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Abstract:

Degradation of ecosystem services may be a major component of climate change damage, and incorporation of this factor could significantly alter the significance of uncertainty in climate-economy modeling. However, this aspect has been little investigated by economic analyses of climate change and uncertainty. We apply standardized numerical techniques of stochastic optimization to this research question. The model results show that the effects of uncertainty are different with different levels of agent's risk aversion. Also, uncertainty exhibits different effects on mitigation policy and capital investment according to the availability of ecosystem services. Importantly, both the risk aversion and the availability of ecosystem services can change the effects of uncertainty on mitigation not only in level but also in sign. In other words, mitigation could both increase and decrease with climatic uncertainty. The model would provide hints for policymaking in finding a balance between economic growth, climate protection, and the conservation of ecosystems.

Keywords: climate change, decision making under uncertainty, stochastic control, renewable resource, ecosystem services

JEL classification: C63, Q54, D81

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1 Introduction

Cost-benefit assessments about climate change provide many insights about policy possibilities dealing with the problem. Although climate change is a genuinely multi-sector, multi-country problem, simple single-agent climate-economy models can still shed light on a rich set of questions, as exemplified by the Stern-Nordhaus debate on the discount rate (Stern (2007), Nordhaus (2008)). An area in which simple models are useful is the assessment of how potentiality of long-term damage of climate change should be translated into the current policy decisions. The damage function of climate change is an essential ingredient for for such simple climate-economy models. However, many open-ended questions remain about the formulation (e.g., Heal (2009)). The DICE model (Nordhaus (2008), perhaps the most widely used climate-economy model, uses a damage function proportional to the level of the atmospheric carbon stock incurring a fractional damage to the gross output. But its choice of functional form is also important and challenged by a number of economists, indicating that the use of alternative forms could radically change the behavior of the system and therefore policy recommendations (e.g., Sterner and Persson (2008)).

Such unknownness of climate damage poses an interesting question for the ones seeing climate change as a problem of economic decision-making under uncertainty[1]. So far, most economic modeling studies of climate change and uncertainty simply adopt the standard DICE-like damage function (major studies include Peck and Teisberg (1993), Pizer (1999) and Nordhaus (2008): reviews on this topic can be found in Heal and Kriström (2002), Peterson (2006) and Pindyck (2007)). But for the above reason, it is fair to assume that an alternative form of damage function may produce significantly different qualitative implications as to the effect of uncertainty on the climate-economy modeling. As such, it is worth examining the impact of stochasticity on the behavior of a climate-economy model with an alternative form of damage function, and that is the focus of the paper.

Specifically, we choose a damage function associating climate change and a renewable

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[1] This is essentially the question that Weitzman (2009) addresses as well, although his approach is very different from ours.

resource. The idea comes from the argument that the effects on the ecosystem could be a major component of climate change damage, as it is considered that up to 40% of species worldwide may face extinction with 2C of warming (Stern (2007)). There is a range of qualitative evidence that damages of ecosystems due to climate change could greatly affect human welfare (MEA (2005)), but this factor is rarely specifically formulated in the existing studies of climate-economy modeling. A renewable resource in the model serves as a simple representation of ecosystem services, and this formulation would also provide implications about the policy need to balance conservation of ecosystems and climate change mitigation. Here, we use a simple framework of resource economic model. The economy is dependent on the depletion of natural resource, which has a potential to regenerate. Climate change is considered to damage the carrying capacity of the resource, as climate change is likely to harm ecosystems more by diminishing habitats than by directly killing species

Our study is an extension of Lontzek and Narita's (2009) study, which found that by using a DICE-like damage function, the effect of damage uncertainty on optimal mitigation varies even in sign with different degrees of risk aversion. This result arises because economies endowed with different levels of the capital stock have different preferences about the tradeoff between consumption smoothing and climate change mitigation.

Our study adds a new dimension to the analysis, namely climate impacts on the ecosystem, and aims to draw a richer set of implications. In addressing different research questions, a number of authors have identified the feature that uncertainty can induce both more precautionary and less precautionary policies depending on the configurations of parameters. Discussions of this feature in renewable-resource modeling include Pindyck (1984), Olsen and Shortle (1985) and Ohta (2005). There are also some studies discussing this effect in a broad context of environmental policy, e.g.: Viscusi (1985). However, in the literature of climate change economics this characteristic is little emphasized and therefore not well investigated.

Our results confirm that the effects of uncertainty on the optimal decisions of climate mitigation, capital investment and resource extraction are not clear-cut, in other words, uncertainty of climate change does not necessarily lead to precautionary actions against the problem. A particularly noteworthy finding is that the ecosystem richness may affect the impact of uncertainty on climate mitigation not only in degree but also in sign i.e., an increase of mitigation with a poor ecosystem but a reduction of climate mitigation with a rich ecosystem. This would have an implication for the present world seeking sustainable development, which is to balance economic growth, and climate and ecosystem protections. Meanwhile, the results show that the effects of uncertainty on the model behavior significantly change according to the level of risk aversion.

We proceed as follows: In section 2 we describe the modeling framework and explain the solution method. Section 4 presents the main results of our model and provides a discussion. Section 5 concludes

2 Modeling Framework & Solution Method

We consider an economy which produces a single consumption good using a decreasing returns to scale technology with capital services K and q units of a renewable resource[2]. The production function has the following properties: $Y_K > 0$, $Y_{KK} < 0$, $Y_q > 0$, $Y_{qq} < 0$ and $Y_{Kq} > 0$. The production process generates emissions $\epsilon \cdot Y$, where ϵ denotes the emissions coefficient of output[3]. With additional expenditure, the amount of emissions is reduced; m represents the fraction of carbon emissions which is under control, i.e. not emitted in the atmosphere. Consequently, the atmospheric stock of carbon S evolves with

$$dS = (\epsilon \cdot Y(K,q) \cdot (1-m) - \beta \cdot S) \cdot dt \tag{1}$$

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[2] For simplicity, we consider only a renewable resource as input and do not explicitly model non-renewable resources. This formulation is valid when the limitation of non-renewable resources is negligible. This could in fact be a fair assumption considering, say, the immense global size of coal reserves.

