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**Four variations on fair wages and  
the Phillips curve**

**by Andrea Vaona**

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Keywords: efficiency wages, money growth, long-run Phillips curve, trend inflation, wage staggering.

JEL classification: E3, E20, E40, E50.

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# Four variations on fair wages and the Phillips curve

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## Abstract

The present paper explores the connection between inflation and unemployment in different models with fair wages both in the short and in the long runs. Under customary assumptions regarding the sign of the parameters of the effort function, more inflation lowers the unemployment rate, though to a declining extent. This is because firms respond to inflation - that spurs effort by decreasing the reference wage - by increasing employment, so to maintain the effort level constant, as implied by the Solow condition. Under wage staggering this effect is stronger because wage dispersion magnifies the impact of inflation on effort. A stronger effect of inflation on unemployment is also produced under varying as opposed to fixed capital, given that in the former case the boom produced by a monetary expansion is reinforced by an increase in investment. Therefore, we provide a new theoretical foundation for recent empirical contributions finding negative long- and short-run effects of inflation on unemployment.

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# 1 Introduction

The economic literature has recently witnessed a flourishing of contributions nesting an efficiency wages framework into business cycle models. Earlier models were proposed within the real business cycle (RBC) realm. Danthine and Donaldson (1990), for instance, showed that efficiency wages within a RBC model can produce structural unemployment, but not wage stickiness over the economic cycle. With difference to Danthine and Donaldson (1990), which focused on a gift exchange model, Uhlig and Xu (1995) and Gomme (1999) adopted a shirking model. However, in a rather similar way, they found that wages tend to be too volatile and employment not enough so over the cycle. In Kiley (1997) efficiency wages generate completely a-cyclical real wages, but not a greater endogenous price stickiness, because the a-cyclical real-wage requires countercyclical effort and hence a procyclical marginal cost.

Collard and de la Croix (2000) showed that, once including past compensations into the reference wage, an efficiency wages/RBC model can replicate wage acyclicity. Along similar lines, Danthine and Kurmann (2004) proposed a model combining efficiency wages of the gift exchange variety - also termed fair wages - with sticky prices, showing that it can well account for the low correlation between wages and employment, also displaying a greater internal propagation of monetary shocks than standard New Keynesian models.

Alexopoulos (2004, 2006, 2007) developed a model in which shirkers are not dismissed once detected. They, instead, forgo an increase in compensation. Under these assumptions it was showed that an efficiency wage model can well replicate empirical evidence regarding the response of the economic system to technological, fiscal and monetary shocks.

The present paper, instead, focuses on the long-run and short-run implications of efficiency wages for the connection between unemployment and inflation under trend money growth within a dynamic general equilibrium

framework. In so doing, we extend a literature that so far investigated the long-run and, to a lesser extent, the short-run effects of money growth by recurring only to models with wage/price stickiness. Pioneering contributions on this issue were King and Wolman (1996) and Ascari (1998). The former study considered a model with a shopping time technology and it obtained a number of different results, among which there is that long-run inflation reduces firms' markup, boosting the level of output. Ascari (1998), instead, showed that in wage-staggering models money can have considerable negative non-superneutralities once not considering restrictively simple utility and production functions. Deveraux and Yetman (2002) focused on a menu cost model. An analysis of dynamic general equilibrium models under different contract schemes in presence of trend inflation was offered in Ascari (2004). Graham and Snower (2004), instead, examined the microeconomic mechanisms underlying this class of models. In presence of Taylor wage staggering, in a monopolistically competitive labour market, they highlighted three channels through which inflation affects output: employment cycling, labour supply smoothing and time discounting. The first one consists in firms continuously shifting labour demand from one cohort to the other according to their real wage. Given that different labour kinds are imperfect substitutes, this generates inefficiencies and it tends to create a negative inflation-output nexus. The second one is that households demand a higher wage in presence of employment cycling given that they would prefer a smoother working time. This decreases labor supply and aggregate output. Finally under time discounting the contract wage depends more on the current (lower) level of prices than on the future (higher) level of prices and, therefore - over the contract period - the real wage will be lower the greater is the inflation rate, spurring labour demand and aggregate output. The time discounting effect dominates at lower inflation rates, while the other two effects at higher inflation rates, producing a hump-shaped long-run Phillips curve. The ultimate goal of Graham and Snower (2004) is questioning the customary assumption

to identify aggregate demand and supply shocks, namely that the former ones would be temporary and the latter ones not so. As a consequence also the concept of the NAIRU would be unsuitable for a fruitful investigation of the dynamics of the unemployment rate.

Graham and Snower (2004) was extended in a number of different directions. Graham and Snower (2008) showed that under hyperbolic time discounting positive money non-superneutralities are more sizeable than under exponential discounting. Vaona and Snower (2007, 2008) showed how the shape of the long-run Phillips curve depends on the shape of the production function. Finally, Vaona (2010) extended the model by Graham and Snower (2004) from the inflation-output domain to the inflation-real growth one.

We here propose four variations on the theme of efficiency wages and the Phillips curve. In the first one, efficiency wages of the gift exchange variety are coupled with trend money growth, once specifying the reference wage as a function of the unemployment rate, the current individual real wage, the current aggregate real wage and of the current real value of the past aggregate wage. After Becker (1996), this specification has been termed in the literature as social norm case.

In our second variation, the reference wage is not a function of the current real value of the past aggregate wage, rather of that of the past individual one, as in the personal norm case. Our third model combines Taylor wage stickiness with fair wages of the social norm variety. In this setting, positive money non-superneutralities turn out to be stronger than under flexible wages<sup>1</sup>. Finally, the fourth variation extends the first one by considering varying instead of fixed capital.

With difference to Graham and Snower (2004, 2008) we provide not only a long-run analysis but also a short-run one, because we think that, even if one cannot identify demand and supply shocks on the basis of their tran-

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<sup>1</sup>Also Fan (2007) proposed to merge sticky and efficiency wages, but not in an intertemporal optimization framework as we do here.

sience, it will be interesting to investigate how the economic system reacts to temporary monetary shocks. In other words, transition dynamics does not lose interest.

Our results can offer a new theoretical foundation for the results obtained in various recent contributions, such as Karanassou et al. (2005, 2008a, 2008b) - surveyed in Karanassou et al. (2010). An estimated value of the long-run elasticity of inflation with respect to unemployment about  $-3.5$  was there explained by resorting to frictional growth, namely the interplay between frictions (lagged adjustments) and growth in economic variables. In the light of our models, this result can be also interpreted as the outcome of efficiency wages mechanisms as explained below.

The rest of this paper is structured as follows. The next section introduces the households' problem and the government budget constraint, which are common to most of the models here presented. Afterwards, we will introduce the firms' problem for the social norm case with flexible wages, the personal norm case under flexible wages, the social norm case with wage staggering and the social norm case with varying capital. In all the cases, we show what is the impact of money growth on both the unemployment and the inflation rates both in the short and in the long runs and we discuss the plausibility of our models in order to detect our preferred ones. The last section concludes.

## **2 The households' problem and the government budget constraint**

We follow Danthine and Kurmann (2004), by supposing the economy to be populated by a continuum of households normalized to 1, each composed by a continuum of individuals also normalized to 1. Households maximize their discounted utility



$$\max_{\{c_{t+i}(h), B_{t+i}(h), M_{t+i}(h), e_{t+i}(h)\}} \sum_{i=0}^{\infty} \beta^{t+i} E \left( U \left\{ c_{t+i}(h), n_{t+i}(h) G[e_{t+i}(h)], V \left[ \frac{M_{t+i}(h)}{P_{t+i}} \right] \right\} \right) \quad (1)$$

subject to a series of income constraints

$$c_{t+i}(h) = \frac{W_{t+i}(h)}{P_{t+i}} n_{t+i}(h) + \frac{T_{t+i}(h)}{P_{t+i}} - \frac{M_{t+i}(h)}{P_{t+i}} + \frac{M_{t+i-1}(h)}{P_{t+i}} - \frac{B_{t+i}(h)}{P_{t+i}} + \frac{B_{t+i-1}(h)}{P_{t+i}} i_{t+i} + q_{t+i}(h)$$

where  $\beta$  is the discount factor,  $E$  is the expectation operator,  $U$  is the utility function,  $c_{t+i}(h)$  is consumption of household  $h$  at time  $t+i$ ,  $B_{t+i}(h)$  are the household's bond holdings,  $i_{t+i}$  is the nominal interest rate,  $n_{t+i}(h)$  is the fraction of employed individuals within the household,  $G[e_{t+i}(h)]$  is the disutility of effort -  $e_{t+i}(h)$  - of the typical working family member,  $V \left[ \frac{M_{t+i}(h)}{P_{t+i}} \right]$  is the utility arising from nominal money balances -  $M_{t+i}(h)$  - over the price level -  $P_{t+i}$ .  $W_{t+i}(h)$  and  $T_{t+i}(h)$  are the household's nominal wage income and government transfers respectively. Finally,  $q_{t+i}(h)$  are profits that households receive from firms.

