# 1 Appendix A. Individualistic Bargaining

This Appendix shows the derivations for the competitive economy under individualistic bargaining. By comparing the competitive economy with the planner solution we show that the efficient wage cannot be replicated by the Nash-bargained wage. While the collective bargaining results in a single wage process, the individualistic bargaining features a distribution for the wage schedule which depends upon $\varepsilon$.

## 1.1 Decentralized Firm under Individualistic Bargaining

Atomistic firms in the competitive economy solve the following maximization problem:

$$
Max_{\{n_t, v_{f,t}, v_{h,t}\}} \Pi_t = E_t \sum_{j=t}^{\infty} \Delta_{t,j} \left[ \begin{array}{c} n_{t-1} \int_{-\infty}^{v_{f,t}} \left( a_t - w_t^I(\varepsilon_t) - \varepsilon_t \right) g(\varepsilon_t) d\varepsilon_t \\ -n_{t-1}\phi\left(v_{f,t}^{CE}\right) f + s_t \int_{-\infty}^{v_{h,t}} \left( a_t - w_t^E(\varepsilon_t) - \varepsilon_t - h \right) g(\varepsilon_t) d\varepsilon_t \end{array} \right],
$$

s.t.

$$
n_t = n_{t-1} \left( 1 - \phi\left(v_{f,t}^{CE}\right) \right) + \eta\left(v_{h,t}^{CE}\right) s_t.
$$

The profit maximization yields the following first order conditions:

$$
\mu_t = E_t \left[ \begin{array}{c} \Delta_{t,t+1} a_{t+1} \left( 1 - \phi\left(v_{f,t+1}^{CE}\right) \right) + \Delta_{t,t+1} \int_{-\infty}^{v_{f,t+1}} \left( -w_{t+1}^I(\varepsilon_{t+1}) - \varepsilon_{t+1} \right) g(\varepsilon_{t+1}) d\varepsilon_{t+1} \\ -\Delta_{t,t+1}\phi\left(v_{f,t+1}\right) f + \Delta_{t,t+1}\mu_{t+1} \left( 1 - \phi\left(v_{f,t+1}^{CE}\right) \right) \end{array} \right],
$$

$$
-f = a_t - \left( w_t^I(v_{f,t}^{CE}) + v_{f,t}^{CE} \right) + \mu_t,
$$

$$
h = a_t - \left( w_t^E(v_{h,t}^{CE}) + v_{h,t}^{CE} \right) + \mu_t.
$$

## 1.2 Social Planner Solution

The social planner solves the following maximization problem:

$$
Max_{\{n_t, v_{f,t}, v_{h,t}\}} \Pi_t = E_t \sum_{j=t}^{\infty} \Delta_{t,j} \left[ \begin{array}{c} n_{t-1} \int_{-\infty}^{v_{f,t}} \left( a_t - \varepsilon_t \right) g(\varepsilon_t) d\varepsilon_t - \\ n_{t-1}\phi\left(v_{f,t}^{PE}\right) f + (1 - n_{t-1}) \int_{-\infty}^{v_{h,t}} \left( a_t - \varepsilon_t - h \right) g(\varepsilon_t) d\varepsilon_t \end{array} \right],
$$

s.t.

$$
n_t = n_{t-1} \left( 1 - \phi\left(v_{f,t}^{PE}\right) - \eta\left(v_{h,t}^{PE}\right) \right) + \eta\left(v_{h,t}^{PE}\right),
$$

