Reproducing business cycle features in Germany - an evaluation for the need of nonlinear models

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Abstract

There exists a wide literature starting by Hess and Iwata (1997) on the issue whether non-linear time series models are needed to reproduce business cycle features. While almost all studies are about the properties of US data, this paper adopts this kind of analysis to address the German case. The results show strong evidence for the existence of non-linearities in data generating process for German GDP.

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1 Introduction

At least since the work of Burns and Mitchell (1946) economic activity has been analyzed in terms of distinct business cycle phases. The output series is divided into phases of recession and expansion and a business cycle consists of one of two types of phases. This categorization of economic activity is prominently advocated by the National Bureau of Economic Research (NBER). The NBER maintains a history of turning points for the US economy. Given these turning points, business cycle features like the number of cycles, the duration of expansions and recessions can be deduced. Since the main economic indicator is GDP the reported business cycles correspond directly to the time series of GDP and GDP growth rates respectively. Hess and Iwata (1997) assume that time series models produce specific business cycle features like number of recessions or duration of cycles and that time series models can be distinguished via these specifics. They analyze whether time series models of GDP, which fit some moments of the GDP series, also fit the business cycle features and compare them. This kind of analysis is an alternative model evaluation technique next to the common (often residual based) specification tests. The reproduction of business cycle features as a evaluation method dates back to Adelman and Adelman (1959) who applied it on economic models. For further applications with economic models see King and Plosser (1994) and Simkins (1994).

This paper adopts the approach of reproducing business cycle features for analyzing different time series models from the German perspective and poses the question which time series model is the best (or the worst) to reproduce the features of the German business cycle. The analysis is especially done with the perspective to discuss the need for non-linearities to model the German business cycle, whereby possible non-linearities show up with important consequences for economic modelling beyond time series models. Thus, a linear reduced form specification of the GDP growth corresponds also to linear multivariate systems with GDP growth like a VAR model. For this reason the analysis whether a linear model even in reduced form can reproduce the business cycle is connected to issue whether linear or linearized models are not just locally but globally interpretable, see Kiani and Bidarkota (2004).

So far studies on non-linearities for the German GDP growth or business cycle show little evidence against the assumption of linear models. Inspection of business cycle features and the reproduction of them add an additional evaluation method. Overall, the results provide strong evidence against the assumption of linearity.

The paper is structured as follows. Section 2 discusses different business cycle features and measures for non-linearities. The competing time series models are introduced in Section 3. In

Section 4 the design of the evaluation analysis and the corresponding results are presented. Section 5 concludes.

2 Business cycles features and non-linearities of the German GDP

2.1 Data

Within this paper we use quarterly, seasonally adjusted German real GDP spanning 1970:I trough 2006:II. The data is taken from the Federal Statistical Office of Germany (Statistisches Bundesamt Deutschland) taking the revisions of national accounting standards published in April 2005 into account.[1] In order to account for the German reunification, up to 1991 growth rates of the West German GDP are used and after 1991 the corresponding growth rates for reunified Germany.

2.2 Business cycle features and asymmetry measures

For the measurement of business cycle features a definition of the business cycle is needed in the first place. Based on Burns and Mitchell (1946) the NBER maintains a history of business cycle turning points which divide a cycle into expansions and contractions. This definition shall be employed in this analysis, too. However, as there is no history of business cycle turning points for Germany as it is for the US and as the procedure of the NBER cannot be reproduced for simulated GDP series an other criterion or method to define the cycle is needed. Bry and Boschan (1971) propose a non-parametric algorithm for monthly data.[2] Its results are very similar to the turning point decisions of the NBER committee. Watson (1994) reformulates the Bry-Boschan algorithm such that it fits for quarterly data (in the following BBQ-algorithm).[3] This reformulation has been widely applied for analyzing time series models and their abilities to reproduce business cycle features, see Harding and Pagan (2002), Engel et al. (2005) or Morley and Piger (2004).

The BBQ algorithm is used to identify business cycle turning points given the German GDP time series. One cycle thereby consists of an expansion and a recession (further subdivisions are neglected). Based on the turning points the following features are gained:[4]

[1] Data are taken from Fachserie 18, Reihe p. 28 for 1970 to 1991 and from Fachserie 18, Reihe 1.3. for 1990 to 2006.

[2] In the context of turning point dating the NBER monitors four monthly time series: employment, industrial production, retail sales and income figures.

[3] Figure 1 shows the result of the BBQ applied on German data.

[4] Figure 2 illustrates the following business cycle features.

1. Number of expansions
2. Duration of expansions in quarters
3. Standard deviation of expansion durations
4. Mean growth rate during expansions (height of expansion)
5. Standard deviation of growth rate during expansions
6. Number of recessions
7. Duration of recessions in quarters
8. Standard deviation of recession durations
9. Mean growth rate during recessions (depth of recession)
10. Standard deviation of growth rate during recessions

A subset of these features are taken into account in the studies of Hess and Iwata (1997), whereby they use a different algorithm to identify turning points. Studies applying the BBQ are e.g. Harding and Pagan (2002) or Morley and Piger (2004). Figures for all features are calculated for German data and deal as benchmarks for the subsequent evaluation of time series models.

