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Stochastic Integrated Assessment of Ecosystem Tipping Risk

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Abstract One of the major potential consequences of climate change is damage to earth's ecosystems, damage which could manifest itself in the form of tipping risks. We establish an economic growth model of ecosystem tipping risks, set in the context of possible forest dieback. We consider different specifications of impacts arising from the forest dieback tipping point, specifications such as changes in the system dynamics of the forests, changes in the forest mass, and impacts on economic output. We also consider endogenous and exogenous tipping point probabilities. For each specification we compute the optimal policies for forest management and emission control. Our results show qualitative differences in patterns of post-tipping event, optimal forest harvest, and either precautionary or aggressive pre-tipping event harvest patterns, a feature consistent with the findings of the existing literature. Optimal control of deforestation and carbon dioxide emission reduction also exhibits varied patterns of post- and pre-tipping levels depending on the nature of the tipping risk. Still, today's optimal policy is one of more stringent emissions control in presence of a potential forest dieback tipping point.

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1 Introduction

One of the major potential consequences of climate change is damages to earth's ecosystems, damage which could manifest itself in the form of tipping risks (e.g., World Bank 2012)—or, put differently—regime shifts that are hard to be predicted[1]. Climate change is known to be one of the major drivers of ecosystem changes, and of habitat change, over-exploitation, pollution, and invasive species (Millennium Ecosystem Assessment 2005). There is evidence that ecosystem tipping events may exist on a local scale (e.g., Folke et al. 2004; Lenton et al. 2008) and also speculation that they may exist on a planetary scale Barnosky et al. (2012). Given the influence of ecosystem services on economic activity, examination of these tipping risks could help inform debates regarding optimal emission control (e.g., Stern 2013) and to guide the optimal reduction of emissions from deforestation.

There already exists an extensive literature of analytical studies of the economic analysis of environmental regime shifts (which could also be conceived as tipping events as considered in Lenton et al. 2008, etc.). Recent contributions include those made by Margolis and Naevdal (2008), Brozović and Schlenker (2011), Polasky et al. (2011), de Zeeuw and Zemel (2012), Zemel (2012), and Ren and Polasky (2014). These studies focus, using restrictive models, on finding clear-cut conditions for precautionary and aggressive exploitation of renewable resources. A drawback of these models is that they are rather parsimonious and might not be able to model relatively complex but realistic features of potential regime shifts relevant to climate change, such as, for example, the characteristic that both risk-subjected ecosystem functions and emissions from economic activities affect the climate system. On the other hand, with the exception of Cai et al. (2015b), an emerging literature of climate-policy and tipping-risk modeling studies focuses on climate tipping events but does not consider tipping risks in ecosystems (Bahn et al. 2008; Cai et al. 2015a; Crost and Traeger 2011; Keller et al. 2004; Lemoine and Traeger 2014; Lontzek et al. 2015).

This paper is an attempt to fill this current gap in research. We consider a simple, dynamic, stochastic integrated assessment model and compute optimal policies under climate change and ecosystem risks, specifically in the form of large-scale forest dieback. Forest dieback is considered to be a major tipping risk associated with climate change and could significantly affect the global carbon cycle through the release of carbon stored in woody biomass and forest soils (Lenton et al. 2008; Kriegler et al. 2009). Widespread dieback of forests has already been observed in Canada (Kurz et al. 2008a, b), and many general circulation and vegetation models forecast an extensive forest dieback both in tropical and boreal regions in

[1] The words "tipping points" and "regime shifts" are often used in similar contexts without clarifying whether the one necessarily accompanies the other or not. A definition distinguishing the two concepts is that by Biggs et al. (2011): they term "regime shifts" "large, abrupt persistent changes in the structure and function of ecosystems" entailing "the shift of a system from one basin of attractor to another when a critical threshold or tipping point is exceeded." By this definition, a regime shift could occur only in the presence of a tipping point, but the reverse is not necessarily the case. In the following, we mainly use the term "tipping points" even in the cases also involving a regime shift, as that term is the one frequently used in the literature of the economics of climate change, such as in Lemoine and Traeger (2014), Cai et al. (2015b), or Lontzek et al. (2015). In discussing our model, however, we make clear which of our examined cases could be also seen as "regime shifts" by Biggs et al.'s definition and which are not.

the future (Lucht et al. 2006; Cook and Vizy 2008; Lenton et al. 2008; Kriegler et al. 2009; Rammig et al. 2010; Hirota et al. 2011; Staver et al. 2011; Good et al. 2013). Forest dieback could, for example, occur due to climate-related changes in precipitation and temperature patterns but its onset is not precisely predictable (Lenton et al. 2008; Kriegler et al. 2009). In the context of climate change impacts, forest dieback is often interpreted as an irreversible loss of forest areas, but in our analysis we consider both the case of an irreversible area loss and an alternative case in which forest dieback involves a recoverable loss of forest stock. While the impacts of forest dieback on human welfare are wide-ranging including loss of biodiversity and of other ecosystem services, in our model analysis we focus on two relatively well-quantifiable functions of forests that affect human welfare—namely, the input for the production of economic output in the form of wood as a resource, and the function of carbon sequestration. Treating the forest as a renewable resource, our model could be seen as an extension to Polasky et al. (2011), an extension that additionally accounts for climate change dynamics. In addition to the use of the forest resource and the control of production-related carbon dioxide emissions, we also explicitly consider the control of deforestation through measures to reduce emissions from deforestation and forest degradation (REDD/REDD+), a mechanism operating under the auspices of the UN Framework Convention on Climate Change (UNFCCC) and one of the key topics of climate change policy debate.

The model computes optimal policy choices when the economy produces output by using reproducible capital and a renewable resource (forest), a resource which has a potential to regenerate and is also subject to a tipping risk. Climate change may raise the probability of tipping, and is in turn determined by carbon dioxide emission reduction and deforestation control, both of which influence the carbon stock. However, motivated by existing studies of regime shifts (discussed by Polasky et al. 2011), we also consider alternative cases of an exogenous probability of tipping. We focus on analyzing the basic trade-offs of policy choices on a global level, and the simple settings of the model miss many detailed features. But this in turn means that our model's framework has some degree of generality and is also potentially applicable to other problems of ecosystem risks induced by climate change. By modeling the growth and optimal harvest of global forest stock, we compute the time paths of optimal control policy for deforestation (i.e., REDD policy) and also of optimal policy for carbon dioxide emission reduction and compute the effects of tipping risks on those policy decisions. We calibrate our model parameters with representative values drawn from the existing literature, but given the scarcity of empirical information on this problem, we conduct an extensive sensitivity analyses as well.

In line with the existing economic literature of regime shifts, especially with Polasky et al. (2011), we examine different cases characterized by whether a tipping event changes the system dynamics or the level of the forest stock. A reason for a systematic change in forest stock dynamics could for example be a permanent habitat loss due to a change of climate-related hydrological conditions. An abrupt reduction in the level of the forest stock could be induced by extreme weather, fire, diseases, or pests (e.g., bark beetle attack on boreal forests).

The results show qualitative differences in patterns of post-event optimal resource harvesting, leading to either precautionary or aggressive pre-event harvest patterns. The optimal control of deforestation and carbon dioxide emission reduction also exhibit varied patterns of post- and pre-tipping levels depending on the nature of the tipping risk. In particular, when the tipping risk triggers a drop in the amount of forest mass (in the way of "stock collapse" modeled by Polasky et al. (2011), although here, the stock does not vanish but is only reduced by a certain amount), the emission reduction rate (optimal deforestation rate) exhibits a positive (negative) post-tipping jump, which so far has been only found in the case of multiple interacting tipping points in Cai et al. (2016b). In our case, this is a reflection of a weak or absent risk-reduction effect: the tendency to conduct precautious climate and resource policies to decrease the probability of the tipping event. In contrast, if the tipping event induces a systematic change in the resource stock dynamics (in the way of "changed system dynamics" modeled by Polasky et al. 2011), the risk reduction effect is much stronger and optimal emission reduction (deforestation control) exhibits a post-tipping drop (positive jump) as a reflection of reduced incentives to delay the tipping by aggressive climate protection policies. We find further evidence for these effects studying a comparable version of the model with an exogenous (i.e., temperature-independent) tipping point probability. In contrast to the case with endogenous tipping risk, optimal emission reduction is lower today and exhibits a positive post-tipping jump, since it is no longer possible to delay the expected timing of the tipping point event. These findings suggest that it is crucial to distinguish between endogenous and exogenous tipping risks when assessing optimal climate and resource management policies.

The initial-year emission reduction is enhanced with any forms of tipping risk, suggesting the influence of the consumption smoothing effect. In this sense, while analysis of our model reveals differences in the effects of different tipping risks on optimal policy choices, it thus still supports the conclusion of existing studies on climate policy and tipping risk that the presence of tipping risks generally raises the stringency of the optimal current climate policy (see, e.g., Lemoine and Traeger (2014) and Cai et al. (2015a), as well as Lontzek et al. (2015)).

We also examine the combined effect where the tipping risk also has a direct effect on output. In line with Cai et al. (2015a) we find that the risk reduction effect is much stronger in this setting. This implies significantly larger pre-tipping climate and resource protection policies to delay the tipping event and hence a pre-tipping reduction (increase) of emission reduction (deforestation control). Furthermore, we find that separating risk aversion from the intertemporal elasticity of substitution (IES) can significantly change optimal policy responses. This result is in line with Ha-Duong and Treich (2004) or Cai et al. (2015a), contributions to the literature that find that, by assuming standard preferences of constant relative risk aversion (CRRA), the sensitivity of optimal climate policies with regard to risk aversion may be misleading. Therefore we consider Epstein–Zin–Weil preferences (Epstein and Zin (1989); Weil (1989)) that allow for a separation of the relative risk aversion coefficient and the IES. Recent findings in the literature of long-run risk (see Bansal and Yaron 2004; Bansal et al. 2012) find a calibration of a risk aversion of 10 and an IES of 1.5 to be consistent with many empirical features of financial markets. We compare the model's results for CRRA preferences (where risk aversion is given by the reciprocal of the IES) to the Epstein–Zin–Weil specification with disentangled coefficients.

