Kiel
Working Papers

Kiel Institute
for the World Economy

Matching labor's share in a search and matching model

by Christopher Phillip Reicher

No. 1733 | September 2011

Kiel Working Paper No. 1733 | September 2011

Matching labor's share in a search and matching model *

Christopher Phillip Reicher

Abstract:

In the United States, labor's share of income falls after a positive disturbance to productivity growth or inflation, and it remains low for some time. Previous researchers have argued that the negative relationship between productivity growth and labor's share is puzzling. I argue otherwise. A search and matching model with infrequently bargained nominal wages would predict the observed behavior of labor's share after a productivity disturbance, and it also predicts the observed behavior of labor's share after an inflationary disturbance. Wages at the macroeconomic level seem to be sticky in a way which is consistent with microeconomic evidence; much of the ongoing discussion about the real effects of sticky wages seems to be well-motivated, while sticky price models fail to match the data.

Keywords: Sticky wages, sticky prices, staggered Nash bargaining, inflation, productivity, search and matching, labor share.

JEL classification: E24, E25, J23, J31.

Christopher Phillip Reicher
Kiel Institute for the World Economy
24100 Kiel, Germany
Telephone: +49 (0)431 8814 300
E-mail: christopher.reicher@ifw-kiel.de

* I wish to thank Henning Weber, Wolfgang Lechthaler, Sanjay Chugh, Susanto Basu, Alessandro Barattieri, and Christian Merkl as well as seminar participants in Kiel for their helpful advice. All errors are mine.

Matching labor's share in a search and matching model

Christopher Phillip Reicher [a]
Kiel Institute for the World Economy

This version: September 14, 2011

Abstract:

In the United States, labor's share of income falls after a positive disturbance to productivity growth or inflation, and it remains low for some time. Previous researchers have argued that the negative relationship between productivity growth and labor's share is puzzling. I argue otherwise. A search and matching model with infrequently bargained nominal wages would predict the observed behavior of labor's share after a productivity disturbance, and it also predicts the observed behavior of labor's share after an inflationary disturbance. Wages at the macroeconomic level seem to be sticky in a way which is consistent with microeconomic evidence; much of the ongoing discussion about the real effects of sticky wages seems to be well-motivated, while sticky price models fail to match the data.

---

[a] *Address:* Institut für Weltwirtschaft, Hindenburgufer 66, 24105 Kiel, Germany. Email: christopher.reicher@ifw-kiel.de, Telephone: +49 431 8814 300.

I wish to thank Henning Weber, Wolfgang Lechthaler, Sanjay Chugh, Susanto Basu, Alessandro Barattieri, and Christian Merkl as well as seminar participants in Kiel for their helpful advice. All errors are mine.

JEL: E24, E25, J23, J31.

Keywords: Sticky wages, sticky prices, staggered Nash bargaining, inflation, productivity, search and matching, labor share.

Matching labor's share in a search and matching model

1. Introduction

In this paper I show that labor's share of income in the United States falls after a positive shock to productivity or inflation. The size and persistence of these movements are consistent with a search and matching model with a microeconomically realistic degree of nominal wage stickiness. Previous authors have considered this pattern puzzling. Recent empirical work by Choi and Ríos-Rull (2009), Ríos-Rull and Santaeulàlia-Llopis (2010), and Shao and Silos (2011), among others, has found that after a positive productivity shock, labor's share falls on impact and then seems to positively "overshoot" its long run value. Choi and Ríos-Rull argue that a search and matching model cannot generate realistic movements in labor's share because such models have no mechanism to generate a large and persistent wedge between real wages and labor productivity. To them, the behavior of labor's share is a puzzle. I argue that the behavior of labor's share is in fact not a puzzle if one takes nominal wage stickiness into account. Staggered Nash bargaining over nominal wages can generate the missing wedge between productivity and wages; this is because nominal wages take time to adjust to movements in either productivity or prices. The staggered bargaining model, when calibrated to fit the microeconomic facts regarding wage formation, fits the macroeconomic data as well.

First, I establish some basic empirical facts on the labor share. According to a VAR, a positive innovation to productivity or inflation results in a substantial and persistent fall in labor's share. The negative effect of productivity growth on labor's share seems to be a robust feature of the data, while previous findings regarding "overshooting" seem to be sensitive to the choice of VAR specifications and to the sample period. The negative relationship between inflation and labor's share appears to be an entirely new result. Then I lay out a search and matching model with staggered bargaining over nominal wages, based on the model of Gertler and Trigari (2009). Based on an impulse response analysis, the model predicts a realistic fall in labor's share after a positive shock to productivity or prices, and the model gives clear intuition as to why this happens. After a positive innovation to productivity or inflation, nominal wages take a long time to adjust to their new value, while output and prices adjust quickly. As a result, labor's share falls, and it remains low while firms and workers rebargain their wage contracts. The model fits the data very well in this regard.

Then I explore the degree to which sticky price models are consistent with the data, since sticky prices are also a popular ingredient in macroeconomic models. Sticky prices alone generate counterfactual movements in labor's share, while sticky prices combined with sticky wages generate behavior which is driven by the sticky wage component of the model. Either way, the staggered Nash bargaining model over wages appears to be a good model of wages at a macroeconomic level; it manages to match two important features of the data in an intuitive way. Sticky price models rely upon a mechanism which is severely at odds with the data on labor's share.

In a sense, applying the Gertler-Trigari model to study the short-run dynamics of labor's share complements older ideas about the role of wage stickiness in explaining labor's share. Boldrin and Horvath (1995) and Gomme and Greenwood (1995) look at the implications of the types of sticky wages which arise from implicit contracts between risk-averse workers and less-risk-averse firms. They find that real wages should vary smoothly over the cycle since workers prefer smoother wages to higher wages; this would imply that labor's share should fall after an increase in productivity while it should not move in response to inflation. While the effects of both productivity and inflation on labor's share specifically support the staggered bargaining approach, the basic idea that sticky wages may explain some patterns in labor's share has strong support in the data.

This paper also addresses an issue from the search and matching literature. Yashiv (2006) finds that a suitably calibrated search and matching model can explain many aspects of the macroeconomic data, but it fails at capturing the appropriate cyclicality of wages—wages are much less volatile in the data than they are in the model. Hall (2005) and Shimer (2005) introduce sticky wages into a search and matching model in order to generate more employment volatility in response to productivity shocks, and Gertler, Sala, and Trigari (2008) and Gertler and Trigari (2009) discuss the implications of microfounding sticky wages in a staggered Nash bargaining process. Putting the issues of volatility and propagation aside, it turns out that the approach taken by Gertler, Sala, and Trigari addresses the problems that the search and matching model had with matching aggregate wages. If the goal of a search and matching model is to match the behavior of aggregate wages (or of labor's share), then the staggered Nash bargaining approach appears to be successful. Sticky wages are an important macroeconomic phenomenon and that the behavior of labor's share has much to say about the microeconomics of the wage-setting process.

2. What happens after a rise in productivity or prices?

2.1 The data

To investigate the relationship among productivity, inflation, and labor's share, I run a three-variable Bayesian VAR using a simple identification scheme. I use a Bayesian VAR primarily for the purpose of constructing exact confidence intervals for the impulse responses, not because I have any interesting prior information about the system. The first variable is log nonfinancial corporate labor productivity from the BLS, taken in first differences, followed by nonfinancial corporate price inflation (also from the BLS) and then labor's share of income. Labor's share comes from the NIPA and it equals the share of gross factor income (i.e. gross value added minus taxes and production and imports) accruing to employees in the nonfinancial corporate business sector.

Labor's share and productivity growth are detrended using a Hodrick-Prescott (HP) filter with a smoothing parameter of 100,000; and inflation is detrended using an HP filter with a parameter of 10,000.[1] The raw data and their HP trends are shown in Figure 1. The data are quarterly and run from the second quarter of 1958 through the second quarter of 2011. The filtered data have a standard deviation of 0.0083 for quarterly productivity growth, 0.0044 for quarterly inflation, and 0.0176 for the labor share.

[1] Other combinations of smoothing parameters give extremely similar results, with a higher smoothing parameter naturally giving somewhat more persistent impulse responses. Likewise, using different measures of productivity, labor's share, and inflation give very similar results.

