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No. 1942 | July 2014

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Risk Preferences may be Time Preferences:
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Ulrich Schmidt*

1 Introduction

In an intensively discussed paper, Andreoni and Sprenger (2012), henceforth A&S, present an experiment where subjects can allocate money between two different points of time under the condition of risk. A&S claim that their results refute discounted expected utility (DEU) as well as prospect theory and other models relying on probability weighting. In this note I will show that the theoretical analysis of A&S is inappropriate and, therefore, that their claims are not valid. It turns out, that the experimental results of A&S are fully in line with DEU. The main problem of A&S´s analysis is that is confounds income with consumption. There exist several other comments on A&S (Miao and Zhong, 2012; Epper and Fehr-Duda, 2014 and Cheung, 2014) which discuss interesting aspects of the analysis of A&S but have not identified the theoretical implications of equalizing consumption and income.

2 The Analysis of Andreoni and Sprenger (2012)

In the experiment of A&S subjects can allocate a constant amount m of experimental currency between two different time points, t and t + k. Each token allocated to t + k results in a payment of \$0.20 whereas tokens allocated to t have a return varying between \$0.20 and \$0.14. However, both payments may be subject to risk, i.e. the payment in t will be only realized with probability p_1 whereas the payment in t+ k will be realized with p_2. The main treatment of A&S compares allocations under $p_1 = p_2 = 1$ with those under $p_1 = p_2 = 0.5$ where both lotteries are played out independently and subjects receive zero if lotteries are lost. A&S observe that allocations under both conditions are significantly different and conclude that this difference refutes DEU as well as models which include probability weighting. This conclusion relies on the following theoretical analysis. Let v(c) denote the utility from consumption c with v(c) = 0. If δ is the discount factor then DEU is given by

(1) $DEU = p_1\delta^t v(c_t) + p_2\delta^{t+k}v(c_{t+k})$.

Maximizing DEU subject to the budget constraint

(2) $(1+ r)c_t + c_{t+k} = m$

yields

(3) $v'(c_t)/ \delta^k v'(c_{t+k}) = (1 + r)p_2/p_1$.

This condition shows that the optimal allocation should just depend on the ratio of probabilities. Moreover, if $p_1 = p_2$ (what I will assume in the sequel), this result also holds when probability weighting is introduced. In contrast to the theoretical result, A&S find that chosen allocations depend on the probability ratio as depicted in Figure 1: If r = 0, y_t is under certainty (i.e. $p_1 = p_2 = 1$) significantly higher that under risk (i.e. $p_1 = p_2 = 0.5$). If r increases, y_t under certainty decreases

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stronger than under risk such that there is a crossover point and already for $r = 0.05$ y_t under certainty is significantly lower than y_t under risk.

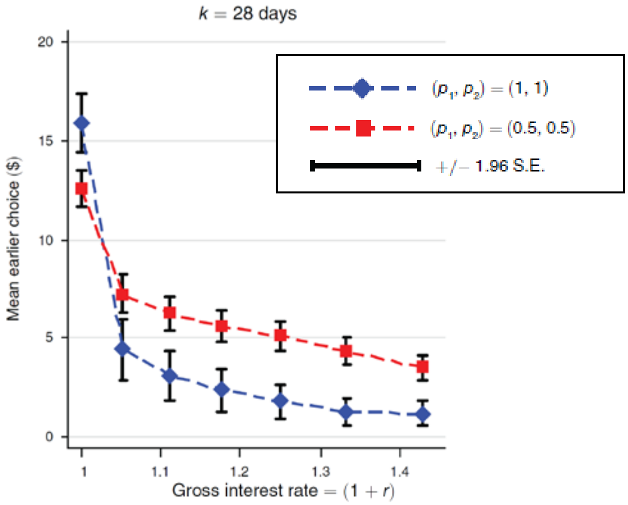

Figure 1: The results of A&S (taken from Andreoni and Sprenger, 2012, p. 3367.)

