

**Kiel Institute for the World Economy**  
Duesternbrooker Weg 120  
24105 Kiel (Germany)

**Kiel Working Paper No. 1292**

**The Effect of Low-Wage Subsidies  
on Skills and Employment**

by

**Frank Oskamp and Dennis J. Snower**

September 2006

*The responsibility for the contents of the working papers rests with the authors, not the Institute. Since working papers are of a preliminary nature, it may be useful to contact the authors of a particular working paper about results or caveats before referring to, or quoting, a paper. Any comments on working papers should be sent directly to the authors.*

# The Effect of Low-Wage Subsidies on Skills and Employment

Frank Oskamp<sup>a</sup> and Dennis J. Snower<sup>a,b</sup>

<sup>a</sup>*Kiel Institute for the World Economy* <sup>b</sup>*Christian-Albrechts-University, Kiel*

---

## Abstract

We explore the far-reaching implications of low-wage subsidies on aggregate employment. Low-wage subsidies have three important effects. First, they promote employment of unskilled workers (who tend to be the ones who earn low wages). Second, by raising the payoff of unskilled work relative to skilled work, low-wage subsidies reduce the incentive to become skilled, so that there are more unskilled workers associated with a relatively low employment rate. Third, the government budget constraint has to be taken into account, which is supposed to cause an additional tax burden for the skilled workers. This amplifies the negative effect of low-wage subsidies on the incentive to acquire human capital. Thus, the first effect on the one hand and the second and third effect on the other hand pull in opposite directions in terms of employment.

This paper presents a theoretical model of the labor market in which these effects can be analyzed. We then calibrate the model with respect to the German labor market to shed light on the relative strengths of these effects and thereby assess the degree to which low-wage subsidies encourage or discourage employment.

*Keywords:* low-wage subsidies; training incentives; employment; unemployment; skill acquisition

*JEL classification:* I29, J21, J24, J31, J38

---

## Address:

Kiel Institute for the World Economy  
Duesternbrooker Weg 120  
24105 Kiel  
Germany

Telephone: +49 431 8814-272  
Fax: +49 431 8814-525  
E-Mail: frank.oskamp@ifw-kiel.de  
dennis.snower@ifw-kiel.de

# 1 Introduction

In many OECD countries, the relative position of employees at the bottom of the wage distribution has deteriorated over the past decades. Whereas in the US this worsening has taken the form of lower relative real wages, in a number of continental European countries the deterioration appears in higher relative unemployment rates for unskilled people. Confronted with these problems, policy makers have been searching for labor market instruments that reduce unemployment while avoiding large disparities in income.<sup>1</sup> A popular tool are low-wage subsidies (LWSs), which have been widely advocated; this case has been made particularly eloquently by Phelps (1997a).<sup>2</sup> The central policy problem posed by unskilled workers is that they are associated to low-wages or low employment opportunities or both. Raising their wages would reduce firm's demand for them, while lowering their wages would be socially unacceptable. LWSs respond to this policy problem by driving a wedge between the incomes these workers receive and their labor costs.<sup>3</sup> These subsidies, in various guises, have been implemented in various countries, including e.g. Canada (Self-Sufficiency Project)<sup>4</sup>, Germany (Kombi-Lohn)<sup>5</sup>, Great Britain (Working Families Tax Credit)<sup>6</sup> and the United States (Earned Income Tax Credit, EITC)<sup>7</sup>.

Pioneered by the work by Pigou (1932) and Kaldor (1936), a huge strand of theoretical and empirical literature has focused on the impact and optimal design of LWSs.<sup>8</sup> Many theoretical papers use static analytical frameworks and thus have the strong drawback that they can only analyze the short-run impact of the policy but not the dynamic long-run effects.<sup>9</sup> The existing dynamic frameworks for evaluating subsidies are mainly deterministic and not well suited to analyze the impact of the policy, such as Hoon and Phelps (2003).<sup>10</sup> Mortensen and Pissarides (2003) explore the effects of taxes and subsidies on job creation, job destruction, employment and wages in a search and matching equilibrium model. However, in their model, migration between skill groups, which is the essential component in our model, is not possible.<sup>11</sup> Orszag and Snower (2003) examine the relative performance of LWSs and unemployment vouchers. However, the effects on the incentives to acquire human capital are not part of their analysis.

One exception is the paper by Heckman et al. (2003). It is closest to ours as they examine the

---

<sup>1</sup>See, for instance, Ventry (2001) for a detailed survey of the political history of the Earned Income Tax Credit.

<sup>2</sup>See, furthermore, Phelps (1994a, 1997b). With respect to Germany, especially Sinn et al. (2002, 2006) argue for a wage subsidy, which is a core element of their policy proposal "activating social support" ("Aktivierende Sozialhilfe"). Another proposal is Riphan et al. (1999).

