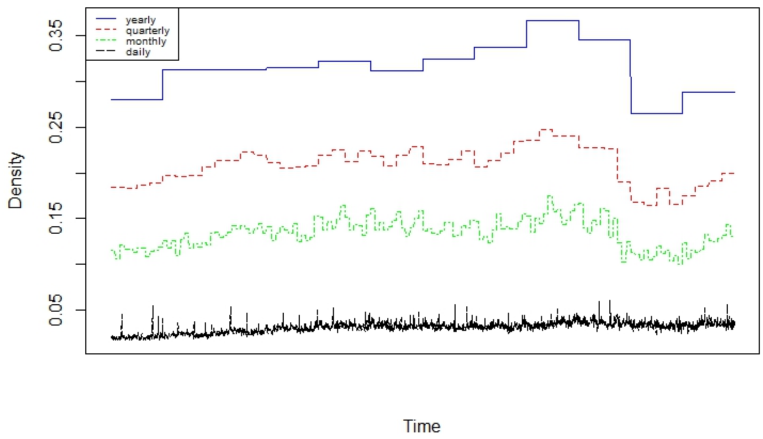

Figure 5: The density for yearly (blue), quarterly (red), monthly (green) and daily (black) aggregated networks. A Chow-test and an additional CUSUM-test indicate a structural break for quarter 39 (month 117) at the 1% significance level, but not for the yearly or daily networks.

experiments work as follows: For each year, we aggregate the daily networks and plot the resulting density in dependence of the aggregation period, from one day up to one year (roughly 250 days). For the random networks, we aggregate artificial Erdös-Renyi and scale-free networks for each day with the same number of active banks and density as the observed daily network. The results are the average values for 100 runs for the Erdös-Renyi and the scale-free networks.[15] We find very similar qualitative results for all 12 years.

attributed to the nodes by ranking those according to the observed out-degrees, considering only active banks during the particular day. Note that if we did not account for the ordering of the observed degree sequences, we would end up with very similar aggregation properties as the Erdös-Renyi case. The in-degrees are distributed in a random uniform way, ruling out self-links and counting each link at most once. For a detailed analysis of the degree distributions for this data set see Fricke *et al.* (2012).

[15] Note that the density of aggregated Erdös-Renyi networks can be written as

$$\rho_T^r = 1 - \prod_{t=1}^{T}(1 - \rho_t^r),$$

where $(1-\rho_t^r)$ is the probability of observing no links in the network at time t whatsoever, but since we adjust the number of active banks on a daily basis using tese probabilities would not constitute a completely satisfactory approach in our case.

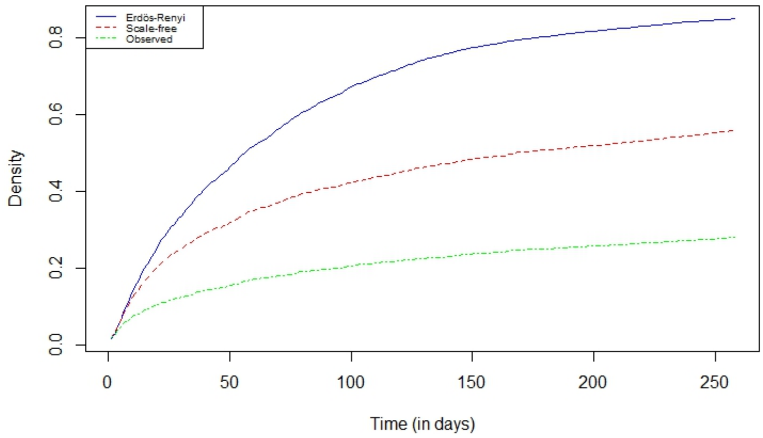

Figure 6: Data for 1999. Density for the aggregated Erdös-Reny (blue), Scale-Free (red) with $\alpha = 2.3$, and observed networks (green). Aggregation period in days. Note: we do not plot standard deviations, since these are negligible.

As an example, Figure 6 illustrates the results for 1999. For all three networks, there appears to be a saturation level for the density, however at different levels. The Erdös-Renyi networks always show the highest density (up to .851), followed by the Scale-Free networks (up to .559) and the observed networks (only up to .280). Apparently, it is much more likely for the empirical data that the same link gets activated several times than for the randomized data of Erdös-Renyi and scale-free networks (where the overall number of links is the same by construction). This is supported by the fact that in the observed networks a total number of $2,757$ links are observed only once in the year, while for the scale-free and random networks these values are $5,040$ and $5,746$, respectively. Hence, these results indicate the existence of lasting (preferential) lending relationships in the actual banking network.