First order conditions read as follows:

$$\mu_t = E_t \left[ \begin{array}{c} \Delta_{t,t+1} a_{t+1} \left(1 - \phi\left(v_{f,t+1}^{PE}\right) - \eta\left(v_{h,t+1}^{PE}\right)\right) + \Delta_{t,t+1} \int_{-\infty}^{v_{f,t+1}} (-\varepsilon_{t+1})\, g(\varepsilon_{t+1}) d\varepsilon_t \\[3mm] -\Delta_{t,t+1} \int_{-\infty}^{v_{h,t+1}} (-\varepsilon_{t+1})\, g(\varepsilon_{t+1}) d\varepsilon_t + \Delta_{t,t+1} h\eta\left(v_{h,t+1}^{PE}\right) \\[3mm] -\Delta_{t,t+1}\phi\left(v_{f,t+1}\right) f + \Delta_{t,t+1}\left(1 - \phi\left(v_{f,t+1}^{PE}\right) - \eta\left(v_{h,t+1}^{PE}\right)\right)\mu_{t+1} \end{array} \right],$$

$$-f = a_t - v_{f,t}^{PE} + \mu_t,$$
$$h = a_t - v_{h,t}^{PE} + \mu_t.$$

## 1.3 Comparison Decentralized and Planner Economy

To decentralize the efficient solution, we need to set $v_{h,t}^{PE} = v_{h,t}^{CE}$. First, we write the optimal job creation/destruction conditions, both in the competitive and in the planner economy, in terms of the firing thresholds:

$$-a_t + w_t^I(v_{f,t}) + v_{f,t} - f = E_t\Delta_{t,t+1}\int_{-\infty}^{v_{f,t+1}} \left(-w_{t+1}^I(\varepsilon_{t+1}) - \varepsilon_{t+1}\right) g(\varepsilon_{t+1})d\varepsilon_t \tag{1}$$
$$-E_t\Delta_{t,t+1}\phi\left(v_{f,t+1}\right) f + E_t\Delta_{t,t+1}\left(1 - \phi\left(v_{f,t+1}\right)\right)\left(w_{t+1}^I(v_{f,t+1}) + v_{f,t+1} - f\right),$$

$$-a_t + v_{f,t} - f = E_t\Delta_{t,t+1}\int_{-\infty}^{v_{f,t+1}} (-\varepsilon_{t+1})\, g(\varepsilon_{t+1})d\varepsilon_t - E_t\Delta_{t,t+1}\int_{-\infty}^{v_{h,t+1}} (-\varepsilon_{t+1})\, g(\varepsilon_{t+1})d\varepsilon_t + E_t\Delta_{t,t+1}h\eta\left(v_{h,t+1}\right) \tag{2}$$
$$-E_t\Delta_{t,t+1}\phi\left(v_{f,t+1}\right) f + E_t\Delta_{t,t+1}\left(1 - \phi\left(v_{f,t+1}\right) - \eta\left(v_{h,t+1}\right)\right)\left(v_{f,t+1} - f\right).$$

Let's subtract (1) from (2):

$$-w_t^I(v_{f,t}) = E_t\Delta_{t,t+1}\int_{-\infty}^{v_{f,t+1}} \left(w_{t+1}^I(\varepsilon_{t+1})\right) g(\varepsilon_{t+1})d\varepsilon_t - E_t\Delta_{t,t+1}\int_{-\infty}^{v_{h,t+1}} (-\varepsilon_{t+1})\, g(\varepsilon_{t+1})d\varepsilon_t + \Delta_{t,t+1}h\eta\left(v_{h,t+1}\right)$$
$$-E_t\Delta_{t,t+1}\eta\left(v_{h,t+1}\right)\left(-f + v_{f,t+1}\right) - E_t\Delta_{t,t+1}\left(1 - \phi\left(v_{f,t+1}\right)\right)w_{t+1}^I(v_{f,t+1}).$$

Thus:

$$w_t^I(v_{f,t}) = E_t\Delta_{t,t+1}\left(1 - \phi\left(v_{f,t+1}\right)\right)\left(w_{t+1}^I(v_{f,t+1}) - \tilde{w}_{t+1}^I\right) + E_t\Delta_{t,t+1}\eta\left(v_{h,t+1}\right)\left(-f + v_{f,t+1} - h - \Xi_{t+1}^e\right), \tag{3}$$

with $\tilde{w}_{t+1}^I = \dfrac{\displaystyle\int_{-\infty}^{v_{f,t+1}} \left(w_{t+1}^I(\varepsilon_{t+1})\right) g(\varepsilon_{t+1})d\varepsilon_t}{\left(1 - \phi\left(v_{f,t+1}\right)\right)}$. Notice the similarity between this equation and the efficient wage under collective bargaining (see equation 34 in the main text). The only difference between

both equations is that the efficient wage under individualistic bargaining has to take account of the gap between the wage of the marginal entrant and the wage of the average entrant (which, naturally, is zero under collective bargaining).