In this framework, households, and not individuals, make all the decisions regarding consumption, bond holdings, real money balances and effort. Individuals are identical ex-ante, but not ex-post, given that some of them are employed - being randomly and costlessly matched with firms independently from time - and some other are unemployed. The fraction of the unemployed is the same across all the families, and so their ex-post homogeneity is preserved.

Note that in our model no utility arises from leisure, therefore individual agents inelastically supply one unit of time for either work or unemployment related activities. Furthermore, after Akerlof (1982), workers, though disliking effort, will be ready to exert it as a gift to the firm if they receive some other gift in exchange, such as a real compensation above some reference level.

Similarly to Danthine and Kurmann (2004), on the basis of the empirical evidence produced by Bewley (1998), we specify the effort function,  $G[e_{t+i}(h)]$ , as follows

$$G[e_{t+i}(h)] = \left\{ e_{t+i}(h) - \left[ \begin{aligned} &\phi_0 + \phi_1 \log \frac{W_{t+i}(h)}{P_{t+i}} + \phi_2 \log u_{t+i}(h) + \\ &\quad + \phi_3 \log \frac{W_{t+i}}{P_{t+i}} + \phi_4 \log \frac{W_{t+i-1}(h)}{P_{t+i}} \end{aligned} \right] \right\}^2$$

in the personal norm case and as follows

$$G[e_{t+i}(h)] = \left\{ e_{t+i}(h) - \left[ \begin{aligned} &\phi_0 + \phi_1 \log \frac{W_{t+i}(h)}{P_{t+i}} + \phi_2 \log u_{t+i}(h) + \\ &\quad + \phi_3 \log \frac{W_{t+i}}{P_{t+i}} + \phi_4 \log \frac{W_{t+i-1}}{P_{t+i}} \end{aligned} \right] \right\}^2$$

in the social norm case<sup>2</sup>.  $W_{t+i}$  is the aggregate nominal wage and  $u_{t+i}(h) = 1 - n_{t+i}(h)$  is the unemployment rate. Note that, with difference to Danthine and Kurmann (2004), the nominal (either individual or aggregate) wage at time  $t + i - 1$  is assessed at the prices of time  $t + i$ . This assumption does not entail any money illusion. On the contrary, its underlying intuition is that households are aware of the damages that inflation can produce to their living standards and so they are ready to exchange more effort for a pay policy that allows nominal wages to keep up with inflation. More briefly, a higher inflation rate reduces the reference wage.

Throughout the paper, similarly to Danthine and Kurmann (2004), we assume  $\phi_1, \phi_2 > 0$  and  $\phi_3, \phi_4 < 0$ . In words a higher household's real wage and a higher unemployment rate induce more effort. On the other hand, a higher reference wage - be it due to either a higher aggregate wage or a higher real value of past compensation - depresses effort.

Note that, under the hypothesis of an additively separable utility function,

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<sup>2</sup>An alternative approach to the effort function is the one pursued by Campbell (2006, 2008a and 2008b), which entails a more general functional specification to be linearized at a later stage. However, calibration is less straightforward in this context and economic theorizing is usually followed by a number of numerical exercises where parameters and results display a somewhat large variation. For this reason we prefer to follow Danthine and Kurmann (2004).

utility maximization implies that

$$G' [e_{t+i}(h)] = 0 \quad (2)$$

and, therefore, that in the personal norm case

$$e_{t+i}(h) = \phi_0 + \phi_1 \log \frac{W_{t+i}(h)}{P_{t+i}} + \phi_2 \log u_{t+i} + \phi_3 \log \frac{W_{t+i}}{P_{t+i}} + \phi_4 \log \frac{W_{t+i-1}(h)}{P_{t+i}} \quad (3)$$

and in the social norm case

$$e_{t+i}(h) = \phi_0 + \phi_1 \log \frac{W_{t+i}(h)}{P_{t+i}} + \phi_2 \log u_{t+i} + \phi_3 \log \frac{W_{t+i}}{P_{t+i}} + \phi_4 \log \frac{W_{t+i-1}}{P_{t+i}} \quad (4)$$

Assuming that  $c_{t+i}(h)$  and  $\frac{M_{t+i}(h)}{P_{t+i}}$  enter (1) in logs, utility maximization implies

$$\frac{1}{c_{t+i}(h)} = E \left( \frac{i_{t+i}}{\pi_{t+i+1}} \frac{1}{c_{t+i+1}(h)} \beta \right) \quad (5)$$

$$\left( \frac{\mu_{t+i}}{\pi_{t+i}} \right)^{-1} = \frac{c_{t+i-1}(h)}{c_{t+i}(h)} \left( 1 - \frac{1}{i_{t+i}} \right) / \left( 1 - \frac{1}{i_{t+i-1}} \right) \quad (6)$$

The government rebates its seigniorage proceeds to households by means of lump-sum transfers,  $T_t(h)$ :

$$\int_0^1 \frac{T_t(h)}{P_t} dh = \int_0^1 \frac{M_t(h)}{P_t} dh - \int_0^1 \frac{M_{t-1}(h)}{P_t} dh$$

### 3 First variation: the social norm case

#### 3.1 The long-run

Firms in the perfectly competitive product market hire individuals belonging to the all the households to produce their output. Firms maximize their profits -  $P_{t+i}y_{t+i} - \int_{h=0}^1 W_{t+i}(h)n_{t+i}(h)dh$  - subject to their production function -  $y_{t+i} = \left[ \int_0^1 e_{t+i}(h)^{\frac{\theta_n-1}{\theta_n}} n_{t+i}(h)^{\frac{\theta_n-1}{\theta_n}} dh \right]^{\frac{\theta_n}{\theta_n-1}}$  - and to (4), by choosing  $n_{t+i}(h)$  and  $W_{t+i}(h)$ . Note that the production function displays decreasing marginal returns to each labour type and constant returns to scale.

The first order condition with respect to  $n_{t+i}(h)$  equates the marginal cost of labour to its marginal product. All households are symmetrical, so we can drop the  $h$  index and write<sup>3</sup>

$$\frac{W_{t+i}}{P_{t+i}} = \frac{y_{t+i}}{n_{t+i}} \quad (7)$$

whereas the first order condition with respect to  $\frac{W_{t+i}}{P_{t+i}}$  equates the marginal cost of rising the real wage to the benefit that this induces by increasing effort

$$\frac{W_{t+i}}{P_{t+i}} \frac{n_{t+i}}{y_{t+i}} = \frac{\phi_1}{e_{t+i}} \quad (8)$$

By substituting (7) into (8), one obtains the well known Solow condition

$$e_{t+i} = \phi_1 \quad (9)$$

Therefore, firms, maximizing their profits, demand the same effort to all households, across time and independently from the rate of inflation. Furthermore, (9) and the production function, under the condition of households'

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<sup>3</sup>Equation (7) implies that  $q_t(h) = 0$ .

symmetry, imply

$$\frac{W_{t+i}}{P_{t+i}} = \frac{y_{t+i}}{n_{t+i}} = \phi_1 \quad (10)$$

Substitute (9) and (10) into (4) and consider that trend inflation is equal to steady state money growth,  $\mu$ , to obtain

$$\log u = \frac{\phi_0 - \phi_1}{-\phi_2} + \frac{(\phi_1 + \phi_3 + \phi_4)}{-\phi_2} \log \phi_1 - \frac{\phi_4}{-\phi_2} \log \mu \quad (11)$$

which, together with our standard assumptions on the sign of  $\phi_4$  and  $\phi_2$  implies that the elasticity of the unemployment rate with respect to inflation is negative

$$\frac{d \log u}{d \log \mu} = \frac{\phi_4}{\phi_2} < 0 \quad (12)$$

The intuition underlying this result is the following. An increase in inflation produces a decrease in the reference wage, by reducing the current real value of the past compensation. This would spur effort, but the firms' optimal level of effort does not depend on inflation. As a consequence firms increase employment (and decrease unemployment) to keep the level of effort constant. Following the results by Karanassou et al. (2005, 2008a, 2008b), one could calibrate  $\frac{\phi_4}{\phi_2} \approx -0.29$ .