4.3 What is a Sensible Aggregation Period?

After showing that longer than daily aggregation tends to reveal non-random structures for the Italian banking network, we are concerned with determining the 'correct' aggregation period in more detail in this section. This question is crucial for extracting relevant information, since the banking network cannot be observed at a given point in time, but always has to be approximated by aggregating trades over a certain period.[16] Most studies use daily aggregates (daily networks), which seems justified by the fact that the underlying loans are (mostly) overnight. Economically, however, overnight loans can be seen as longer-term loans, where the lender can decide every day whether to prolong the loan or not. Aggregating over a longer period is only preferable, if it can reveal a non-random structure of the banking network. The existence of preferential relationships would imply that daily transactions are not determined myopically, but that a virtual network of longer lasting relationships exists. Daily transactions would then be akin to random draws from this underlying network with the realizations depending on current liquidity needs and liquidity overhang. Aggregation over a sufficiently long time horizon might reveal more and more of the hidden links, rather than adding up purely random draws from all possible links. The relatively fast saturation of the empirical density that we observe in figure 6 is consistent with this interpretation. To shed more light on this issue we consider the consistency of yearly, quarterly, monthly and daily aggregated networks. The main finding is that we observe a much higher degree of structural stability for monthly or quarterly networks, depending on the application, rather than daily networks.

The use of a 'sensible' aggregation period should ensure that we extract stable features (if they exist) of the banking network rather than noisy trading patterns at different points in time. In this regard, it is important to investigate the stability of the link structure, in order to assess whether subsequent occurrences of the network share many common links. In order to do this, we rely on the Jaccard Index (JI),[17] which can be used to quantify the similarity of two sample sets in general. Here it is defined as

$$J = \frac{S_{11}}{S_{01} + S_{10} + S_{11}}, \tag{2}$$

where S_{xy} counts the number of relations having status $a_{ij} = x$ at the first instance and $a_{ij} = y$ at the second. The JI measures links which survive as a fraction of links which are established at any of the two points in time. Hence, it also takes into account those banks which are active in only one of the two periods. Figure 7 shows that the JI is very stable over time for longer aggregation periods, but not for the daily level. As expected, the JI tends to

[16] The literature on interbank networks is surprisingly silent about the choice of the aggregation period. We are aware of only one paper (Kyriakopoulos *et al.* (2009)) investigating this issue.

[17] The so-called graph correlation, see e.g. Butts and Carley (2001), shows qualitatively very similar results, but is not able to cope with banks entering or exiting market. The correlation of both measures is always above .9 irrespective of the aggregation period.

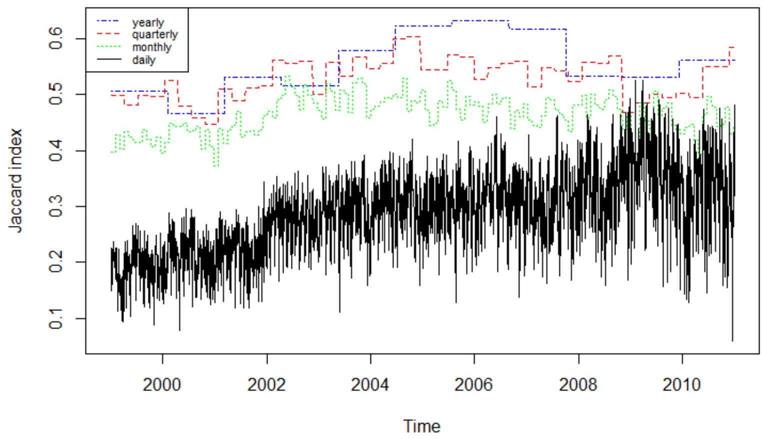

Figure 7: Jaccard Index for daily (black), monthly (green), quarterly (red) and yearly (blue) networks.

be higher for longer aggregation intervals. The daily measures are much more unstable and increase substantially until the GFC. More problematic than the smaller average level are however the extreme outliers on the downside. As a rule-of-thumb, in social network analysis one considers networks with JI values above .3 as substantially stable.[18]

Table 1 shows the mean, minimum, 10th percentile and standard deviation of the JIs for different aggregation periods. Again, the most evident observation is that the daily networks are rather special: the minimum and the 10th percentile of the JI are significantly smaller, indicating that we ob-

[18]See Snijders *et al.* (2009).