We find that if the tipping risk has a direct effect on output and there is uncertainty about the magnitude of the tipping event, risk aversion has a significant influence on optimal climate and resource policies. In particular an increase in risk aversion amplifies the risk reduction effect, which implies larger investments in pre-tipping emission reduction and deforestation control. This effect can not be modeled by standard CRRA preferences, which suggests that it is important to consider Epstein–Zin–Weil preferences for the analysis of stochastic tipping events. In contrast, if the tipping point event does not directly affect output, Epstein-Zin preferences with a larger risk aversion compared to the corresponding CRRA have only a marginal influence on model outcomes.

We describe our model in Sect. 2 and present results in Sect. 3. Section 4 concludes.

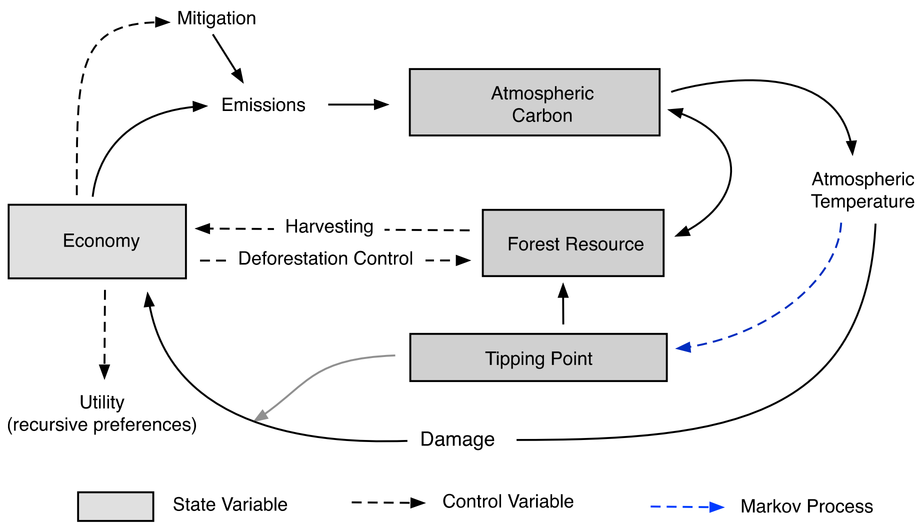

Fig. 1 Schematic of the model economy

2 Description of the Model

Our model builds on the standard Ramsey-type economic model with exogenous growth but we include a renewable resource (forest mass) as an additional factor of production.[2] Figure 1 presents a schematic of the model economy.

We assume that the production of output generates carbon emissions, which can be controlled. Uncontrolled emissions increase the atmospheric stock of carbon. The latter causes climate change resulting in damage to economic output. The atmospheric stock of carbon also interacts with the carbon content of the renewable forest mass. Since forests store carbon, harvesting and uncontrolled deforestation will release carbon into the atmosphere. At the same time, growth in the forest mass absorbs atmospheric carbon.

In addition to these features we include the possibility of a forest dieback tipping event. The determinants and impacts of the tipping event are not perfectly known and we choose a stochastic formulation of both to represent this lack of knowledge. We study different cases of the tipping event, each representing different characteristics of the determinants and impacts of a possible forest dieback. More specifically we study the case of (a) a tipping point which reduces the size of the forest resource, and (b) a tipping point which leads to a permanent reduction in the carrying capacity of the forest resource, which in turn reduces its growth rate. Figure 2 depicts schematics of each case. We investigate those two distinct cases partly because it allows for comparison of our analysis with the existing studies (especially Polasky et al. 2011), but also because both of these two mechanisms are possible to exist in reality as there is still scarcity of knowledge about the ecological mechanisms of forest dieback. Note that in the first case (left panel in Fig. 2), the loss of resource is partial, and therefore the resource stock could recover. This is a more conservative formulation than Polasky et al.'s total collapse of resource and also is not a regime shift by the definition of Biggs et al. (2011), in the sense of not involving a shift to a different state of the system. As we see below,

[2] In some economies, the forestry sector provides a significant proportion of economic output. For example, 22 countries have over 3 % of GDP coming from the forestry sector in 2011 (FAO 2014).

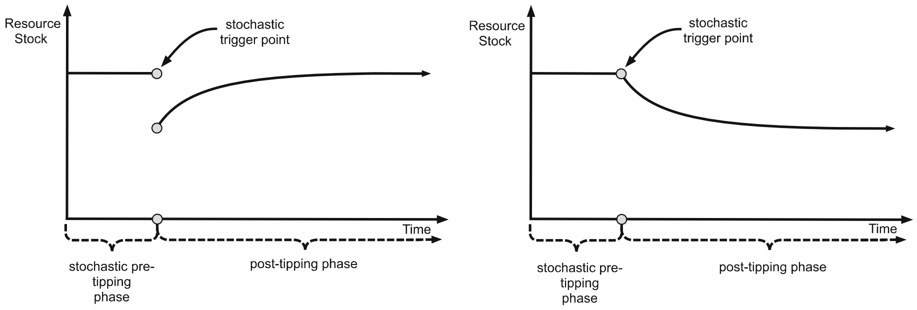

Fig. 2 Schematic of how a tipping point affects the level of the resource. Left tipping point reduces the size of the forest resource. Right tipping point leads to a permanent reduction in the carrying capacity of the forest resource, which reduces its growth rate

however, the modeling results for this case show consistent characteristics with Polasky et al's, implying that a risk of partial loss, not of total collapse, is sufficient for obtaining some effects on optimal resource management. Meanwhile, the second case (right panel in Fig. 2) does involve a permanent shift of the system and so is a regime shift according to Biggs et al. (2011).

Because of the imprecise knowledge about the nature of a forest tipping point we also include the possibility of an additional damage to economic output. This type of tipping risk is a representation of the general climate tipping risk as considered in Cai et al. (2015a). In this case, the tipping point affects directly both the economic output and the forest stock.

The social planner's objective is to maximize expected welfare which is the sum of discounted utility flows over an infinite horizon. We assume non-separable preferences are of Epstein–Zin–Weil type, and that the time horizon is infinite. The timing of the forest tipping point event is unknown to the social planner, leading to substantial risk regarding future welfare. The social planner facing this risk must determine in each time period the optimal choice of consumption, emission reductions, harvesting of the forest resource, and controlling the deforestation of the forest resource

We first present the structure of the general model and set up the social planner's optimization problem. Afterward, we describe how the structure of the model is altered when we study different sub-versions of the general model.

2.1 The General Model

We denote by Y_t total economic output in nominal units, and by $A_t = A_0(1 + g^A)^t$ the level of total factor productivity with an annual exogenous growth rate g^A. We therefore convert output into efficiency units, and define $y_t = \frac{Y_t}{A_t}$. We also convert other, infinitely growing variables into efficiency units. The conversion of the model is presented in Section 5 in the Appendix. Furthermore, output is a function of the capital stock (in efficiency units) $k_t = \frac{K_t}{A_t}$, and the input of the forest resource. We assume that the effective input of the forest resource is given by $Q_t = q_t(1 + g^q)^t$, where $q_t \geq 0$ is the controlled level of the forest resource harvested. Furthermore, we assume an harvesting technology which grows at a constant rate g^q. The harvesting technology advances at the same rate as total factor productivity, implying $g^q = g^A$. We can then write economic output in efficiency units as

$$y_t = \frac{Y_t}{A_t} = k_t^{\nu}(q_t/A_0)^{\mu}$$

where $\frac{q_t}{A_0} = \frac{Q_t}{A_t}$. The dynamics of the forest resource R_t are represented by

$$R_{t+1} = R_t + G\left(R_t, \omega^G_{t+1}\right) - q_t - \xi(1-\varphi_t) + \omega^D_{t+1} \tag{1}$$

where $G\left(R_t, \omega^G_{t+1}\right) = g_R R_t\left(1 - \frac{R_t}{\omega^G_{t+1} R_{max}}\right)$ is the natural growth rate of the forest mass. Here, g_R is a growth rate parameter. Equation (1) allows for two possible types of tipping point events to be studied. First, $0 < \omega^G_{t+1} \leq 1$ denotes a stochastic shock to the maximum sustainable yield of the forest mass R_{max} with $\omega^G_{t+1} = 1$ before a tipping event has occurred. Second, we include the possibility of a tipping point that leads to a drop in the level of the forest resource, denoted by $\omega^D_{t+1} \leq 0$ with $\omega^D_{t+1} = 0$ before the tipping event. Details of both tipping point cases are presented in Sect. 2.2. As such, Eq. (1) serves the purpose of a general description of the forest dynamics. In addition, we assume a constant level of deforestation ξ. The latter can be controlled by, for example, a REDD policy and we denote by φ_t the rate of the deforestation control with $0 \leq \varphi_t \leq 1$. Furthermore, we denote by $\Lambda_t = \zeta\varphi_t^2$ the share of output spent on REDD measures.

Equation (2) describes the dynamics of the atmospheric carbon stock. The first term of the right-hand denotes the amount of carbon absorbed by other carbon sinks and δ^S is the absorption rate.

$$S_{t+1} = \left(1-\delta^S\right) S_t + \omega^y_{t+1}\sigma_t A_0 y_t(1-m_t) - \underbrace{G\left(R_t, \omega^G_{t+1}\right) + q_t + \xi(1-\varphi_t) - \omega^D_{t+1}}_{R_t} \tag{2}$$

The second term represents the emission inflow from the output sector. Here, $0 \leq m_t \leq 1$ is the controlled fraction of emissions and $M_t = \theta_1 m_t^{\theta_2}$ denotes the share of output spent on mitigation. Similar to Nordhaus (2008) we assume a constant rate of decarbonization of output, g^I. The emissions-output ratio is given by $\sigma_t = \frac{\sigma_0}{(1+g^I)^t}$. By assuming $g^I = g^A$, the term $\sigma_0 A_0 y$ denotes nominal units.[3] Furthermore, we assume that a forest tipping point could induce a permanent reduction in economic productivity given by $\omega^y_{t+1} \leq 1$ with $\omega^y_{t+1} = 1$ before the tipping point has occurred. Finally, the remaining (underbraced) terms represent the inflow of carbon from the forest resource, either from harvesting and deforestation control or from the natural growth process of the forest resource.

To model the impact of climate change on the economy and potential forest dieback tipping points we implicitly assume that the global average temperature T_t is a linear function of cumulative carbon emissions—that is to say, $T_t = \tau(S_t - S_{PI})$, where S_{PI} denotes the preindustrial stock of carbon and τ is a parameter. With the former implicit assumption we follow the specification in Cai et al. (2016a) and denote the endogenous factor of damage to output by

$$D^K_t = 1 + \kappa_1(\tau(S_t - S_{PI}))^2 + \kappa_2(\tau(S_t - S_{PI}))^{6.754}.$$

With $\kappa_2 = 0$ in our benchmark case, the damage factor is that of the DICE model in Nordhaus (2008) but for a calibrated value of κ_2 we can also incorporate (in a sensitivity analysis) a much more convex temperature-to-damage relationship as has been recently suggested by Dietz et al. (2013).