[3] Note that emissions could be proportional to the output even if Y is a function of the renewable resource which originates from the natural system and does not involve fossil fuel combustion itself. For example, wood does not produce emissions itself (it is carbon-neutral) but the use of timber accompanies emissions, as a form of either logging and transportation of products or of enhanced activities of housing construction, etc.

where β is the net constant removal rate of atmospheric carbon into other carbon sinks (e.g. the ocean). At this point we assume that the atmospheric stock of carbon causes a rise in the level of global mean temperature. Let $T(S)$ be the increase of global mean temperature from the pre-industrial level with $T_S > 0$ and $T_{SS} \geq 0$. We assume that rising levels of global mean temperature cause damage to output. We denote the damage by $D(T)$ and assume $D_T > 0$, $D_{TT} > 0$ and $D(0) = 1$.

Thus, the output balance condition reads

$$\frac{Y(K, q)}{D(T(S))} = I + c + M(m) \tag{2}$$

The left-hand side of (3) is the net output inclusive of damage. The net output is in balance with the sum of the following: (i) consumption c which yields utility $U(c)$ with $U_c > 0$ and $U_{cc} < 0$; (ii) $M(m, Y)$, the emission control costs with $M_m > 0$, $M_{mm} > 0$, $M_Y > 0$, $M_{mY} > 0$ and $M_{YY} \geq 0$; (iii) capital accumulation via investment I. The stock of capital K evolves according to

$$dK = (I - \delta \cdot K) \cdot dt \tag{3}$$

where δ is the capital depreciation rate. Finally, we represent the dynamics of the renewable resource stock as:

$$dR = (\gamma \cdot R \cdot (1 - \frac{R}{CAP/D(T(S))}) - q) \cdot dt + \sigma \cdot R \cdot dB \tag{4}$$

As for the growth function of renewable resource, we adopt a simple logistic function with the carrying capacity parameter CAP. The renewable resource is a representation of ecosystem services, such as the provision of fresh water, food, wood, fiber and fuel. A fraction of resource q is subtracted from the stock for human use at each time. Since the purpose of our study is to examine the effect of alternative climate damage representation, here we consider that the climate change damage negatively influences the level of carrying capacity. This is a logical assumption given the fact that climate change is likely to cause damaging effects more to the habitat of species directly than to the flow of ecosystem service, through changes in river flows or ocean acidification (e.g., MEA (2005)). We add stochasticity as a geometric brownian motion to the dynamics of resource stock since it is conceivable that the renewable resource is more susceptible to natural variabilities than other parameters.

Our purpose is to dynamically investigate the optimal choice of consumption, emissions control, capital investment and the use of the renewable resource given uncertainty about the availability of the eco-system services. To this end, we formulate the problem from the social planner's perspective. Given the uncertainty over R, the social planner maximizes the expected present value welfare.

$$\max_{c_t > 0, 0 \leq m_t \leq 1, 0 \leq q_t} E \int_0^\infty e^{-\rho t} \left[U(c_t) \right] dt \tag{5}$$

subject to (1)-(4) and $S(0) = S_0$, $K(0) = K_0$ and $R(0) = R_0$. To solve (5) we perform stochastic control, the continuous time version of dynamic programming. The corresponding Hamilton-Jacobi-Bellman (HJB) equation is [4]

$$\begin{aligned} 0 = \max_{c > 0, 0 \leq m \leq 1, 0 \leq q_t} \{ U(c) &+ V_S(K,S,R)(\epsilon \cdot Y(K,q) \cdot (1-m) - \beta \cdot S) \\ &+ V_K(K,S,R)(\frac{Y(K,q)}{D(\eta,S))} - c - M(m,Y) - \delta \cdot K) \\ &+ V_R(K,S,R)(\gamma \cdot R \cdot (1 - \frac{R}{CAP/D(T(S))}) - q) \\ &+ \frac{1}{2}\sigma^2 R V_{RR}(K,S,R) - \rho V(K,S,R) \} \end{aligned} \tag{6}$$

where V is the value function and V_K, V_S and V_R are the derivatives of the value function

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[4] Notice that by setting up the maximization problem as in (5), we do not restrict capital investments I to be positive. In fact, for some areas of the state and parameter space optimal investment is negative.

w.r.t. the state variables, i.e. the shadow values. The Euler equations are obtained from differentiating the Bellmann equation w.r.t. the control variables. They are:

$$U_c = V_K(K,S,R) \tag{7}$$

$$M_m = -\frac{V_S(K,S,R)\cdot\epsilon\cdot Y(K)}{V_K(S,K,R)} \tag{8}$$

$$Y_q = \frac{V_R(S,K,R)}{V_K(S,K,R)(D^{-1}-M_Y)+V_S(S,K,R)(1-m)\epsilon} \tag{9}$$

Equation (7) states that the marginal utility from consumption should be equal to the derivative of the value function with respect to capital, i.e. the shadow price of capital. From (8) it can be easily seen that $V_S \leq 0$. The optimal choice of m, the emissions control rate, thus positively depends on the shadow price of atmospheric carbon (in absolute terms) and instant emissions. It negatively depends on the shadow price of capital.