Jaccard Index	Year	Quarter	Month	Day
mean	.5543	.5302	.4638	.2861
standard deviation	.0535	.0368	.0333	.0740
min	.4652	.4479	.3735	.0603
10th percentile	.5072	.4835	.4183	.1904

Table 1: Jaccard Index for daily, monthly, quarterly and yearly networks. Calculations were carried out for all subsequent networks at the different aggregation periods. Standard deviations based on all observations.

Reciprocity	Year	Quarter	Month	Day
mean	.4264	.2085	.0829	.0042
standard deviation	.0580	.0423	.0244	.0060

Table 2: Reciprocity of the Italian Banking network. Calculations were carried out for all networks at the different aggregation periods. Standard deviations based on all observations.

serve values below .2 in at least 10% of the sample, which is not a rare event.[19] These results suggest a high degree of randomness in the daily networks.

Obviously, higher values of the JI are no guarantee that we are closer to the 'real' network per se. Note that in a network with randomly drawn connections, the index should be positively related to the length of the aggregation period. Thus, it is important to show that other network measures also take on values significantly different from random networks for longer aggregation periods. In the following, we will therefore have a closer look at the reciprocity of the network.

Reciprocity is a global concept for directed networks that measures how many of the existing links are mutual. It can be calculated by adding up all loops of length two, i.e. reciprocal links, and dividing them by the total number of links.

Table 2 shows higher levels of reciprocity for longer aggregation periods.[20] In the case of daily networks we observe very few mutual links, as Iori *et al.* (2008) stated this is a very plausible finding, since banks rarely borrow and lend money from the same bank within a particular day. However, the values for longer aggregation periods show that the banking network is not one-sided, supporting the evidence on the inability of daily networks to represent the 'true' underlying (directed) banking network. The left panel of Figure 8 illustrates the results for 1999, where we perform a similar analysis as for the density above, by comparing the observed network reciprocity to those of Erdös-Renyi and scale-free random networks.[21] The actual values are, again, always the lowest. The right panel of Figure 8 shows that the reciprocity (after 19 days for the real network) exceeds the density for all

[19] We should note that, as apparent from Figure 7, the reason for the 10th percentile to be below the .2 threshold is not the GFC.

[20] A structural break (after the GFC) is detected by a Chow-test as well as an additional CUSUM test for the 10th year, the 39th quarter and the 117th month respectively, but not for daily networks. For the yearly networks only the Chow-test indicates a structural break. Note that the yearly analysis involves only 12 data points.

[21] Again the results are qualitatively very similar for the other years as well.

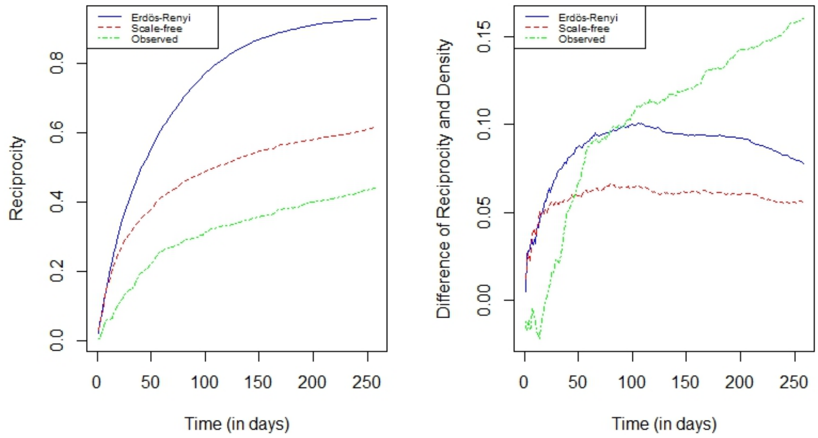

Figure 8: Data for 1999. Left: reciprocity for the Erdös-Renyi (blue), Scale-Free (red) with $\alpha = 2.3$, and observed networks (green). Right: difference between reciprocity and density for the respective networks. Aggregation period in days.

three networks. Note that for random networks one would expect the two measures to be almost identical. However, different banks are active at different days and if many banks are often simultaneously active the chance of forming a reciprocal link is higher (remember that we used the actually active banks of each day in the Monte Carlo exercise). However, more important for our analysis is that for the real network the difference between reciprocity and density increases steadily and exceeds the difference of the random networks. Hence, using longer than daily aggregation is not only capable of taking mutual credit relationships into account, but even indicates a preference of banks to form them. On the other hand, the saturation of the reciprocity indicates that banks will have mutual credit relationships with most of their counterparties.[22] Thus, the noise level of networks with longer aggregation periods is smaller and the directed version of the networks contains a substantial amount of information.