[3] Note that emissions can be proportional to output even if Y is a function of the renewable resource that originates from the natural system and does not involve fossil fuel combustion itself. For example, the burning of wood does not produce net emissions itself (it is carbon-neutral) but the use of timber accompanies emissions as a form of either logging and transportation of products or of enhanced activities in housing construction, etc.

The accumulation dynamics of capital (in efficiency units) are then given by

$$k_{t+1} = \left[\left(1 - \delta^K\right) k_t + \omega^y_{t+1} y_t \frac{1 - M_t - \square_t}{D^K_t} - c_t\right] (1 + g^A)^{-1} \tag{3}$$

where δ^K is the rate of depreciation of capital and non-depreciated capital and output net of mitigation and afforestation costs can be invested in capital accumulation and consumption c_t (in efficiency units). See Appendix 5 for details of obtaining Eq. 3.

2.2 Specification of the Forest Tipping Point

We use the general model specified above to study different cases of impacts from a tipping point in the forest resource. Regarding the impacts on the forest resource itself we distinguish between the cases of a permanent reduction of the carrying capacity of the forests, which affects its growth rate (growth rate tipping point) and a single-event abrupt dieback of the forests (level tipping point). Furthermore, we also include the possibility that in addition to the impacts on the forest resource, the tipping point will also reduce economic output. We also study a deterministic version without any tipping risk as a benchmark case. In the following we specify the model equations for these different cases.

2.2.1 Deterministic (No Tipping) Benchmark Case

To set up the deterministic benchmark case, we impose: $\omega^G_{t+1} = 1$ and $\omega^D_{t+1} = 0\ \forall \theta^R_{t+1}$ where θ^R_t is a discrete variable indicating the state of the tipping point event. In the deterministic benchmark case, the only source of damage is D^K_t, the smooth damage factor to economic output. Therefore, we also set $\omega^y_{t+1} = 0\ \forall \theta^R_{t+1}$.

2.2.2 Growth Rate Tipping Point

The conditional probability of the tipping point event not occurring at time t is given by

$$p(T_t) = \exp(-p^\omega * \max(0, T_t - T_0)) \tag{4}$$

where p^ω is a hazard rate parameter and T_0 denotes the degree of global warming at the initial time. Thus, the tipping probability is endogenous, depending on the level of global warming. Given this dependence, the state of the tipping point event is described as

$$\theta^R_{t+1} = h(T_t, \theta^R_t). \tag{5}$$

Using Equ. (4) we specify the Markov probability matrix for θ^R_{t+1} as

$$P = \begin{bmatrix} p(T_t) & 1 - p(T_t) \\ 0 & 1 \end{bmatrix}$$

with $\theta^R_{t+1} = 1$ for the pre-tipping state and $\theta^R_{t+1} = 2$ for the post-tipping and irreversible state. The transition probability matrix is set up, so that the probability of remaining in the pre-tipping state is $p(T_t)$ and there is a probability of $1 - p(T_t)$ for the tipping event to occur (going from state 1 to state 2). Once the tipping event has occurred, the system remains in the post tipping state.

For the magnitude of the tipping point impact, we assume that, in case of tipping, the carrying capacity of forests drops by 25 %, corresponding to a change in ω^G_{t+1} from 1 to 0.75 as specified in Equ. (6)—note that the 25 % loss specification does not result from rigorous calibration with empirical data but is rather taken as a benchmark of a significant loss, as is used by Rammig et al. (2010). We show in Sect. 3.2 that changing the level of reduction does not change the qualitative features of our computed results.

After a tipping point event in the carrying capacity the growth rate of the forest mass $G\left(R_t, \omega^G_{t+1}\right)$ is substantially reduced for any positive level of the forest resource mass.

$$\omega^G_{t+1} = \begin{cases} 1 & \text{if } \theta^R_{t+1} = 1 \\ 0.75 & \text{if } \theta^R_{t+1} = 2 \end{cases} \tag{6}$$

Since in this case, the tipping point does not affect directly the level of the resource stock, we set $\omega^D_{t+1} = 0 \; \forall \theta^R_{t+1}$. In addition, we also set $\omega^y_{t+1} = 0 \; \forall \theta^R_{t+1}$ as we assume that the tipping point event has no direct impact on economic output.

2.2.3 Level Tipping Point

In this case, we assume that the tipping point event leads to an abrupt dieback of the forest resource. We specify the immediate dieback to be 30 % of the forests above a minimum level R_{min}. As in the case of a growth rate tipping point, this 30 % level should be seen as a benchmark of a significant loss. In fact, some modeling studies even forecast well over 50 % of forest loss toward the end of the 21st century under climate change (e.g., Cook and Vizy 2008; Rammig et al. 2010), though the reduction is generally expected to be gradual (Lenton et al. 2008). For this case, we also show in Sect. 3.2 that changing the level of reduction does not change the qualitative features of our computed results.

To model this case, there are three different states for ω^D_{t+1}

$$\omega^D_{t+1} = \begin{cases} 0 & \text{if } \theta^R_{t+1} = 1 \\ -0.3(R_t - R_{min}) & \text{if } \theta^R_{t+1} = 2 \\ 0 & \text{if } \theta^R_{t+1} = 3 \end{cases} \tag{7}$$

where $\theta^R_{t+1} = 1$ denotes the pre-tipping state, $\theta^R_{t+1} = 2$ denotes the state in the period of the tipping point event, and $\theta^R_{t+1} = 3$ is the post-tipping state needed to ensure that the tipping point event can only occur once—in this study, we restrict the number of tipping events to one for the reasons of the clarity of discussion and of conser.

Furthermore, in line with the specification for the growth rate tipping point, the Markov transition probabilities for θ^r_t are given by

$$P = \begin{bmatrix} p(T_t) & 1 - p(T_t) & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \end{bmatrix}$$

where $p(T_t)$ is given by Eq. (4). Again, the transition probability matrix is set up, so that the probability of remaining in the pre-tipping state is $p(T_t)$ and there is a probability of $1 - p(T_t)$ for the tipping event to occur. State 2 describes the tipping state with the drop in the forest mass. Once the system is in state 2, it will deterministically move to state 3 in the subsequent period to prevent that a tipping point may occur twice.

For this case we set $\omega^G_{t+1} = 1 \; \forall \theta^R_{t+1}$ and $\omega^y_{t+1} = 0 \; \forall \theta^R_{t+1}$ as we again assume that the tipping point event has no direct impact on economic output.

2.2.4 Additional Reduction of Output

For both tipping scenarios, the growth rate tipping point and the level tipping point, we also consider cases, in which the tipping point event will on average induce a 2.5 % permanent reduction of output, which is a representation of the general climate tipping risk as first studied in Cai et al. (2015a). We analyze the influence of different volatilities of the tipping shock to output. In particular, we consider a case, in which the tipping event will, with certainty, induce a 2.5 % drop to output ($\sigma^y = 0$). Furthermore, we study a case, in which the reduction of output will be 3.75 or 1.25 % (both with 50 % probability) and a case in which there the reduction of output will be 5 or 0 % (both with 50 % probability). The two latter cases are denoted by $\sigma^y = 0.0125$ and $\sigma^y = 0.025$ respectively. We choose those parameter levels only as benchmarks rather than rigorously calibrated values, and we examine a single, not multiple, tipping risk only for the clarify of discussion. For a technical formulation of these cases see Section 6 in the Appendix.

2.3 The Social Planner's Optimization Problem

We assume the social planner has Epstein–Zin–Weil preferences (Epstein and Zin 1989; Weil 1989) and write the value function as

$$V(S_t) = \max_{C_t,q_t,m_t,\varphi_t} \left\{ \frac{C_t^{1-\frac{1}{\psi}}}{1-\frac{1}{\psi}} + \beta \frac{1}{(1-\frac{1}{\psi})} \left[E[(1-\frac{1}{\psi})V(S_{t+1})]^{\frac{1-\gamma}{1-\frac{1}{\psi}}} \right]^{\frac{1-\frac{1}{\psi}}{1-\gamma}} \right\}$$

where $S_t = \left(k_t, S_t, R_t, \theta_t^R, \theta_t^y\right)$ is the vector of the five state variables of the model, $\beta = \frac{1}{1+\rho}$ is the utility discount factor and ρ being the pure rate of time preference. Furthermore, γ is the coefficient of relative risk aversion and ψ is the IES. By defining $V^*(S_t) = \frac{V(S_t)}{A_t^{1-\frac{1}{\psi}}}$ we can rewrite the value function as

$$V^*(S_t) = \max_{c_t,q_t,m_t,\varphi_t} \left\{ \frac{c_t^{1-\frac{1}{\psi}}}{1-\frac{1}{\psi}} + \beta \frac{(1+g^A)^{1-\frac{1}{\psi}}}{1-\frac{1}{\psi}} \left[E[(1-\frac{1}{\psi})V^*(S_{t+1})]^{\frac{1-\gamma}{1-\frac{1}{\psi}}} \right]^{\frac{1-\frac{1}{\psi}}{1-\gamma}} \right\}.$$

The social planner maximizes expected welfare. Solving the social planner's problem is equivalent to solving the Bellman equation of the problem together with the state dependent equality constraints. The advantage of the infinite horizon model is that the time index can be dropped and the decisions depend solely on the current state. The dynamic programming problem is given by

$$\begin{aligned} V^*(S) &= \max_{c,q,m,\varphi} \left\{ \frac{c^{1-\frac{1}{\psi}}}{1-\frac{1}{\psi}} + \beta \frac{(1+g^A)^{1-\frac{1}{\psi}}}{1-\frac{1}{\psi}} \left[E[(1-\frac{1}{\psi})V^*(S^+)]^{\frac{1-\gamma}{1-\frac{1}{\psi}}} \right]^{\frac{1-\frac{1}{\psi}}{1-\gamma}} \right\}. \\ \text{s.t.}\, k^+ &= \left(\left(1-\delta^K\right) k + \omega^{y+} y \frac{1-M-}{D^K} - c \right) (1+g^A)^{-1} \\ S^+ &= \left(1-\delta^S\right) S - \left(G\left(R, \omega^{G+}\right) - q \right) + \xi(1-\varphi) + \omega^{y+} \theta A_0 y(1-m) - \omega^{D+} \\ R^+ &= R + \left(G\left(R, \omega^{G+}\right) - q \right) - \xi(1-\varphi) + \omega^{D+} \\ \theta^{R+} &= h(T, \theta^R). \end{aligned} \tag{8}$$

We solve the model numerically by value function iteration using a collocation projection method. For a detailed exposition on value function iteration, see Judd (1992 and 1998). We approximate the value function by tensor product Chebychev polynomials of degree 6 together with 7 Chebychev nodes in each dimension and fit the unknown solution coefficients for the value function by least squares. We iterate over the value function until the maximum absolute approximation error is less than 10^{-8}. We have found that increasing the polynomial degree, the approximation range or the number of collocation nodes does not significantly change the results, suggesting accuracy of our computational method.