Figure 2 shows the cross correlations among the three variables over a two year window. In that figure, each row denotes the time '*t*' variable and each column denotes the variable at time '*t*+*k*'. Productivity growth shows very little persistence, and it has a slight negative correlation with current and future inflation. It is negatively correlated with labor's share for about a year and a half after the disturbance. Inflation shows a slight negative correlation with contemporaneous productivity growth, but high inflation is not strongly correlated with future productivity growth. Inflation is highly autocorrelated, and it has an unconditional positive correlation with the current and future labor share. The labor share is negatively correlated with current productivity growth and has a weak positive correlation with future productivity growth. It slightly leads inflation, and it is very highly autocorrelated.

In general, the strongest unconditional cross correlation comes between inflation and productivity growth, followed by the positive correlation between inflation and the labor share and the negative correlation between productivity and the labor share. At first glance, it seems like the sticky wage model would run into a problem at matching the behavior of labor's share in response to inflation. As a VAR analysis will show, this fear turns out to be unfounded. Simply looking at the correlation between labor's share and inflation misses out on the fact that inflation and productivity growth are negatively correlated. To look just at cross correlations is misleading and somewhat confusing; in fact, conditional on productivity growth, inflation and labor's share are negatively correlated. Looking at a VAR helps to establish this point much more clearly.

2.2 The VAR coefficients – some information about dynamics

The Akaike Information Criterion strongly suggests a parsimonious VAR specification with one lag. The resulting system is easy to interpret. Table 1 contains the maximum likelihood (OLS) estimates of the coefficient matrices of the VAR along with the exact posterior probability, to within simulation error, that each element is positive. Based on the first row of Table 1, it seems like log productivity almost (but not quite) follows a random walk. There is a slight negative relationship between past inflation and current productivity growth, and there is a slight positive relationship between the past labor share and current productivity growth. An F test on the null hypothesis that the coefficients on all three lagged variables are zero gives a p value of 0.0032. Productivity does not seem to strictly follow a random walk when looking at the data over the sample, but in terms of magnitudes, it approximately does so.

A look at Figure 1 reveals a series of extremely large productivity disturbances as measured by the BLS productivity index during and right after the Great Recession; these disturbances seemed to coincide with and follow a severe deflationary episode. Restricting the sample to the period which ends at the end of 2007 brings these same coefficients to 0.012, -0.160, and 0.068. The estimated effect of past inflation on productivity growth falls to half of its full sample value, and an F test on the null hypothesis that productivity growth is a random walk with drift now generates a p value of 0.0664. The strength of a possible lead-lag relationship between past inflation and current productivity growth seems to some extent to be an artifact of the unusual observations observed during the period of Great Recession.

Inflation seems to be positively related to its own lag and nothing else, and the same is true of labor's share. In the full sample, F tests on these propositions have p values of 0.1776 and 0.7220, respectively. The inference for these propositions is not sensitive to restricting the subsample to the pre-2007 period. In terms of magnitudes, it makes some sense to treat productivity as following a random walk and inflation as not responding to either lagged productivity or the lagged labor share. The effect of lagged inflation on productivity seems to be somewhat precisely measured but economically small.

2.3 Identification and impulse response analysis

Identification of the contemporaneous disturbances in the VAR works as follows: Disturbances to productivity and to inflation are orthogonal to each other, but either may contemporaneously affect labor's share. When talking about productivity disturbances, this is equivalent to saying that productivity disturbances may only feed through into labor share disturbances. When talking about inflation disturbances, this is equivalent to saying that inflation disturbances may only feed through into labor share disturbances.[2]

Confidence intervals (technically speaking, Bayesian credible intervals) are obtained by running through 100,001 Markov Chain Monte Carlo iterations and calculate the impulse responses, discarding the first 10,000 draws and redrawing whenever I encounter an unstable system; this only occurs about 0.02% of the time. The prior distribution for the coefficients is an uninformative normal distribution (that is, with a mean of zero and a covariance matrix which goes toward infinity times the identity matrix), and the prior for the covariance matrix of the shocks is an inverse Wishart distribution equivalent to having observed no observations with a sum of squared errors of zero. The shocks are multivariate Gaussian. The identification scheme corresponds with the same ordering of variables used by Ríos-Rull and Santaeulàlia-Llopis (2010) and others.

Figure 3 shows the posterior median impulse responses of productivity, inflation, and labor's share to a unit productivity disturbance, along with confidence bands bounded at the 2.5th, 5th, 95th, and 97.5th percentiles. After a unit productivity disturbance which has no immediate effect on inflation, labor's share falls by about 0.77 points in log terms. Labor's share takes considerable time to revert back to its long-run level while productivity more or less follows a random walk; this means that real wages lag productivity. The posterior probability that labor's share falls upon impact is near one; there were no MCMC draws that show a positive effect of an innovation to productivity on labor's share. The evidence that real wages take some time to catch up to productivity in the postwar United States is extremely strong.

Figure 4 shows what happens after a unit inflation disturbance which raises the price level by 1% upon impact but has no immediate effect on productivity. Inflation seems to have a similar effect on labor's share to productivity—the median estimate of the effect

[2] This identification scheme answers the respective questions, "Holding the disturbance to inflation constant, what effect does a disturbance to productivity have on labor's share?", and, "Holding the disturbance to productivity constant, what effect does a disturbance to inflation have on labor's share?"

upon impact is -0.57%. As with productivity disturbances, labor's share takes a long time to recover to its initial value. High inflation tends to erode away at real wages in the short run; it takes some time for nominal wages to adjust toward their longer-run values, so in the meantime, labor's share will remain depressed. The statistical confidence behind this finding is also extremely strong; none of the MCMC draws show an initial impact of a shock to inflation as having a positive effect on labor's share. Nominal wages lag the price level to a strong degree, just as nominal wages lag productivity.

The findings from the VAR do not completely coincide with the inferences that one would make from the unconditional correlations shown in Figure 2. This is because innovations to productivity and inflation are negatively correlated—they have a contemporaneous correlation of -0.385 based on the covariance matrix of the VAR residuals. High inflation tends to coincide with low productivity growth, and labor's share may either rise or fall depending on whether the productivity effect or inflation effect dominates. To look simply at a bivariate relationship between inflation and labor's share would give a misleading result.

2.4 The "overshooting" question

The empirical literature on the relationship between productivity and labor's share has uncovered an apparent anomaly—that the short run relationship between productivity and labor's share is negative, but that the medium run relationship is highly positive—that labor's share positively "overshoots" its long run value after a productivity shock. Ríos-Rull and Santaeulàlia-Llopis (2010) and Shao and Silos (2011) run a VAR which delivers these unusual impulse responses of labor's share to a shock in productivity. They run their bivariate VAR on the linearly detrended *level* of productivity and labor's share using data through 2004, with one lag. A VAR on data in levels using a sample which ends in 2004 (not shown in the figures) replicates their results. Upon a rise in productivity, labor's share falls by somewhat less than in the other VARs and recovers somewhat more quickly. The VAR also delivers the "overshooting" behavior described by these authors several years after the initial shock.

A bivariate or trivariate VAR in first differences does not deliver behavior like this, and neither does a VAR which includes observations from the period after 2004. The "overshooting" result appears sensitive to the choice of VAR specification and time period. However, the initial negative impact of productivity on labor's share seems to be highly robust. In fact, the previous literature might have slightly understated the true effect of productivity disturbances on labor's share by omitting inflation from the discussion. Either way, labor's share negatively lags innovations to both productivity and inflation in a robust way, and if productivity is nonstationary, it does so persistently. The rest of this paper shows that a search and matching model with infrequently bargained nominal wages will deliver a result very similar to those obtained from the baseline VAR.

3. The model

This section lays out a model similar to that of Hagedorn and Manovskii (2008) but adds the possibilities of sticky prices and sticky wages in order to see what happens after a permanent real or nominal shock. Cooley and Quadrini (1999), Walsh (2002, 2005), Trigari (2009), and many others present different models of job creation and destruction in the presence of nominal rigidities, building upon the search and matching models of Mortensen and Pissarides (1994) and den Haan, Ramey, and Watson (2000). Following Gertler and Trigari (2009), wages are determined according to staggered Nash bargaining, although I follow Gertler, Sala, and Trigari (2008) in having workers and firms bargain over nominal wages rather than real wages.