Besides business cycle features discussed in the previous subsection this analysis considers also measures regarding possible asymmetries over the business cycle.[5] It is often argued that asymmetries impose the need for non-linear modelling of GDP growth rates as linear models have symmetric properties. For a discussion on business cycle models and asymmetries, see Knüppel (2004). The following three asymmetry measures are taken into account:

11. **Deepness**: This kind of asymmetry assumes that recessions are seldom but exhibit very strong reductions of the level of GDP. A process is called deep, if the cyclical component of the GDP is negatively skewed, see Sichel (1993).[6]
12. **Steepness**: Growth rates in recessions deviate relatively strongly from the mean growth compared to growth rates in expansions. A process is called steep, if the growth rates of the GDP are negatively skewed, see Sichel (1993).

[5]Kim et al. (2005) and Morley and Piger (2004) consider asymmetries besides common business cycle features, too.
[6]Trend-cycle decomposition is done via the Hodrick-Prescott filter, see Hodrick and Prescott (1997).

13. **Sharpness**: A process is called sharp, if the transition from an expansion to a recession is rather smooth while the opposite transitions are accompanied by a strong growth, see McQueen and Thorley (1993). To get a measure for this phenomenon the growth rates before and after a turning point are subtracted from each other. For all transitions from expansion to recession the absolute mean of these differences is subtracted from the absolute mean of the differences corresponding to the transition in the opposite direction.[7]

Non of the asymmetries discussed here can be found significant for German data.[8] This is in line with the results of Razzak (2001) or Belaire-Franch and Contreras (2003) testing for deepness, steepness and sharpness.[9] This finding on its own does allow to maintain the assumption of linear models for the GDP. However, the asymmetry measures are not ruled out in the further analysis as they may provide a tool to distinguish between non-linear models.

3 Time series models

The paper assesses the capability of different time series models to reproduce the features of the German business cycle. While some of them are already introduced into the discussion on the time series properties of German GDP growth, most of them have been formulated or initially used for U.S. GDP growth.

3.1 Linear autoregressive model

A univariate linear representation of the GDP growth series is given by the class of ARMA models. Model selection analysis in terms of the Akaike information criterium (AIC) showed that an autoregressive model with 4 lags is the best characterization of the German GDP growth.[10] The resulting model can be represented as follows:

[7] See also Clements and Krolzig (2003) for a discussion of all three kinds of asymmetry.

[8] The skewness of the cyclical component is found positive with 0.345 and the skewness of the growth rates amounts to -0.01 with a corresponding p value of 0.536 in a test with the null of a non-steep process. Additionally, there is no evidence for sharpness as the mean difference between the differences of the growth rates at the turning points amounts to 0.0483 accompanied by a standard deviation of 1.053.

[9] Belaire-Franch and Contreras (2003) apply test methods based on Markov-Switching models proposed in Clements and Krolzig (2003).

[10] For all Markov-Switching models and the SETAR model also 4 lags are assumed.

$$y_t = \mu + \sum_{\ell=1}^{k} \phi_\ell y_{t-\ell} + \epsilon_t, \qquad \epsilon_t \sim \mathcal{N}(0, \sigma^2), \tag{1}$$

whereby y_t represents the quarterly growth rates of German GDP. In the following this model will be abbreviated AR.

3.2 Markov-Switching models

A popular non-linear model is the Markov-Switching model, where a latent and discrete state variable s_t is assumed to govern the distribution of the observable variable y_t.[11]

$$y_t = \mu_{s_t} + \sum_{\ell=1}^{k} \phi_{s_t,\ell} y_{t-\ell} + \epsilon_t, \quad \epsilon_t \sim \mathcal{N}(0, \sigma^2_{s_t}). \tag{2}$$

The state variable itself evolves via a Markov Chain with transition probabilities:

$$P(s_{t+1} = i | s_t = j) = p_{ij}. \tag{3}$$

Throughout this analysis Markov-Switching models with two states are considered. Several Markov-Switching models are taken into account differing with respect to the parameters, which are subject to switching. While in Equation 2 all parameters are labelled by the state variable s_t in an Markov-Switching intercept model (MSI) only $\mu_1 \neq \mu_2$, while all other parameters are not affected by the state variable (e.g. $\sigma_1^2 = \sigma_2^2$).[12] Within a MSI the different states can be interpreted depending on the parameter estimates as different business cycle phases or different growth phases. In an alternative version of the Markov-Switching model it is assumed that the autoregressive parameters are subject to the switching states (MSA), see Fritsche and Kuzin (2005). In addition to the MSI and the MSA, models with contemporaneously switching variances, $MSIH$ and $MSAH$, are assumed.

An extension of the Markov-Switching model is the Bounce-Back model (BB) proposed by Kim et al. (2005). It is assumed that at the end of a recession an additional effect takes place which increases the growth. In the aftermath of a recession a bounce back with higher growth rate occurs to compensate the losses of GDP during a recession.

$$y_t = \mu_{s_t} + \sum_{\ell=1}^{k} \phi_\ell y_{t-\ell} + \gamma \sum_{j}^{m} (1 - s_{t-j}) y_{t-j} + \epsilon_t, \quad \epsilon_t \sim \mathcal{N}(0, \sigma^2). \tag{4}$$

[11] See Krolzig (1997) for a detailed description of Markov-Switching models.

[12] In the seminal paper Hamilton (1989) the Markov-Switching affected the mean (MSM). With modest autoregressive parameters as in the German case the MSI yields almost the same results.

The bounce back effect is governed via the parameter γ. In case of a recession s_t takes the value 0 and the bounce back regressor is activated.