3 Consumption versus Income

The theoretical analysis of A&S relies on consumption whereas in their experiment subjects distribute income. Obviously, income received at time t does not need to equal consumption in t as subjects can save or borrow. Let us for convenience assume they can do both with the same interest rate i. Then we get $c_t = y_t - s$ and $c_{t+k} = y_{t+k} + (1 + i)s$ where y_t and y_{t+k} are the two incomes subjects receive in the experiment of A&S, s denotes savings in t and $s < 0$ indicates that the subject is borrowing money in t from the money which will be received in $t + k$. For our analysis it is sufficient that subjects can distribute by saving only the money received in the experiment, i.e. $y_t \geq s \geq - y_{t+k}/(1 + i)$. The budget constraint is as in A&S given by

(4) $(1+ r)y_t + y_{t+k} = m$

but utility now depends on the correlation structure. If payments are safe or risks in t and $t + k$ are perfectly positively correlated, the analysis is similar to the one in A&S and utility equals

(5) $DEU = p\delta^t v(y_t - s) + p\delta^{t+k} v(y_{t+k} + (1 + i)s)$.

The following proposition shows that with the possibility of saving only corner solutions are optimal.

Proposition 1:

If preferences are represented by DEU we get for $p = 1$ and in the case of perfectly positive correlation for all p with $0 < p \leq 1$:

(i) $y_t = m/(1 + r)$ and $y_{t+k} = 0$ if $i > r$
(ii) $y_t = 0$ and $y_{t+k} = m$ if $r > i$.

Now assume that risks in t and $t + k$ are uncorrelated, as they are in the experiment of A&S. In this case there are in principle four states of the world (see Table 1) and if the subject wins, as in states 2 and 3, in only one point of time she can transfer money to the other point by saving (state 2) or borrowing (state 3). Hence, consumption is characterized as in the fourth row of the table where it is assumed that both, y_t and y_{t+k}, are strictly positive. Note that in state 2 savings have to be positive, as nothing can be borrowed from the zero income in t + k (see fifth row). Analogously, savings have to be negative in state 3.

state	1	2	3	4
probability	p^2	$p(1-p)$	$(1-p)p$	$(1-p)^2$
income				
t	y_t	y_t	0	0
t+k	y_{t+k}	0	y_{t+k}	0
consumption				
t	$y_t - s$	$y_t - s$	$-s$	0
t+k	$y_{t+k} + (1+i)s$	$(1+i)s$	$y_{t+k} + (1+i)s$	0
saving	$y_t \geq s \geq - y_{t+k}/(1+i)$	$y_t \geq s \geq 0$	$0 \geq s \geq - y_{t+k}/(1+i)$	$s = 0$
consumption if $y_{t+k} = 0$				
t	$y_t - s$	$y_t - s$	0	0
t+k	$y_{t+k} + (1+i)s$	$(1+i)s$	0	0
consumption if $y_t = 0$				
t	$y_t - s$	0	$-s$	0
t+k	$y_{t+k} + (1+i)s$	0	$y_{t+k} + (1+i)s$	0

Table 1: Uncorrelated Risks

Suppose the subject would choose corner solutions as under perfect correlation. If $y_t = 0$, this implies an income and a consumption of zero in both points of time for states 2 and 4, as nothing can be saved in t if $y_t = 0$ and also nothing can be borrowed as y_{t+k} is not paid out in this state (see sixth row). Conversely, $y_{t+k} = 0$ implies a zero income and consumption in both points of time for states 3 and 4 (see last row). Altogether, this means that corner solutions imply a zero consumption in both points of time with probability $1 - p$. If the subject, however, chooses an interior solution this zero consumption is only incurred in state 4, i.e. with probability $(1 - p)^2$. This shows that there is a risk-reducing portfolio effect in the presence of independent risks such that risk averse subjects will avoid corner solutions. If we for instance consider a power function $v(c) = c^\gamma$ with $0 < \gamma < 1$ marginal utility becomes infinity if c converges to zero. Here it is evident that $y_t = 0$ can never be optimal, as a slightly positive y_t along with a positive saving rate leads to an infinite marginal utility increase in both points of time in state 2. Analogously, $y_{t+k} = 0$ can never be optimal.

3 Discussion

The theoretical analysis of A&S relies on consumption whereas their experimental results rely on income. Due to the possibility of saving, consumption does not need to equal income, however, and therefore I have adjusted the theoretical analysis accordingly. It turns out that DEU is well compatible with the experimental results presented by A&S. First, as i should not be negative, for r = 0 and p = 1 most subjects should choose $y_{t+k} = 0$ (see Proposition 1), whereas y_{t+k}. should be always positive for p < 1 and uncorrelated risks. If r is increasing, for p = 1 more and more subjects should switch to $y_t = 0$ whereas y_t should also be always positive for p < 1. This gives exactly the picture depicted in Figure 1 and is compatible with the result that A&S observe 80.7% corner solutions for p = 1 but only 26.1% for p = 0.5. Interestingly, Miao and Zhong (2012) replicated the experiment of A&S with perfectly positively correlated risks and find that allocations here are identical for p = 1 and p = 0.5. According to Proposition 1, also this behavior is implied by DEU. Altogether, while I have shown that time-separable preferences are able to explain the evidence presented in A&S, intertemporal portfolio considerations as discussed in Epper and Fehr-Duda (2014) and Cheung (2014) should influence choice in many situations.