The model has two structural disturbances: A shock to total factor productivity and a shock to price inflation in a cashless economy. On the household side, there is a continuum of infinitely lived consumers. Production and hiring take place in a firm-worker match, with wages reset in a staggered manner. A retail sector aggregates output from the wholesale sector and resets retail prices in a staggered manner. The monetary authority follows a simple autoregressive inflation stabilization rule, and there are no government and foreign sectors.

3.1 The household sector

Individuals within households, indexed by *j*, supply labor indivisibly; they either work for a set number of hours per week or do not work at all. They also have the choice between consuming in a given period and saving in nominal bonds to consume in the future. They each seek to maximize the objective function:

$$E_t \sum_{i=0}^{\infty} \beta^i \left[\ln(C_{j,t+i}) - A n_{j,t+i} \right], \tag{1}$$

where $C_{j,t+i}$ equals the household's period-by-period real consumption; and $n_{j,t+i}$ is the proportion of workers in the household who work at the end of a given period. For the sake of tractability, households are large and pool consumption efficiently. Households buy vacancies from firms; the vacancy-posting process has a microeconomic structure which will be discussed in more detail during the discussion of the wholesale sector.

The household's budget constraint is the usual one with a couple of additions. B_t equals the number of bonds that households can buy; households also can consume, invest, or buy vacancies out of beginning-of-period wealth and gross income Q_t. Bonds earn the gross nominal interest rate R_t. The budget constraint is a familiar one, with the addition of vacancies:

$$B_{t+1} + P_t C_t + P_t I_t + p_t^v v_t = P_t Q_t + R_{t-1} B_t . \tag{2}$$

The household's first-order conditions also end up looking familiar. Optimization in bonds generates the usual intertemporal asset pricing relationship:

$$\lambda_t = E_t \beta R_t \frac{P_t}{P_{t+1}} \lambda_{t+1}, \tag{3}$$

where the household's marginal utilities of consumption and wealth are equal:

$$\frac{1}{C_t} - \lambda_t = 0. \tag{4}$$

Households own capital outright, and they rent it at a rate ρ_t^K to the production sector. The capital accumulation equation is given by:

$$K_{t+1} = (1-\delta)K_t + I_t. \tag{5}$$

The price of capital in consumption units is constant and normalized to one; there are no investment-specific shocks. Capital depreciates at a constant rate δ. Capital pricing comes from the household's optimal choice of capitalholdings going into the next period:

$$1 = \beta E_t \frac{\lambda_{t+1}}{\lambda_t}\left[1 + \rho_{t+1}^K - \delta\right]. \tag{6}$$

A transversality condition which eliminates the possibility of Ponzi schemes closes the household side of the model.

3.2 The retail sector and sticky prices

Monopolistically competitive retailers buy output competitively from the wholesale sector and resell it at a markup to households. Households aggregate it according to a Dixit-Stiglitz aggregator. Retailers buy their products y_{jt} competitively from wholesale producers who produce homogeneous intermediate goods. Aggregate output equals:

$$Y_t = \left[\int_0^1 y_{jt}^{\frac{\theta-1}{\theta}} dj \right]^{\frac{\theta}{\theta-1}}, \quad (7)$$

for some substitutability parameter θ greater than one. From this expression, each individual retail firm faces a demand curve:

$$y_{jt} = \left(\frac{p_{jt}}{P_t} \right)^{-\theta} Y_t, \quad (8)$$

where the aggregate price level P_t equals the CES price index:

$$P_t = \left[\int_0^1 p_{jt}^{1-\theta} dj \right]^{\frac{1}{1-\theta}}. \quad (9)$$

The retailers buy unfinished output from the wholesalers at a price P_t^W and sell it at an aggregate markup $\mu_t \equiv P_t / P_t^W$.

Each retailer, in the spirit of Calvo (1983), only changes its price with a probability 1 - ω. Optimization and aggregation result in standard New Keynesian aggregate supply relationship, which when linearized around a zero-inflation steady state, is given by:

$$(1-\omega)(1-\omega\beta)\hat{\mu}_t = -\omega\hat{\pi}_t + \omega\beta E_t \hat{\pi}_{t+1}. \quad (10)$$

Here, hats over variables denote log deviations of gross values from steady state.

3.3 The wholesale sector

The wholesale sector produces output which originates in a set of worker-firm matches. Workers and firms separate for exogenous reasons, and new firms search for workers based on expectations of future profitability. Using standard notation, the number of searchers U_t, equals 1 - N_t, with the labor force normalized to one. There is a constant probability ρ that a match will end, for a constant survival rate of $\varphi = 1 - \rho$. The remaining φN_t matches must decide to continue or separate. If the relationship continues, the match produces $y_{it} = z_t^{1-\alpha} k_{it}^{\alpha}$ which is sold at the competitive wholesale price P_t^w to the retailers. If the relationship separates, production equals zero; the job is destroyed; and the worker becomes unemployed.

Matches rent capital from households in a competitive rental market at a rate ρ_t^k after all aggregate shocks are realized. The surplus of a match at period *t* equals the real value of the match's production, minus the disutility of work in product terms, plus the expected discounted continuation value of the match (denoted by q_{it}), minus the match's capital rental payments:

$$s_{it} = \frac{z_t^{1-\alpha} k_{it}^{\alpha}}{\mu_t} - \frac{A}{\lambda_t} + q_{it} - \rho_t^k k_{it} .$$

The value of k_{it} is determined optimally by the match, so that:

$$k_{it} = z_t \left[\frac{\alpha}{\mu_t \rho_t^K} \right]^{\frac{1}{1-\alpha}} . \tag{11}$$

As a result, firms and workers bargain efficiently over the worker's marginal product:

$$s_{it} = \frac{(1-\alpha) z_t^{1-\alpha} k_{it}^{\alpha}}{\mu_t} - \frac{A}{\lambda_t} + q_{it} ,$$

so substituting in the firm's capital demand, one can find the reduced form of the surplus:

$$s_{it} = \frac{(1-\alpha)z_t}{\mu_t}\left[\frac{\alpha}{\mu_t \rho_t^K}\right]^{\frac{\alpha}{1-\alpha}} - \frac{A}{\lambda_t} + q_{it}.$$

After consolidating terms some more, one obtains the expression:

$$s_{it} = \frac{(1-\alpha)z_t}{\mu_t^{\frac{1}{1-\alpha}}}\left[\frac{\alpha}{\rho_t^K}\right]^{\frac{\alpha}{1-\alpha}} - \frac{A}{\lambda_t} + q_{it}. \tag{12}$$

A high retail markup rate and a high rental rate on capital reduce the surplus of a match in equilibrium, and a persistently high markup rate or high rental rate will result in fewer vacancies posted and fewer matches created.

Entrepreneurial firms owned directly by the households can post vacancies at a marginal cost p_t^v but face no other barriers to entry. These vacancies get filled at a gross rate k_t^f and the wage for the worker is drawn from the prevailing distribution of wages. A firm's portion of the surplus at any given date is given by s_{it}^f, with a cross-sectional average of s_t^f. Free entry in vacancies equates the present surplus value of a vacancy with the cost of vacancy posting:

$$p_t^v = (1-\rho)k_t^f \beta E_t \frac{\lambda_{t+1}}{\lambda_t} s_{t+1}^f, \tag{13}$$

The probability of an unemployed worker actually finding a match equals k_t^w. After doing some algebra, the continuation value of the surplus is given by:

$$q_t = (1-\rho)\beta E_t \frac{\lambda_{t+1}}{\lambda_t}(1-k_t^w)s_{t+1} + \frac{k_t^w p_t^v}{k_t^f}. \tag{14}$$

Firms have a simple vacancy production technology. They can either produce their output or produce $1/\gamma$ vacancies using their rented labor and capital, which they then sell to households. Households buy these vacancies competitively. In equilibrium the price of a vacancy, in output units, equals γ times the marginal revenue product of labor:

$$p_t^v = \frac{\gamma Y_t}{\varphi N_t \mu_t}. \tag{15}$$

3.4 Wage formation

Wages are formed according to staggered Nash bargaining. With probability 1 - υ, wages in an existing match are bargained such that the worker receives a share η of the bilateral surplus, and the firm receives the remainder. Otherwise, the nominal wage remains unchanged. Firms and workers separate only when it is efficient to do so; otherwise they stay together. New firm-worker matches choose a wage at random from the prevailing wage distribution, so when wages are "too high" relative to their flexible values, new matches will become less profitable for firms, and hiring will fall. Gertler and Trigari (2009) discuss the mechanics of the sticky wage model, including its propagation mechanism, in some detail. Unlike them, I concentrate on the dynamics of aggregate wages themselves rather than on the spillover effects of sticky wages into the real economy. Those real effects depend on the extent to which the wages of new hires are flexible, and that issue is not the main focus of this paper.