On the base of the Jaccard index, monthly and quarterly networks appear most stable as they have a high index with a very low standard deviation, i.e., the highest degree of structural stability of lending relationships over time. Yearly aggregation levels, in contrast, have somewhat higher variation in their

[22]This may of course be affected by the time-varying composition of active banks. See below.

JI and might be problematic, because the banking network (in particular during unstable times) is likely to evolve much faster. Somewhat related is the change in the composition of banks, i.e. banks leaving and entering the market, since we consider a bank as active for the whole year even if it leaves the market after the first trading day.[23] Concerning the monthly level, Iori *et al.* (2008) discovered intradaily and -monthly seasonalities which may affect our results. In everything that follows, we will therefore mostly focus on the quarterly networks.

4.4 Transitivity

Here we are interested in transitive relations between three banks. The concept of transitivity states that a specific relationship $\diamond$ is transitive if from $i \diamond j$ and $j \diamond k$ it follows that $i \diamond k$ holds. Equality is a transitive relation, but inequality is not. From $i = j = k$ follows $i = k$, yet $i \neq j \neq k$ does not imply $i \neq k$.

The relation we are interested in is i has a link to j or $a_{ij} = 1$. The relationship is obviously not transitive, since i has a link to j and j has a link to k does not strictly imply that k also has a link to i. However, it is interesting to investigate how many such closed triplets occur. More generally speaking, transitivity measures whether the existence of certain links depend both on the relation between the two counterparties and on the existence of links with a third party. The measure most prominently used for this

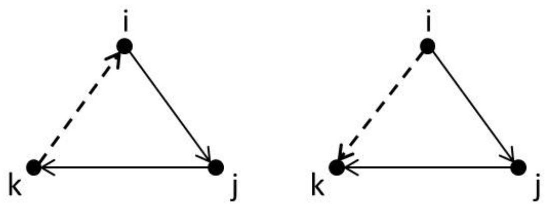

Figure 9: The two possibilities how the directed path of length two (solid lines) between i and k can be closed. On the left hand side the path is closed into a loop of length three (CC^1). On the right hand side the triplet is interconnected but not in the same single direction (CC^2).

purpose is the (directed) clustering coefficient[24], which, despite its name, has

[23]This problem occurs for each aggregation period, but is likely to become more pronounced for longer frequencies.

[24]For more detailed definitions of clustering coefficients see Zhou (2002).

no relation to cluster identification whatsoever.[25] It measures the number of (directed) paths of length two in the network and takes the fraction of these which are closed.[26] Figure 9 illustrates two ways to close the triplet i, j, k in a directed network. First, the directed path may be closed into a loop as shown on the left hand side of the figure. The function of such closure is given by the coefficient CC^1:

$$CC^1 = \frac{\sum_{j \neq i \neq k} a_{ij} a_{jk} a_{ki}}{\sum_{j \neq i \neq k} a_{ij} a_{jk}} \tag{3}$$

Second, the link from i to k may be reversed. The function of such closure is given by the coefficient:

$$CC^2 = \frac{\sum_{j \neq i \neq k} a_{ij} a_{jk} a_{ik}}{\sum_{j \neq i \neq k} a_{ij} a_{jk}} \tag{4}$$

An important distinction is that the nodes in the case of CC^1, as apparent from Figure 9, have each one in- and one outgoing link and therefore show no hierarchical ordering. Figure 10 shows that the results for the two coefficients are very different. The mean for CC^1 is .164 and .571 for CC^2. This is further evidence for the non-random character of the banking network, since the probability of an 'average' link to exist is just equal to the density of .208. Hence, the existence of a path of length two between i and k via j makes it 2.75 times more likely that the link from i to k exists compared to a random link[27], but reduces the probability that there is a link from k to i by 21%. The huge difference indicates that the banking network has a hierarchical ordering on the triadic level.