<sup>3</sup>See, for an analysis, Hamermesh (1978) as well as Haveman and Palmer (1982).

<sup>4</sup>See, for example, Michalopoulos et al. (2005) for a description of the project and an analysis based on a randomized social experiment.

<sup>5</sup>Analyses of different proposals and existing models have been undertaken by Boss (2006), Dietz et al. (2006), Spermann (2003) as well as Spermann and Strootmann (2005). Buslei and Steiner (1999) survey the theoretical and empirical studies in this field; they also examine the effects of LWSs on labor demand and supply in Germany.

<sup>6</sup>See, for example, Dilnot and McCrae (2000) for a description and analysis of the program.

<sup>7</sup>See Hotz and Scholz (2003) for a detailed description and an exhaustive review of the literature.

<sup>8</sup>With respect to existing subsidy schemes, especially the EITC has been analyzed intensely. See, for example, Eissa and Liebman (1996), Meyer (2002) as well as Eissa and Hoynes (2005) for an analysis of the effects on labor supply. See Liebman (2001) for an analysis of the optimal design. Bassanini et al. (1999) analyze the effects of a simplified version of the EITC on the labor market in different countries.

<sup>9</sup>See, for instance, Layard and Nickell (1980), Layard, Nickell and Jackman (1991: 490-492) and Snower (1994).

<sup>10</sup>They analyse the impact of low-wage subsidies using a labor-turnover model illustrated in Phelps (1994b). The analysis is limited to the impact of subsidies on the worker's decision to quit the firm and thereby on the firm's incentive to invest in firm-specific training.

<sup>11</sup>For recent work using a search and matching-model to evaluate subsidies see, for instance, Cardullo and Van der Linden (2006). Also in their model migration between skill groups is not possible.

impact of wage subsidies on skill formation. Particularly, they focus on the EITC and analyze their impact on the incentives to accumulate skills in two different models of human capital formation. Other than Heckman et al. (2003), our analysis is not based on the EITC structure but on a more general version of a LWS. Furthermore, we do not only focus on skill formation but also on the effects of subsidies on aggregate employment. In this context, we model the wage bargaining process explicitly and thereby also examine the impact of subsidies on wages.

Generally speaking, much of the existing macro literature on subsidizing low-wage employment which mostly corresponds to unskilled employment, has tended to ignore the impact of LWSs on skill formation. Thus, a possible negative effect on the incentives to acquire human capital and thereby on skilled employment is not taken into account. Therefore, it is commonly supposed that since LWSs reduce the labor cost of low wage workers, they must stimulate aggregate employment. This paper calls this presumption into question. As our analysis below shows, the negative effect is of particular importance for an overall assessment of LWSs. In this context, our analysis distinguishes from the existing literature. We explicitly take into account the heterogeneity of the labor market by distinguishing between a skilled and an unskilled labor force and, furthermore, we allow for transition between these two groups. Specifically, we consider three important employment effects of LWSs:

1. *The direct employment effect:* The demand for unskilled labor rises, since the cost of this labor falls.
2. *The skill-acquisition effect:* The incentive to acquire skills falls, because when people acquire skills, their productivities and wages rise and, as result, they lose their entitlement to the LWSs. This effect reduces employment, since unskilled workers have lower employment rates than their skilled counterparts.
3. *The government budget effect:* The LWSs are generally financed through taxes. Higher taxes may lead to lower employment.

This paper presents a theoretical model of the labor market in which these three effects can be analyzed. We apply a simple dynamic model, in which the transition probabilities between the different labor market states are governed by a Markov Process. The transition probabilities are specified as functions of the LWSs. We then calibrate this model with respect to the German labor market in order to shed light on the relative strengths of these effects and thereby assess how LWSs affect employment. Our calibration results suggest that the skill-acquisition effect and the government budget effect are important. In the steady state, they are at least as large as the direct employment effect. Consequently, LWSs do not raise employment; on the contrary, employment falls slightly in the long run.

The paper is organized as follows. Section 2 presents the underlying model. In section 3, the model is calibrated for the German economy. In section 4, we illustrate the impact of LWSs on employment. Section 5 concludes.

## 2 The Underlying Model

### 2.1 Employment and Unemployment

Production takes place in worker-firm pairs. For simplicity, there is no capital.<sup>12</sup> Workers and firms are infinitely lived. Total population is divided into groups: people engaged in training and those in the labor force (either employed or unemployed). Let  $T$  be the inflow into training and  $p$  be the number of periods that training lasts. Thus, the outflow from training in a given year  $t$  is  $T_{t-p}$ . Those in the labor force comprise  $N_s$  skilled employed,  $U_s$  skilled unemployed,  $N_u$  unskilled employed, and  $U_u$  unskilled unemployed. (Here, as well as for other variables below, the subscript  $s$  stands for "skilled"; the subscript  $u$  for "unskilled".)