The average nominal wage rate lags the rebargained wage rate. Average wages per worker are given by the law of motion:

$$P_t W_t = \upsilon P_{t-1} W_{t-1} + (1-\upsilon) P_t W_t^* . \tag{16}$$

To solve for the rebargained real wage, one could note that the only difference between the profits captured by firms is given by the wage that they pay. The surplus of a firm paying w_{it} conditional on the state of the economy x_t is given by the following expression:

$$s^f(w_{it}, x_t) = (1-\alpha)y_t - w_{it}$$
$$+\beta\varphi E_t \frac{\lambda_{t+1}}{\lambda_t}\left[\upsilon s^f\left(w_{it}, x_{t+1}\right) + (1-\upsilon)s^f\left(W_{t+1}^*, x_{t+1}\right)\right]. \quad (17)$$

Subtracting the surplus of a given firm from the surplus of a rebargaining firm gives the following expression:

$$s^f(w_{it}, x_t) - s^f(W_t^*, x_t) = W_t^* - w_{it}$$
$$+\beta\varphi\upsilon E_t \frac{\lambda_{t+1}}{\lambda_t}\left[s^f\left(w_{it}, x_{t+1}\right) - s^f\left(W_t^*, x_{t+1}\right)\right]. \quad (18)$$

The difference between the average firm's surplus and the rebargained surplus is given by iterating (18) forward and averaging over all firms, noting that the firm which pays the rebargained wage has a surplus of $(1 - \eta)s_t$:

$$s_t^f - (1-\eta)s_t = E_t \sum_{i=0}^{\infty} (\beta\varphi\upsilon)^i \frac{\lambda_{t+i}}{\lambda_t} \frac{P_t}{P_{t+i}} \left(W_t^* - W_t\right). \quad (19)$$

The dynamics of labor's share are mainly driven by the partial adjustment equation (16). In response to a one-time permanent shock to the contract wage W^* which might happen because productivity or the price level had permanently risen, nominal wages will only adjust slowly to their new value, and labor's share (wages as a share of output) will fall at the outset. A low labor share will persist until nominal wages catch up with the new contract wage, and this is governed by the frequency of rebargaining $1 - \upsilon$.

3.5 Aggregation and market clearing

Turning to aggregation, the firms' surplus in the aggregate is given by the present value of profits, expressed as a difference equation:

$$s_t^f = \frac{(1-\alpha)z_t^{1-\alpha}k_t^\alpha}{\mu_t} - W_t + (1-\rho)\beta E_t \frac{\lambda_{t+1}}{\lambda_t} s_{t+1}^f . \tag{20}$$

The total number of unemployed in a period equals the starting stock of unemployed plus those who separate at the beginning of the period. Abstracting from labor force entry and exit, this comes out to:

$$u_t \equiv U_t + \rho N_t = 1 - (1-\rho)N_t . \tag{21}$$

Given a constant-returns Cobb-Douglas matching function $m(u_t, v_t) = \varsigma u_t^a v_t^{1-a}$, the vacancy-filling rate equals:

$$k_t^f = \frac{m(u_t, v_t)}{v_t}, \tag{22}$$

and the worker's job-finding rate is given by:

$$k_t^w = \frac{m(u_t, v_t)}{u_t}. \tag{23}$$

The number of matches evolves according to the accounting identity:

$$N_{t+1} = (1-\rho)N_t + m(u_t, v_t), \tag{24}$$

and the gross output of the matched firms and workers is given by:

$$Y_t = (1-\rho)N_t z_t \left[\frac{\alpha}{\mu_t \rho_t^K}\right]^{\frac{\alpha}{1-\alpha}}. \tag{25}$$

Because of market clearing, output equals consumption plus investment and the opportunity cost of posting vacancies, all in output units:

$$Y_t = C_t + I_t + \gamma v_t \left(\frac{Y_t}{\varphi N_t} \right). \tag{26}$$

The analysis which follows uses the national accounts definition of GDP, which in this case equals consumption plus gross capital investment.

3.6 Shocks to technology and inflation

Detrended labor productivity follows a geometric random walk:

$$\ln(z_t) = \ln(z_{t-1}) + \varepsilon_t^z . \tag{27}$$

Inflation is related to its own lag and a disturbance term, which may be correlated with the disturbance term to productivity:

$$\pi_t = \rho_\pi \pi_{t-1} + \varepsilon_t^\pi . \tag{28}$$

It has become popular to model price level determination by adding shocks to an explosive Taylor rule. As described by Cochrane (2011), the price level in such New Keynesian models is determined by the threat of a hyperinflationary policy response to out-of-equilibrium inflation disturbances, and excluding the resulting unstable eigenvalue from the numerical calculations serves to reduce the rank of the model. Instead of jointly examining the price formation and wage formation processes, it makes some sense to tie the structural analysis to the VAR discussed in Section 2. The coefficient matrix from the VAR indicates that (27) and (28) do not provide a bad description of the actual dynamics of productivity and inflation. The discussion which follows takes the evolution of productivity and inflation as given, and it answers the question, “Given what we observe about productivity and inflation, how do different wage-setting models perform at matching the behavior of aggregate wages?"

3.7 Equilibrium

The aggregate household conditions, the New Keynesian retail conditions, the aggregated versions of variables from the wholesale sector, the driving processes, and the appropriate transversality conditions constitute a rational expectations equilibrium for this economy. Based on a linearized version of this system, is possible to obtain feedback coefficients using the gensys.m program of Sims (2002). In this particular situation, the equilibrium exists and is unique in the neighborhood around the steady state.

4. Calibration

Most of the parameter values are related to the calibrated or estimated values used by Walsh (2002), Hagedorn and Manovskii (2008), Gertler, Sala, and Trigari (2008), or Gertler and Trigari (2009), with some guidance from current-vintage data where possible. Inflation and growth are zero in steady state—this is not literally true but most models are linearized around such a steady state, and I wish to facilitate comparison with previous work. The real rate of return R equals one percent per quarter. Investment (including residential structures but excluding consumer durables) is 16% of GDP based on NIPA data, and depreciation is 1.5% per quarter. The gross retail markup μ equals 1.11, for a value of θ of 10. Prices are flexible. Wages have an average duration of one year for a value of υ of 0.75. Gertler and Trigari use a similar parameter value, while Gertler, Sala, and Trigari estimate a slightly larger degree of stickiness. Barratieri, Basu, and Gottschalk (2010) find that the most common duration of a nominal wage is one year, with an average quarterly hazard rate of adjustment somewhat below 0.2. Kahn (1997) also presents evidence on the high degree of nominal wage stickiness in the United States. A value of υ of 0.75 therefore seems somewhat conservative.

The total job separation rate ρ equals 0.12 per quarter, in line with the CPS average employment outflow rate and also with the JOLTS hiring and separation rates. Vacancy posting costs altogether equal 0.5 percent of output, in line with the sources cited by Hagedorn and Manovskii (2008). Hairault (2002) and Andolfatto (1996) use a value of 1.0 percent, value while Hagedorn and Manovskii use a very small value which just includes search costs. Barron, Berger, and Black (1997) compile a number of different survey results concerning hiring activity in the United States. Roughly, the average new hire incurs somewhat less than 16 hours of search and screening time. Given that the average employee works about 450 hours in a quarter based on NIPA data for 2007, and given the number of new hires (12%) relative to total workers, this works out to about 0.36 percent of hours worked. This number is probably somewhat too low as a share of efficiency units of labor going toward recruiting activities, since human resources workers are typically better-paid than other workers. Therefore I round this number up to 0.5 percent, which is still a small number. One should note that training costs are much larger than search costs, but looking only at search costs maintains comparability with previous literature. Including training costs does not change any of my results.