3.3 Self-Exciting-Threshold-Autoregressive model

Potter (1995) introduces the Self-Exciting-Threshold-Autoregressive model (SETAR) for analyzing the GDP growth of the US. Analogously to the MS model two different regimes are assumed governing the GDP growth. Contrary to the MS model the regime switches are not driven by a latent Markov process but rather by the own past of the GDP growth rates. If the past growth crosses some threshold the data generating process changes:

$$y_t = \begin{cases} \mu_1 + \sum_{\ell=1}^{k} \phi_{1,\ell}\ y_{t-\ell} + \sigma_1 \eta_t, & \text{if}\ \ y_{t-q} < r, \\ \mu_2 + \sum_{\ell=1}^{k} \phi_{2,\ell}\ y_{t-\ell} + \sigma_2 \eta_t & \text{otherwise}, \end{cases} \tag{5}$$

where $\eta_t \sim N(0, 1)$. The parameter r is the unobserved threshold. If the threshold variable y_{t-q} crosses the value of r the regime switch occurs.

3.4 Floor-ceiling model

Based on a model of Beaudry and Koop (1993) an extension, the floor-ceiling model (FC), taking some features of the SETAR into account is proposed by Pesaran and Potter (1997). Again, regime switches are driven by threshold variables but here the regimes are connected to the idea that the economy should tend to return to its potential output (trend growth). When the growth rates have been too low the floor regime is activated and growth increases. On the other hand, when growth rates have been too high, the ceiling regime provides some dampening. This kind of modelling shall provide that business cycle shocks have no long lasting impacts on growth and are reduced rapidly. Pesaran and Potter (1997) provide different specifications of their model. The hard ceiling specifications fits better the German data and is used in the following. The model setup is as follows:

$$y_t = \mu + \sum_{\ell=1}^{k} \phi_\ell y_{t-\ell} + \theta_1 CDR_{t-1}(\gamma) + \theta_2 HC_{t-1}(\gamma) + \eta_t h_t(\gamma, \sigma) \tag{6}$$

where

$$h_t(\gamma, \sigma) = \sigma_0(1 - \max\{F_{t-1}, C_{t-1}\}) + \sigma_1 F_{t-1} + \sigma_2 C_{t-1}$$

and $\gamma = (r_F\ r_C)$ as well as $\sigma = (\sigma_0\ \sigma_1\ \sigma_2)$.

Die dummy-variable C_t denotes the ceiling regime and HC_t is the corresponding regressor. The variable F_t is the dummy for the floor regime and CDR_t the corresponding regressor, representing the depth of a recession. These variable are build as follows:

$$F_t = \begin{cases} \mathbf{1}(y_t < r_F), & \text{if } F_{t-1} = 0, \\ \mathbf{1}(CDR_{t-1} + y_t < 0), & \text{if } F_{t-1} = 1. \end{cases} \tag{7}$$

If the growth rate is beneath the threshold, the floor regime is activate. The regime lasts as long as the effect of contraction is still present. The variable CDR_t increases during the floor regimes and is given as:

$$CDR_t(\gamma) = \begin{cases} (y_t - r_F)F_t, & \text{if } F_{t-1} = 0, \\ (CDR_{t-1} + y_t)F_t, & \text{if } F_{t-1} = 1. \end{cases} \tag{8}$$

The ceiling regime is activated when the growth rates are above their mean (as threshold $r_c = \overline{y}$ is assumed):[13]

$$C_t = \mathbf{1}(HC_{t-1} + y_t > r_C) \tag{9}$$

$$HC_t(\gamma) = (HC_{t-1} + y_t - r_C)C_t, \qquad HC_0 = 0. \tag{10}$$

3.5 Switching trend model

DeJong et al. (2005) propose a model (DLR) combining aspects of the Floor-Ceiling model and the Markov-Switching model. Phases of accelerating and decelerating growth follows each other. Within each phase growth follows a linear trend model with stochastic parameters. The idea of switching trends is related to the idea of Markov-Switching models, while the process producing the regime switches is related to the floor-ceiling model. The regime switches are triggered by an observable tension index h_t given as the geometric sum of former deviations from long-run growth:

$$h_t = \sum_{\ell=1}^{\infty} \delta^{\ell}(y_{t-\ell} - y_t^*), \tag{11}$$

where parameter $\delta \in (0, 1)$ represents the persistence of past deviations.[14] The sample mean deals as long-run growth. Like in the floor-ceiling model deviations from long-run conditions lead to adjustment processes in the DLR, too. However, adjustment is not achieved via the consideration of additional regressors but by regime switches.

[13] Pesaran and Potter (1997) propose two specifications for the ceiling effect. Here, only the one providing a better fit for the German data is presented.

[14] Here $\delta = 0.575$ is assumed.

The regime switches follow a Probit model, where the tension index enters as a regressor multiplied by the state variable r_t:

$$\pi_{t+1} = P(r_{t+1} = -r_t | r_t, h_t) = \Phi(\beta_0 + \beta_1 r_t h_t), \tag{12}$$

whereby

$$r_t = \begin{cases} 1, & \text{if } t \text{ is in an accelerating regime} \\ -1, & \text{if } t \text{ is in a decelerating regime} \end{cases} .$$

The state variable on the right hand sight ensures that the effect of the tension index is reverted by the change of the regime. The tension is relieved by the switch.