## 1.4 Efficiency and Nash Bargaining

We shall now verify whether the efficient wage (equation 3) can be replicated under standard Nash bargaining. An incumbent worker's value with a realization of the operating cost $\varepsilon$ is given by:

$$V_t^I\left(\varepsilon_t\right) = w_t^I\left(\varepsilon_t\right) + E_t\Delta_{t,t+1}\left(\left(1 - \phi_{t+1}\right)V_{t+1}^I + \phi_{t+1}V_{t+1}^U\right),$$

where $V^U$ is the value of unemployment. The fall-back position is given by (setting $b = 0$):

$$V_t^U = 0 + E_t\Delta_{t,t+1}\left(\eta_{t+1}V_{t+1}^E + \left(1 - \eta_{t+1}\right)V_{t+1}^U\right).$$

The firm's value for the worker is:

$$\Pi_t^I\left(\varepsilon_t\right) = \left(a_t - w_t^I\left(\varepsilon_t\right) - \varepsilon_t\right) + E_t\Delta_{t,t+1}\left(\left(1 - \phi_{t+1}\right)\Pi_{t+1}^I - \phi_{t+1}f\right).$$

Under disagreement, the firm's fallback position is equal to $-f$, since the firm must pay the firing costs in case of dismissal.

$$\tilde{\Pi}_t^I = -f.$$

The bargaining optimization problem reads as follows:

$$\Lambda_t = \left(V_t^I\left(\varepsilon_t\right) - \tilde{V}_t^I\right)^\gamma \left(\Pi_t^I\left(\varepsilon_t\right) - \tilde{\Pi}_t^I\right)^{1-\gamma}, \tag{4}$$

where $\gamma$ is the worker's bargaining power. After maximizing 4 with respect to $w_t^I\left(\varepsilon_t\right)$, we obtain:

$$\begin{aligned} w_t^I\left(\varepsilon_t\right) &= \gamma\left(a_t - \varepsilon_t + f + E_t\Delta_{t,t+1}\left(\left(1 - \phi_{t+1}\right)\Pi_{t+1}^I - \phi_{t+1}f\right)\right) + \\ &\quad \left(1 - \gamma\right)\left(-E_t\Delta_{t,t+1}\left(\left(1 - \phi_{t+1}\right)V_{t+1}^I - \left(1 - \phi_{t+1} - \eta_{t+1}\right)V_{t+1}^U - \eta_{t+1}V_{t+1}^E\right)\right). \end{aligned}$$

Similarly for entrants:

$$\begin{aligned} w_t^E\left(\varepsilon_t\right) &= \gamma\left(a_t - \varepsilon_t - h + E_t\Delta_{t,t+1}\left(\left(1 - \phi_{t+1}\right)\Pi_{t+1}^I - \phi_{t+1}f\right)\right) + \\ &\quad \left(1 - \gamma\right)\left(-E_t\Delta_{t,t+1}\left(\left(1 - \phi_{t+1}\right)V_{t+1}^I - \left(1 - \phi_{t+1} - \eta_{t+1}\right)V_{t+1}^U - \eta_{t+1}V_{t+1}^E\right)\right). \end{aligned}$$

Thus, the marginal incumbent worker earns:

$$\begin{aligned} w_t^I\left(v_{f,t}\right) &= \gamma\left(a_t - v_{f,t} + f + E_t\Delta_{t,t+1}\left(\left(1 - \phi_{t+1}\right)\Pi_{t+1}^I - \phi_{t+1}f\right)\right) \\ &\quad + \left(1 - \gamma\right)\left(-E_t\Delta_{t,t+1}\left(\left(1 - \phi_{t+1}\right)V_{t+1}^I - \left(1 - \phi_{t+1} - \eta_{t+1}\right)V_{t+1}^U - \eta_{t+1}V_{t+1}^E\right).\right) \end{aligned}$$