The unemployment share a of the matching function equals 0.4. Walsh cites Blanchard and Diamond (1989, 1991) who use postwar CPS data to derive an estimate of 0.4. The steady-state unemployment rate u (end-of-period) equals 0.06 which is just above the average postwar CPS unemployment rate; this value allows for some slight underreporting of unemployment on average. The vacancy rate $v/(v + N)$ equals 0.027 and the quarterly job-finding rate k^w equals 0.60, the former based on JOLTS data and the latter from monthly CPS data. These imply that the vacancy-filling rate k^f equals 1.21 in the steady state. A vacancy-filling rate in excess of one does not pose a problem for the model. Vacancies are a stock and hires are a flow, so the unit for the hiring rate is hires per quarter per vacancy. Workers have a bargaining power η of 0.5. Detrended inflation has a quarterly autocorrelation of 0.5276, which gives the value of ρ_π.

5. Quantitative properties of the model

5.1 The effects of productivity disturbances – the role of wage stickiness

Figure 5 shows the impulse response of labor's share to a once-and-for-all rise in total factor productivity *z*. The flexible wage model shows almost no movements in labor's share. As productivity rises by one percent, consumption nearly rises by one percent, so labor's outside option A/λ_t (equal to AC_t) rises by close to one percent, and wages move in tandem with productivity and consumption. Labor's share does fall ever so slightly since consumption rises more slowly than output on impact (the difference going to investment). However, this mechanism does not generate large enough movements in labor's share to mirror those obtained from the VAR. Even setting the bargaining power for workers to zero would generate a maximum impact effect of productivity on labor's share of -0.17 versus the impact of -0.77 seen in reality.

Figure 5 also shows what happens when one introduces wage stickiness with a value of υ of 0.75—that is, wages are bargained once a year on average. Now the model fits the data much better. Labor's share falls by about half the size of the productivity shock on impact, and it reverts very slowly toward its long run mean. Equation (16) in the model offers some clear intuition as to why this is the case. Wages take some time to catch up to their long run values, so after a productivity shock, wages do not rise by much at first but output does. Wages therefore fall as a share of output. The model does not generate fluctuations in the labor share of quite the same size as those generated by the VAR, but it goes a long way toward doing so. Robustness checks suggest that omitting capital and lowering the worker's bargaining weight η toward zero as suggested by Hagedorn and Manovskii (2008) would further improve the fit of the theoretical model; doing both things would bring the model almost exactly into line with the data.

It seems like the sticky wage model manages to fit at least one important aspect of the data—the strong negative relationship between labor's share and productivity innovations. After a productivity innovation, labor's share falls and takes a long time to recover, and a sticky wage model manages to explain exactly why this should be the case.

Workers mostly earn their old wage when the shock hits, and as a result wages lag productivity, so labor's share falls mechanically. The response of labor's share to productivity is not much of a puzzle when viewed through the lens of a model with staggered Nash bargaining in wages.

5.2 The effects of inflation disturbances: The role of wage stickiness

It turns out that the impulse response of the model to disturbances to inflation also offers support to the sticky wage interpretation of labor share dynamics. In the flexible wage setting, nothing happens to labor's share after an innovation to inflation. Inflation is neutral in its real effects, and nominal wages adjust instantaneously. Figure 6 shows what happens after a disturbance to inflation under sticky wages. The nominal target wage rises but actual wages at first remain where they were. Nominal wages fall as a share of nominal output, so labor's share falls. The model almost exactly predicts the magnitude and persistence of the fall as seen in the data, though the model predicts a slight counterfactual hump shape to the response due to the persistence of inflation.

The intuition regarding the behavior of labor's share after an inflation shock mirrors that regarding a productivity shock. Nominal wages are rebargained only infrequently, so they will lag the wages which would prevail under flexible wages. The sticky wage model accords with the microeconomic data on wage-setting behavior, and it accords with the macroeconomic data on wages as well. Investigations which rely upon an exotic production function, exogenous movements in labor's share, or instantaneous Nash bargaining do not capture the dynamics of labor's share in response to inflation which are predicted by the sticky wage model. Those models predict that labor's share should be constant in response to inflation, while the sticky wage model provides a parsimonious explanation for the negative effect of both productivity growth and inflation on labor's share.

5.3 Comparing sticky price and sticky wage models

It is a common device to include sticky prices in models of the labor market in order to study the propagation and amplification of nominal shocks. Walsh (2002, 2005), Trigari (2009), and many others have used the device of a sticky price retail sector in order to generate a link between the nominal and real sides of the economy. Figure 7 compares the impulse response from the data after an inflationary shock with the impulse responses from the sticky wage model, the sticky price model, and the model with sticky prices and sticky wages. In the sticky price specifications, price persistence ω is set to 0.5 as suggested by Bils and Klenow (2004), and the other parameters are left unchanged. As mentioned before, the basic sticky wage model fits the data extremely well after a shock to inflation. The model with both sticky prices and sticky wages actually overpredicts the negative effect of inflation on labor's share, though it does not do badly in a qualitative sense. The model with sticky prices alone predicts that high inflation should raise labor's share, which is greatly at odds with the data.

For the sticky price model with sticky wages, equation (10) offers some intuition as to what goes wrong. For inflation which is not highly explosive, markups are a decreasing function of current inflation since inflation erodes firms' monopoly power. Labor's share in a standard off-the-shelf New Keynesian model is proportional to the inverse of the markup, so it is possible to write the deviation of labor's share as a function of inflation:

$$(1-\omega)(1-\omega\beta)\hat{l}_t = \omega\hat{\pi}_t - \omega\beta E_t\hat{\pi}_{t+1} = \omega(1-\beta\rho_\pi)\hat{\pi}_t \qquad (29)$$

In New Keynesian models, low markups and higher wages create an incentive for more work or more hiring; a low markup in the retail sector means more revenue for the wholesale sector and a larger bargaining surplus as given by equation (12). But low markups, a large surplus, and the resulting high wages imply a high labor share in response to anything but the most permanent inflationary disturbances. In the data, inflation is associated with a lower labor share. The New Keynesian idea that inflation raises real wages and creates an incentive to work or hire does not seem to match the macroeconomic facts. This finding goes beyond the findings of Rudd and Whelan (2005) in casting doubt on the main mechanism behind the New Keynesian Phillips Curve.

There seems to be an interesting but unintuitive interaction between sticky prices and sticky wages in that sticky prices seem to slightly amplify the effects of sticky wages on labor's share, while sticky prices alone cause the model to fit the data worse than a model with no nominal rigidities at all. In order for sticky price models to match the data, the model needs an offsetting degree of wage stickiness, and the wage stickiness provides the main mechanism through which the model matches wages. A sticky price model by itself is not a promising model of the dynamics of labor's share. While staggered Nash bargaining can generate reasonable labor share dynamics, sticky prices do not seem to contribute much.

6. Conclusion

According to recent research, the behavior of labor's share after a productivity shock is puzzling. In fact, labor's share behaves in a manner which is consistent with the microeconomic facts on wage-setting. A search and matching model with staggered Nash bargaining over nominal wages does a relatively good job at fitting the dynamics of labor's share after a change in productivity, and it does so in a parsimonious, intuitive way. Additionally, it generates predictions regarding the relationship between labor's share and inflation which fit the data. Labor's share falls after a disturbance to either productivity growth or inflation, and theory suggests that this is because it takes time to rebargain nominal wages. As a model of aggregate wage dynamics, the search and matching model with sticky wages performs well. Sticky price models by themselves perform poorly; the dynamics of labor's share are inconsistent with the mechanism through which sticky prices are supposed to affect real aggregates in the absence of sticky wages.

My results suggest that the staggered Nash bargaining model of Gertler and Trigari (2009), when applied to nominal wages using a microeconomically realistic wage formation process, provides a reasonably good description of aggregate wage formation as well. Wages are sticky at a macroeconomic level. The debate regarding the implication of sticky wages for the transmission of macroeconomic shocks is well-motivated, and these results also indicate that sudden movements in labor's share could indicate a disturbance to productivity or prices.