Figure 11 illustrates that for the Erdös-Renyi networks the evolution of the clustering coefficients is almost identical (correlation above .999). The exact numbers of the clustering coefficients for the observed networks change with the aggregation, but CC^1 is always much smaller than CC^2 for all aggregation levels as shown by figure. CC^2 is at the beginning even higher for the observed network than for the random networks, but saturates after a steep increase relatively quickly on a much lower level (up to .624). CC^1 on the other hand is almost zero at the beginning. Note that a loop of length three at a single day implies that each involved bank would get back some

[25] See Fricke and Lux (2012) and Fricke (2012) for detailed approaches of cluster identification in the e-MID market.

[26] Any connection along directed links between two nodes i and j is called a path and the length of the path is defined as the number of edges crossed. There are no restrictions on visiting a node or link more than once alongside a path.

[27] The probability of random link is exactly the density of .208.

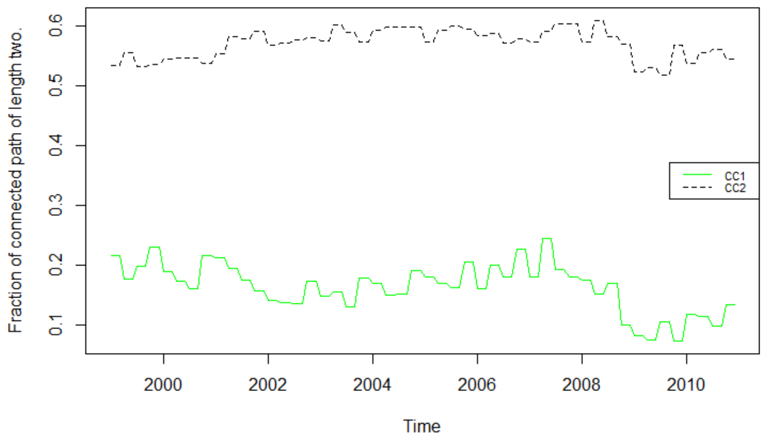

Figure 10: The two clustering coefficients of quarterly networks: CC^1 (green) is the fraction of path of length two which are closed into a loop by a third link. CC^2 charracterizes links in which the triangle is closed in a hierarchical way.

of its own lending via an intermediate bank, which appears very unlikely. However, CC^1 increases (up to .386) for longer aggregation periods showing that such relations do exist. Note that the daily or undirected networks are not capable of taking this into account.

4.5 Small-World Property

Another very prominent measure in the network literature is the average shortest path length (ASPL). Interest in this measure stems from the remarkable finding that in many 'real world' networks the ASPL is quite small, also known as the small-world phenomenon.[28] Here we focus on the undirected version of the network.

Watts and Strogatz (1998) show that completely random networks already have a very small ASPL, but at the same time a relatively low (undirected) clustering coefficient (CC) equal to its density. On the other hand, a regular network, where all nodes have connections to their s nearest neigh-

[28]Small ASPLs have been detected for social, information, technological and biological networks. The first empirical finding dates back to the chain letter experiments conducted by Milgram (1967). His finding that on average only six acquaintances are needed to form a link between two random selected persons led to the famous phrase of 'six degrees of separation'.

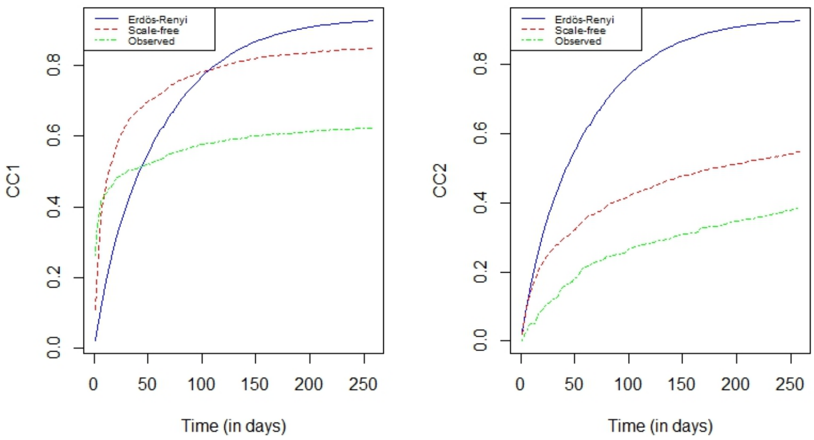

Figure 11: Left: CC^1 for the Erdös-Renyi (blue), Scale-free (red) with $\alpha = 2.3$, and observed networks (green), while the are aggregated on a daily basis up to one year(1999). Right: CC^2.

bours, has a CC of 1, but a very high ASPL. Interestingly, already the introduction of a few 'random' links reduces the ASPL significantly, because these 'long range' links connect different clusters of the network. The authors argue that the interesting 'real world' networks are neither purely regular nor random. Therefore Watts and Strogatz (1998) define small-world networks to additionally have a higher CC than random networks. Hence, to further investigate if the banking network exhibits the small-world property, we calculate the ASPL and the CC and compare the values to random networks.