The growth rates of GDP in the DLR model are given as follows

$$y_t = m_t + \nu h_{t-1} + \gamma y_{t-1} + \epsilon_t, \quad \epsilon_t \sim \mathcal{N}(0, \sigma^2). \tag{13}$$

The variable m_t represents the local trend specification:

$$m_t = a_j + b_j r_t \sum_{v=1}^{t-t(j-1)-1} d^{v-1}, \quad b_j > 0, \quad t = t(j-1)+1, ..., t(j), \tag{14}$$

where j $(j : 1 \rightarrow J)$ is the prevailing regime at time t and $t(j)$ denotes the time of the regime switch from j to $j+1$ ($t(j)$ is the last period of regime j where $t(0) \equiv 0$). The parameter a_j denotes the intercept of the regime j while b_j is the gradient of the trend and r_t determines the direction of the trend. The parameter d steers the curvature of the trend. Deviating from DeJong et al. (2005) only linear trends are assumed ($d = 1$). The trend parameters a_j and b_j are stochastic:

$$a_j \sim N(\mu_a, \sigma_a^2), \qquad b_j \sim LN(\mu_b, \sigma_b^2). \tag{15}$$

If regime j is decelerating $\mu_a = \mu_{a,1}$ and in accelerating regimes is $\mu_a = \mu_{a,2}$. For b_j a log-normal distribution with parameters μ_b and σ_b is assumed to guarantee that b_j is positive. A further deviation from the originally proposed model is the homoscedastic specification of the variance. [15] For the estimation of the DLR model the conditional maximum likelihood approach described in DeJong et al. (2005) is applied. See there for details.

3.6 Existing results for German data

Some of these time series models have already be applied on German data. However, the evidence for the need of non-linear models is mixed and depends on the data set. All of the following

[15] DeJong et al. (2005) specify a GARCH equation for the variance.

approaches compare models via likelihood or residual based tests and do not rely on the reproduction of business cycle features. Goodwin (1993) uses data spanning from 1960 to 1990 and cannot reject the null-hypothesis of a linear AR model compared to a Markov-Switching model at a common level. Bidarkota and Kiani (2004) find evidence for a SETAR model over an AR model for data from 1960 to 2000, while Cancelo and Mourelle (2005) applying a generalized model which nests the SETAR as a special case cannot reject the null of an AR model for data from 1970 to 2002.

Together with the results on asymmetry test in Razzak (2001) and Belaire-Franch and Contreras (2003) evidence against a linear representation of the German GDP growth based on the evaluation strategies used so far is weak and only present if data before 1970 is regarded. In the next section the design for an alternative evaluation method is explained.

4 Reproducing business cycle features

4.1 Design of the analysis

In a first step all models are estimated via Maximum-Likelihood with exception of the DLR, where the conditional maximum likelihood algorithm is applied.[16] Next, given the parameter estimates of the models for each model 10,000 time series are simulated. One simulation run can be interpreted as a hypothetical time series of GDP growth. In the following step business cycle turning points for the hypothetical time series have to be determined. The method to determine the turning points needs to be replicable and should work without inspection, so the Bry-Boschan-Algorithm is used. However, instead of the algorithm designed for monthly data by Bry and Boschan (1971) a modified version proposed by Watson (1994) is applied which can cope with quarterly data. Before applying the modified algorithm, the hypothetical GDP in levels is calculated from the simulated growth rates.

Given the turning points, the business cycle features of each hypothetical GDP series are calculated. Accordingly, for each model 10,000 time series and correspondingly 10,000 sets of business cycle features are simulated. The result is a simulated distribution of each business cycle feature for each time series model, which can be compared to the business cycle features observed in reality.

Next to the analysis of the business cycle features one by one a joint analysis seems desirable. For this reason Hess and Iwata (1997) advocate the use of a Q-test where all features are combined to one test statistic that follows an asymptotic χ^2 distribution. However, they already point at

[16] The parameter estimates are given in Tables 1 through 3.

the drawbacks of such an asymptotic test. In this context the drawback is even more severe as the number of business cycles extracted from the German GDP is much smaller the the number of business cycles Hess and Iwata (1997) find in their study.[17] Therefore the concept of acceptance regions is applied. A definition for the firstly applied concept of acceptance region is given below.

4.2 Results

Table 4 shows the simulation results for the different time series models and compares them to the observed business cycle features. For each feature two rows are given. The first row reports the median of the simulated distribution for each of the models and the second row reports the value of the simulated cumulative distribution function (cdf) evaluated at the observed business cycle feature. Very low and very high values of the cdf indicate that the observed feature is in the tails of the distribution and that it is rather unlikely that the observed feature is produced from the corresponding time series model.

The results show that the reproduction of some business cycle features is problematic for all models. Especially, the number of recessions and expansions as well as the deepness reveals particular high cdf values for all models under consideration, where the corresponding cdfs are above 0.9. On the other hand some features are easy replicable by all models like the depth of recessions or the observed values of the sharpness measure. These features are not able to distinguish between the models.

Comparing the different time series models on striking results becomes obvious. The AR as well as the MSIH model is not able to reproduce the number of recessions and expansions at all. The cdfs take the value 1. Both models produce much too long expansions. The median of the mean duration of expansions for the AR model is 69 quarters, while the observed value is just 17. All models have the tendency to produce longer lasting expansions than the observed ones. However, the median of the AR and the MSIH models are at least twice the value the other models exhibit. Besides the MSIH such a strong rejection can not be found for any other model than the AR model. Based on these results it seems already possible to make the strong conclusion that linear AR model is not able to reproduce the dynamics of the German business cycle.