A solution to (5) requires finding a value function V(K,S,R) and policy functions $c(K,S,R)$, $m(K,S,R)$ and $q(S,K,\eta)$ which satisfy the Bellmann and Euler equations (6)-(9). We determine the policy functions and the value function numerically using a projection method. Projection methods work very well with continuous-time, continuous-state problems (Judd (1992), Judd (1998)). In particular, we apply the Chebyshev collocation method using a linear combination of basis functions whose coefficients approximate the solution to (6)-(9) at specific collocation nodes. The approximated value function and policy functions are given by:

$$\tilde{V}(K,S,R) = \sum_i \sum_j \sum_k w^V_{ijk} T_i(x_K) T_j(y_S) T_k(z_R) \tag{10}$$

$$\tilde{c}(K,S,R) = \sum_i \sum_j \sum_k w^c_{ijk} T_i(x_K) T_j(y_S) T_k(z_R) \tag{11}$$

$$\tilde{m}(K,S,R) = \sum_i \sum_j \sum_k w^m_{ijk} T_i(x_K) T_j(y_S) T_k(z_R) \tag{12}$$

$$\tilde{q}(K,S,R) = \sum_i \sum_j \sum_k w^q_{ijk} T_i(x_K) T_j(y_S) T_k(z_R) \tag{13}$$

We define $T_i(x_K), T_j(x_S)$ and $T_k(z_R)$ as n_i, n_j, n_k-degree Chebyshev polynomials which are evaluated at the states with x_K, x_S, x_R being a mapping $[K^{\min}, K^{\max}] \times [S^{\min}, S^{\max}] \times [R^{\min}, R^{\max}] \longmapsto [-1,1] \times [-1,1] \times [-1,1]$. The collocation coefficients $w^V_{ijk}, w^c_{ijk}, w^m_{ijk}$ and w^q_{ijk} are then determined in order to minimize the Bellman and Euler errors and to deliver a good approximation of (6)-(9). In order to solve the model numerically we assume the following functional forms:

$$\begin{aligned} Y(K,q) &= A \cdot \mu K^{\nu} \cdot q^m u \\ D(T(S)) &= 1 + \kappa \cdot T(S)^2 \\ T(S) &= \tau \cdot (S - S_{PI}) \\ M(m) &= \psi \cdot \epsilon \cdot Y \cdot m^2 \\ U(c) &= \frac{c^{1-\alpha}}{1-\alpha} \end{aligned}$$

The parameter values are: $\nu = .5$, $\kappa = .005$, $\tau = .003$, $\psi = .1$, $\epsilon = .3$, $S_{PI} = 400$, $\alpha = .8$, $\rho = .01$, $A = 1$, $CAP = 500$, $\gamma = .05$ and $\sigma = .05$. With the lack of solid empirical data, we choose these parameter levels by considering computational constraints and usefulness for illustration. The choice of level for some parameters in fact does not have influence on the general model dynamics as it only determines how the model is normalized. Meanwhile, the model behavior is significantly influenced by the level-setting of some other parameters, most importantly of the elasticity of marginal utility α. For this reason, we later examine the model characteristics with different levels of α as well

3 Results and Discussion

We compute numerically the deterministic steady state and obtain $\tilde{K} = 50$, $\tilde{S} = 1500$ and $\tilde{R} = 175$. Given these values we set up the projection grid by discretizing the spate space around the steady state. We choose $K \in [25, 75]$, $S \in [800, 2500]$ and $R \in [100, 250]$. The Chebyshev polynomials are of degree 6 in all states i.e.: $n_i, n_j, n_k = 6$. We formulate the optimization problem in AMPL and use KNITRO's active set algorithm to solve it.

3.1 Basic results: optimal values and policy functions

First, we show the basic results of the model. Figure 1 reports the value function for three

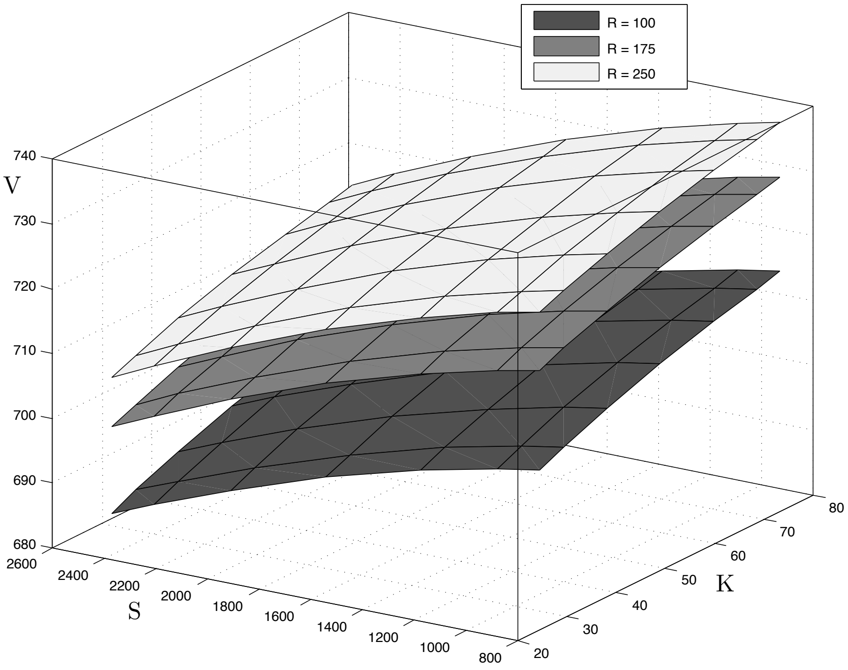

Figure 1: Value function for different $R's$ and $\sigma = 0.05$

different levels of the renewable resource in $K-S$ space[5]. Note that the graph shows feasible sets of K-S with respective levels of R and V and not the dynamics of K and S (as R changes over time) We observe that for each resource level the value function is smooth and concave in both, the carbon stock and the capital stock. This implies ceteris paribus that an increase in capital leads to a less than proportional increase in the value function. This is due to diminishing returns to capital investments. On the contrary, an increase in the carbon stock leads to a more than proportional loss in welfare which reflects the quadratic structure of the damage function. Finally, since in Figure 1 the three levels of the resource state ($R = 100, 175, 250$) are equally spaced, we deduct that the value function is also concave in the level of the resource stock. Again, this result is mainly driven by the diminishing returns to resource input.