The (symmetric) matrix $\mathbf{G}$ of dimension $N \times N$ contains the geodesic distances between any two nodes, i.e. each element g_{ij} is the length of the geodesic path between node i and j. The ASPL is calculated by dividing the sum of all (existing) geodesic path lengths by the total number of (existing) geodesic paths.[29] The CC is calculated similar to the directed case (CC^1 and CC^2), but ignores the directedness of the links.

In this case a comparison of the networks aggregated day by day is not suitable, since the density of both random networks as visible in figure 6 after

[29]It is not necessary for two nodes to have a shortest path, since there might be no link leading from one to the other. In this case the two nodes lie in different components of the network and by convention the length of these non existing geodesic paths are set to infinity. The undirected banking network consists always of only one component and the same is true for the investigated random networks, because of the high density.

	CC	ASPL
Erdös-Renyi	.3736	1.6470
	(.0058)	(.0073)
Scale-free	.4131	1.6259
	(.0103)	(.0134)
Observed	.5422	1.6486
	(.0434)	(.0373)

Table 3: The average CC and ASPL for the observed network and for 100 Erdös-Renyi and scale-free (with $\alpha = 2.3$) random networks. Standard deviations in brackets.

a quarter (around 63 days) is much higher. Therefore the simulated random networks correspond to the aggregated quarterly networks with respect to their density (and their out-degree distribution for scale-free networks). Table 3 summarizes the results: The ASPL for all networks is small and almost identical, in which the very low value is due to the high density. The CC for the Erdös-Renyi networks is by construction close to the density, since all links have per quarter exactly this probability to occur. The CC for the scale-free is with .413 higher, but is exceeded by the observed network (.542). This indicates the higher regularity in the link structure. Hence, the banking network lies midway between regular and completely random graphs. As can be seen from the last column of Table 3, the ASPL does not provide much scope for distinguishing between the benchmark Erdös-Renyi and scale-free networks and the empirical ones. The reason might be that the relatively high density leads to relatively short path lengths anyway.[30]

4.6 Effects of the Global Financial Crisis

Finally, we take a closer look at the effects of the GFC on the banking network.[31] The start of the GFC is not easy to determine, but we have seen that the collapse of Lehman Brothers in quarter 39 (2008 Q4) was a major shock for the global financial market in general and the Italian interbank market as well.[32] The effects of this event were twofold: first, the counter-

[30]The results are qualitatively the same if we consider the directed version of the network. The ASPL for the quarterly networks is 1.912 against 1.802 for the random networks. The CC^2 (.571) is significantly higher than for Erdös-Renyi networks with .208, whereas CC^1 (.164) is even smaller. However, in this case the network consists not only of one giant component which makes the interpretation of the ASPL more difficult.

[31]See also Fricke and Lux (2012).

[32]See Brunnermeier (2008).

parties of Lehman Brothers realized huge loses. Second, the perception of risks changed, since Lehman had been considered to be 'too-big-too-fail' before. The resulting dramatic increase of perceived counterparty risk reduced the willingness of banks to lend to each other, which ultimately affected the real economy due to tighter lending restrictions. The monetary authorities and governments around the world injected substantial amounts of capital into the financial system to prevent interbank markets from freezing in the following weeks. We have seen, that important network measures such as density and reciprocity, were significantly affected by these events as well.

Quarter	36	37	38	39	40
Banks	101	100	100	98	97
Volume	445,991	409,340	435,338	404,353	385,819
Trades	20,984	20,078	19,963	18,160	16,477
Links	2,425	2,253	2,249	2,153	1,768
Trades per Link	8.65	8.91	8.88	8.43	9.32
Quarter	41	42	43	44	45
Banks	96	96	94	94	94
Volume	234,102	267,057	197,021	227,076	196,503
Trades	14,184	13,981	12,525	11,636	11,577
Links	1,533	1,506	1,599	1,449	1,530
Trades per Link	9.25	9.28	7.83	8.03	7.57

Table 4: The table summarizes the number of Italian banks, total volume (million Euros), number of trades, number of links and links per trade for the quarters 36-45, while the Lehman collapse has been in quarter 39.