Note that this mechanism does not imply that hyperinflation will produce large decreases in unemployment. In order to understand this point we focus on the semielasticity of the unemployment rate with respect to the money growth rate. In our context, the advantage of the semi-elasticity versus the elasticity is that it is a measure of the reactiveness of the unemployment rate to absolute, and not percentage, changes in the money growth rate, mirroring, under this respect, the results provided by, among others, Ascari (1998, 2004) and Graham and Snower (2004, 2008). The semielasticity of the unemployment rate with respect to money growth is

$$\frac{d \log u}{d \mu} = \frac{\phi_4}{\phi_2} \frac{1}{\mu} < 0 \quad (13)$$

which is still negative, given that  $\mu \geq 1$ , but  $\lim_{\mu \rightarrow \infty} \frac{d \log u}{d \mu} = 0$ .

### 3.2 The short-run

In order to analyze the short run dynamics of the present economic model, consider first that the only steady state condition we imposed to obtain (11) is the equality of money growth and inflation. Out of steady state one can write (11) as  $\log u_{t+i} = \frac{\phi_0 - \phi_1}{-\phi_2} + \frac{(\phi_1 + \phi_3 + \phi_4)}{-\phi_2} \log \phi_1 - \frac{\phi_4}{-\phi_2} \log \pi_{t+i}$ . The other equations of the system are (5), (6), the aggregate resource constraint,  $y_t = c_t$ , the production function and the condition  $n_t = 1 - u_t$ . The equilibrium for this model is a sequence  $\{u_{t+i}, \pi_{t+i}, n_{t+i}, y_{t+i}, i_{t+i}, c_{t+i}\}$  satisfying households' utility maximization and firms' profit maximization.

This system of equations, after log-linearization around the steady state, can be expressed as a second order difference equation in inflation, which in its turn can be re-arranged to obtain the following system of first order difference equations

$$E(\hat{x}_{t+i+1}) = \left[ i_{ss} + \frac{\frac{u_{ss}}{n_{ss}} \phi_{\pi}}{\left(1 + \frac{u_{ss}}{n_{ss}} \phi_{\pi}\right)} \right] \hat{x}_{t+i} - \frac{\frac{u_{ss}}{n_{ss}} \phi_{\pi} i_{ss}}{\left(1 + \frac{u_{ss}}{n_{ss}} \phi_{\pi}\right)} \hat{\pi}_{t+i} \quad (14)$$

$$E(\hat{\pi}_{t+i+1}) = \hat{x}_{t+i} \quad (15)$$

where hats denote deviations from steady state,  $\phi_{\pi} \equiv -\frac{\phi_4}{\phi_2}$  and  $i_{ss}$ ,  $u_{ss}$  and  $n_{ss}$  are the steady state values of the nominal interest rate, of the unemployment rate and of the employment rate respectively. In order to investigate the stability of (14)-(15) we need to calibrate not only  $\frac{\phi_4}{\phi_2}$  as above, but also  $i_{ss}$ ,  $u_{ss}$  and  $n_{ss}$ . In order to do so we take as reference the post-second-world-war US time series and we set  $u_{ss} = 0.056$ ,  $n_{ss} = 1 - u_{ss}$  and  $i_{ss} = 1.02 * (1 + \mu)$ . We compute the roots of (14)-(15) for various values of trend inflation and the results are showed in Figure 1. As it is possible to see the system is always saddle-path stable, being one root outside and the other within the unit circle.

It is possible to wonder what are the effects of trend inflation on the stable arm of the system. The answer to this question is showed in Figure 2 where, following Shone (2001), different trajectories along the stable arm are projected on the  $\{\pi_t, \pi_{t+1}\}$  plane for trend inflation equal to 2%, 20% and 80%. The higher is trend inflation and the flatter is the stable arm. In other words, the higher is trend inflation and the sharper should inflation reductions be in order to achieve stability.

## 4 Second variation: the personal norm case

In the personal norm case, firms reckon that wage setting has intertemporal consequences. A wage increase will induce more effort in the first period by rising the household's real wage, but it will decrease effort in the second period by rising the household's reference wage. The firms' profit maximization problem will therefore be

$$\begin{aligned} & \max_{\{n_{t+i}(h), W_{t+i}(h)\}} \sum_{j=0}^{\infty} \Delta_{t,t+i} \left[ P_{t+i} y_{t+i} - \int_{h=0}^1 W_{t+i}(h) n_{t+i}(h) dh \right] \\ s.t. \quad y_{t+i} &= \left[ \int_0^1 e_{t+i}(h)^{\frac{\theta_n-1}{\theta_n}} n_{t+i}(h)^{\frac{\theta_n-1}{\theta_n}} dh \right]^{\frac{\theta_n}{\theta_n-1}} \\ e(h) &= \phi_0 + \phi_1 \log \frac{W_t(h)}{P_t} + \phi_2 \log u + \phi_3 \log \frac{W_t}{P_t} + \phi_4 \log \frac{W_{t-1}(h)}{P_t} \end{aligned}$$

where  $\Delta_{t,t+i}$  is the firm discount factor.

Hereafter, we drop the  $h$  index, being all the households symmetric. In the present setting (8) turns out to be

$$\Delta_{t,t+i} n_{t+i} = \Delta_{t,t+i} \frac{y_{t+i}}{e_{t+i}} \left( \frac{\phi_1}{\frac{W_{t+i}}{P_{t+i}}} \right) + E \left[ \Delta_{t,t+i+1} \frac{y_{t+i+1}}{e_{t+i+1}} \left( \frac{\phi_4}{\frac{W_{t+i}}{P_{t+i} \mu_{t+i+1}}} \right) \right]$$

In words, firms equate the discounted marginal cost of increasing the real wage to the sum of its discounted marginal revenues, which are composed by a positive effort effect in period  $t + i$  and a negative effort effect in period  $t + i + 1$ .

Consider that households and firms have access to a complete set of frictionless security markets, which, after Collard and de la Croix (2000) and Lucas (1978), implies that, at equilibrium,  $\Delta_{t,t+i}$  will be proportional to the discounted marginal value of wealth, which, assuming a logarithmic separable utility function in consumption and knowing that  $c_{t+i} = y_{t+i}$ , will be equal to  $\beta^{t+i}/y_{t+i}$ .

Substituting (7) – which holds also for the present model - into the previous equation and re-arranging one has

$$1 = \frac{\phi_1}{e_{t+i}} + \frac{\beta}{e_{t+i+1}} [\phi_4 \mu_{t+i+1}]$$

In steady state this implies a modified Solow condition

$$e = \phi_1 + \beta\phi_4\mu$$

Firms still demand the same effort level to all the households across time, but not independently from money growth, given that they now keep into account the discounted future effect of rising wages on effort. This produces a negative impact of trend inflation on effort. This is because trend inflation, equal to trend money growth, has a negative impact on marginal revenues, given that it has a positive but declining marginal effect on effort. Along the lines followed in the previous section it is easy to show that in the present model (13) turns out to be

$$\frac{d \log u}{d\mu} = \frac{\beta\phi_4}{\phi_2} - \frac{(\phi_1 + \phi_3 + \phi_4)}{\phi_2} \frac{\beta\phi_4}{(\phi_1 + \beta\phi_4\mu)} + \frac{\phi_4}{\phi_2\mu}$$

As a consequence  $\lim_{\mu \rightarrow \infty} \frac{d \log u}{d\mu} \neq 0$ , namely the effect of money growth on



inflation does not vanish at high inflation rates. This is unrealistic and we will not develop the present model any further.