Noting that the firing threshold is defined as $v_{f,t} = a_t - w_t^I\left(v_{f,t}\right) + f + E_t\Delta_{t,t+1}\left(\left(1 - \phi_{t+1}\right)\Pi_{t+1}^I - \phi_{t+1}f\right)$, this equation can be simplified to:

$$w_t^I\left(v_{f,t}^{CE}\right) = -E_t\Delta_{t,t+1}\left(\left(1 - \phi_{t+1}\right)V_{t+1}^I - \left(1 - \phi_{t+1} - \eta_{t+1}\right)V_{t+1}^U - \eta_{t+1}V_{t+1}^E\right)$$

After substituting this into the efficient wage, we obtain

$$-E_t\Delta_{t,t+1}\left(\left(1-\phi_{t+1}\right)V_{t+1}^I-\left(1-\phi_{t+1}-\eta_{t+1}\right)V_{t+1}^U-\eta_{t+1}V_{t+1}^E\right) \tag{5}$$
$$=\ E_t\Delta_{t,t+1}\left(1-\phi\left(v_{f,t+1}\right)\right)\left(w_{t+1}^I(v_{f,t+1})-\tilde{w}_{t+1}^I\right)+E_t\Delta_{t,t+1}\eta\left(v_{h,t+1}^{PE}\right)\left(-f+v_{f,t+1}-h-\Xi_{t+1}^e\right).$$

Further substitution of workers' present value functions yields:

$$-E_t\Delta_{t,t+1}\left(1-\phi_{t+1}\right)\left[\frac{\displaystyle\int_{-\infty}^{v_{f,t+1}}\left(w_{t+1}^I(\varepsilon_{t+1})\right)g(\varepsilon_{t+1})d\varepsilon_{t+1}}{1-\phi_{t+1}}+\delta\left(\left(1-\phi_{t+2}\right)V_{t+2}^I+\phi_{t+2}V_{t+2}^U\right)\right]+$$

$$+E_t\left(1-\phi_{t+1}-\eta_{t+1}\right)\left[\Delta_{t,t+2}\left(\eta_{t+2}V_{t+2}^E+\left(1-\eta_{t+2}\right)V_{t+2}^U\right)\right]+$$

$$+E_t\Delta_{t,t+1}\eta_{t+1}\left[\frac{\displaystyle\int_{-\infty}^{v_{h,t+1}}\left(w_{t+1}^E(\varepsilon_{t+1})\right)g(\varepsilon_{t+1})d\varepsilon_{t+1}}{\eta_{t+1}}+\Delta_{t+1,t+2}\left(\left(1-\phi_{t+2}\right)V_{t+2}^I+\phi_{t+2}V_{t+2}^U\right)\right]=$$

$$=\ E_t\Delta_{t,t+1}\left(1-\phi\left(v_{f,t+1}\right)\right)\left(w_{t+1}^I(v_{f,t+1})-\tilde{w}_{t+1}^I\right)+\eta\left(v_{h,t+1}^{PE}\right)\left(-f+v_{f,t+1}-h-\Xi_{t+1}^e\right)$$

Simplifying yields:

$$E_t\Delta_{t,t+1}\left(1-\phi_{t+1}-\eta_{t+1}\right)\left[\Delta_{t+1,t+2}\left(-\left(1-\phi_{t+2}\right)V_{t+2}^I+\left(1-\eta_{t+2}-\phi_{t+2}\right)V_{t+2}^U\eta_{t+2}V_{t+2}^E\right)\right]$$
$$=\ E_t\Delta_{t,t+1}\left(1-\phi\left(v_{f,t+1}\right)\right)w_{t+1}^I(v_{f,t+1})+E_t\Delta_{t,t+1}\eta\left(v_{h,t+1}^{PE}\right)\left(-f+v_{f,t+1}-h-\Xi_{t+1}^e\right)$$
$$-E_t\Delta_{t,t+1}\eta\left(v_{h,t+1}^{PE}\right)\tilde{w}_{t+1}^E$$