Let  $h_i$  be the probability that an unemployed (either skilled or unskilled) is hired, and let  $f_i$  be the probability that an employee (either skilled or unskilled) is fired. The number of skilled employees,  $N_s$ , is the sum of three components: (a) skilled employees from the previous period who have not been fired, (b) skilled unemployed persons from the previous period who are hired and (c) people who finished training and are hired:

$$N_{s,t+1} = (1 - f_s)N_{s,t} + h_s U_{s,t} + h_s T_{t+1-p} \quad (1)$$

Regarding the skilled unemployed, we take the possibility of deskilling into account. We model this phenomenon quite simply, assuming that in each period an exogenous proportion  $\lambda$  of the skilled unemployed loses its human capital, turning them into unskilled unemployed people. The number of skilled unemployed comprises (a) those unemployed from the previous period who are not hired and have not lost their human capital, (b) skilled employees who have been fired and (c) those who finished their training but are not hired:

$$U_{s,t+1} = (1 - h_s - \lambda)U_{s,t} + f_s N_{s,t} + (1 - h_s)T_{t+1-p} \quad (2)$$

Furthermore, in each period a fraction  $\tau^N$  of the unskilled employees and a fraction  $\tau^U$  of the unskilled unemployed enters training. (Here, as well as for other variables below, the superscript  $N$  stands for "previously employed"; the superscript  $U$  for "previously unemployed".) Thus unskilled employment is

$$N_{u,t+1} = (1 - f_u - \tau^N)N_{u,t} + h_u U_{u,t} \quad (3)$$

and unskilled unemployment is

$$U_{u,t+1} = (1 - h_u - \tau^U)U_{u,t} + f_u N_{u,t} + \lambda U_{s,t} \quad (4)$$

The inflow into the training phase is calculated as

$$T_{t+1} = \tau^N N_{u,t} + \tau^U U_{u,t} \quad (5)$$

The dynamic structure of the model is summarized in figure 1.<sup>13</sup> In short, the unskilled employed and unemployed ( $N_u$  and  $U_u$ ) must go through training in order to become skilled employed and unemployed ( $N_s$  and  $U_s$ ). A fraction  $\lambda$  of the skilled unemployed becomes unskilled. Unemployed

<sup>12</sup>Insofar as high-skilled labor is more complementary with capital than is low-skilled labor, the inclusion of capital in our model would strengthen our result that LWSs reduce employment.

<sup>13</sup> $T^p$  ( $T^1$ ) represents the age cohort being in training for  $p$  (1) periods. In a given period  $t$ , the total stock of people being in training is calculated as  $T_t + T_{t-1} + T_{t-2} + \dots + T_{t+1-p}$ .

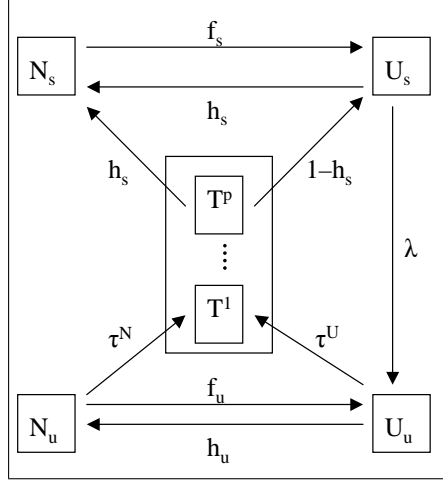


Figure 1: The dynamic structure of the model

people (skilled  $U_s$  or unskilled  $U_u$ ) who are hired (at rates  $h_s$  and  $h_u$ , respectively) become employed, and employed people (skilled  $N_s$  or unskilled  $N_u$ ) who are fired (at rates  $f_s$  and  $f_u$ , respectively) become unemployed.

In what follows, we proceed to provide the microfoundations for the transition probabilities between the five labor market states ( $N_u$ ,  $N_s$ ,  $U_u$ ,  $U_s$  and  $T$ ), with the exception of the exogenous deskilling parameter  $\lambda$ . Then, we examine the employment influence of LWSs by deriving their effect on these transition probabilities.