## 5 Third variation: the social norm case with wage staggering

### 5.1 The long-run

In the present section we combine efficiency wages with Taylor wage staggering. In order to do so we assume households to belong to different cohorts, whose labour services are not perfect substitutes. This assumption is necessary because if different labour kinds were perfect substitutes, labour demand for cohorts whose wage is reset would go to zero. The wage is not set by households, as usual in wage staggering model, but by firms, as customary in fair wages models.

Note that, due to the existence of wage staggering, households belonging to different cohorts have different income levels. However, as customary, we assume they have access to complete asset markets, which allows them to consume all the same amount of the final good as implied by the first order condition with respect to consumption in problem (1).

Following Graham and Snower (2004), one can write the firms' profit maximization problem as follows

$$\begin{aligned}
 & \max_{\{n_{t+i}(h), W_{t+Nj}(h)\}} \sum_{j=0}^{\infty} \sum_{i=jN}^{(j+1)N-1} \Delta_{t,t+i} \left[ y_{t+i} - \int_{h=0}^1 \frac{W_{t+Nj}(h)}{P_{t+i}} n_{t+i}(h) dh \right] \\
 s.t. \quad y_{t+i} &= \left[ \int_0^1 e_{t+i}(h)^{\frac{\theta_n-1}{\theta_n}} n_{t+i}(h)^{\frac{\theta_n-1}{\theta_n}} dh \right]^{\frac{\theta_n}{\theta_n-1}} \\
 e(h) &= \phi_0 + \phi_1 \log \frac{W_{t+Nj}(h)}{P_{t+i}} + \phi_2 \log u_{t+i} + \phi_3 \log \frac{W_{t+i}}{P_{t+i}} + \phi_4 \log \frac{W_{t+i-1}}{P_{t+i}}
 \end{aligned}$$

where  $N$  is the contract length and  $\theta_n$  is the elasticity of substitution among different labour types. The first order conditions with respect to  $n_{t+j}(h)$  and  $W_{t+Nj}(h)$  and the recursiveness of the problem above imply

$$\frac{W_t(h)}{P_{t+i}} = y_{t+i}^{\frac{1}{\theta_n}} e_{t+i}(h)^{\frac{\theta_n-1}{\theta_n}} n_{t+i}(h)^{-\frac{1}{\theta_n}} \quad (16)$$

$$\begin{aligned} \sum_{i=0}^{N-1} \Delta_{t,t+i} n_{t+i}(h) &= \sum_{i=0}^{N-1} \Delta_{t,t+i} \left[ \int_0^1 e_{t+i}(h)^{\frac{\theta_n-1}{\theta_n}} n_{t+i}(h)^{\frac{\theta_n-1}{\theta_n}} dh \right]^{\frac{\theta_n}{\theta_n-1}-1} \cdot \\ &\cdot e_{t+i}(h)^{\frac{\theta_n-1}{\theta_n}-1} n_{t+i}(h)^{\frac{\theta_n-1}{\theta_n}} \left[ \frac{\phi_1}{W_t(h)/P_{t+i}} \right] \end{aligned} \quad (17)$$

Substituting (16) into (17) one obtains

$$\sum_{i=0}^{N-1} \Delta_{t,t+i} \frac{n_{t+i}(h)}{P_{t+i}} \left[ 1 - \frac{\phi_1}{e_{t+i}(h)} \right] = 0$$

which, given that  $\Delta_{t,t+i}, n_{t+i}(h), P_{t+i} > 0$ , leads to the Solow condition

$$e_{t+i}(h) = \phi_1 \quad (18)$$

Substituting (18) into  $y_{t+i} = \left[ \int_0^1 e_{t+i}(h)^{\frac{\theta_n-1}{\theta_n}} n_{t+i}(h)^{\frac{\theta_n-1}{\theta_n}} dh \right]^{\frac{\theta_n}{\theta_n-1}}$  one has

$$1 = \frac{1}{\phi_1} \left\{ \int_0^1 \left[ \frac{W_t(h)}{P_{t+i}} \right]^{1-\theta_n} dh \right\}^{\frac{1}{1-\theta_n}} \quad (19)$$

and in steady state

$$\frac{W^*}{P} = \phi_1 \left[ \frac{1}{N} \frac{1 - \mu^{N(\theta_n-1)}}{1 - \mu^{\theta_n-1}} \right]^{\frac{1}{\theta_n-1}}$$

where  $W^*$  is the reset wage.

Further, substitute the Solow condition into (4) and aggregate across

households keeping in mind that  $\frac{P_{t+i}}{P_{t+i-1}} = \mu$  to obtain

$$\phi_1 = \phi_0 + \phi_1 \sum_{j=0}^{N-1} \frac{\log\left(\frac{W^*}{P} \mu^{-j}\right)}{N} + \phi_2 \log u + (\phi_3 + \phi_4) \log \frac{W}{P} - \phi_4 \log \mu$$

and

$$\log u_{WS} = \frac{\phi_1 - \phi_0}{\phi_2} - \frac{\phi_1}{\phi_2} \log \left\{ \phi_1 \left[ \frac{1}{N} \frac{1 - \mu^{N(\theta_n-1)}}{1 - \mu^{\theta_n-1}} \right]^{\frac{1}{\theta_n-1}} \right\} + \quad (20)$$

$$+ \frac{\phi_1(N-1)}{\phi_2} \log \mu + \frac{\phi_4}{\phi_2} \log \mu - \frac{(\phi_3 + \phi_4)}{\phi_2} \log \frac{W}{P} \quad (21)$$

where the subscript  $WS$  stays for wage-staggering.

Subtracting (20) from (11) and taking the first order derivative with respect to  $\mu$ , one can compute the semielasticity of the percentage deviation of the unemployment rate under wage staggering from its level with flexible wages

$$\begin{aligned} \frac{\partial(\log u_{WS} - \log u)}{\partial \mu} &= -\frac{\phi_1}{-\phi_2} \frac{N}{[1 - \mu^{N(\theta_n-1)}]} \mu^{N(\theta_n-1)-1} + \\ &+ \frac{\phi_1}{-\phi_2} \frac{1}{[1 - \mu^{(\theta_n-1)}]} \mu^{(\theta_n-1)-1} - \frac{\phi_1}{-\phi_2} \frac{(N-1)}{2} \frac{1}{\mu} \end{aligned}$$

If  $\frac{\partial(\log u_{WS} - \log u_{ff})}{\partial \mu}$  is negative, it will mean that unemployment will be more responsive to absolute changes in money growth under wage staggering than under flexible wages. In order to explore this issue, it is necessary to check that the following condition holds<sup>4</sup>

$$\Omega(\mu) = -\frac{N}{[1 - \mu^{N(\theta_n-1)}]} \mu^{N(\theta_n-1)} + \frac{1}{[1 - \mu^{(\theta_n-1)}]} \mu^{(\theta_n-1)} - \frac{(N-1)}{2} > 0 \quad (22)$$

We do so for different values of  $N$  and  $\theta_n$  in Figures 3 and 4 respectively.

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<sup>4</sup>Recall that  $\frac{\phi_1}{-\phi_2} < 0$ .

In both the cases (22) is verified.

The intuition for this result is that wage staggering has two effects on effort. On the one hand, wage dispersion increases with inflation, leading to a higher ratio between the wage of the resetting cohort and the aggregate wage index. On the other hand, a higher inflation rate means that, over the contract period, the real wage of not-resetting cohorts will decline faster. The former effect has a positive impact on effort, while the latter a negative one. However, the former prevails on the latter. As a matter of consequence firms have to increase employment and decrease unemployment to a greater extent than under flexible wages in order to keep effort at their constant desired level. Increasing  $N$  and  $\theta_n$  boosts wage dispersion, decreasing the slope of the long-run Phillips curve.