which, after substituting for $w_{t+1}^E\left(v_{h,t+1}\right)$, can then be written as:

$$E_t\Delta_{t,t+1}\left(1-\phi_{t+1}-\eta_{t+1}\right)w_{t+1}^E\left(v_{h,t+1}\right)$$
$$=\ E_t\Delta_{t,t+1}\left(1-\phi\left(v_{f,t+1}\right)\right)w_{t+1}^I(v_{f,t+1})+E_t\Delta_{t,t+1}\eta\left(v_{h,t+1}\right)\left(-f+v_{f,t+1}-h-\Xi_{t+1}^e-\tilde{w}_{t+1}^E\right) \tag{6}$$

The wages corresponding to the hiring and firing thresholds are the same (i.e., $w_t^I\left(v_{f,t}\right)=w_t^E\left(v_{h,t}\right)$; this is so since the threat-points are the same). After collecting for wages and and using the relation $-f+v_{f,t}-h=v_{h,t}$, we obtain:

$$E_t\Delta_{t,t+1}\eta_{t+1}\left(\tilde{w}_{t+1}^E-w_{t+1}^E\left(v_{h,t+1}\right)\right)=E_t\Delta_{t,t+1}\eta\left(v_{h,t+1}\right)\left(v_{h,t+1}-\Xi_{t+1}^e\right). \tag{7}$$

Finally, notice that the difference between the two wages can be written as:

$$E_t\Delta_{t,t+1}\left(\tilde{w}_t^E-w_{t+1}^E\left(v_{h,t+1}^{CE}\right)\right)=E_t\Delta_{t,t+1}\left(\gamma\left(v_{h,t}^{CE}-\Xi_t^e\right)\right).$$

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Substituting this in equation (7) yields the equation governing the difference between the competitive and the planner economy:

$$E_t \Delta_{t,t+1} \gamma \left( v_{h,t+1}^{CE} - \Xi_{t+1}^e \right) = E_t \Delta_{t,t+1} \left( v_{h,t+1} - \Xi_{t+1}^e \right) \tag{8}$$

Given that $E_t \Delta_{t,t+1} v_{h,t+1}^{CE} < E_t \Delta_{t,t+1} \Xi_{t+1}^e$, the last condition can only holds if $\gamma = 1$.

The condition under which the efficient solution can be decentralized, namely under $\gamma = 1$, shall be interpreted as a limiting case in which the bargaining process prevents firms' externalities from affecting the wage process. By giving firms no role in the bargaining process and thus making workers' reservation wage decisive for the hiring and firing thresholds, efficiency can be restored. The case of unitary bargaining is clearly an unrealistic one.[1] Likely, bargaining would not take place if the worker would appropriate the whole surplus. We therefore conclude that decentralizing efficiency is also not possible under individualistic bargaining.

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[1]Nash bargaining restricts the bargaining power to be between zero and one ($0 < \gamma < 1$).

# 2 Appendix B. Efficient Wages and Firing Costs

## 2.1 Proof Lemma 1.

We need to derive the derivative of wages with respect to productivity shocks. The efficient wage reads as follows:

$$w_t^* = E_t \Delta_{t,t+1} \eta \left( v_{h,t+1}^{CE} \right) \left[ v_{h,t+1}^{CE} - \Xi_t^e (v_{h,t+1}^{CE}) \right].$$

After opening up the integrals, it reads as follows:

$$w_t^* = E_t \Delta_{t,t+1} \left[ v_{h,t+1}^{CE} \int_{-\infty}^{v_{h,t+1}^{CE}} g\left(\varepsilon_{t+1}\right) d\varepsilon_{t+1} - \int_{-\infty}^{v_{h,t+1}^{CE}} \varepsilon_{t+1} g\left(\varepsilon_{t+1}\right) d\varepsilon_{t+1} \right].$$