2.4 Calibration of the Model

Functional forms The functional forms for the production function (Y_t), deterministic damage (D_t^K), and mitigation costs (M_t) are similar to those of the DICE model (Nordhaus 2008) with the exception of the resource as an input in the production function. For simplicity, we assume a quadratic function for the cost of reducing deforestation ($\square_t$). The growth of forest mass (G_t) is a common specification for a renewable resource stock, except for the stochastic component (ω^G).

Output and cost parameters The share of input of forest products in the production function (μ) is set at 0.002, which comes from FAO's (2010) estimate that the annual value of forest product use is around USD 100 billion. With $\zeta = 0.0008572$, the cost function of deforestation control is set to yield USD 180 per tC at 0.5 GtC, which is relatively high but still at a realistic level based on the review of REDD modeling studies by Lubowski and Rose (2013). The capital elasticity in the production function (ν) is 0.3. The capital stock at the initial time is set to USD 160 trillion ($K_0 = 160$) and A_0 is set to 47.88 to yield a world GDP of USD 70 trillion. Capital depreciates at 10 % per year ($\delta^K = 0.1$). We furthermore assume an annual growth rate of total factor productivity of 1.5 % ($g^A = 0.015$), which is consistent with the DICE model (Nordhaus, 2013). Regarding the mitigation costs, we assume $\square_1 = 0.15$ and $\square_2 = 2.5$. This implies costs of 0.3 % of GDP for a 25 % reduction in emissions, 2.2 % of GDP for a 50 % emission reduction, and 15 % of GDP for a 100 % emission reduction.

Carbon stock and damage function parameters The emissions–output ratio ($\square_0 = 0.00139$) is set in line with the DICE model (Nordhaus 2013). The rate of decarbonization of output is set at 1.25 % per year ($g_I = g_A = 0.0125$). We specify the preindustrial stock of carbon to be 600 GtC ($S_{PI} = 6$) and in line with the DICE model (Nordhaus, 2013) we assume that today's atmospheric stock of carbon is 830 GtC ($S_0 = 8.3$).

With $\delta^S = 0.0004$ we model an annual net uptake of carbon by the oceans, and thus partly capture the dynamics of the carbon cycle (at $S_0 = 8.3$, our assumption implies that the annual net carbon uptake is about 0.33 GtC). With this specification we calibrate $\tau = 0.45$ such that our emission-to-temperature relationship is in line with that of the default specification of MAGICC 6.0 (Meinshausen et al. 2011) for all four representative concentration pathways.

An alternative specification of the emissions-to-temperature relationship would be to set $\delta^S = 0$, update S_0 and then to interpret S as the total cumulative stock of carbon emissions. The latter could be calibrated according to Matthews et al. (2009) who propose a linear relationship between cumulative carbon emissions and global warming. However, in this study we chose to explicitly model the atmospheric carbon stock as we specifically focus on the two-directional carbon flows between the resource stock (with carbon content) and the atmospheric carbon stock. Finally, the damage coefficient $\kappa_1 = 0.003$ is in accordance with the DICE model (Nordhaus 2008).

Resource stock parameters Using information from the Global Forest Resources Assessment 2010 (FAO 2010), we assume that the current size of global forests is 300 GtC ($R_0 = 3$), and that the current global harvest rate of forest mass and the global rate of uncontrolled deforestation are 1.5 and 0.5 GtC ($\xi = 0.005$), respectively. From the Millennium Ecosystem Assessment (2005) we infer that global forests have been reduced by about 50 % from their pristine levels. Therefore, we assume that the maximum possible size of forest mass is 600 GtC ($R_{max} = 6$), and the current annual growth of forests is 1.5 GtC (the amount of current human forest use excluding the factor of deforestation). These assumptions imply $g_R = 0.0135$.

Preference parameters The pure rate of time preference is set at 1 % per year ($\rho = 0.01$). Furthermore, we choose the risk aversion parameter ($\gamma = 10$) and the IES ($\psi = 1.5$) to be in line with the parameter estimates obtained from the long-run risk literature that is consistent with many empirical findings of investor behavior in financial markets (see, e.g., Bansal and Yaron 2004; Bansal et al. 2012). We use these parameters as our default setting of preferences. For the influence of different parameters of risk aversion and the intertemporal elasticity of substitution on optimal climate policies, see, for example, Ha-Duong and Treich (2004) and Cai et al. (2015a).

Tipping point event parameters We set the hazard rate parameter $p^{\omega} = 0.007$, which implies that for an additional warming of 1 Celsius, the conditional probability of tipping increases by 0.7 %. This parametrization will produce cumulative tipping probabilities for the forest resource that are in line with recent probability assessment studies, such as those in Kriegler et al. (2009). We also use the Kriegler et al. (2009) definition of the magnitude of impacts from the tipping point events as an orientation for our study. In our growth rate tipping point case, we assume that the tipping point event reduces the carrying capacity of the forest mass by 25 % (from $R_{max} = 6$ to $R_{max} = 4.5$). For the single-event level tipping point we assume that the stock of the forest mass in excess of 100 GtC ($R_{min} = 1$) is instantly reduced by 30 %. For the cases in which the tipping point event also affects economic output, we assume an expected permanent output damage of 2.5 % per year and study model variants for which $\sigma^y \in \{0, 0.0125, 0.025\}$.

3 Results and Discussion

Given the optimal solution to the social planner's problem of Equ. (8), we conduct a Monte Carlo analysis with 1000 simulated time paths of the model, ranging from 2014 to 2200. In Sects. 3.1–3.3 we consider tipping point events that only affect the dynamics of the forest resource. In Sect. 3.4 we study cases in which the tipping point event also affects economic output. In Sect. 3.5 we present the results of a sensitivity analysis on preferences and impact on output.

3.1 Optimal Harvesting and Climate Policies Under Forest Tipping Risks

We first analyze versions of the model without deforestation control, implying $\varphi_t = 0\ \forall t$. We assume that the tipping point event only affects the dynamics of the forest resource and has no direct impact on economic output. Figure 3 shows the time paths for the atmospheric carbon stock (S_t), the forest stock (R_t), the emission reduction rate (m_t), and the forest harvesting rate (q_t).

For comparison, the left panel of Fig. 3 shows the time paths of these variables obtained from our deterministic benchmark case without any tipping risk. The center panel in Fig. 3

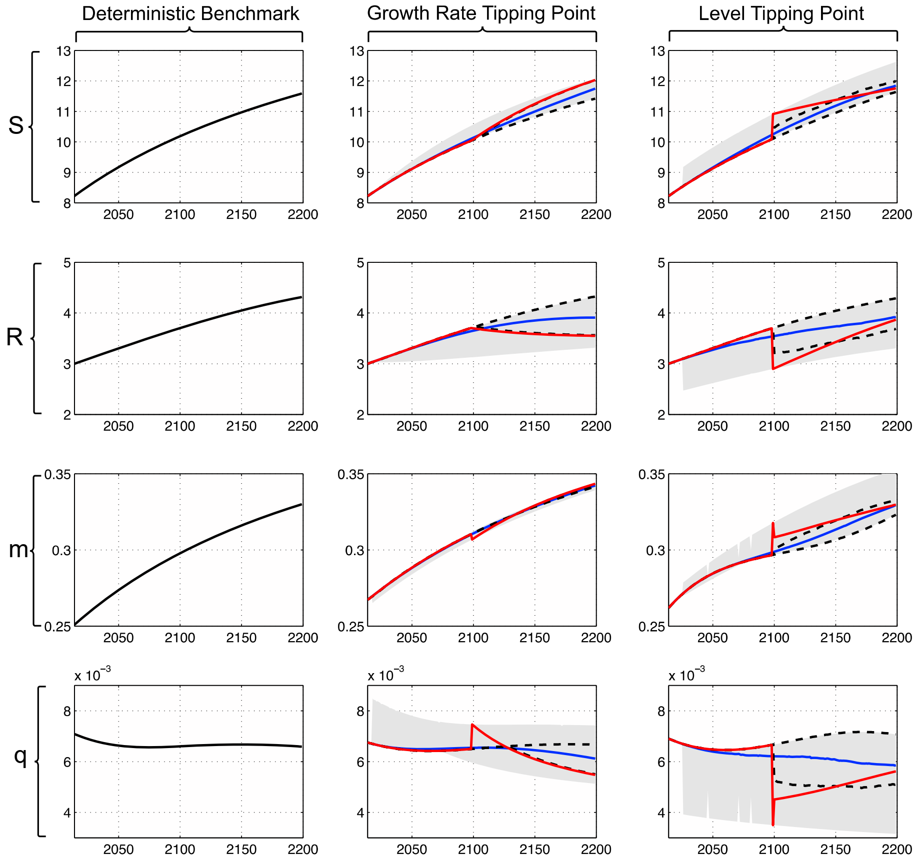

Fig. 3 Time paths of the carbon stock (S_t), the forest stock (R_t), the emission reduction rate (m_t), and the forest harvesting rate (q_t) over the years 2014–2200. These cases are without deforestation control ($\varphi_t = 0\ \forall t$) and the tipping point event does not directly affect output, ($\omega^y = 0$). We show the deterministic paths (solid black lines), the expected paths (blue lines), 25 and 75 % quantiles (dashed black lines), a sample path producing a tipping point in the the year 2100 (red lines), and the maximum ranges (grey areas) obtained from 1000 Monte Carlo simulations. The variables S, R and q are in units of 100 GtC. (Color figure online)

shows the results of the case of a tipping point event that affects the growth rate of the resource by permanently reducing its carrying capacity by 25 % (growth rate tipping point). The right panel shows the results of the case of a tipping point event that leads to a single-event, abrupt 30 % dieback in the forest stock above $R_{min} = 1$ (Level Tipping).