Figure 2 depicts the contour plot for the optimal mitigation policy in $K-S$ space. The solid contour lines relate to a low resource stock (R=140) while the dashed contour lines represent a high resource stock (R=210).

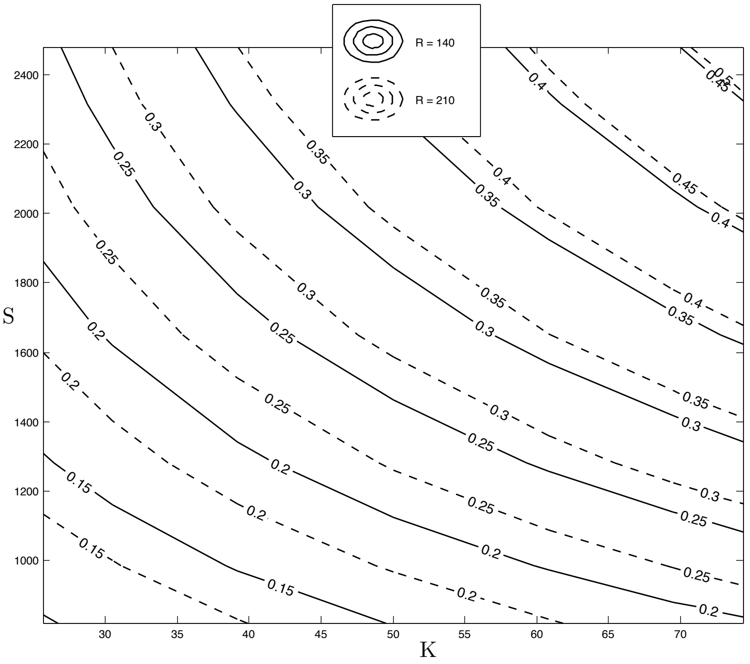

Figure 2: Optimal mitigation for $\sigma = 0.05$ and different levels of R

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[5] The approximation residuals are in the order of 10^{-9}

The first observation from Figure 2 is the concavity of the contour plots. Optimal mitigation rises with north-eastern movement in the state grid. This result seems intuitive since (i) for any given level of the capital stock higher carbon levels call for more mitigation to prevent larger damage and (ii) for any given level of the carbon stock higher capital availability leads to more output which can be allocated to mitigation activities. Finally, for any $K - S$ combination higher availability of the renewable resource affects mitigation positively. This is mainly due to higher extraction levels which lead to more output. The contour plots for optimal consumption are shown in Figure 3 below. Consumption increases with higher capital availability since the latter also generates more output. For any given capital stock level, consumption slightly falls when the stock of carbon is higher. This effect intensifies for higher levels of the capital stock which can be seen from the stronger curvature of the contours when going to the right along the horizontal axis. This behavior occurs because mitigation serves as a stronger substitute for consumption when capital availability is high. Thus, the dynamic trade-off between decreasing marginal utility and increasing marginal prevention of damage moves in favor of mitigation activities. Figure 3 also shows

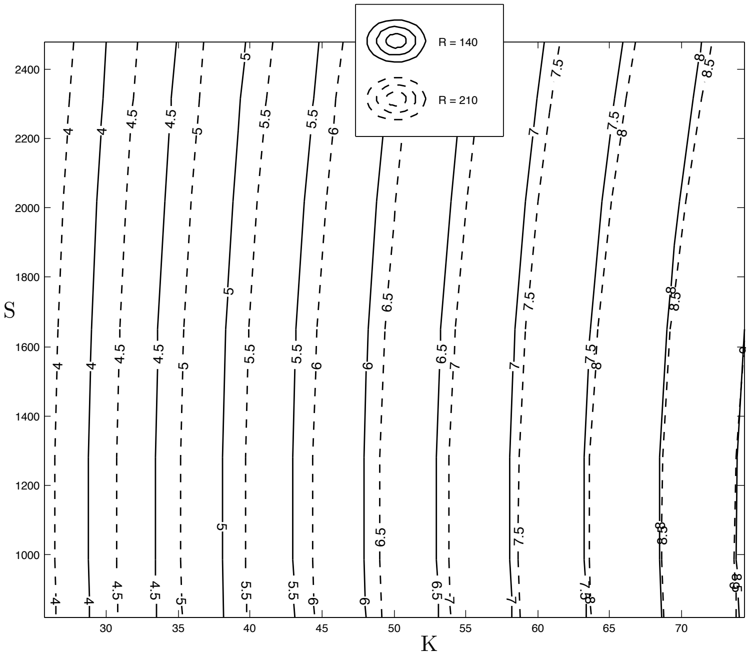

Figure 3: Optimal consumption for $\sigma = 0.05$ and different levels of R

that consumption is intensified with rising availability of eco-system services. This effect results from the substitutability of resource for capital in the production function, leaving more capital idle for e.g.: consumption.