This can be further simplified as:

$$w_t^* = E_t \Delta_{t,t+1} \left[ \int_{-\infty}^{v_{h,t+1}^{CE}} \left( v_{h,t+1}^{CE} - \varepsilon_{t+1} \right) g\left(\varepsilon_{t+1}\right) d\varepsilon_{t+1} \right].$$

We can now obtain the derivative of wages with respect to future productivity shocks:

$$
\begin{aligned}
\frac{\partial w_t^*}{\partial E_t a_{t+1}} &= E_t \Delta_{t,t+1} \left[ g\left(v_{h,t+1}^{CE}\right) \frac{\partial v_{h,t+1}^{CE}}{\partial a_{t+1}} v_{h,t+1}^{CE} + \int_{-\infty}^{v_{h,t+1}^{CE}} g\left(\varepsilon_{t+1}\right) d\varepsilon_{t+1} \frac{\partial v_{h,t+1}^{CE}}{\partial a_{t+1}} - g\left(v_{h,t+1}^{CE}\right) \frac{\partial v_{h,t+1}^{CE}}{\partial a_{t+1}} v_{h,t+1}^{CE} \right] \\
&= E_t \Delta_{t,t+1} \left[ \eta \left( v_{h,t+1}^{CE} \right) \frac{\partial v_{h,t+1}^{CE}}{\partial a_{t+1}} \right].
\end{aligned}
$$

The expression for the hiring threshold in the competitive economy, $E_t v_{h,t+1}^{CE}$, evaluated at the efficient wage reads as follows:

$$
\begin{aligned}
E_t v_{h,t+1}^{CE} &= E_t \left( a_{t+1} - w_{t+1}^* - \Delta_{t,t+2} \left[ \phi \left( v_{f,t+2}^{CE} \right) f + \left( 1 - \phi \left( v_{f,t+2}^{CE} \right) \right) \Xi_{t+2}^i (v_{f,t+2}) \right] \right) + \\
&\quad + E_t \mu_{t+2}^{CE} \Delta_{t,t+2} \left( 1 - \phi \left( v_{f,t+2}^{CE} \right) \right) - h.
\end{aligned}
$$

Thus: $\frac{\partial E_t v_{h,t+1}^{CE}}{\partial E_t a_{t+1}} = 1 - \frac{\partial E_t w_{t+1}^*}{\partial E_t a_{t+1}}$ Since the efficient wage does not depend on contemporaneous productivity, $\frac{\partial E_t v_{h,t+1}^{CE}}{\partial E_t a_{t+1}} = 1$.

Thus:

$$\frac{\partial w_t^*}{\partial E_t a_{t+1}} = E_t \Delta_{t,t+1} \eta \left( v_{h,t+1}^{CE} \right) > 0.$$

Dividing by the wage and multiplying with the future productivity yields equation 37 in the main text.

## 2.2 Proof Lemma 3

The elasticity of the wage with respect to aggregate productivity reads as follows:

$$\xi_{w_t^*,a_{t+1}} = E_t \frac{a_{t+1}}{\Delta_{t,t+1}^{CE}\left[v_{h,t+1}^{CE} - \Xi_t^e(v_{h,t+1}^{CE})\right]}.$$

where we have cancelled $\eta$ and substituted $\mu$. The derivative of the wage elasticity with respect to firing costs reads as:

$$\frac{\partial \xi_{w_t^*,a_{t+1}}}{\partial f} = -E_t \Delta_{t,t+1}^{CE} \frac{\partial\left[v_{h,t+1}^{CE} - \Xi_t^e(v_{h,t+1}^{CE})\right]}{\partial v_{h,t+1}^{CE}} \frac{a_{t+1}}{\left[v_{h,t+1}^{CE} - \Xi_t^e(v_{h,t+1}^{CE})\right]^2} \frac{\partial v_{h,t+1}^{CE}}{\partial f}. \qquad (9)$$

The first term is the difference between marginal operating cost and the average operating cost, which is increasing in the threshold, i.e. the term is positive. The second term, $\frac{a_{t+1}}{\left[v_{h,t+1}^{CE} - \Xi_t^e(v_{h,t+1}^{CE})\right]^2}$, is also positive by construction.