We find that in the deterministic benchmark case without any tipping risk, conducting optimal climate policies would lead to an increase in the atmospheric carbon stock from 822 GtC today to 1018 GtC in 2100 and 1160 GtC in 2200. That implies global temperature levels of 1 today, 1.88 in 2100, and 2.52 in 2200.[4] Accounting for the risk of tipping events in the resource changes the optimal paths of the model variables fundamentally and those

[4] For comparison, we have also computed a business-as-usual scenario (not reported in the graphs) where q_t remains at its level of today ($q_t = 0.0152$) and there is no mitigation control ($m_t = 0$). In such a case temperature would rise much higher with 2.94 in 2100 and 4.46 in 2200. Thus, we see that optimally controlling emissions (on average about 30 % over the next two centuries) and optimally reducing the harvesting of the forest resource (an average reduction of 50 %) will have a strong effect on reducing global warming.

changes differ with respect to the type of tipping point event under consideration. We consider each case separately.

Level tipping point In the case of a level tipping point, the forest stock R_t is reduced when the tipping point occurs. Because of the interactions between the resource stock and the atmospheric carbon stock, we observe that the latter, (S_t), also increases immediately after the level tipping point event occurs. The optimal forest harvesting rate q_t exhibits a sharp drop after the tipping event as a consequence of the lower resource stock. Two effects could explain this optimal response. First, the incentive to build up the resource stock and compensate for the negative shock. Second, because a higher atmospheric stock of carbon leads to higher damages to output, it appears optimal to engage in reducing the accumulation of atmospheric carbon. A higher net growth rate of the resource is one way to achieve this. At the same time, increasing emission control in response to the tipping point event will also dampen the accumulation of carbon in the atmosphere. Consequently, the slope of the carbon stock in the atmosphere is significantly lower post-tipping, as can be seen in Fig. 3.

The sharp drop in q_t in the tipping period is partly offset in the subsequent period, which can be explained by considering the timing of the system dynamics. A sudden dieback of the forests will increase the growth rate of forests in the period after the tipping point occurs but not in the period in which the tipping point event occurs (this is the case because the growth rate is monotonically decreasing for $R \geq 3$). Therefore, a much larger reduction in the harvesting of the resource is required in the tipping event period to compensate for this delay. As we show in Fig. 4, it appears that, from a decision making point of view, the essential variable is the net growth rate of the resource $G\left(R_t, \omega^G_{t+1}\right) - q_t - \xi$ and as that figure shows, the pattern of the net growth rate is smooth post-tipping.

Growth rate tipping point In the case of a growth rate tipping point, the growth rate of the forest stock is reduced and the expected buildup of R_t is slowed down, while the actual sample path shows a sharp decrease in the level of R_t. This is because the growth rate is substantially lower for any level of R_t and the new steady-state level of R_t will be lower as well. In contrast to the level tipping case, q_t temporarily increases after the tipping point event. This is because the system adjusts to a lower carrying capacity and exploits an "excess" amount of the forest stock. Such an increase is temporary, however, and q_t eventually converges to a lower steady state compared to the deterministic case. Figure 4 clarifies this somewhat paradoxical pattern. It shows the net growth rate of the resource stock $G\left(R_t, \omega^G_{t+1}\right) - q_t - \xi$ that, in fact, exhibits a sharp decrease in the event of a growth rate tipping point but slightly recovers thereafter. Because of the interactions between the resource stock and the atmospheric carbon stock, we observe that the latter, (S_t), gradually rises after the tipping point event occurs.

Also, contrary to the level tipping case, we observe that emission control is reduced, albeit only slightly. There are two major effects driving this result. First, in anticipation of the tipping point event emission control is higher than in the deterministic benchmark case. This is because the likelihood of tipping is endogenous, rising with higher carbon content in the atmosphere. The optimal policy is to delay the expected tipping time. Once the tipping point event occurs, this risk-reduction effect will vanish as we assume that the tipping point can only occur once. Second, after the tipping point event occurs there will be a desire to increase emission control to optimally respond to the more rapidly accumulating stock of carbon. As our sample path shows, it appears that the former effect is stronger than the latter.

We find further evidence for this pattern by running a comparable model version with exogenous tipping point probabilities, which mean that the tipping probabilities are independent of the temperature change or climate change, unlike in the previous cases, with which the occurrence of a tipping event is not influenced by the reduction efforts of carbon dioxide emissions. Such an analysis corresponds to the analysis of exogenous risks conducted by the

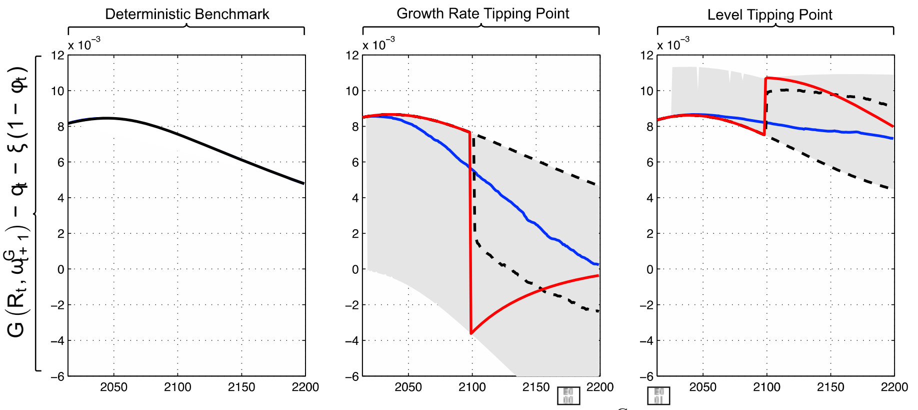

Fig. 4 Time paths of the net growth rate of the resource stock $G\left(R_t, \omega^G_{t+1}\right) - q_t - \xi\,(1 - \varphi_t)$ (in units of 100 GtC) over the years 2014–2200. This model case does not allow for deforestation control ($\varphi_t = 0\ \forall t$) and the tipping point event does not directly affect output ($\omega^y = 0$). We show the deterministic paths (solid black line), the expected paths (blue lines), 25 and 75 % quantiles (dashed black lines), a sample path producing a tipping point in the the year 2100 (red lines), and the maximum ranges (grey areas) obtained from 1000 Monte Carlo simulations. (Color figure online)

previous studies on regime shifts such as Polasky et al. (2011). In the context of our analysis, examining the case with the exogenous tipping probabilities is useful in that it allows us to see the pure effects of risk-averse resource emission reduction decisions, separated from the effects of random future reduction in the forest resource. Figure 12 in Section 7 in the Appendix shows our results. We find that, in contrast to the case with endogenous tipping risk, optimal emission reduction will increase after the growth rate tipping point occurs, since it is no longer possible to delay the expected timing of the tipping point event and there is no risk-reduction effect. Furthermore, assuming an exogenous tipping probability results in a higher optimal initial-year harvesting compared to the deterministic benchmark, in particular in the case of a growth rate tipping point. Hence, a tipping risk does not necessarily justify precautionary actions. This finding is consistent with Polasky et al. (2011) and other existing studies.

These results show, that it is crucial to distinguish between endogenous and exogenous tipping risks as well as tipping risks that change the system dynamics or the level of the forest stock as they can have quantitatively and qualitatively different impacts on optimal climate and forest harvesting policies. We next introduce the possibility of deforestation control and study how it affects optimal policy.

3.2 Sensitivity Analysis on the Benchmark Case

In this section we study whether the effects described above hold qualitatively for different parameter settings. In particular, we consider changes in the damage function D^K_t, the growth rate of total factor productivity g^A and the growth rate of the resource g^R.

Figure 5 shows the robustness of our findings for a much steeper damage factor D^K_t. We consider the original specification as in the benchmark model with $\kappa_2 = 0$ (solid lines) and the damage factor specification of Dietz et al. (2013) with $\kappa_2 = 0.000004$ where the overall damage of an increase in the carbon stock on output is much larger. We find that in the deterministic case investments into emission control m_t are increased and resource extraction q_t is reduced compared to the benchmark specification. These optimal policies are

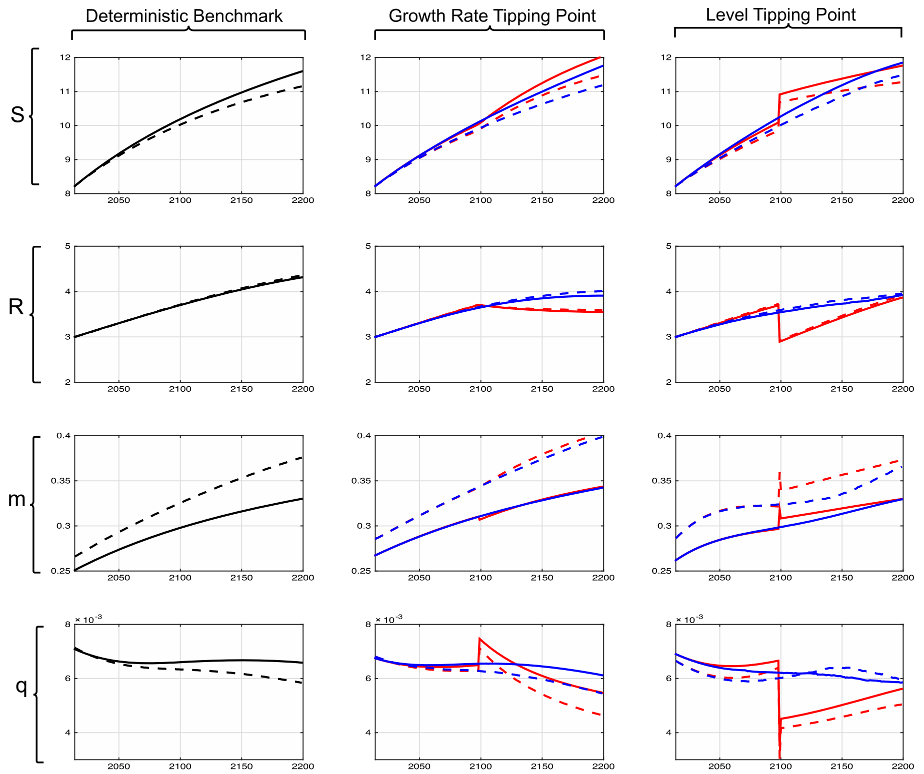

Fig. 5 Time paths of the carbon stock (S_t), the forest stock (R_t), the emission reduction rate (m_t), and the forest harvesting rate (q_t) over the years 2014–2200. These cases are without deforestation control ($\varphi_t = 0\ \forall t$) and the tipping point event does not directly affect output, ($\omega^y = 0$). Solid lines show the benchmark model specification as shown in Fig. 3 with $\kappa_2 = 0$ while dashed lines depict the case with $\kappa_2 = 0.000004$. We show the deterministic paths (black lines), the expected paths (blue lines) and sample paths producing a tipping point in the the year 2100 (red lines). The variables S, R and q are in units of 100 GtC. (Color figure online)

conducted in order to reduce the additional damage to output by lowering the overall level of the carbon stock. This effect also occurs in the two cases with the tipping risks. However, the optimal policy responses to the tipping risks don't change qualitatively and the effects described above also hold for the new damage function specification.