## 5.2 The short run

In order to analyze the short run dynamics of the present economic model, we set  $N = 2$ . The equation for the log of the unemployment rate can be obtained integrating the effort function over  $h$  and keeping in mind equation (19) :

$$\log u_{t+i} = \frac{\phi_0 - \phi_1}{-\phi_2} + \frac{\phi_1}{-\phi_2} \int_0^{1/2} \log \frac{W_{t+i}(h)}{P_{t+i}} dh + \frac{\phi_1}{-\phi_2} \int_{1/2}^1 \log \frac{W_{t+i-1}(h)}{P_{t+i-1}\pi_{t+i}} dh - \frac{\phi_4}{-\phi_2} \log \pi_{t+i}$$

The other equations of the system are (5), (6), (19), the aggregate resource constraint -  $y_t = c_t$  -, the definition of unemployment rate  $\int_0^{1/2} n_t(h) dh + \int_{1/2}^1 n_t(h) dh = 1 - u_t$ , and the demands for the labour services of the households belonging to the two cohorts

$$\begin{aligned}\frac{W_{t+i}(h)}{P_{t+i}} &= \left[ \frac{y_{t+i}}{n_{t+i}(h)} \right]^{\frac{1}{\theta_n}} \text{ for } h \in \left[ 0, \frac{1}{2} \right] \\ \frac{W_{t+i-1}(h)}{P_{t+i-1}\pi_{t+i}} &= \left[ \frac{y_{t+i}}{n_{t+i}(h)} \right]^{\frac{1}{\theta_n}} \text{ for } h \in \left[ \frac{1}{2}, 1 \right]\end{aligned}$$

Finally, the autoregressive process for money growth is

$$\mu_t = \mu^{1-\zeta} \mu_{t-1}^\zeta \exp(\epsilon_t) \quad (23)$$

The equilibrium for this model is a sequence  $\left\{ \frac{W_{t+i}(h)}{P_{t+i}}, \mu_{t+i}, u_{t+i}, \pi_{t+i}, n_{t+i}(0), n_{t+i}(1), y_{t+i}, i_{t+i}, c_{t+i} \right\}$  satisfying households' utility maximization and firms' profit maximization. We log-linearized the system around a steady state with  $u_{ss} = 0.056$  on the basis of the US post-WWII experience. We calibrated the system parameters as customary in the New-Keynesian literature (see for instance Ascari, 2004):  $\beta = 1.04^{-\frac{1}{2}}$ ,  $\mu = 1.02^{\frac{1}{2}}$ ,  $\theta_n = 5$ ,  $\frac{\phi_1}{\phi_2} = 0.29$ ,  $\zeta = 0.57^{\frac{1}{2}}$ . In order to attach a value to  $\frac{\phi_1}{\phi_2}$  we note that it can be considered as the inverse of the elasticity of households' wages with respect to the unemployment rate and so we set it to  $0.07^{-1}$  after Nijkamp and Poot (2005).

Figure 5 plots the percentage deviations from steady state of the inflation rate against those of the unemployment rate. As it is possible to see, wage staggering imply a flatter Phillips curve than flexible wages not only in the long-run but in the short run too. Note that increasing  $\theta_n$  from 5 to 15 would not change our results markedly<sup>5</sup>. Instead, increasing N from 2 to 4 has a considerable impact on the dynamics of inflation and unemployment. As showed in Figure 6, their reactiveness increases, however, unemployment first declines and then increases before going back to its steady state value. A shortcoming of this model is that, with difference to the other models

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<sup>5</sup>Further results are available from the author upon request.

presented in this work, a monetary expansion can cause a contraction in output due to the inefficiencies arising from firms shifting labour demand from one cohort to the other, given that different labour kinds are imperfect substitutes. For  $N=4$  and  $\theta_n = 5$  a one percentage shock in money growth produces a 0.18 percent decline in output. This is implausible and for this reason the model presented in this section is not our preferred one.

## 6 Fourth variation: the social norm case with varying capital

Once considering varying capital within the model, we assume the existence of capital adjustment costs after Bernanke et al. (1999) and Gertler (2002). The households' budget constraint changes to

$$c_{t+i}(h) = \frac{W_{t+i}(h)}{P_{t+i}}n_{t+i}(h) + \frac{T_{t+i}(h)}{P_{t+i}} - \frac{M_{t+i}(h)}{P_{t+i}} + \frac{M_{t+i-1}(h)}{P_{t+i}} - \frac{B_{t+i}(h)}{P_{t+i}} + \frac{B_{t+i-1}(h)}{P_{t+i}}i_{t+i-1} + \frac{R_{t+i}}{P_{t+i}}K_{t+i}(h) - \frac{Q_{t+i}}{P_{t+i}}[K_{t+i}(h) - (1 - \delta)K_{t+i-1}(h)] + q_{t+i}(h)$$

where  $K_{t+i}(h)$  is the capital held by household  $h$ ,  $\delta$  is the capital depreciation rate,  $R_{t+i}$  is the capital rental rate and  $Q_{t+i}$  is the nominal Tobin's  $q$ . Furthermore, households maximize utility with respect to capital too and interacting the first order conditions for capital and consumption leads, under households' symmetry, to the following equation

$$E(c_{t+i+1}\frac{Q_{t+i}}{P_{t+i}}) = \frac{R_{t+i}}{P_{t+i}}E(c_{t+i+1}) + c_{t+i}\beta(1 - \delta)\frac{Q_{t+i+1}}{P_{t+i+1}} \quad (24)$$

As in the New-Keynesian tradition, we assume the existence of an intermediate labour market, where labour intermediaries hire households' horizontally differentiated labour inputs to produce homogeneous labour to be sold

to firms operating on the final product market. In the intermediate labour market we assume productivity to depend on effort. The profit maximization problem of labour intermediaries is

$$\begin{aligned} & \max_{\{n_{t+i}(h), W_{t+i}(h)\}} W_{t+i} n_{t+i} - \int_0^1 W_{t+i}(h) n_{t+i}(h) dh \\ & s.t. \ n_{t+i} = \left[ \int_0^1 e_{t+j}(h)^{\frac{\theta_n-1}{\theta_n}} n_{t+j}(h)^{\frac{\theta_n-1}{\theta_n}} dh \right]^{\frac{\theta_n}{\theta_n-1}} \end{aligned}$$

The solution of this problem and households' symmetry imply

$$\frac{W_{t+i}(h)}{W_{t+i}} = \frac{n_{t+i}}{n_{t+i}(h)} = \phi_1 = e_{t+i} = 1 \quad (25)$$

Firms in the final product market maximize profits hiring labour and capital and adopting a Cobb-Douglas production function. The solution of their problem leads to two customary demand functions for labour and capital

$$(1 - \alpha) \frac{y_{t+i}}{\frac{W_{t+i}}{P_{t+i}}} = n_{t+i} \quad (26)$$

$$\alpha \frac{y_{t+i}}{\frac{R_{t+i}}{P_{t+i}}} = K_{t+i} \quad (27)$$

Substituting these two equations into the production function one has

$$\frac{W_{t+i}}{P_{t+i}} = \left( \frac{\frac{R_{t+i}}{P_{t+i}}}{\alpha} \right)^{\frac{\alpha}{\alpha-1}} (1 - \alpha) \quad (28)$$

Finally, capital producer  $j$  has the following production function

$$Y_{t+i}^k(j) = \phi \left[ \frac{I_{t+i}(j)}{K_{t+i-1}(j)} \right] K_{t+i}(j)$$

where  $Y_{t+i}^k(j)$  is new capital,  $I_{t+i}(j)$  is raw output used as material input at time  $t+i$  and  $\phi'(\cdot) > 0$ ,  $\phi''(\cdot) < 0$ ,  $\phi(0) = 0$  and  $\phi\left(\frac{I}{K}\right) = \frac{I}{K}$ , with  $\frac{I}{K}$  being the steady state investment-capital ratio.  $K_{t+i}(j)$  is capital rented after it has been used to produce final output within the period. The profits of the  $j$ -th capital producer can be written as  $\frac{Q_{t+i}}{P_{t+i}}\phi\left[\frac{I_{t+i}(j)}{K_{t+i-1}(j)}\right]K_{t+i}(j) - I_{t+i}(j) - Z_{t+i}^k K_{t+i}(j)$  where  $Z_{t+i}^k$  is the rental price of capital used for producing new capital. The first order condition for  $I_{t+i}(j)$  is, under a symmetry condition:

$$\frac{Q_{t+i}}{P_{t+i}}\phi'\left(\frac{I_{t+i}}{K_{t+i-1}}\right) - 1 = 0 \quad (29)$$

where  $I_{t+i} = \int_0^1 I_{t+i}(j) dj$  and  $K_{t+i-1} = \int_0^1 K_{t+i-1}(j) dj$ . One can show that the first order condition with respect to  $K_{t+i}(j)$ ,  $\phi\left(\frac{I}{K}\right) = \frac{I}{K}$  and (29) imply that  $Z_{t+i}^k$  is approximately zero near the steady state and so it can be ignored.