References

Andolfatto, David, 1996. "Business Cycles and Labor-Market Search." American Economic Review 86(1), 112-132.

Barratieri, Alessandro, Susanto Basu, and Peter Gottschalk, 2010. "Some Evidence on the Importance of Sticky Wages." NBER Working Paper 16130.

Barron, John, Mark Berger, and Dan Black, 1997. "Employer Search, Training, and Vacancy Duration." Economic Inquiry 35(1), pages 167-192.

Bils Mark, and Peter Klenow, 2004. "Some Evidence on the Importance of Sticky Prices." Journal of Political Economy 112(5), 947-985.

Blanchard, Olivier, and Peter Diamond, 1989. "The Beveridge Curve." Brookings Papers on Economic Activity 1989(1), 1-60.

Blanchard, Olivier, and Peter Diamond, 1991. "The Aggregate Matching Function." NBER Working Paper 3175.

Boldrin, Michele, and Michael Horvath, 1995. "Labor Contracts and Business Cycles." Journal of Political Economy 103(5), pages 972-1004.

Calvo, Guillermo, 1983. "Staggered Prices in a Utility-Maximizing Framework." Journal of Monetary Economics 12(3), 383-398.

Choi, Sekyu, José Víctor Ríos-Rull, 2009. "Understanding the Dynamics of Labor Share: The Role of Noncompetitive Factor Prices." Annales d'Economie et de Statistique 95/96, pages 251-278.

Cochrane, John, 2011. "Determinacy and Identification with Taylor Rules: A Critical Review." Journal of Political Economy 119(3), pages 565-615.

Cooley, Thomas, and Vincenzo Quadrini, 1999. "A Neoclassical Model of the Phillips Curve Relation." Journal of Monetary Economics 44(2), 165-193.

Den Haan, Wouter, Garey Ramey, and Joel Watson, 2000. "Job Destruction and Propagation of Shocks." American Economic Review 90(3), 482-498.

Gertler, Mark, Luca Sala, and Antonella Trigari, 2008. "An Estimated Monetary DSGE Model with Unemployment and Staggered Nominal Wage Bargaining." Journal of Money, Credit and Banking 40(8), 1713-1764.

Gertler, Mark, and Antonella Trigari, 2009. "Unemployment Fluctuations with Staggered Nash Wage Bargaining." Journal of Political Economy 117(1), 38-86.

Gomme, Peter, and Jeremy Greenwood, 1995. "On the Cyclical Allocation of Risk." Journal of Economic Dynamics and Control 19(1-2), pages 91-124.

Hagedorn, Marcus, and Iourii Manovskii, 2008. "The Cyclical Behavior of Equilibrium Unemployment and Vacancies Revisited." American Economic Review 98(4), 1692-1706.

Hairault, Jean-Olivier, 2002. "Labor-Market Search and International Business Cycles." Review of Economic Dynamics 5(3), 535-558.

Hall, Robert, 2005. "Employment Fluctuations with Equilibrium Wage Stickiness." American Economic Review 95(1), 50-65.

Kahn, Shulamit, 1997. "Evidence of Nominal Wage Stickiness from Microdata." American Economic Review 87(5), 993-1008.

Mortensen, Dale, and Christopher Pissarides, 1994. "Job Creation and Job Destruction in the Theory of Unemployment." Review of Economic Studies 61(3), 397-415.

Ríos-Rull, José Víctor, and Raul Santaeulàlia-Llopis, 2010. "Redistributive Shocks and Productivity Shocks." Journal of Monetary Economics 57, pages 931-948.

Rudd, Jeremy, and Karl Whelan, 2005. "Does Labor's Share Drive Inflation?" Journal of Money, Credit, and Banking 37(2), pages 297-312.

Shao, Enchuan, and Pedro Silos, 2011. "Accounting for the cyclical dynamics of factor shares." Federal Reserve Bank of Atlanta Working Paper 2011-09.

Shimer, Robert, 2005. "The Cyclical Behavior of Equilibrium Unemployment and Vacancies." American Economic Review 95(1), 25-49.

Sims, Christopher, 2002. "Solving Linear Rational Expectations Models." Computational Economics 20(1-2), 1-20.

Trigari, Antonella, 2009. "Equilibrium unemployment, job flows and inflation dynamics." Journal of Money, Credit and Banking, 41(1), 1-33.

Walsh, Carl, 2002. "Labor Market Search and Monetary Shocks." Working paper version. In: Altuĝ, S., Chadha, J., Nolan, C. (Eds.), 2004. Elements of Dynamic Macroeconomic Analysis. Cambridge: Cambridge University Press.

Walsh, Carl, 2005. "Labor Market Search, Sticky Prices, and Interest Rate Policies." Review of Economic Dynamics 8(4), 829-849.

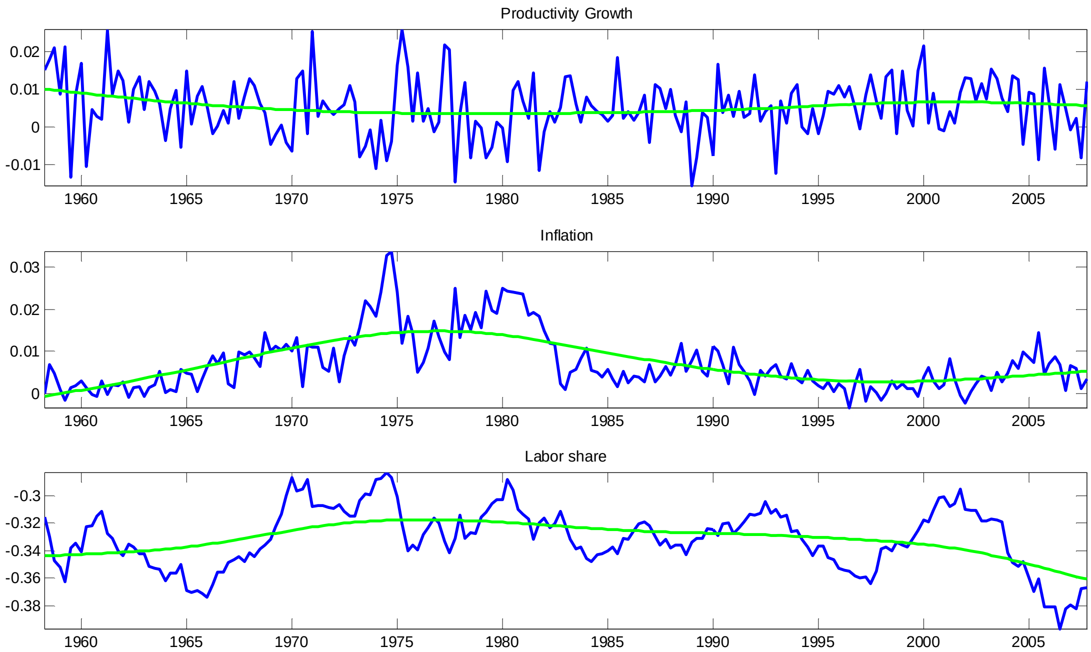
Yashiv, Eran, 2006. "Evaluating the performance of the search and matching model." European Economic Review, 50(4), 909-936.