In Fig. 6 we show the influence of different values for the average growth rate of total factor productivity g^A. The deterministic case shows, that a lower growth rate (dashed line) leads to less investments in emission control and less extraction of the resource. The opposite is true for a larger growth rate (dotted line). One possible explanation for this is that a lower growth rate implies less output growth in the economy and hence there is less capital left for investments in carbon emissions. To account for the resulting increase in the carbon stock, it is optimal to extract less of the resource to reduce emissions. Again, this finding is consistent with the two tipping scenarios and the implications obtained from the previous section about the impacts of the tipping risks don't change qualitatively.

For a third robustness check we analyze different variations for the resource dynamics. In particular we analyze if slow resource growth effects qualitatively change our results. Therefore, we decrease the growth rate of the resource from $g_R = 0.0135$ as in the benchmark case

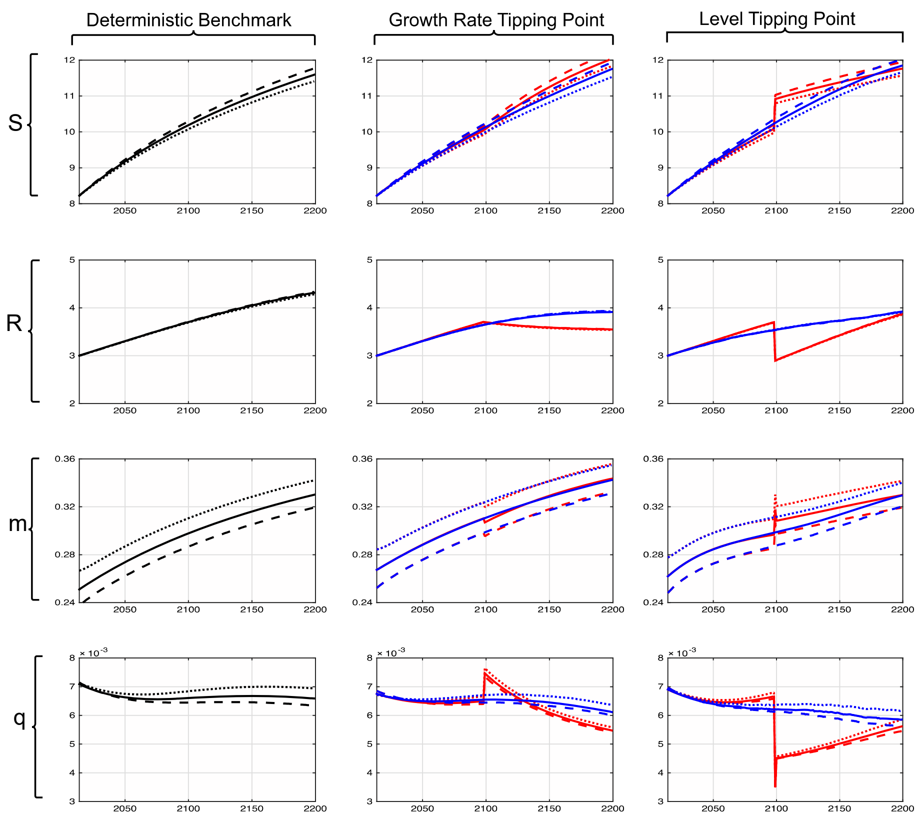

Fig. 6 Time paths of the carbon stock (S_t), the forest stock (R_t), the emission reduction rate (m_t), and the forest harvesting rate (q_t) over the years 2014–2200. These cases are without deforestation control ($\varphi_t = 0\ \forall t$) and the tipping point event does not directly affect output, ($\omega^y = 0$). Solid lines show the benchmark model specification as shown in Fig. 3 with $g^A = 0.0125$, dashed lines depict the case with $g^A = 0.01$ and dotted lines the case with $g^A = 0.015$. We show the deterministic paths (black lines), the expected paths (blue lines) and sample paths producing a tipping point in the the year 2100 (red lines). The variables S, R and q are in units of 100 GtC. (Color figure online)

to $g_R = 0.01$ and $g_R = 0.0065$. Figure 7 shows the results. A lower growth rate leads to a smaller optimal extraction rate q_t and hence lowers the stock of the resource. To understand the dynamics of the carbon stock and the optimal emission control we need to analyze the induced changes on the net growth rate of the resource $G(R_t, \omega^G_{t+1}) - q_t - \xi$ which enters the carbon stock as emissions (see Fig. 8). We find that the net growth rate decreases with g_R (the reduction in $G(R_t, \omega^G_{t+1})$ is larger than the induced reduction in optimal q_t) which implies more carbon inflows to S_t ($G(R_t, \omega^G_{t+1}) - q_t - \xi$ negatively enters the carbon stock). Hence the optimal response implies an increase in the emission control m_t. Again, we find these patterns for the benchmark as well as the two tipping scenarios and the qualitative effects of the tipping point events do not change. Although, no details are reported in this study, we have found that the results presented in this section are qualitatively robust to changes in the magnitude of both tipping point cases so we can conclude that the qualitative effects induced by the tipping risks are fairly robust with regard to changes in the model parameters.

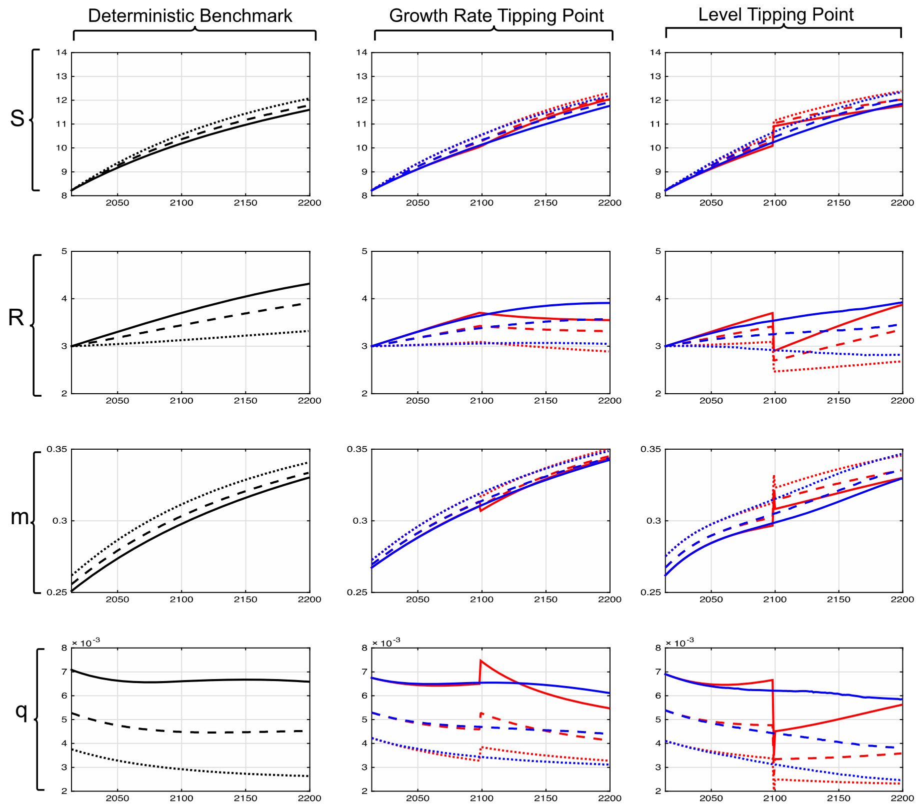

Fig. 7 Time paths of the carbon stock (S_t), the forest stock (R_t), the emission reduction rate (m_t), and the forest harvesting rate (q_t) over the years 2014–2200. These cases are without deforestation control $(\varphi_t = 0\ \forall t)$ and the tipping point event does not directly affect output, $(\omega^y = 0)$. Solid lines show the benchmark model specification as shown in Fig. 3 with $g_R = 0.0135$, dashed lines depict the case with $g_R = 0.01$ and dotted lines the case with $g_R = 0.0065$. We show the deterministic paths (black lines), the expected paths (blue lines) and sample paths producing a tipping point in the the year 2100 (red lines). The variables S, R and q are in units of 100 GtC. (Color figure online)

3.3 Optimal Deforestation Control under Forest Tipping Risks

Until now, we have restricted our model by excluding the option of deforestation control $(\varphi_t = 0\ \forall t)$. For the remainder of this study, we allow for optimal deforestation control. Figure 9 shows the climate system dynamics as well as the optimal deforestation control rate (φ_t) for the deterministic case, the growth rate tipping case, and the level tipping case.

We find that resource harvesting in the initial period increases to $q_{2014} \approx 0.008$ (0.8 GtC/yr) in the case with the possibility of deforestation control, from about $q_{2014} \approx 0.007$ (0.7 GtC/yr) in the case without deforestation control. Hence, the ability to control some fraction of the natural deforestation rate allows for higher harvesting quantities. This result holds for both the deterministic case and the cases with tipping risks.