The system of equations is therefore composed by (5), (6), the aggregate resource constraint  $y_{t+i} = c_{t+i} + I_{t+i}$ , the law of motion of capital  $K_{t+i} = \phi_K\left(\frac{I_{t+i}}{K_{t+i-1}}\right)K_{t+i} - (1 - \delta)K_{t+i-1}$ , the definition of unemployment  $n_t = 1 - u_t$ , (23), (24), (26), (27), (28), (29) and (4), which imposing (25) and after rearranging becomes

$$\log u_{t+i} = \frac{\phi_0 - \phi_1}{-\phi_2} - \frac{\phi_4}{\phi_2} \log \pi_{t+i} + \frac{(\phi_1 + \phi_3)}{-\phi_2} \log \frac{W_{t+i}}{P_{t+i}} + \frac{\phi_4}{\phi_2} \log \frac{W_{t+i-1}}{P_{t+i-1}} \quad (30)$$

The equilibrium of this system is a sequence  $\left\{ \frac{R_{t+i}}{P_{t+i}}, \frac{W_{t+i}}{P_{t+i}}, y_{t+i}, n_{t+i}, K_{t+i}, c_{t+i}, u_{t+i}, \mu_{t+i}, \pi_{t+i}, i_{t+i}, I_{t+i}, \frac{Q_{t+i}}{P_{t+i}} \right\}$  satisfying utility and profit maximization problems.

Regarding the long-run we note that in steady state the real Tobin's  $q$  is equal to one and therefore that  $\frac{R}{P}$  and  $\frac{W}{P}$  are pinned down by (24) and (28) independently from money growth. On the basis of (30) and of the steady state equality of inflation and money growth, this entails that (12) and (13) hold also for the present model.

Regarding the short-run, we do not change the calibration of the parame-



ters that already appeared in the previous sections of the present work, with the only exception that, given that we have flexible wages here, we do not rise them to the power of  $\frac{1}{2}$ . Following the same reasoning above regarding the elasticity of the wage to the unemployment rate we set  $\frac{(\phi_1 + \phi_3)}{\phi_2} = 0.07^{-1}$ . Furthermore, as customary,  $\alpha = 0.33$ ,  $\delta = 1 - 0.92$  and, after Bernanke and Gertler (1999),  $\eta = -\frac{\phi''[\frac{I}{K}]\frac{I}{K}}{\phi'[\frac{I}{K}]} = 0.5$ . We log-linearize the system around the steady state. The short-run Phillips curve with fixed and varying capital are plotted in Figure 7. As it is possible to see, varying capital implies a flatter short run Phillips curves than under fixed capital, given that the boom following a monetary expansion is reinforced by an increase of investments, which rise upon impact by 0.08%<sup>6</sup>.

## 7 Conclusions

In the present paper, we explored the relationship between inflation and unemployment in different models with fair wages. We showed that, under customary assumptions regarding the parameters of the effort function, they have a negative long- and short-run nexus, which is motivated by the fact that firms respond to inflation - which spurs effort via a decrease in the reference wage - by increasing employment in order to maintain the effort level constant, as implied by the Solow condition. Under wage staggering this effect is stronger because wage dispersion magnifies the impact of inflation on effort. This effect is also stronger in the short-run once considering varying instead of fixed capital as booms generated by monetary expansions are

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<sup>6</sup>Changing  $\eta$  and  $\mu$  would only have negligible effects on the Phillips curve. Further results are available from the author on request. It is worth noting that our model does not produce a persistent reaction of either the unemployment or the inflation rate after a monetary shock. This accords well with the empirical evidence produced by the inflation persistence network, whose main result is that, once allowing for structural breaks in the mean of the inflation time series, inflation has low persistence (Altissimo et al., 2007). Empirical evidence of a fast adjustment of unemployment after a monetary shock was produced by Karanassou et al. (2007, p. 346) where the unemployment rate takes just two periods to hit its new long-run level after a permanent monetary shock.

reinforced by greater investment.

Once considering the personal norm case, the model produces an unrealistic negative impact of hyper-inflation on unemployment. Furthermore, under wage-staggering the model can produce output contractions in response of monetary expansions. For these reasons, our preferred variation is the social norm case with flexible wages and, possibly, varying capital.

Our results can offer new theoretical insights into the evidence produced by recent empirical contributions finding a negative long-run relationship between unemployment and inflation.

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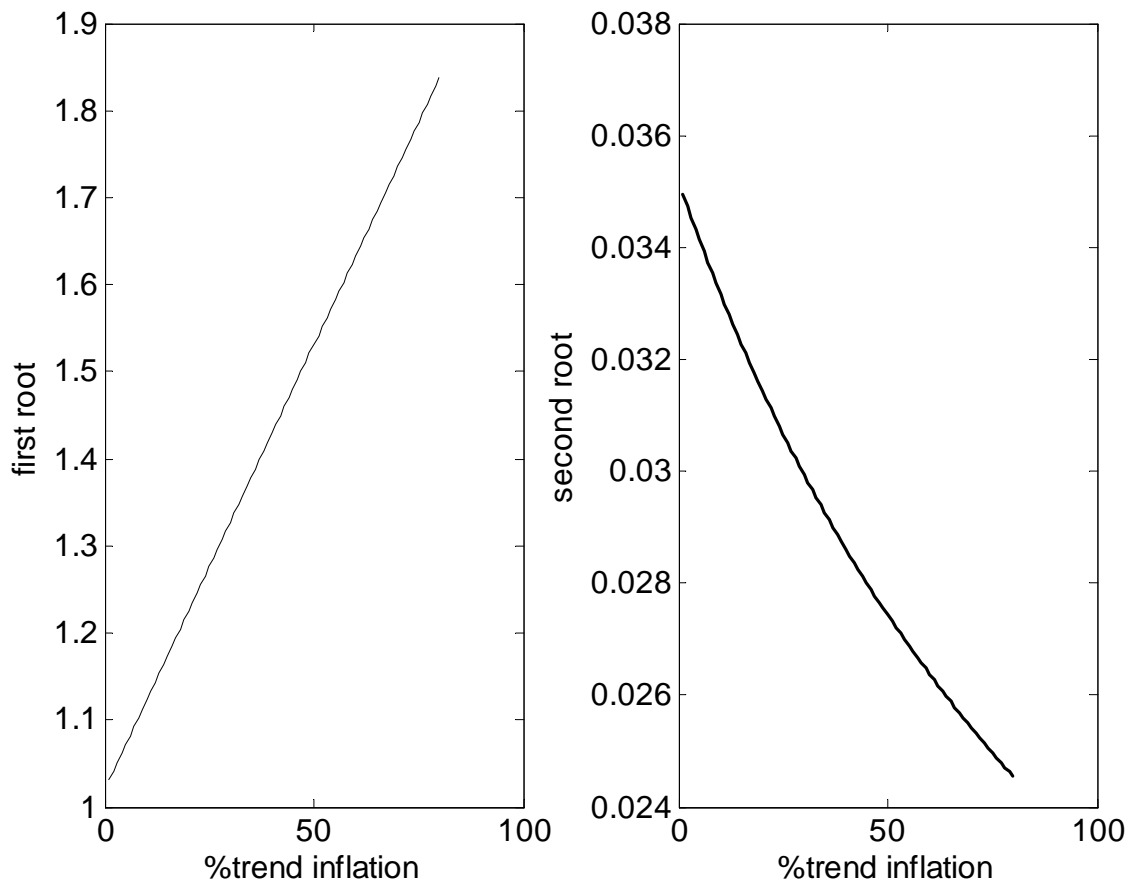
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**Figure 1 – The roots of the system for different trend inflation rates**