The optimal usage of eco-system services for generating output is depicted in Figure 4. Higher levels of the carbon stock and thus, higher damage reduces the extraction of the resource stock. Extraction levels also fall with higher availability of the capital stock, which results from the substitutability between capital and eco-system services in the production function. In Figure 4 one can observe a flattening of the contour lines for higher levels of capital. This can be explained by the decreasing (negative) elasticity of extraction with respect to the carbon stock. When capital is largely available, higher damage reduces extraction much faster when compared to cases in which capital is scarce. The comparison of the dashed and solid lines in Figure 4 leads to an intuitive conclusion that the optimal extraction rises with a higher stock of the renewable resource (in other words, the stock potential for ecosystem services). Meanwhile, uncertainty influences the resource extraction in a more complex fashion. Figure 5 illustrates the impact of stochasticity on the patterns of optimal resource extraction. Each plot in Figure 5 shows the percentage change in optimal extraction when uncertainty is included ($\sigma = 0$ to $\sigma = 0.05$). The three plots

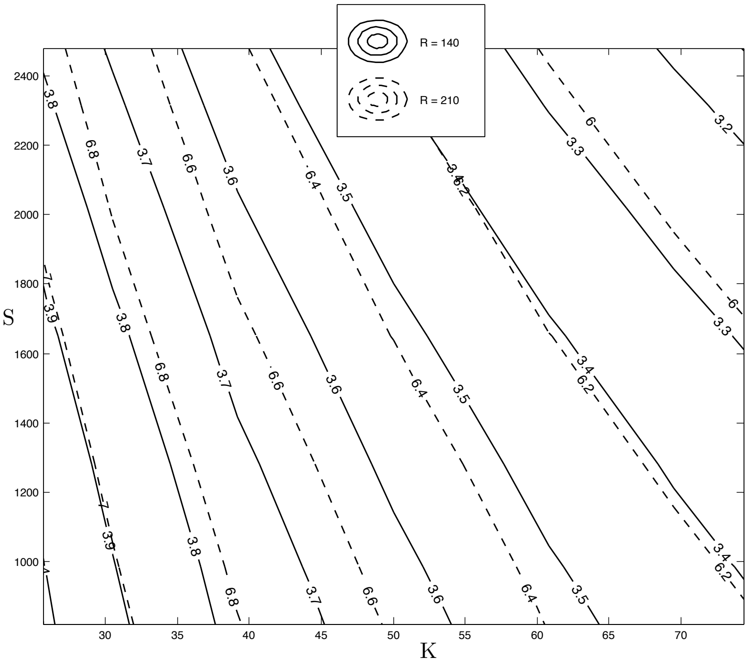

Figure 4: Optimal extraction for $\sigma = 0.05$ and different levels of R

differ with respect to the size of the resource stock (100, 175 and 250).

While the shape of all surface plots is quite intuitive (increasing in K and decreasing in S), we observe a change in sign for the effect of uncertainty on the percentage change in extraction. For lower levels of the resource stock uncertainty about the size of the resource stock leads to lower extraction. On the contrary, high availability of the resource makes the effect of including uncertainty on extraction positive. This change in sign can be explained as follows. Generally, uncertainty may influence the optimal decision in case that today's action makes a lasting effect on the utility flow in the future (i.e., in presence of irreversibility). If the extraction of resource has a chance to cause negative irreversible effects on the wealth, uncertainty itself has an effect of reducing the present extraction level. This is the case for R=100 and 175, in which the current resource extraction would overall have an diminishing effect on the future wealth through a size reduction of the resource base or an increase of the carbon stock (climate damage). Meanwhile, the capital stock is also an determinant of the utility or consumption level. The resource extraction enhances the output and consequently the capital accumulation. Hence, under certain conditions, a way to hedge against uncertainty is rather an increase of resource use and accumulation of capital. This is the case for R=250, a consequence of the ample resource size with which a fractional loss of resource at present does not have grave long-term implications on the

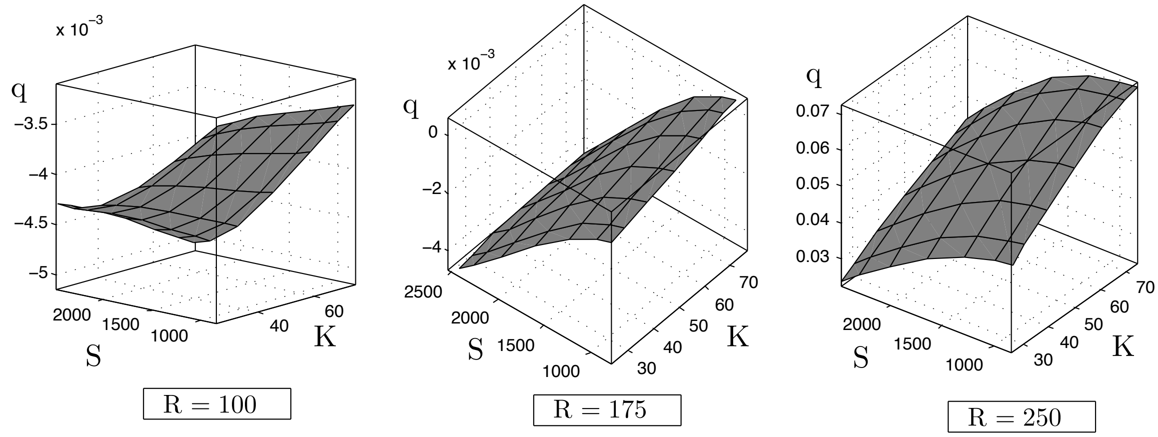

Figure 5: Optimal extraction - % change from $\sigma = 0$ to $\sigma = 0.05$

(regeneration of) resource base.