Inclusion of deforestation control does not fundamentally change the time profiles of the other variables. In comparison to the deterministic benchmark case we find that tipping risks do not necessarily result in distinctively stronger initial-year effects on φ_t and q_t. This is

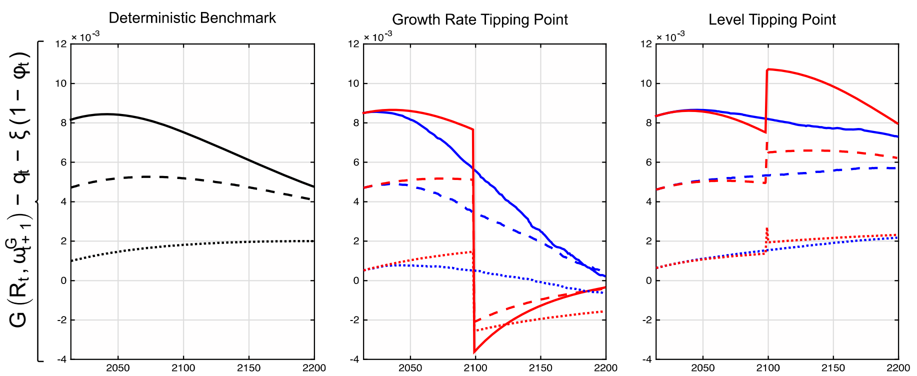

Fig. 8 Time paths of the net growth rate of the resource stock $G(R_t, \omega^G_{t+1}) - q_t - \xi(1 - \varphi_t)$ (in units of 100 GtC) over the years 2014–2200. This model case does not allow for deforestation control ($\varphi_t = 0\ \forall t$) and the tipping point event does not directly affect output ($\omega^y = 0$). Solid lines show the benchmark model specification as shown in Fig. 4 with $g_R = 0.0135$, dashed lines depict the case with $g_R = 0.01$ and dotted lines the case with $g_R = 0.0065$. We show the deterministic paths (black lines), the expected paths (blue lines) and sample paths producing a tipping point in the the year 2100 (red lines). (Color figure online)

in contrast with the pattern of the emission reduction rate, m_t, as discussed in the previous subsections, which robustly induces more stringent policy.

Post-tipping patterns of the deforestation control rate φ_t, a decline for the growth rate tipping case, and an increase for the level tipping case are qualitatively different from the patterns of the forest harvesting rate q_t. Since, now, the net growth of the resource is given by $G\left(R_t, \omega^G_{t+1}\right) - q_t - \xi(1 - \varphi_t)$ deforestation control serves as an additional policy for controlling the exchange of carbon between the atmosphere and the forest mass. Consequently, as Fig. 9 shows, the responses of the harvesting rate q_t are slightly dampened.

3.4 Additional Tipping Impacts on Economic Output

In this section we analyze the model's implications for cases in which the tipping point event causes a permanent reduction in economic output in addition to its impacts on the resource stock. Regarding the latter, we again distinguish between the cases of a growth rate tipping point and a level tipping point. As for the impact on output, we assume that the tipping point event will also induce a 2.5 % permanent reduction in output ($\sigma^y = 0$). Figure 10 shows the results for the core variables.

We find that in both cases (growth rate tipping and level tipping) mitigation control m_t in the initial period increases strongly (from about 0.26 to about 0.38) when also including the tipping risk to output. Also resource harvesting, q_t, drops sharply (from 0.84 to 0.35 GtC/yr) and optimal deforestation, φ_t, increases (from 70 to 100 %).

In contrast to the previous cases studied, we see that the pre- and post-tipping patterns of each variable are qualitatively the same, irrespective of the tipping point event under consideration. Clearly the prospect of a permanent reduction in output of 2.5 % induces a stringent policy with the purpose of delaying the expected occurrence of the tipping point event and thus reducing risks regarding future output. This risk-reduction effect significantly intensifies the post-tipping responses of m_t, q_t, and φ_t in the case of the growth rate tipping point. In the case of a level tipping event, the risk-reduction effect even changes the direction of post-tipping responses in those control variables. This risk-reduction effect has been observed in Cai et al. (2013) and is also the prime driver of our results in these cases.

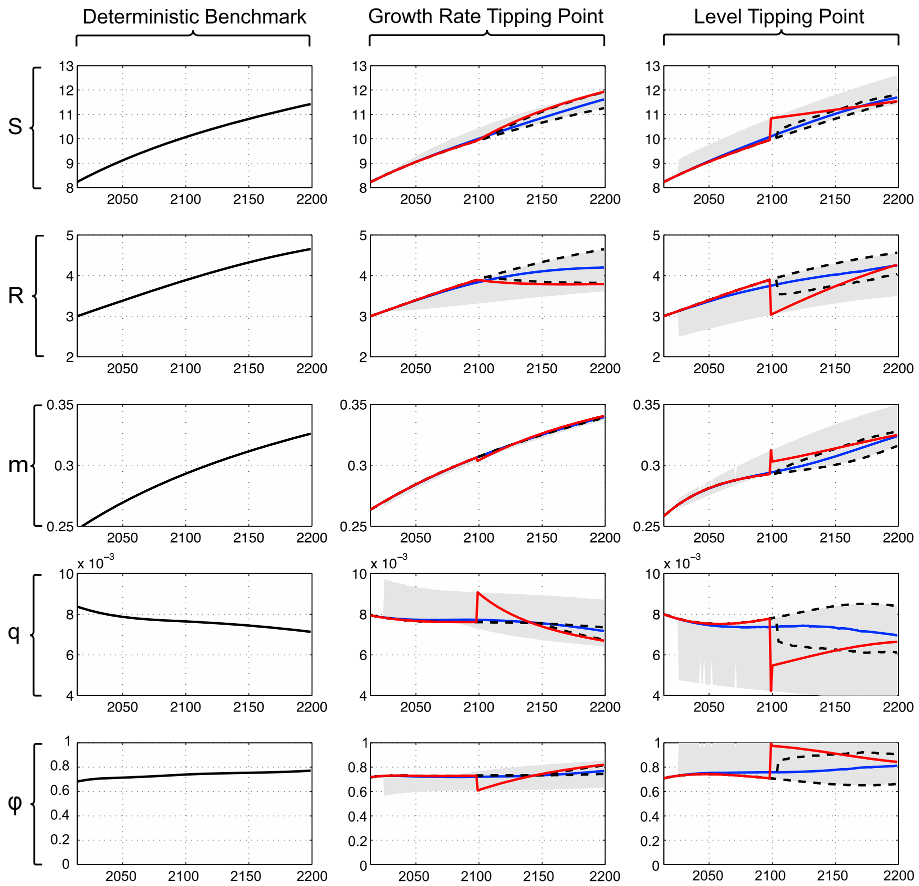

Fig. 9 Time paths of the carbon stock (S_t), the forest stock (R_t), the emission reduction rate (m_t), the forest harvesting rate (q_t) and the deforestation control rate (φ_t) over the years 2014–2200. In these cases the tipping point event does not directly affect output ($\omega^y = 0$). We show the deterministic paths (solid black lines), the expected paths (blue lines), 25 and 75 % quantiles (dashed black lines), a sample path producing a tipping point in the the year 2100 (red lines), and the maximum ranges (grey areas) obtained from 1000 Monte Carlo simulations. The variables S, R and q are in units of 100 GtC. (Color figure online)

Studying the statistical distribution of the optimal paths, we observe that the 25 % quantile is roughly at the year 2150, implying that there is a 75 % chance that the tipping point event will occur after the year 2150 (see Figure 14 and in Appendix 7.3 for a plot of the likelihood of tipping). In the cases without the reduction in output the 25 % quantiles were at around the year 2100 (see Figure 11 in Section 7.1 in the Appendix) and our exogenous probability case produced a 25 % chance of tipping by the year 2030 (see Figure 13 in Section 7.2 in the Appendix). Thus, in the case of an endogenous probability and the possibility of output reduction, optimal policy is quite effective in delaying the expected occurrence of the tipping point event.

Our tipping likelihood is calibrated quite conservatively, but in conclusion—despite this—we observe severe efforts to reduce emissions as suggested by higher emission control, lower harvesting levels, and much higher reduction in deforestation.

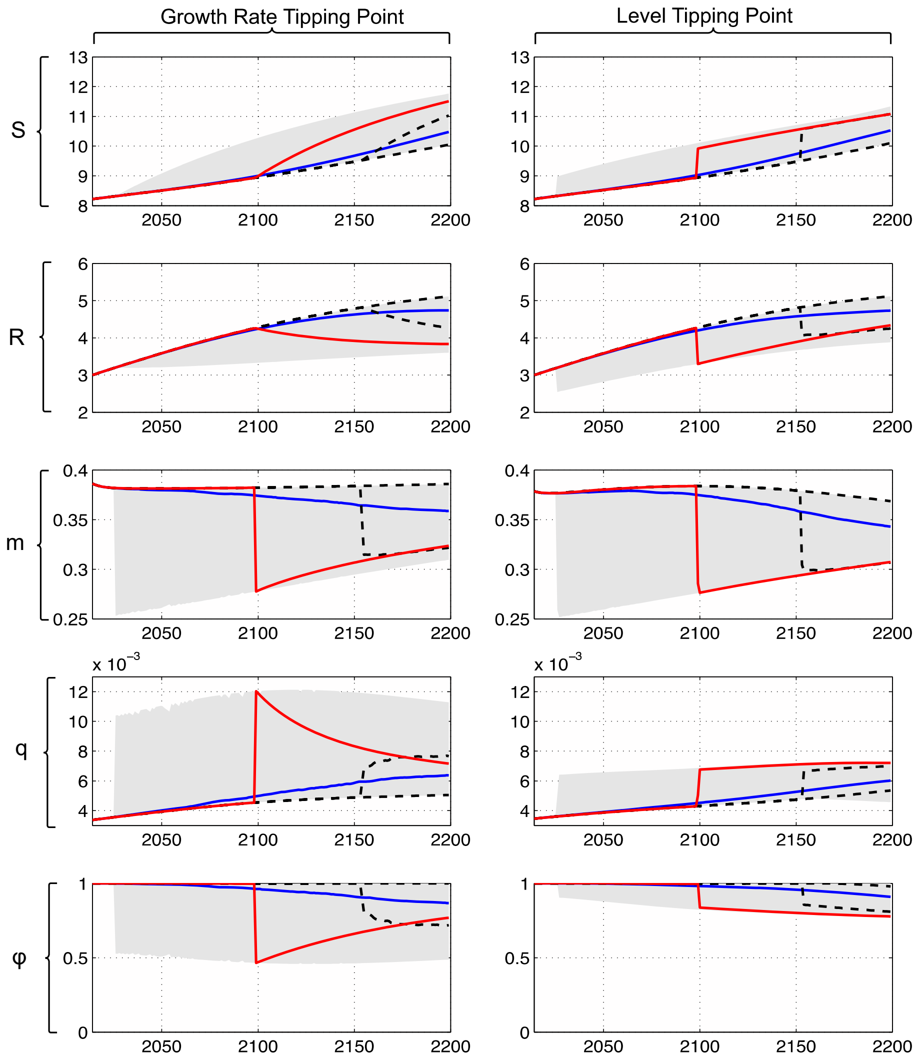

Fig. 10 Time paths of the carbon stock (S_t), the forest stock (R_t), the emission reduction rate (m_t), the forest harvesting rate, (q_t) and the deforestation control rate (φ_t) over the years 2014–2200. The tipping event has a direct effect on output as specified in Section 6.1 in the Appendix and there is no uncertainty about the magnitude of the output shock ($\sigma^y = 0$). We show the expected paths (solid black lines), 25 and 75 % quantiles (dashed black lines), a sample path producing a tipping point in the the year 2100 (red lines), and the maximum ranges (grey areas) obtained from 1000 Monte Carlo simulations. The variables S, R and q are in units of 100 GtC. (Color figure online)