To determine the sign of $\frac{\partial \xi_{w_t^*,a_{t+1}}}{\partial f}$ it remains to assess the sign of the term $\frac{\partial v_{h,t+1}^{CE}}{\partial f}$. Recall that $E_t\mu_{t+1}^{CE} = E_t v_{h,t+1}^{CE} + h$. It follows that $\frac{\partial E_t v_{h,t+1}^{CE}}{\partial f} = \frac{\partial E_t \mu_{t+1}^{CE}}{\partial f}$. For convenience we rewrite $E_t\mu_{t+1}^{CE}$, evaluated at the efficient wage, as follows:

$$
\begin{aligned}
E_t\mu_{t+1}^{CE} &= E_t\left(a_{t+1} - w_{t+1}^* - \Delta_{t+1,t+2}\left[\phi\left(v_{f,t+2}^{CE}\right)f + \left(1 - \phi\left(v_{f,t+2}^{CE}\right)\right)\Xi_t^i(v_{f,t+2})\right]\right) + \\
&\quad E_t\mu_{t+2}^{CE}\Delta_{t+1,t+2}\left(1 - \phi\left(v_{f,t+2}^{CE}\right)\right).
\end{aligned}
$$

Using the integral signs:

$$E_t\mu_{t+1}^{CE} = E_t\left(a_{t+1} - w_{t+1}^* - \Delta_{t+1,t+2}\left[\int_{v_{f,t+2}^{CE}}^{\infty} fg(\varepsilon_{t+2})d\varepsilon_{t+2} + \int_{-\infty}^{v_{f,t+2}^{CE}}\left(\varepsilon_{t+2} - \mu_{t+2}^{CE}\right)g(\varepsilon_{t+2})d\varepsilon_{t+2}\right]\right)$$

The derivative of $E_t\mu_{t+1}^{CE}$ with respect to $f$ is:

$$
\begin{aligned}
\frac{\partial E_t\mu_{t+1}^{CE}}{\partial f} &= -\frac{\partial E_t w_{t+1}^*}{\partial f} - E_t\Delta_{t+1,t+2}\left[\int_{v_{f,t+2}^{CE}}^{\infty} g(\varepsilon_{t+2})d\varepsilon_{t+2} - \left(-f + v_{f,t+2}^{CE} - \mu_{t+2}^{CE}\right)g(v_{f,t+2}^{CE})\frac{\partial v_{f,t+2}^{CE}}{\partial f}\right] + \\
&\quad + E_t\Delta_{t+1,t+2}\frac{\partial \mu_{t+2}^{CE}}{\partial f}\int_{-\infty}^{v_{f,t+2}^{CE}} g(\varepsilon_{t+2})d\varepsilon_{t+2} = \\
&= -\frac{\partial E_t w_{t+1}^*}{\partial f} \\
&\quad - E_t\Delta_{t+1,t+2}\int_{v_{f,t+2}^{CE}}^{\infty} g(\varepsilon_{t+2})d\varepsilon_{t+2} + E_t\Delta_{t+1,t+2}\frac{\partial \mu_{t+2}^{CE}}{\partial f}\int_{-\infty}^{v_{f,t+2}^{CE}} g(\varepsilon_{t+2})d\varepsilon_{t+2}, \qquad (10)
\end{aligned}
$$

because $\mu_{t+2}^{CE} = v_{f,t+2}^{CE} - f$. Expected future wages are:

$$E_t w_{t+1}^* = E_t\Delta_{t+1,t+2}\left[v_{h,t+2}^{CE}\int_{-\infty}^{v_{h,t+2}^{CE}} g\left(\varepsilon_{t+2}\right)d\varepsilon_{t+2} - \int_{-\infty}^{v_{h,t+2}^{CE}}\varepsilon_{t+2}g\left(\varepsilon_{t+2}\right)d\varepsilon_{t+2}\right].$$