The consecutive question is which of the remaining time series models does the best job or is the most likely to reproduce the German business cycle features. Based on the cdfs the MSAH model reveals good results. For all problematic features (# of expansions, # of recessions and deepness)

[17] Note, next to a different data set Hess and Iwata (1997) use a different method to determine the turning points.

it shows up with the best cdf values with the DLR and the MSA as successors. However, in Table 4 each feature is presented on its own. Correlation between the features is neglected. Maybe all simulations which exhibit well fitting values for the deepness have a poor number of cycles. Therefore, an inspection only looking at the feature one by one is insufficient.

Thus a second evaluation approach is performed. Acceptance regions based on the observed business cycle features and corresponding confidence intervals are defined. A simulation run which lies inside all confidence intervals of all features is accepted. If the run lies outside one or more confidence intervals it is rejected. Table 5 compares the time series models via different acceptance regions. The acceptance regions differ in terms of the length of the confidence intervals. The confidence intervals are central around the observed value. Each metric measure for a feature is assumed to be at least asymptotically normally distributed and the length of the central confidence intervals is taken from a normal distribution given the estimate for its standard deviation based on the observable business cycle and GDP growth. Confidence intervals for three different levels are reported: 90 %, 95 % and 99 %. However, for the number of recessions or business cycles the confidence intervals are defined in terms of counts as an approximation via the normal distribution seems not suitable for this kind of count data. The confidence intervals contain the observed number of business cycle phases plus/minus one through three business cycle phases.

The results of the acceptance region exercise underline the former findings that the linear AR model is not capable to reproduce the German business cycle properly. Only four from 10,000 draws entered the most conservative acceptance region (90 % and $\pm$ 1). The best model in this exercise is the DLR. In the most conservative acceptance region 4,31 % of the draws are accepted, almost twice as many draws compared to the second best model, the MSA model. The ranking of the first and second best model is true for all acceptance regions even when the asymmetry measures are neglected (second part of Table 5).

Concerning the question, which model is the best to reproduce the German business cycle, the joint inspection of all business cycle features shows up with a somewhat different result compared to the inspection of the cdfs only. Here, the DLR appears to be the best one although it is less capable to reproduce the observed deepness (or better non-deepness) than competing models like the MSAH. Interestingly, the MSAH reveals for the most distinguishing features the best cdfs, but shows worse results when the features are considered jointly.

Interestingly, the rather complicated DLR model performs better than the linear and the simpler non-linear models like the popular MSI. One might assume that the properties of the DLR, like stochastic trends, correspond closely to the observed features of the German business cycle.

5 Conclusion

Residual or likelihood based evaluation methods show mixed results for the need of non-linearities for modelling German GDP. Especially, with data starting after 1970 non-linear models seem to provide no improvement compared to linear models. The approach of reproducing business cycle features regards additional "moments" that have been neglected in the analysis of German data so far. Thus, this paper analyzes the capability of different univariate time series models to reproduce the features of the German business cycle. This issue is of particular interest with respect to the question whether non-linearities are present in the German business cycle and what kind of non-linearities. All models tend to produce too little cycles. However, with the AR model not one out of 10,000 simulation draws provided as many cycles as observed. The best fitting model in this direction is the MSAH. Inspecting all features jointly shows that the DLR model is the best to reproduce the German business cycle features. The results of the joint analysis of all features also underline that the AR model is a rather bad approximation for the business cycle dynamics in Germany. As the AR model can be interpreted as a reduced form model of a VAR model, the findings in this paper casts some doubt on the use of VAR models, when dealing with German GDP. Results of these VAR studies have to be interpreted with care and might be only justified as local approximations. The rejection of the linear model is much more pronounced than in the case of the US.

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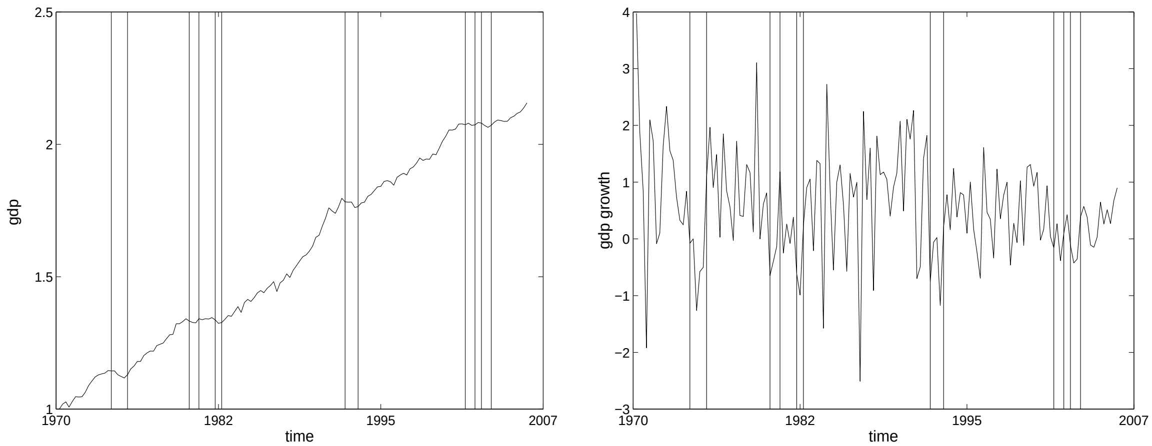
Figure 1: Business cycle turning points with GDP and GDP growth