Here we investigate the change in banks' behavior during and after the breakdown of Lehman Brothers. To begin with, Table 4 contains several basic network statistics for ten quarters around the breakpoint (quarter 39). Interestingly, the number of active Italian banks remained relatively stable during this period, and in fact 86 banks were active in all of the ten quarters. The stability of this composition is important, since under these circumstances changes in the behavior of the banks on the aggregated or individual level should be mainly driven by their response to this exogenous shocks.

We also see that the total trading volume, the number of trades and the number of links are all decreasing over this period, but the exact patterns are distinct. Surprisingly, the volume is quite stable until the 40th quarter, but drops by 39.3 percent in the 41st quarter. In contrast, the number of trades starts to fall in the 39th quarter and decreases further until quarter 44. The total number of links already starts to decrease in quarter 37, but overall tends to develop in a very similar way to the number of trades.[33] In the end, the most immediate reaction to the crisis was that banks traded similar total volumes but in fewer trades and with a smaller number of counterparties in order to minimize their (perceived) counterparty risks.[34]

Interestingly, the trades per link are the highest for the three quarters after the Lehman collapse. This indicates that the banks relied stronger on their preferred counterparties. Preferred in this context might simply means that the banks had more reliable information about these banks, which however should coincide with former trading relationships. Eventually, we conclude that the breakdown of Lehman Brothers significantly affected the behavior of individual banks and thus had a clear impact on the structure of the network. Quite surprisingly, we find that the link structure of subsequent networks remained rather stable during this period, since no structural break is detected for the Jaccard Index. Furthermore, despite the significant impact of the GFC, we do not find evidence for a complete drying up of the e-MID market, even at the daily level.[35]

5 Conclusions and Outlook

In this paper, we have investigated the interbank lending activity as documented in the e-MID data from 1999 until the end of 2010 from a network perspective. Our main finding is that daily networks feature too much randomness to be considered a representative statistic of some underlying latent network. The JI shows the higher consistency over time for longer aggregation periods and the very low density compared to (aggregated) random networks indicates the existence of preferred trading relations. In general the evolution of all global network measures for longer aggregation periods (month, quarter, year) is very similar in their deviation from the Erdös-Enyi and scale-free benchmarks. Moreover, the monthly and quarterly networks are characterized by a significantly higher than random clustering coefficient, and thus reveal some regularity in the link structure. The (almost) zero reciprocity and CC^1 of daily networks proves the inability of this aggregation level to reveal information on such structural elements. However, quarterly networks consistently exhibit a non-random structure and allow us to con-

[33] The correlation between both is .963 for this period and .957 over the complete sample.

[34] Fricke and Lux (2012) show that most of these changes were in fact driven by behavioral changes of core banks.

[35] As noted above, this is not true for loans longer than overnight. These markets essentially collapsed completely, which is not surprising given that the loans are unsecured.

sider the mutuality of the relations and are therefore a preferable subject of study, especially if one is interested in the evolution of the network over time.

Essentially, these results show that it is far from trivial to map a given data structure into a 'network'. While daily records of the interbank trading system can be arranged in an adjacency matrix and treated with all types of network statistics, they provide probably only a very small sample of realizations from a richer structure of relationships. Just like daily contacts of humans provide very incomplete information of networks of friendship and acquaintances, the daily interbank data might only provide a small selection of existing, dormant established trading channels. Hence, inference based on such high-frequency data may be misleading while a higher level of time aggregation might provide a more complete view on the interbank market. What level of aggregation is sufficient for certain purposes is an empirical question depending on the research questions at hand. However, saturation of certain measures may be a good indicator that most dormant links have been activated at least once over a certain time horizon. At the same time, such dependence of statistics on the time horizon serves to sort out a number of simple generating mechanisms (i.e. completely randomly determined networks in every period) and reveal interesting dynamic structure.

Another interesting result is that the network is asymmetrical in many respects. For the quarterly network the fraction of reciprocal links is very similar to the density. Furthermore, the two directed clustering coefficients are very different. The probability for path of length two to be closed into a loop is 3.48 times smaller than the other way. Additionally, the correlation between in- and out-degree is merely .12 for the complete sample. Therefore, the information that a bank has a a large number of incoming links is a surprisingly poor indicator of how many outgoing links the bank has.