Table 1 – Point estimates of the VAR coefficients

	Constant	ΔProductivity$_{t-1}$	Inflation$_{t-1}$	Labor Share$_{t-1}$
ΔProductivity$_t$	0.000	0.011	-0.325	0.088
(pr>0)	0.496	0.585	0.011	0.998
Inflation$_t$	0.000	0.041	0.544	0.022
(pr>0)	0.521	0.887	1.000	0.937
Labor Share$_t$	0.000	-0.025	0.075	0.881
(pr>0)	0.350	0.360	0.695	1.000

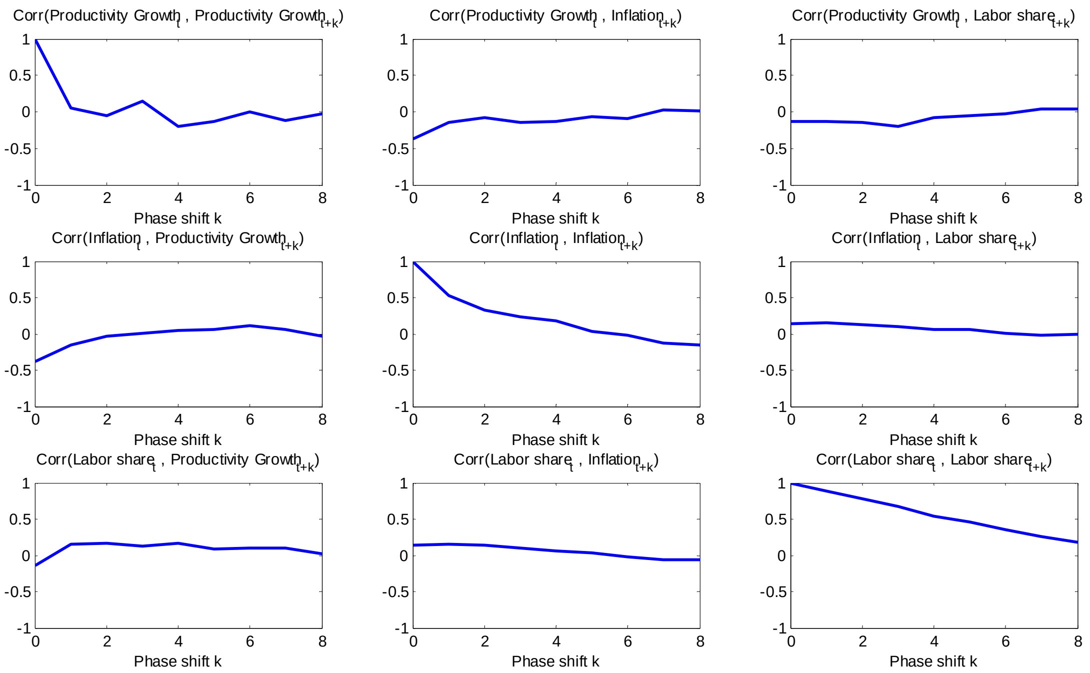
This table contains the maximum likelihood (OLS) estimates of the VAR coefficients in the one-lag, three-variable system. Below the point estimates are the posterior probabilities that these VAR coefficients are positive.

Figure 1 – Productivity growth, inflation, and labor share, with trends



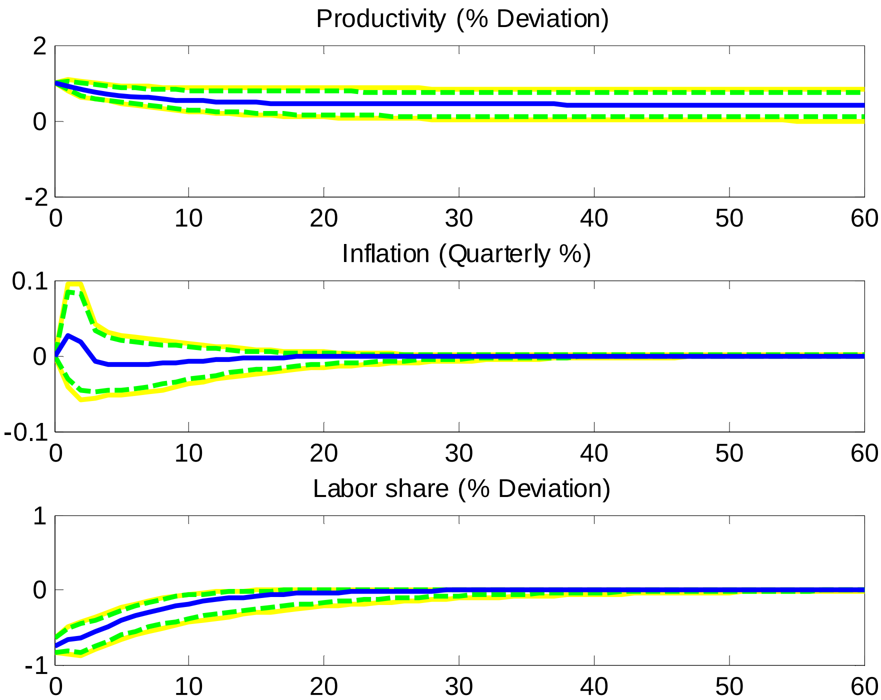
Sources: BLS (nonfinancial corporate output per hours worked and implicit price deflator for output), NIPA (labor share, defined as the share of compensation of employees as a share of gross value added less taxes on production and imports, for the nonfinancial corporate business sector). Trends (the smooth series) are calculated using a Hodrick-Prescott (HP) filter with a smoothing parameter of 100,000, except for inflation, whose trend is calculated using a parameter of 10,000. Time is in quarters.

Figure 2 – Cross correlations among productivity growth, inflation, and labor share



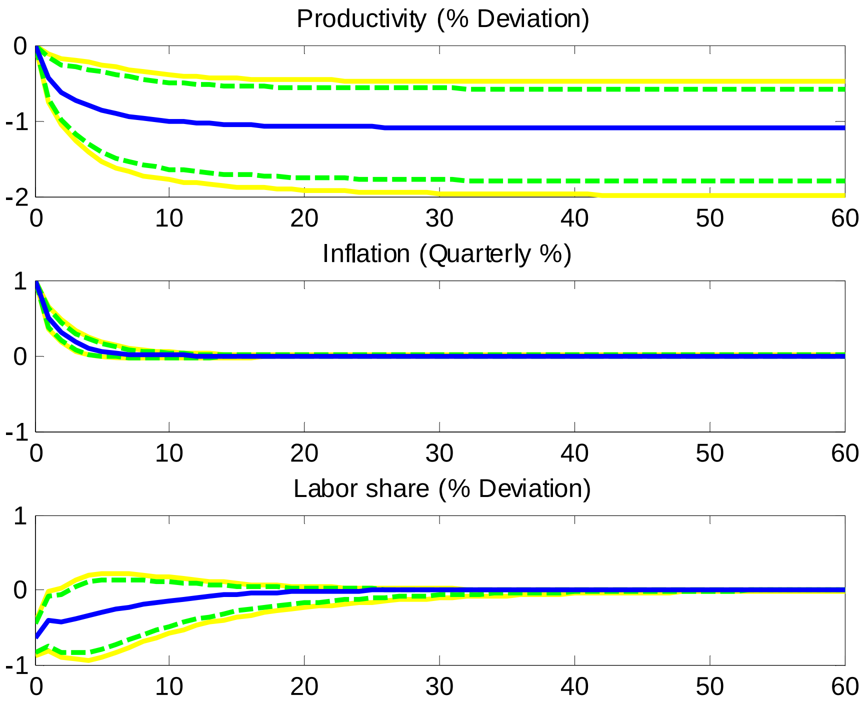
This figure shows the pattern of cross-correlation in the data among the three detrended quarterly series. The rows contain correlation coefficients centered on contemporaneous productivity growth, inflation, and the labor share, respectively. The columns contain cross correlation coefficients on leads of the same variables for up to two years.