Note: horizontal lines mark business cycle turning points according to BBQ

Figure 2: Business cycle features

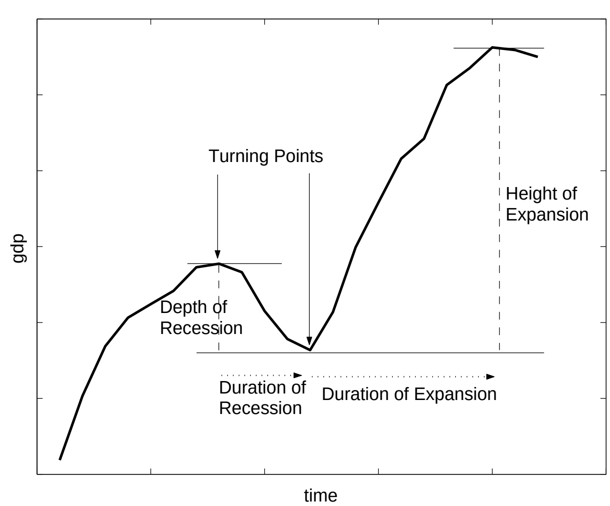


Table 1: Estimation results: AR, MSA, and MSAH model

AR model		MSA model		MSAH model	
μ	$2,161$ 4,937	p_{11}	$0,858$ 9,365	p_{11}	$0,815$ 8,952
ϕ_1	$-0,017$ −0,210	p_{22}	$0,817$ 6,332	p_{22}	$0,679$ 4,066
ϕ_2	$0,031$ 0,389	μ	$1,359$ 3,403	μ	$1,136$ 3,439
ϕ_3	$0,083$ 1,040	$\phi_{1,1}$	$-0,033$ −0,290	$\phi_{1,1}$	$-0,132$ −1,409
ϕ_4	$0,236$ 3,101	$\phi_{1,2}$	$0,016$ 0,142	$\phi_{1,2}$	$0,059$ 0,605
σ^2	11,998	$\phi_{1,3}$	$-0,027$ −0,228	$\phi_{1,3}$	$0,076$ 0,751
		$\phi_{1,4}$	$0,548$ 5,565	$\phi_{1,4}$	$0,529$ 5,597
				σ_1^2	$11,142$ 6,379
		$\phi_{2,1}$	$0,129$ 0,964	$\phi_{2,1}$	$0,402$ 4,585
		$\phi_{2,2}$	$0,115$ 0,871	$\phi_{2,2}$	$0,048$ 0,565
		$\phi_{2,3}$	$0,249$ 1,873	$\phi_{2,3}$	$0,180$ 2,244
		$\phi_{2,4}$	$-0,372$ −2,574	$\phi_{2,4}$	$-0,431$ −5,495
		σ^2	$8,355$ 6,935	σ_2^2	$2,573$ 3,235
R^2	0,075	R^2	0,125	R^2	0,105
$\overline{R^2}$	0,048	$\overline{R^2}$	0,058	$\overline{R^2}$	0,036
logLik	-372,696	logLik	-366,268	logLik	-360,002
AIC	5,357	AIC	5,366	AIC	5,291
BIC	5,462	BIC	5,617	BIC	5,563
$Q_1(\epsilon)$	$0,334$ 0,564	$Q_1(\epsilon)$	$0,013$ 0,908	$Q_1(\epsilon)$	$0,027$ 0,870
$Q_4(\epsilon)$	$3,315$ 0,507	$Q_4(\epsilon)$	$0,523$ 0,971	$Q_4(\epsilon)$	$0,562$ 0,967
$Q_1(\epsilon^2)$	$2,619$ 0,106	$Q_1(\epsilon^2)$	$2,790$ 0,095	$Q_1(\epsilon^2)$	$2,153$ 0,142
$Q_4(\epsilon^2)$	$5,504$ 0,239	$Q_4(\epsilon^2)$	$4,768$ 0,312	$Q_4(\epsilon^2)$	$5,415$ 0,247

Note: Beneath parameter estimates t values are reported. Q_1 denotes the Ljung-Box statistic for one lag , Q_4 for four lags. Beneath the Ljung-Box statistics p values are reported. $\overline{R^2}$ denotes the adjusted R^2.