References

Andreoni, J., and C. Sprenger (2012): "Risk Preferences Are Not Time Preferences," American Economic Review, 102(7), 3357–3376.

Cheung, S. (2014): "On the Elicitation of Time Preference under Conditions of Risk," American Economic Review (forthcoming).

Epper, T., and H. Fehr-Duda (2014): "Balancing on a Budget Line: Comment on Andreoni and Sprenger's `Risk Preferences Are Not Time Preferences´" American Economic Review (forthcoming).

Miao, B., and S. Zhong (2012): "Separating Risk Preference and Time Preference," Working Paper, National University of Singapore.

Appendix: Proof of Proposition 1

We have to maximize (5) under the budget constraint (4) and the four following conditions:

(i) $y_t \geq 0$, (ii) $y_{t+k} \geq 0$, (iii) $y_t – s \geq 0$, and (iv) $s + y_{t+k}/(1 + i) \geq 0$. The Lagrangian becomes

(A1) $L = p\delta^t v(y_t – s) + p\delta^{t+k}v(y_{t+k} + (1 + i)s) + \lambda((1+ r)y_t + y_{t+k} – m) + \mu_1 y_t + \mu_2 y_{t+k} + \mu_3(y_t – s) + \mu_4(s + y_{t+k}/(1 + i))$.

We get the following first-order conditions

(A2) $p\delta^t v'(y_t – s) + (1 + r)\lambda + \mu_1 + \mu_3 = 0$

(A3) $p\delta^{t+k}v'(y_{t+k} + (1 + i)s) + \lambda + \mu_2 + \mu_4/(1 + i) = 0$

(A4) $–p\delta^t v'(y_t – s) + (1 + i)p\delta^{t+k}v'(y_{t+k} + (1 + i)s) – \mu_3 + \mu_4 = 0$

(A5) $y_t \geq 0$, $\mu_1 \geq 0$ and $y_t\mu_1 = 0$

(A6) $y_{t+k} \geq 0$, $\mu_2 \geq 0$ and $y_{t+k}\mu_2 = 0$

(A7) $y_t – s \geq 0$, $\mu_3 \geq 0$ and $(y_t – s)\mu_3 = 0$

(A8) $s + y_{t+k}/(1 + i) \geq 0$, $\mu_4 \geq 0$ and $(s + y_{t+k}/(1 + i))\mu_4 = 0$.

(A2) and (A3) yield

(A9) $p\delta^t v'(y_t – s) + \mu_1 + \mu_3 = (1 + r)[p\delta^{t+k}v'(y_{t+k} + (1 + i)s) + \mu_2 + \mu_4/(1 + i)]$,

whereas (A4) implies

(A10) $p\delta^t v'(y_t – s) + \mu_3 = (1 + i)p\delta^{t+k}v'(y_{t+k} + (1 + i)s) + \mu_4$.

If we insert (A10) in (A9) we get

(A11) $(1 + i)p\delta^{t+k}v'(y_{t+k} + (1 + i)s) + \mu_4 + \mu_1 = (1 + r)[p\delta^{t+k}v'(y_{t+k} + (1 + i)s) + \mu_2 + \mu_4/(1 + i)]$

and hence

(A12) $(i - r)[p\delta^{t+k}v'(y_{t+k} + (1 + i)s) + \mu_4/(1 + i)] = (1 + r)\mu_2 - \mu_1$.

Since $p\delta^{t+k}v'(y_{t+k} + (1 + i)s) > 0$ and $\mu_4 \geq 0$ we can only have $y_t > 0$ and $y_{t+k} > 0$, i.e. $\mu_1 = 0$ and $\mu_2 = 0$ if $i = r$. If $i > r$, the right-hand side of (A12) has to become positive which implies $\mu_2 > 0$ and, therefore, $y_{t+k} = 0$ and $y_t = m/(1 + r)$. Conversely, $i < r$ implies $\mu_1 > 0$ and, therefore, $y_t = 0$ and $y_{t+k} = m$.