**Figure 2 - The stable arm for different trend inflation rates**

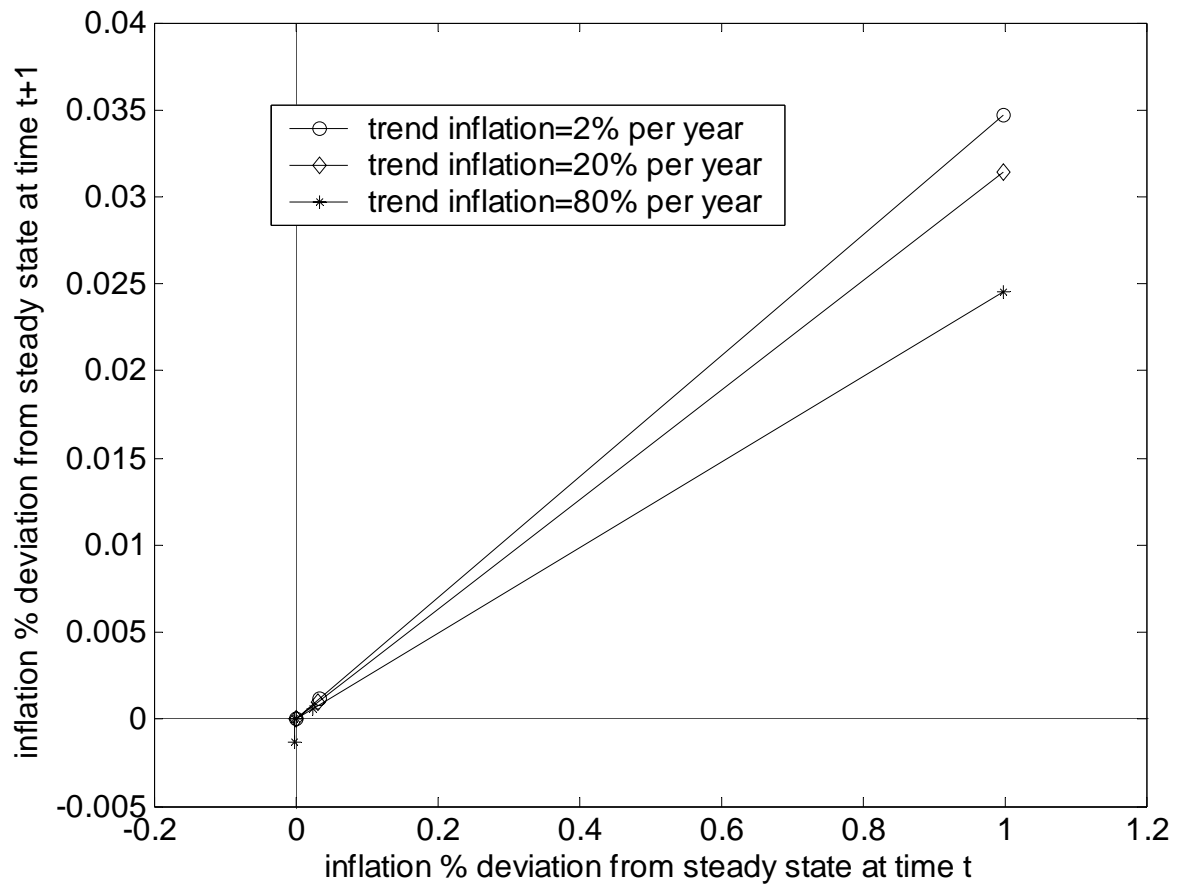
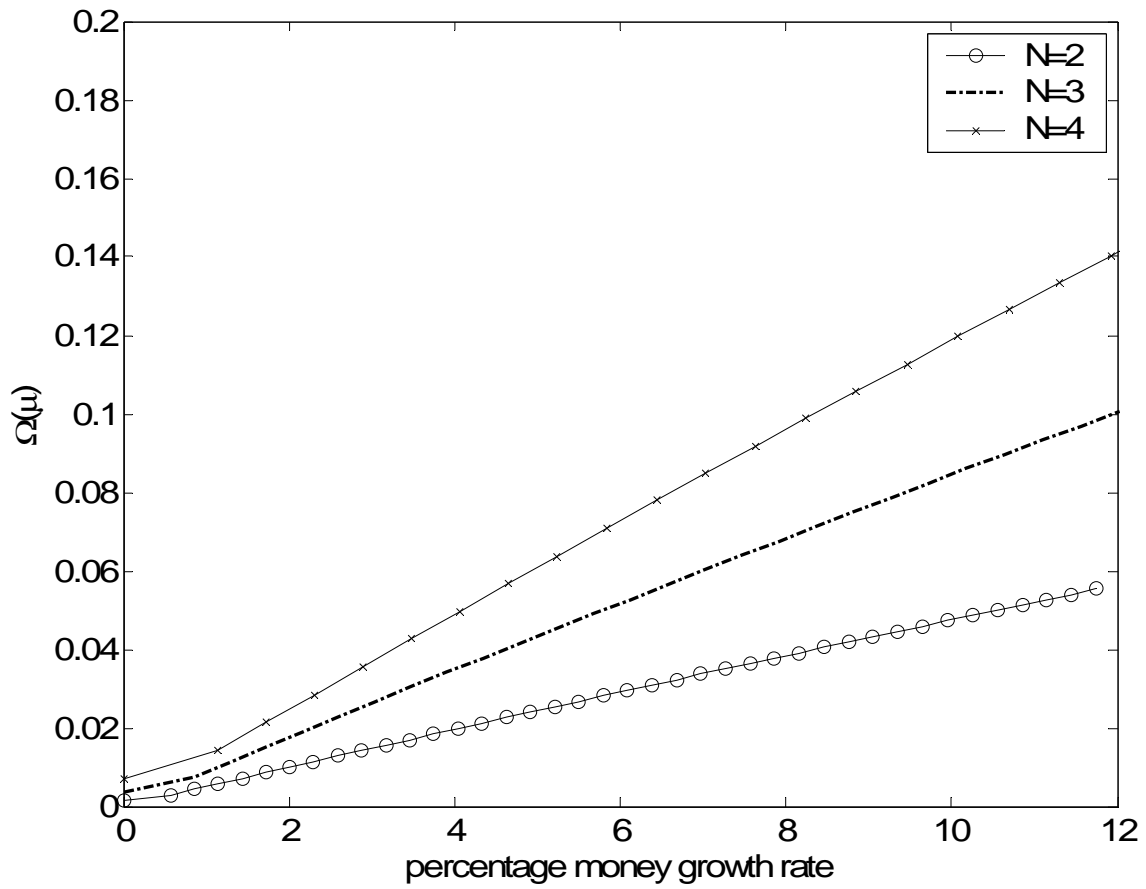


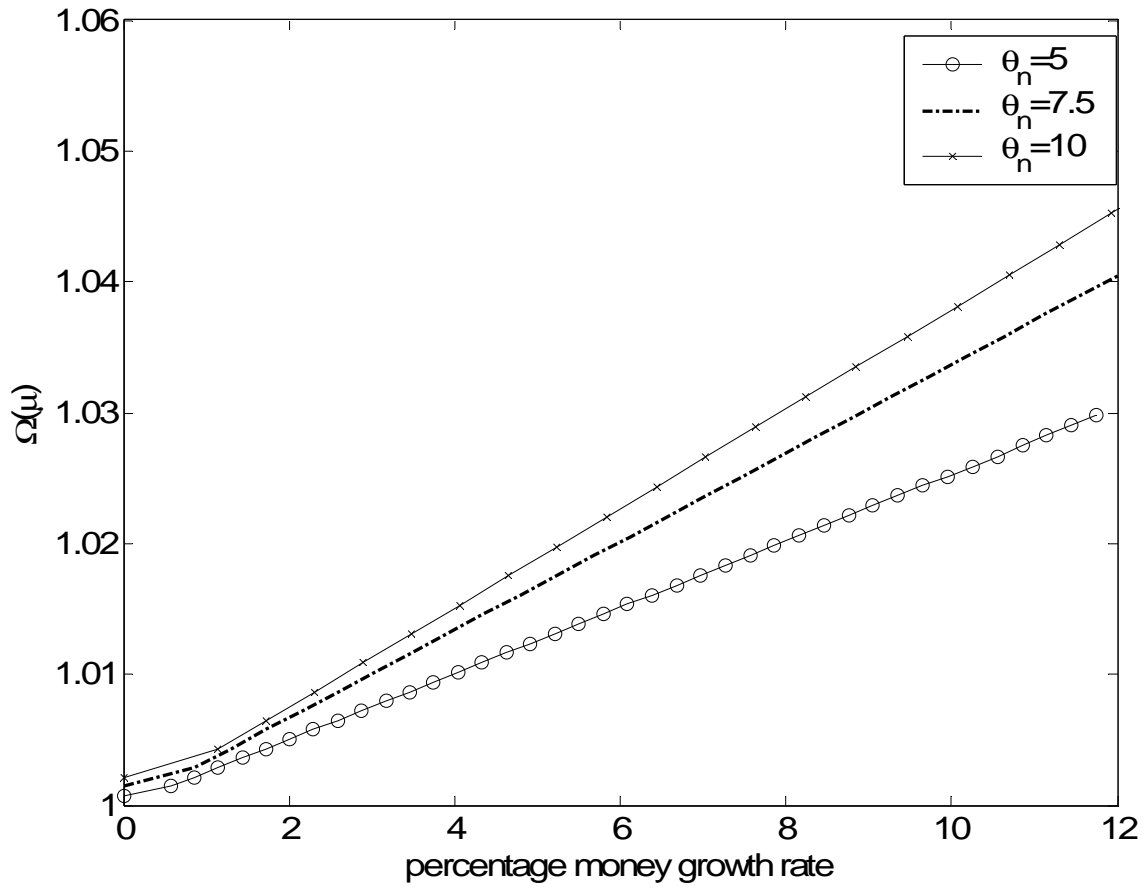
Figure 3 –  $\Omega(\mu)$  for different money growth rates and contract lengths