Table 2: Estimation results: MSI, MSIH, and BB model

MSI model		MSIH model		BB model	
p_{11}	$0,850$ 7,224	p_{11}	$0,993$ 117,579	p_{11}	$0,915$ 20,667
p_{22}	$0,897$ 8,265	p_{22}	$0,979$ 23,359	p_{22}	$0,336$ 2,034
μ_1	$0,751$ 0,936	μ_1	$0,592$ 1,499	$\mu_1 - \mu_2$	$6,156$ 5,260
$\mu_2 - \mu_1$	$3,455$ 4,136	$\mu_2 - \mu_1$	$1,297$ 2,228	μ_2	$-4,982$ $-3{,}739$
ϕ_1	$-0,148$ $-1{,}700$	ϕ_1	$-0,009$ $-0{,}110$	ϕ_1	$-0,021$ $-0{,}271$
ϕ_2	$-0,065$ $-0{,}744$	ϕ_2	$-0,009$ $-0{,}112$	ϕ_2	$0,099$ 1,305
ϕ_3	$0,022$ 0,260	ϕ_3	$0,053$ 0,692	ϕ_3	$0,149$ 1,970
ϕ_4	$0,202$ 2,569	ϕ_4	$0,190$ 2,634	ϕ_4	$0,315$ 4,013
σ^2	$9,123$ 6,144	σ_1^2	$2,453$ 3,237	γ	$-0,426$ $-3{,}539$
		σ_2^2	$13,157$ 7,957	σ^2	$7,217$ 6,496
R^2	0,094	R^2	0,087	R^2	0,111
$\overline{R^2}$	0,047	$\overline{R^2}$	0,039	$\overline{R^2}$	0,057
logLik	-371,510	logLik	-367,036	logLik	-367,941
AIC	5,397	AIC	5,348	AIC	5,361
BIC	5,586	BIC	5,557	BIC	5,570
$Q_1(\epsilon)$	$0,034$ 0,854	$Q_1(\epsilon)$	$0,155$ 0,694	$Q_1(\epsilon)$	$0,0013$ 0,971
$Q_4(\epsilon)$	$0,315$ 0,989	$Q_4(\epsilon)$	$0,425$ 0,980	$Q_4(\epsilon)$	$0,0771$ 0,999
$Q_1(\epsilon^2)$	$2,639$ 0,104	$Q_1(\epsilon^2)$	$1,143$ 0,285	$Q_1(\epsilon^2)$	$0,4635$ 0,496
$Q_4(\epsilon^2)$	$6,357$ 0,174	$Q_4(\epsilon^2)$	$3,777$ 0,437	$Q_4(\epsilon^2)$	$7,4037$ 0,116

Note: Beneath parameter estimates t values are reported. Q_1 denotes the Ljung-Box statistic for one lag , Q_4 for four lags. Beneath the Ljung-Box statistics p values are reported. $\overline{R^2}$ denotes the adjusted R^2.

Table 3: Estimation results: SETAR, FC and DLR model

SETAR model		FC model		DLR model	
μ_1	$0,858$ 2,053	μ	$1,363$ 3,133	ν	$-0,168$ $-1{,}352$
$\phi_{1,1}$	$0,232$ 2,457	ϕ_1	$0,142$ 1,583	γ	$-0,061$ $-0{,}459$
$\phi_{1,2}$	$-0,042$ $-0{,}300$	ϕ_2	$0,072$ 0,945	β_0	$4,601$ 4,730
$\phi_{1,3}$	$0,020$ 0,196	ϕ_3	$0,066$ 0,842	β_1	$-0,426$ $-3{,}113$
$\phi_{1,4}$	$0,292$ 2,840	ϕ_4	$0,236$ 3,259	$\mu_{a,1}$	$0,479$ 0,645
σ_1^2	$7,261$ 6,734	σ_1^2	$11,694$ 7,541	$\mu_{a,2}$	$1,803$ 1,539
μ_2	$0,996$ 0,865	σ_2^2	$1,359$ 2,683	μ_b	$-2,097$ $-4{,}800$
$\phi_{2,1}$	$-0,321$ $-2{,}528$	θ_1	$-0,506$ $-2{,}745$	σ_a^2	$1,054$ 1,959
$\phi_{2,2}$	$0,319$ 1,587	θ_2	$-0,036$ $-1{,}468$	σ_b^2	$0,025$ 0,035
$\phi_{2,3}$	$0,041$ 0,353	r_f^*	-1,694	σ^2	$9,313$ 9,260
$\phi_{2,4}$	$0,115$ 1,073				
σ_2^2	$13,999$ 11,921				
r^*	2,400				
q^*	2,000				
R^2	0,169	R^2	0,124	R^2	0,285
$\overline{R^2}$	0,112	$\overline{R^2}$	0,085	$\overline{R^2}$	0,242
logLik	-361,325	logLik	-362,668	logLik	-384,708
AIC	5,295	AIC	5,272	AIC	5,599
BIC	5,546	BIC	5,460	BIC	5,808
$Q_1(\epsilon)$	$0,625$ 0,429	$Q_1(\epsilon)$	$0,501$ 0,479	$Q_1(\epsilon)$	$0,334$ 0,564
$Q_4(\epsilon)$	$1,118$ 0,891	$Q_4(\epsilon)$	$0,709$ 0,950	$Q_4(\epsilon)$	$3,315$ 0,507
$Q_1(\epsilon^2)$	$1,660$ 0,198	$Q_1(\epsilon^2)$	$0,032$ 0,857	$Q_1(\epsilon^2)$	$0,385$ 0,535
$Q_4(\epsilon^2)$	$4,005$ 0,405	$Q_4(\epsilon^2)$	$2,473$ 0,649	$Q_4(\epsilon^2)$	$6,657$ 0,155

Note: Beneath parameter estimates t values are reported. Q_1 denotes the Ljung-Box statistic for one lag , Q_4 for four lags. Beneath the Ljung-Box statistics p values are reported. $\overline{R^2}$ denotes the adjusted R^2.