3.2 Effects of uncertainty and risk aversion

In this section, we examine the effects of uncertainty on the model behavior more in depth. In particular, we discuss model results with varying levels of the risk aversion parameter α, which influences the agent's responses to uncertainty. in the published literature of climate change economics, this aspect is hardly explored (an exception is a simple calculation by Heal and Kriström (2002)), and little insight has been obtained.

In the following we present three plots (Figures 6-8), each for one control (mitigation, consumption and extraction). Each plot consists of four subplots which vary with the location in the $K - S$ space (i.e.: low K = 25, low S = 800, high K = 75, high S = 2500). Each subplot depicts the contours of the specific control policy for different combinations of the risk aversion parameter α and the size of the renewable resource.

First, Figure 6 displays optimal mitigation. It can be easily seen that optimal mitigation increases with higher levels of capital, carbon stock and resource stock - intuitive results. Meanwhile, the relationship between the emission abatement and the risk aversion is somewhat more complex, showing that a higher risk aversion is associated with more mitigation with a high capital but less mitigation with a low capital. This is a different form of manifestation of the characteristic discussed in the last section, a competition between a moderation effect (i.e., dealing with the risk of climate change by scaling up mitigation efforts and reducing investment) and a substitution effect (i.e., hedging against loss of climate change damage by enhancing output by more capital investment and therefore less mitigation). A high capital means a small return to investment and thus makes investment relatively unattractive over mitigation, while a low capital has exactly the opposite effect.

A different but similar logic could explain the effects of uncertainty displayed by dashed contours in Figure 6. We see that for larger levels of the risk aversion parameter, uncertainty about the availability of the renewable resource increases mitigation, while the latter is reduced for lower levels of risk aversion. Thus, for any level of the resource stock there

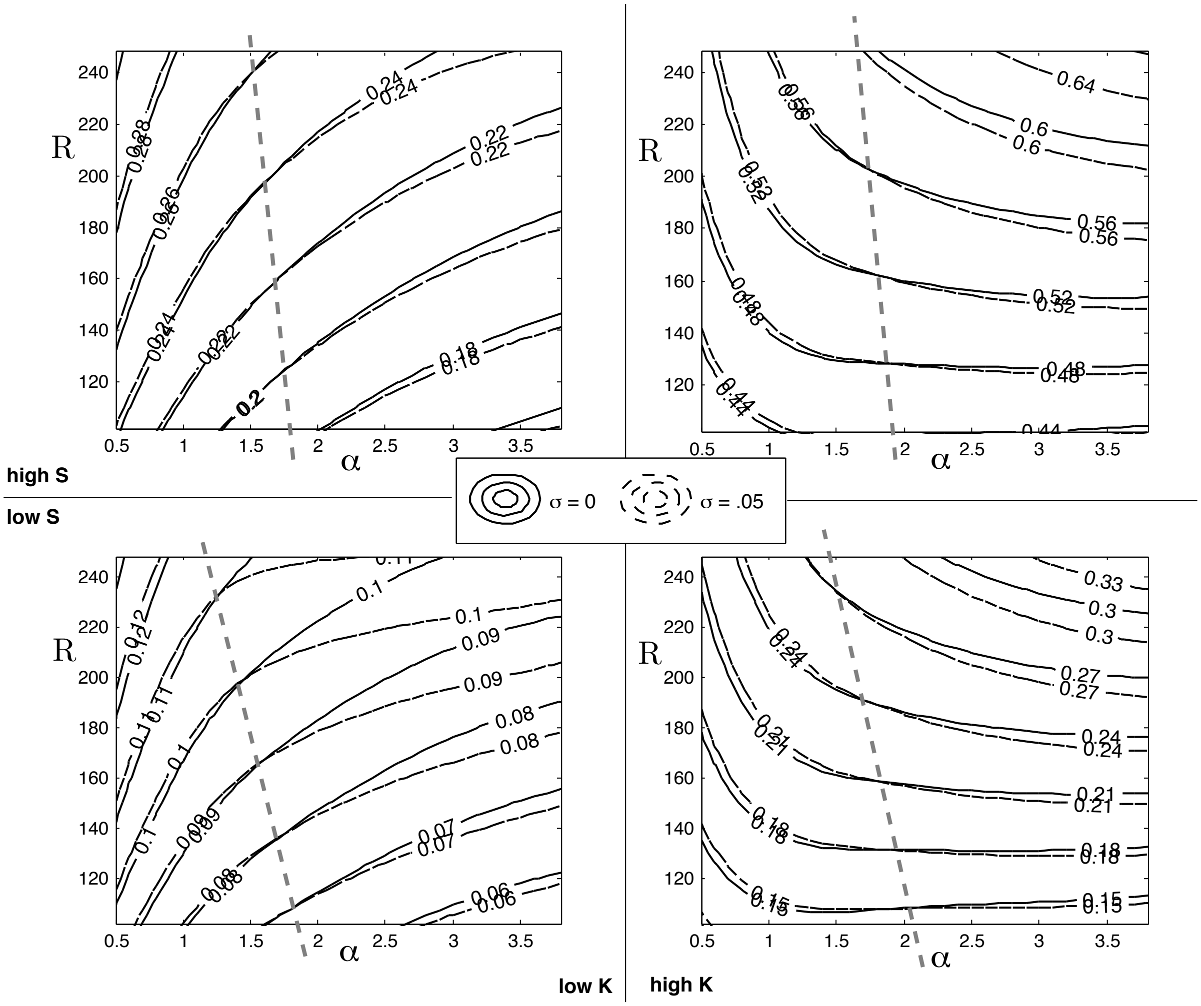

Figure 6: Optimal mitigation in $\alpha - R$ space for different levels of σ and $K - S$ combinations.