3.5 Sensitivity Analysis on Preferences and Impacts on Output

In this section we perform a sensitivity analysis on our results with respect to different preferences and uncertainty about the magnitude of the post-tipping reduction in economic output. For all previous cases we have assumed non-separable preferences with an elasticity of inter-temporal substitution $\psi = 1.5$ and a risk aversion parameter $\gamma = 10$. With only

Table 1 Expected values of emission control rate, m_t, and temperature, T_t in the years 2014, 2100 and 2200

	m_t				T_t			
	Growth rate TP		Level TP		Growth rate TP		Level TP	
	EZ	CRRA	EZ	CRRA	EZ	CRRA	EZ	CRRA
$\sigma^y = 0$								
2014	0.3865	0.3803	0.3788	0.3740	1	1	1	1
2100	0.3742	0.3732	0.3748	0.3719	1.35	1.36	1.36	1.38
2200	0.3589	0.3624	0.3430	0.3448	2.01	2.00	2.04	2.03
$\sigma^y = 0.0125$								
2014	0.3925	0.3806	0.3844	0.3743	1	1	1	1
2100	0.3778	0.3726	0.3801	0.3724	1.33	1.36	1.34	1.38
2200	0.3623	0.3593	0.3496	0.3451	1.97	2.02	1.98	2.03
$\sigma^y = 0.025$								
2014	0.4112	0.3815	0.4014	0.3751	1	1	1	1
2100	0.3926	0.3733	0.3942	0.3745	1.27	1.36	1.28	1.37
2200	0.3731	0.3596	0.3603	0.3454	1.87	2.02	1.89	2.03
No output shock								
2014	0.2635	0.2634	0.2582	0.2582	1	1	1	1
2100	0.3065	0.3066	0.2940	0.2937	1.79	1.79	1.84	1.84
2200	0.3395	0.3394	0.3237	0.3235	2.53	2.51	2.56	2.56
Det. case								
2014	0.2473				1			
2100	0.2926				1.82			
2200	0.3258				2.44			

few exceptions non-separable preferences are not commonly used in stochastic integrated assessment. However, as for example, Cai et al. (2015a) have recently shown, they have a strong effect on optimal climate policy, when compared to standard CRRA preferences for which the elasticity of inter-temporal substitution and the risk aversion parameter are entangled by an inverse relationship. For comparison, we study the CRRA case of $\psi = \frac{1}{\gamma} = 1.5$.

Regarding the uncertainty about the magnitude of the post-tipping reduction in economic output we study the following cases: A case for which the tipping in the resource induces a permanent reduction of output of 2.5 % ($\sigma^y = 0$); a case where once the tipping occurs there is a 50 % probability of a 1.25 % reduction of output and a 50 % probability of a 3.75 % reduction of output ($\sigma^y = 0.0125$); and a case with a 50 % probability that there is no reduction of, and a 50 % probability that there is a 5 % reduction, of output once the tipping occurs ($\sigma^y = 0.025$). Note that in all cases the average reduction of output is 2.5 %. We also report two cases as a reference. First, a case in which the tipping point event has no additional impact on economic output (see Sect. 3.3). Second, our deterministic benchmark case. For all analyses we report the expected values of m_t, q_t, φ_t, and T_t for the years 2014, 2100, and 2200.

Table 1 reports the results of the sensitivity analysis for the emission control rate and the implied degree of global warming. Accounting only for the tipping point (TP) event

Table 2 Expected values of the forest harvesting rate, q_t (in 100 GtC), and the deforestation control rate, φ_t in the years 2014, 2100 and 2200

	q_t				φ_t			
	Growth rate TP		Level TP		Growth rate TP		Level TP	
	EZ	CRRA	EZ	CRRA	EZ	CRRA	EZ	CRRA
$\sigma^y = 0$								
2014	0.0034	0.0035	0.0035	0.0036	1	0.9995	0.9991	1
2100	0.0050	0.0050	0.0045	0.0046	0.9625	0.9639	0.9825	0.9783
2200	0.0064	0.0062	0.0060	0.0060	0.8681	0.8828	0.9094	0.9181
$\sigma^y = 0.0125$								
2014	0.0033	0.0035	0.0034	0.0036	1	0.9995	0.9994	1
2100	0.0049	0.0051	0.0044	0.0046	0.9633	0.9606	0.9854	0.9787
2200	0.0063	0.0064	0.0058	0.0060	0.8755	0.8694	0.9212	0.9183
$\sigma^y = 0.025$								
2014	0.0030	0.0035	0.0031	0.0036	1	0.9996	1	1
2100	0.0045	0.0050	0.0041	0.0046	0.9750	0.9611	0.9882	0.9809
2200	0.0058	0.0063	0.0055	0.0059	0.8978	0.8696	0.9288	0.9196
No output shock								
2014	0.0079	0.0079	0.0080	0.0080	0.7172	0.7175	0.7078	0.7082
2100	0.0077	0.0077	0.0074	0.0074	0.7199	0.7223	0.7577	0.7511
2200	0.0072	0.0072	0.0070	0.0070	0.7684	0.7649	0.8112	0.8052
Det. case								
2014	0.0084				0.6811			
2100	0.0076				0.7384			
2200	0.0071				0.7714			

in the resource (no impacts on output) will slightly increase global warming over the next two centuries. This is because both types of the resource tipping points enhance the carbon flux from the the forest mass to the atmosphere. Emission control also increases slightly in the initial period and, as discussed earlier, is more pronounced for the growth rate tipping point.

In the previous section we have discussed that when, in addition, a reduction of output is included, the effects on emission control and global warming will be severe. We called the motive to delay the expected timing of the tipping point event as risk-reduction effect. Thus, Table 1 provides further indication for the strong risk reduction effect induced by the tipping risk to output compared to the model with only tipping in the resource dynamics. We also see that risk aversion plays a crucial role for optimal policies. We find that, in the case of CRRA preferences the volatility of the shock to output has only a marginal influence (e.g. the results for $\sigma^y = 0$ and $\sigma^y = 0.025$ do not differ substantially). That result does hold for both, the growth rate tipping point and the level tipping point. This changes with the assumption of Epstein–Zin–Weil preferences with higher risk aversion. Here we find that the volatility in the tipping intensifies the risk reduction effect and induces higher emission control rates. These effects also have significantly influence on temperature levels. For example, in the case of CRRA preferences optimal climate policies would lead to a temperature level of about 2.02°C in the year 2200 while with the Epstein–Zin–Weil specification and increased risk aversion temperature would only increase to 1.79° C. Hence, the findings show that focusing on special preferences, such as CRRA, can give misleading policy implications as there is in general a large uncertainty about the magnitude of tipping events.

We observe similar effects on the harvest rate and the deforestation control rate, as shown in Table 2. The much lower (higher) levels of harvesting (deforestation control) are motivated by the risk-reduction effect. In all cases in which the tipping point event also impacts output, deforestation control is about 100 % (compared to about 70 % without the reduction in output) and the harvesting rate is reduced by more than 50 %.

4 Conclusion

We have analyzed optimal resource management and climate policy under tipping risk, set in the context of a possible forest dieback. We have studied several plausible cases of post-tipping impacts, such as changes the system dynamics, reduction of the forest stock, and an additional reduction of economic output. Our results show either precautionary or aggressive pre-tipping harvest patterns depending on the nature of the tipping risk. We also find qualitative differences in patterns of the post-tipping optimal forest harvest, control of deforestation, and carbon dioxide emission.

When the tipping risk concerns a single-event drop in the amount of forest mass, the time trend of the emission reduction rate exhibits a pattern of a post-tipping jump, which is not reported in the previous research, with the exception of Cai et al. (2016b). This is a reflection of a weak or absent risk-reduction effect. Correspondingly, the optimal control rate of deforestation exhibits a post-tipping jump after a tipping event on the amount of forest mass.

In contrast, when the tipping event changes the system dynamics of the forest stock, optimal policy responses are fundamentally different. In the case of endogenous tipping probabilities (depending on the level of climate change), the risk-reduction effect is much stronger, leading to more stringent climate policies. If, on the other hand, the tipping probabilities are exogenous, the risk-reduction effect is absent and a risk of tipping events does not necessarily justify precautionary actions. This finding is consistent with Polasky et al. (2011) and other existing studies.

We also find that the risk-reduction effect is amplified with the degree of risk aversion when there is uncertainty about the magnitude of post-tipping impacts on economic output. This finding suggests that it is crucial to distinguish between the risk aversion and the intertemporal elasticity of substitution for the analysis of optimal climate and resource policies under tipping risks. In contrast, if there is no additional damage of a tipping point on output, the results for CRRA preferences are not significantly different from the corresponding Epstein-Zin case with the same IES and a higher risk aversion.

Finally, our results show that initial-year emission reduction is enhanced with any form of tipping risk. In this sense, the analysis of our model thus still does not change the conclusion of existing studies of climate policy and tipping risk: that the presence of tipping risks generally raises the stringency of the optimal current climate policy. However, our results show that the effects of forest tipping risks on the level of emission control rates are small, with both the CRRA and Epstein-Zin preferences. Although our model calculations are illustrative, small impacts of forest tipping risk across various cases of parameter levels suggest that the risk of forest dieback may not strongly affect the optimal climate policy in practice as well.

In other words, the most important implications of forest dieback are likely to be those for forest harvesting and management.

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