Table 4: Simulation results for the time series models – median and CDF

	AR	MSA	MSAH	MSI	MSIH	BB	FC	SETAR	DLR	GDP
1. # expansions	2,000	5,000	5,000	4,000	2,000	4,000	4,000	4,000	5,000	7,000
$\hookrightarrow$ $F(GDP)$	1,000	0,963	0,944	0,972	1,000	0,980	0,981	0,979	0,954	
2. duration of expansions	69,000	26,200	24,400	30,500	65,000	32,000	31,750	31,000	24,800	17,286
$\hookrightarrow$ $F(GDP)$	0,001	0,110	0,175	0,090	0,002	0,058	0,063	0,074	0,131	
3. std. of duration	22,189	19,312	17,574	20,704	26,163	21,676	20,817	21,407	17,542	13,659
$\hookrightarrow$ $F(GDP)$	0,400	0,254	0,316	0,222	0,278	0,214	0,233	0,215	0,293	
4. height of expansions	3,396	2,561	2,477	2,660	2,991	2,704	2,909	2,612	2,692	2,815
$\hookrightarrow$ $F(GDP)$	0,054	0,779	0,827	0,660	0,416	0,620	0,392	0,732	0,666	
5. std. of height	3,440	3,266	3,282	3,417	3,396	3,310	3,628	3,429	3,489	3,553
$\hookrightarrow$ $F(GDP)$	0,693	0,844	0,783	0,722	0,596	0,815	0,381	0,665	0,614	
	AR	MSA	MSAH	MSI	MSIH	BB	FC	SETAR	DLR	GDP
6. # recessions	1,000	4,000	4,000	3,000	2,000	3,000	3,000	3,000	4,000	6,000
$\hookrightarrow$ $F(GDP)$	1,000	0,945	0,916	0,957	1,000	0,970	0,971	0,966	0,933	
7. duration of recessions	3,000	4,500	4,882	4,400	3,333	4,000	4,250	4,500	4,250	3,333
$\hookrightarrow$ $F(GDP)$	0,617	0,155	0,099	0,197	0,494	0,284	0,222	0,205	0,185	
8. std. of duration	0,000	1,871	2,217	1,862	0,000	1,528	1,633	2,082	1,732	1,033
$\hookrightarrow$ $F(GDP)$	0,813	0,254	0,173	0,271	0,689	0,355	0,325	0,262	0,240	
9. depth of recessions	-2,273	-1,722	-1,601	-1,596	-1,836	-2,066	-1,699	-1,673	-1,781	-1,664
$\hookrightarrow$ $F(GDP)$	0,701	0,552	0,443	0,441	0,569	0,741	0,531	0,508	0,594	
10 std. of depth	6,401	5,100	4,509	4,990	6,222	6,686	5,493	4,922	5,306	4,454
$\hookrightarrow$ $F(GDP)$	0,358	0,329	0,484	0,364	0,318	0,146	0,265	0,395	0,280	
	AR	MSA	MSAH	MSI	MSIH	BB	FC	SETAR	DLR	GDP
11. Deepness	0,004	-0,001	-0,011	0,002	0,002	-0,101	-0,092	-0,076	0,024	0,345
$\hookrightarrow$ $F(GDP)$	0,987	0,974	0,943	0,981	0,970	0,988	0,994	0,984	0,979	
12. Steepness	0,005	0,016	0,113	0,032	0,133	-0,351	0,009	0,038	-0,035	-0,012
$\hookrightarrow$ $F(GDP)$	0,470	0,442	0,307	0,408	0,292	0,940	0,458	0,408	0,548	
13. Sharpness	0,002	-0,060	0,222	-0,036	-0,052	-1,428	-1,104	0,032	-0,350	-0,048
$\hookrightarrow$ $F(GDP)$	0,494	0,502	0,441	0,498	0,501	0,701	0,662	0,483	0,550	

Note: In the first row of each feature 1. through 13. the median of the simulated distribution is denoted, in the second row the value of the cdf of the corresponding measure observed in the German GDP series. The last column denotes the results of the business cycle features for the German GDP series.

Table 5: Simulation results for the time series models – relative frequency of acceptance region in %

	AR	MSA	MSAH	MSI	MSIH	BB	FC	SETAR	DLR
90% / # ± 1	0,04	2,18	1,38	1,61	0,19	0,33	1,24	1,09	4,31
90% / # ± 2	0,23	3,71	2,28	2,71	0,39	0,53	2,19	1,92	6,42
90% / # ± 3	0,23	3,74	2,31	2,73	0,39	0,53	2,19	1,93	6,43
95% / # ± 1	0,04	3,41	2,26	2,58	0,29	0,59	1,96	1,84	6,50
95% / # ± 2	0,41	6,28	3,84	4,89	0,72	1,12	3,99	3,64	10,53
95% / # ± 3	0,41	6,33	3,89	4,94	0,72	1,12	3,99	3,66	10,59
99% / # ± 1	0,10	7,72	5,93	6,20	0,57	1,89	5,02	4,21	12,88
99% / # ± 2	0,74	14,24	9,64	11,77	1,84	3,79	9,90	8,91	21,71
99% / # ± 3	0,74	14,36	9,74	11,88	1,85	3,81	10,04	8,98	21,92
90% / # ± 1 /*	0,25	8,85	8,04	7,84	1,07	8,13	6,70	5,70	14,91
95% / # ± 2 /*	1,32	17,86	15,13	16,57	3,75	18,07	15,10	12,93	27,67

Note: For the results in the last two rows the measures of asymmetry are neglected.