exists a level of risk aversion which marks a switching point for the effect of uncertainty on mitigation. The combination of these switching points in $\alpha - R$ space is depicted by the dashed grey line. It can bee seen that the switch of mitigation response to uncertainty in sign occurs at levels of α roughly around $1.2 - 2.2$. In order to explain this result intuitively we need to answer (1) why do the switching points occur and (2) why the combination of these switching points is downward sloping. As for the first point, we should keep in mind that any dollar spent on mitigation cannot be used for consumption or capital accumulation. The economy must therefore weigh an intensive use of ecosystem services leading to future consumption losses, and a reduction in mitigation, leading to current consumption gains. Therefore, there is less need for mitigation with uncertainty if the degree of risk aversion, and consequently the need for consumption smoothing, is low. After all, mitigation is an instrument whose eventual aim is to ensure higher consumption levels in the future since it substitutes future consumption gains due to less reduced climate change for current consumption losses. Consequently, if the need for consumption smoothing increases (i.e.: α rises), mitigation, as an instrument of consumption smoothing, becomes more effective at achieving the latter and thus, becomes more desirable. Finally, in order to explain the second question, we note that the downward-sloping dashed grey line in Figure 6 implies that with higher levels of the resource stock, the positive effect of uncertainty on mitigation sets in already at lower levels of risk aversion. High levels of the resource stock translate into larger output and therefore consumption levels. Thus, at this stage, the economy can simply pay more attention to sustainable future consumption despite the irreversibility of mitigation expenditure. As a consequence, for larger levels of the resource stock the switching point will occur at lower levels of risk aversion.

Figure 7 displays the optimal consumption policy with a plot setting analogous to Figure 6. Uncertainty has a small negative effect on consumption throughout the state space of the model simulations (result not shown in the graph for the reason of better visualization). In general we observe that higher availability of the capital stock (i.e.: higher output) increases consumption whereas a larger stock of atmospheric carbon lowers consumption. These findings are in line with the ones from Figure 3. The degree of risk aversion has a mixed effect on consumption, depending on the size of the capital stock. With a low availability of the capital stock (in this case below the long run steady state level) increasing risk aversion leads higher consumption, while the reverse effect is observed for high capital levels. In the latter case, consumption is shifted towards the future since the need for consumption smoothing is larger due to higher risk aversion.

The need for consumption smoothing due to increasing risk aversion is also the major driver of optimal extraction levels, as shown in Figure 8. As a consequence the degree of risk aversion also has a mixed effect on extraction, depending on the size of the capital stock. With a low availability of the capital stock increasing risk aversion leads to higher

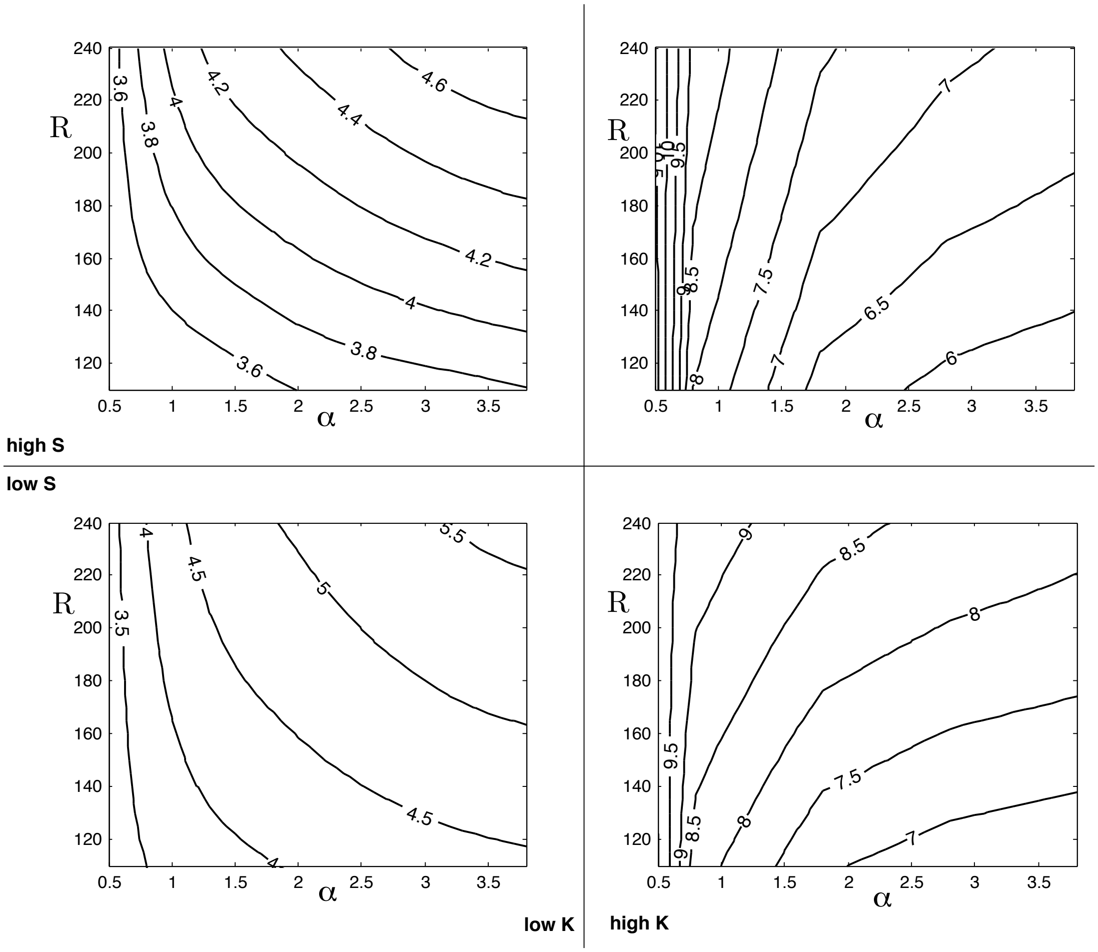

Figure 7: Optimal consumption in $\alpha - R$ space for different levels of σ and $K - S$ combinations.