Notes:  $\theta_n$  was set equal to 5; for a definition of  $\Omega(\mu)$  see equation (22).

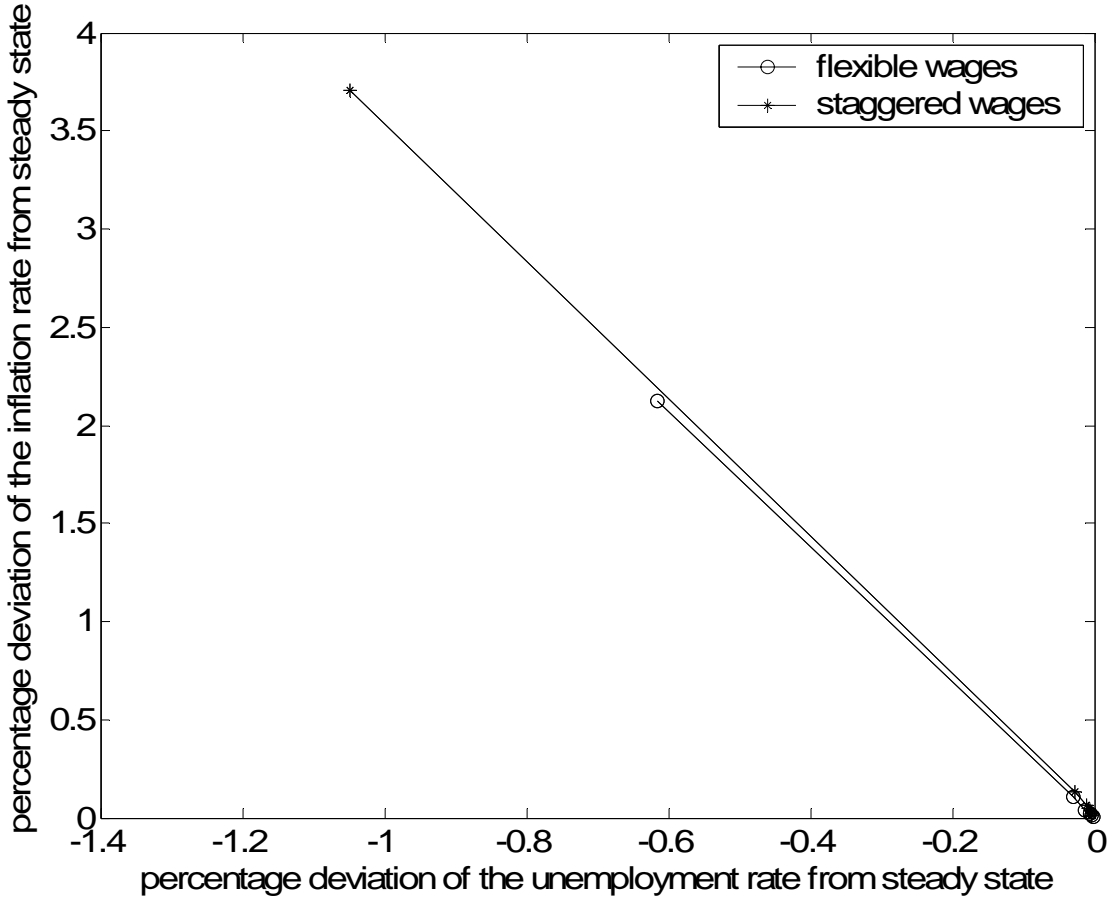


Figure 4 –  $\Omega(\mu)$  for different money growth rates and elasticities of substitution among labour kinds

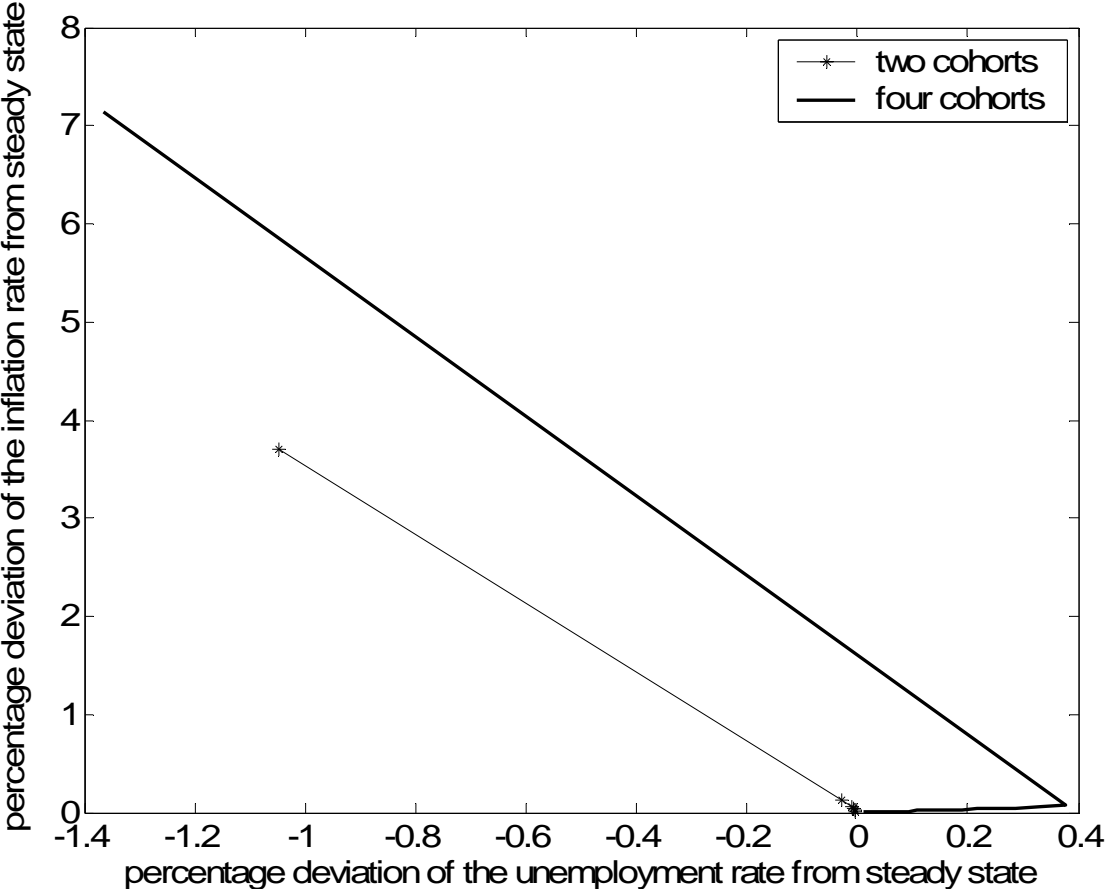


Notes: N was set equal to 2; for a definition of  $\Omega(\mu)$  see equation (22).

**Figure 5 – The short-run Phillips curve with flexible and staggered wages**



**Figure 6 – The short-run Phillips curve with staggered wages and with different number of cohorts**



**Figure 7 – The short-run Phillips curve with fixed and varying capital**