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The derivative of $E_t w^*_{t+1}$ with respect to the firing costs is:

$$
\begin{aligned}
\frac{\partial E_t w^*_{t+1}}{\partial f} &= E_t \Delta_{t+1,t+2} \left[ g\left(v^{CE}_{h,t+2}\right) \frac{\partial v^{CE}_{h,t+2}}{\partial f} v^{CE}_{h,t+2} + \int_{-\infty}^{v^{CE}_{h,t+2}} g\left(\varepsilon_{t+2}\right) d\varepsilon_{t+2} \frac{\partial v^{CE}_{h,t+2}}{\partial f} \right] + \\
&\quad + E_t \Delta_{t+1,t+2} \left[ -g\left(v^{CE}_{h,t+2}\right) \frac{\partial v^{CE}_{h,t+2}}{\partial f} v^{CE}_{h,t+2} \right] \\
&= E_t \Delta_{t+1,t+2} \left[ \eta\left(v^{CE}_{h,t+2}\right) \frac{\partial v^{CE}_{h,t+2}}{\partial f} \right] = E_t \Delta_{t+1,t+2} \left[ \eta\left(v^{CE}_{h,t+2}\right) \frac{\partial E_t \mu^{CE}_{t+1}}{\partial f} \right].
\end{aligned}
$$

Let's substitute this expression in equation 10.

$$
\begin{aligned}
\frac{\partial E_t \mu^{CE}_{t+1}}{\partial f} &= -E_t \Delta_{t+1,t+2} \left[ \eta\left(v^{CE}_{h,t+2}\right) \frac{\partial E_t \mu^{CE}_{t+2}}{\partial f} \right] - \\
&\quad - E_t \Delta_{t+1,t+2} \int_{v^{CE}_{f,t+2}}^{\infty} g(\varepsilon_{t+2}) d\varepsilon_{t+2} + E_t \Delta_{t+1,t+2} \frac{\partial \mu^{CE}_{t+2}}{\partial f} \int_{-\infty}^{v^{CE}_{f,t+2}} g(\varepsilon_{t+2}) d\varepsilon_{t+2} \\
&= -E_t \Delta_{t+1,t+2} \phi\left(v^{CE}_{f,t+2}\right) + E_t \Delta_{t+1,t+2} \frac{\partial \mu^{CE}_{t+2}}{\partial f} \left(1 - \phi\left(v^{CE}_{f,t+2}\right) - \eta\left(v^{CE}_{h,t+2}\right)\right).
\end{aligned}
$$

We can write the last expression as an infinite sum:

$$
\frac{\partial E_t \mu^{CE}_{t+1}}{\partial f} = E_t \sum_{i=t+2}^{\infty} -\Delta_{t+1,i} \phi\left(v^{CE}_{f,i}\right) \prod_{j=t+1}^{i-1} \frac{\left(1 - \phi\left(v^{CE}_{f,j}\right) - \eta\left(v^{CE}_{h,j}\right)\right)}{\left(1 - \phi\left(v^{CE}_{f,t+1}\right) - \eta\left(v^{CE}_{h,t+1}\right)\right)}.
$$

We know that $\left(1 - \phi\left(v^{CE}_{f,t+2}\right) - \eta\left(v^{CE}_{h,t+2}\right)\right) > 0$ and that $-E_t \Delta_{t+1,t+2} \phi\left(v^{CE}_{f,t+2}\right) < 0$. Thus: $\frac{\partial E_t \mu^{CE}_{t+1}}{\partial f} < 0$. It follows that $\frac{\partial E_t v^{CE}_{h,t+1}}{\partial f} < 0$ and that:

$$
\frac{\partial \xi_{w^*_t, a_{t+1}}}{\partial f} = -E_t \Delta^{CE}_{t,t+1} \frac{\partial \left[v^{CE}_{h,t+1} - \Xi^e_t(v^{CE}_{h,t+1})\right]}{\partial v^{CE}_{h,t+1}} \frac{a_{t+1}}{\left[v^{CE}_{h,t+1} - \Xi^e_t(v^{CE}_{h,t+1})\right]^2} \frac{\partial v^{CE}_{h,t+1}}{\partial f} > 0.
$$