Moreover, for many measures the GFC could be identified as a structural break and also the decreasing number of volume, trades and links support that the GFC heavily affected the Italian interbank market. However, the network overall remained surprisingly stable and despite the decrease of its volume (in the beginning of 2010) the e-MID market was never close to drying up completely.

In the future more attention should be given to the analysis of directed banking networks using longer aggregation periods to identify structural commonalities. This has important consequences for the regulation of credit institutions, since at the daily level it is difficult to detect the systemically important institutions. For policymakers and regulators, it would be potentially (dangerously) misleading to focus on the noisy daily networks, even more since the low level of connectivity suggests a low-degree of systemic risk at any point in time. More important, in our view, is to get a better idea on the wider pool of counterparties of all credit institutions, in order to detect possible behavioral changes among the set of relatively active banks. Such changes might then serve as an indicator for funding problems of individual institutions.[36] In the end, it would be important to extend our phenomenological analysis in order to test hypotheses about the behavior of banks at the micro-level that drives the system's properties.

Bibliography

Bech, M., and E. Atalay (2010): "The topology of the federal funds market," *Physica A*, 389(22), 5223–5246.

Boss, M., H. Elsinger, M. Summer, and S. Thurner (2004): "Network topology of the interbank market," *Quantitative Finance*, 4(6), 677–684.

Brandes, U., and D. Wagner (2004): "visone-Analysis and Visualization of Social Networks," in *Graph Drawing Software*, ed. by M. Jünger, and P. Mutzel, pp. 321–340. Springer-Verlag.

Brunnermeier, M. K. (2008): "Deciphering the Liquidity and Credit Crunch 2007-08," Working Paper 14612, National Bureau of Economic Research.

Butts, C., and K. Carley (2001): "Multivariate Methods for Interstructural Analysis," CASOS Working Paper, Carnegie Mellon University.

Cocco, J. F., F. J. Gomes, and N. C. Martins (2009): "Lending relationships in the interbank market," *Journal of Financial Intermediation*, 18(1), 24–48.

De Masi, G., G. Iori, and G. Caldarelli (2006): "Fitness model for the Italian interbank money market," *Phys. Rev. E*, 74(6), 66112.

Fricke, D. (2012): "Trading Strategies in the Overnight Money Market: Correlations and Clustering on the e-MID Trading Platform," *Physica A, forthcoming*.

Fricke, D., K. Finger, and T. Lux (2012): "The Distributional Properties of the e-MID Overnight Money Market," In progress, Kiel Institute for the World Economy.

[36]See Fricke and Lux (2012).

Fricke, D., and T. Lux (2012): "Core-Periphery Structure in the Overnight Money Market: Evidence from the e-MID Trading Platform," Kiel Working Papers 1759, Kiel Institute for the World Economy.

Inaoka, H., T. Ninomyia, K. Taniguchi, T. Shimizu, and H. Takayasu (2004): "Fractal Network derived from banking transaction – An analysis of network structures formed by financial institutions," *Bank of Japan Working Papers*, pages(04-E-04), 1–22.

Iori, G., G. De Masi, O. V. Precup, G. Gabbi, and G. Caldarelli (2008): "A network analysis of the Italian overnight money market," *Journal of Economic Dynamics and Control*, 32(1), 259–278.

Kyriakopoulos, F., S. Thurner, C. Puhr, and S. W. Schmitz (2009): "Network and eigenvalue analysis of financial transaction networks," *The European Physical Journal B - Condensed Matter and Complex Systems*, 71(4), 523–531.

Milgram, S. (1967): "The small-world problem," *Psychol. Today*, 2, 60–67.

Snijders, T. A. B., G. G. van de Bunt, and C. E. G. Steglich (2009): "Introduction to stochastic actor-based models for network dynamics," *Social Networks*, 32(1), 44–60.

Soramaki, K., M. L. Bech, J. Arnold, R. J. Glass, and W. Beyeler (2006): "The topology of interbank payment flows," Staff Reports 243, Federal Reserve Bank of New York.

Watts, D. J., and S. H. Strogatz (1998): "Collective dynamics of small-world networks," *Nature*, 393(6684), 440–442.

Zhou, H. (2002): "Scaling exponents and clustering coefficients of a growing random network," *Phys. Rev. E*, 66, 016125.