Figure 3 – VAR impulse responses to a unit productivity innovation



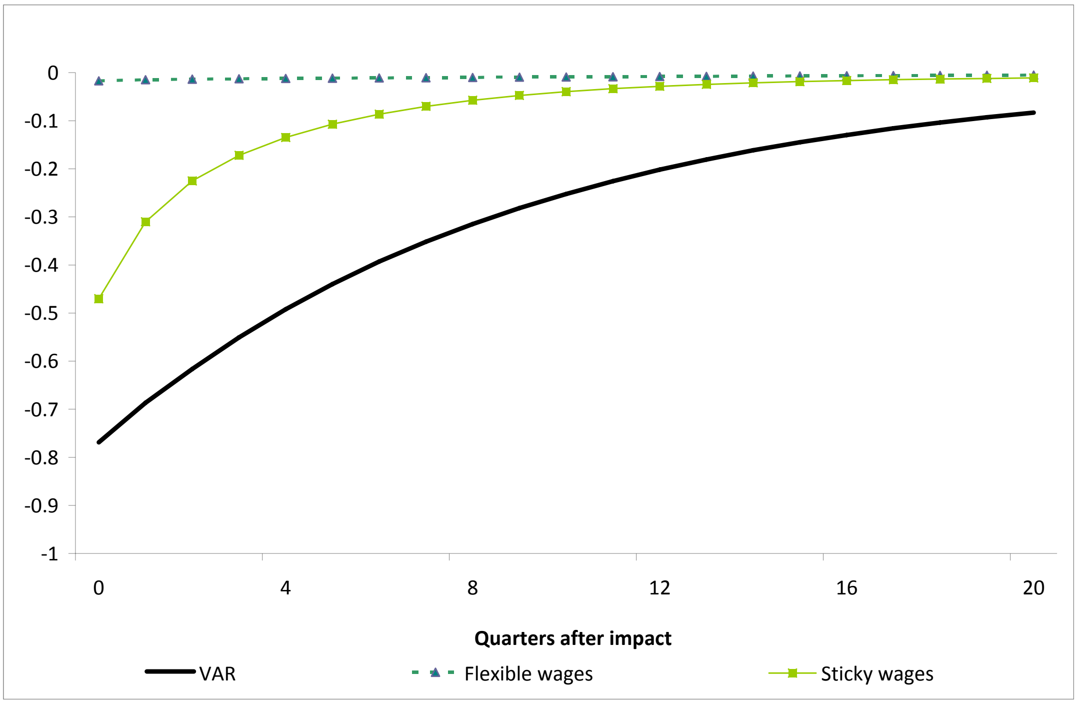
This figure shows impulse responses to a unit productivity innovation under a short-run identification scheme, where a shock has a unit impact effect on the variable of interest. Productivity is assumed to be nonstationary in levels. The panels show the posterior median impulse response, along with 90% and 95% posterior confidence intervals from 90,001 MCMC simulations. The VAR is a one-lag VAR with the ordering: Inflation, productivity, and the labor share. Time is in quarters.

Figure 4 – VAR impulse responses to a unit inflation innovation



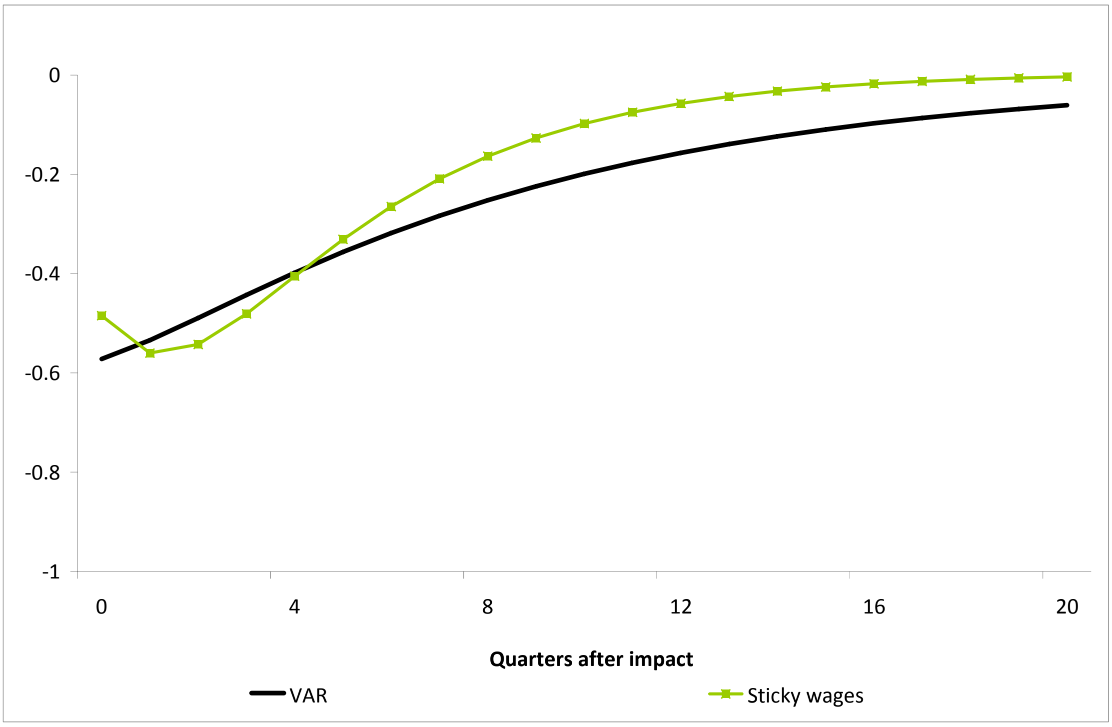
This figure shows impulse responses to a unit inflation innovation under a short-run identification scheme, where a shock has a unit impact effect on the variable of interest. Productivity is assumed to be nonstationary in levels. The panels show the posterior median impulse response, along with 90% and 95% posterior confidence intervals from 90,001 MCMC simulations. The VAR is a one-lag VAR with the ordering: Productivity, inflation, and the labor share. Time is in quarters.

Figure 5 – Impulse responses of labor's share to a unit productivity shock



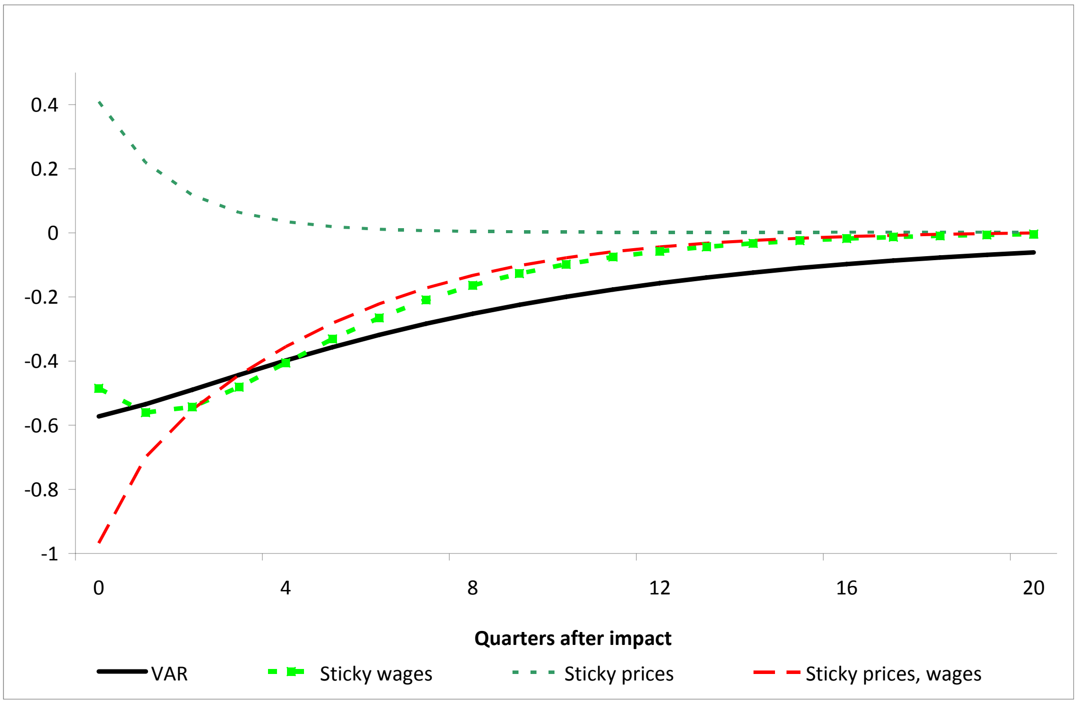
This figure shows impulse responses of labor's share to a unit productivity shock in the data as well as under flexible wages and sticky wages (persistence=0.75).

Figure 6 – Impulse responses of labor's share to a unit inflation shock



This figure shows impulse responses of labor's share to a unit productivity shock in the data as well as under sticky wages (persistence=0.75). Labor's share is constant in this situation under flexible wages, since inflation has no real effect whatsoever.

Figure 7 – Impulse responses of labor's share to a unit inflation shock



This figure shows impulse responses of labor's share to a unit productivity shock in the data as well as under different combinations of sticky wages (persistence=0.75) and sticky prices (persistence=0.75).