consumption and therefore to lower extraction volumes. The reverse effect can be observed for high capital stock levels. In that case, higher risk aversion leads to a postponement of resource extraction in order to smooth out the consumption pattern. Furthermore, we observe that optimal extraction levels decrease with higher levels of the capital stock, which is due to the substitution of the input factors of the production function. Also, extraction levels fall with more carbon stock in the atmosphere. Though, not shown in this figure, uncertainty in general has a negative effect on extraction, except for very high levels of R and K, and S for which the effect becomes positive and can be explained along the lines of Figure 5.

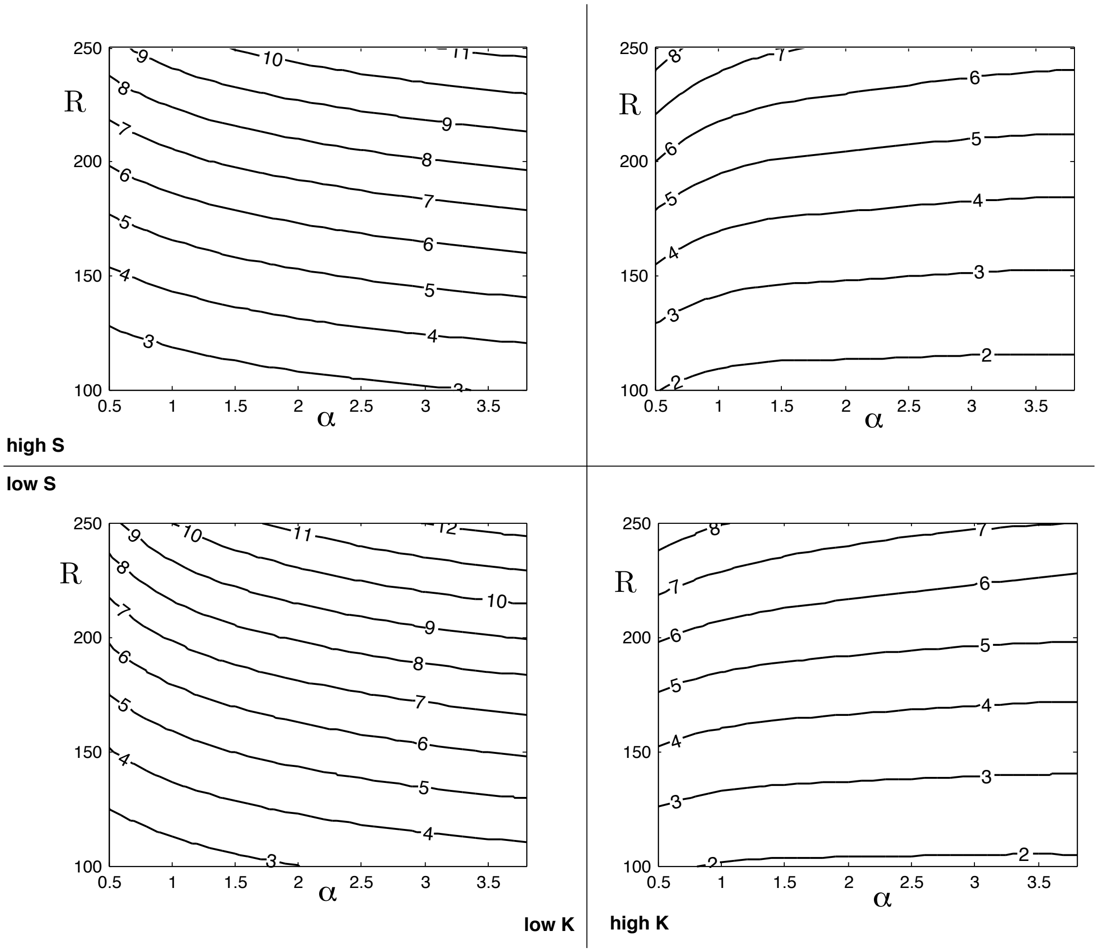

Figure 8: Optimal extraction in $\alpha - R$ space for different levels of σ and $K - S$ combinations.

4 Concluding remarks

This study's results present interesting policy implications for economies seeking sustainable development. The availability of the renewable resource in the model, a simple representation of ecosystem services, significantly influences our optimal policy of climate change mitigation under uncertainty. This would be a useful perspective for the present world having to find the balance between economic growth, climate change mitigation, and ecological conservation. In particular, the model shows that an economy where ecosystems are in danger would need to be very precautionary both in keeping the potential of ecosystem services and in mitigating climate change. On the other hand, the results also provide some ground for a "get rich first" approach by a poor economy in tackling with climate change – in other words, prioritization of capital investment and economic growth at the expense of the climate and ecosystems at the initial stage, and the uncertainty of climate change may in fact amplify this tendency, not deterring it. It should be reminded that this study is not an analysis one and is not conclusive about in what domain the actual world economy is currently located in terms of effects of uncertainty on optimal policy decisions – this question is left to future investigations.

Besides, our results clearly show that with different levels of risk aversion, uncertainty affects the optimal mitigation policy differently, not only in quantity but also in sign. This is an important viewpoint for this framework to be applied in real policymaking, as people's perception about risk is a deep conceptual issue by itself being analyzed from various standpoints.

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