## 2.2 Transition Probabilities

### 2.2.1 Hiring and Firing Rates

Assume that the work-leisure options of an individual worker are discrete, i.e. the worker is either unemployed or employed. If employed, the worker of type  $i$  produces  $a_i$  of output per period. There is a random operating cost  $\epsilon_t$ , iid across workers and time, with a mean normalized to zero and a constant cumulative distribution function  $\Gamma(\epsilon_t)$ . For the producer wage  $w_i$ , the firing rate  $f_i$ , the firing cost  $\varsigma_i$  per worker and the discount factor  $\delta$ , the expected present value of profit generated by an employee is<sup>14</sup>

$$V_{i,t} = (a_i - w_i) - \epsilon_t + \sum_{t=1}^{\infty} \delta^t (1 - f_i)^t (a_i - w_i) - \sum_{t=0}^{\infty} \delta^{t+1} (1 - f_i)^t f_i \varsigma_i \quad \text{with } i = s, u$$

Given the firing cost  $\varsigma_i$  per worker, an employee is fired when  $V_{i,t} < -\varsigma_i$ . Thus, the firing rate is

$$f_i = 1 - \Gamma\left(\frac{a_i - w_i + c_f w_i (1 - \delta)}{1 - \delta(1 - f_i)}\right) \quad \text{with } i = s, u \quad (6)$$

<sup>14</sup>For a derivation of the hiring and firing rates, see Appendix A1.

Given the hiring cost  $\gamma_i$  per worker, an unemployed is hired, when  $V_{i,t} > \gamma_i$ . Thus, the hiring rate is

$$h_i = \Gamma\left(\frac{a_i - w_i - \delta f_i c_f w_i}{1 - \delta(1 - f_i)} - c_h w_i\right) \quad \text{with } i = s, u \quad (7)$$

### 2.2.2 Training Rates

The training rates,  $\tau^N$  and  $\tau^U$ , (i.e. the proportion of unskilled employees and unskilled unemployed, respectively, who enters training) are modeled as functions of the expected income differential which exist between the skilled and unskilled people. Thus, we first describe the relevant income equations, then we derive the training rates.

At the beginning of each period, each unskilled worker decides whether to enter training, (i.e. whether to acquire sufficient human capital to become skilled). This decision is discrete (the person either trains over  $p$  periods or does not train at all). For simplicity, we assume that each worker is indifferent to work and thus his objective is to maximize expected lifetime income,  $Y$ , given the wages as well as the hiring and firing rates when being skilled and unskilled, respectively. We use the notation for incomes as shown in table 1.

variable	expected lifetime income of ...
$Y_s^N$	skilled employee
$Y_s^U$	skilled unemployed person
$Y_u^N$	unskilled employee
$Y_u^U$	unskilled unemployed person
$Y^{T,N}$	person (previously employed) entering training
$Y^{T,U}$	person (previously unemployed) entering training

Table 1: The expected lifetime income

The expected lifetime income of an unskilled unemployed who decides to remain unskilled is

$$Y_{u,t}^U = b_{u,t} + \delta[(1 - h_u)Y_{u,t+1}^U + h_u Y_{u,t+1}^N] \quad (8)$$

In words, in the current period this person receives an unemployment benefit  $b_u$ . In the following period, she faces a probability  $1 - h_u$  of remaining unemployed being associated with an expected lifetime income  $Y_{u,t+1}^U$ ; with a probability  $h_u$  she will get a job and receive an expected lifetime income  $Y_{u,t+1}^N$ .

The lifetime income of a unskilled employee who decides to remain unskilled is

$$Y_{u,t}^N = w_{u,t}^c + \delta[(1 - f_u)Y_{u,t+1}^N + f_u Y_{u,t+1}^U] \quad (9)$$

where  $w_{u,t}^c$  is the consumer wage which is described below.

If this person decides to enter training, the expected lifetime income is

$$Y_t^{T,j} = b_{u,t} \sum_{k=0}^{p-1} \delta^k - e^j + \delta^p [h_s Y_{s,t+1}^N + (1 - h_s) Y_{s,t+1}^U] \quad \text{with } j = N \quad (10)$$

where we assume that the person receives an income  $b_u$  (equal to the unemployment benefit) in each training period; the education costs are given by  $e^j$ . The expected lifetime income of a skilled

unemployed worker is<sup>15</sup>

$$Y_{s,t}^U = b_{s,t} + \delta[(1 - h_s - \lambda)Y_{s,t+1}^U + h_s Y_{s,t+1}^N + \lambda Y_{u,t+1}^U]$$

Finally, the expected lifetime income of a skilled employed worker is

$$Y_{s,t}^N = w_{s,t}^c + \delta[(1 - f_s)Y_{s,t+1}^N + f_s Y_{s,t+1}^U]$$

Workers are assumed to be heterogenous in terms of their exogenously given education costs,  $e^j$ . An unskilled unemployed person enters training if and only if  $Y_t^{T,U} \geq Y_{u,t}^U$ . For the marginal unskilled unemployed who decides to enter training, the following equation is valid:

$$Y_t^{T,U} = Y_{u,t}^U$$

Substituting  $Y_t^{T,U}$  and  $Y_{u,t}^U$  by the corresponding equations (10) for  $j = U$  and (8) and taking into account that  $Y_{u,t}^U$ ,  $Y_{u,t}^N$ ,  $Y_{s,t}^N$  and  $Y_{s,t}^U$  can be expressed by their corresponding steady state equations,<sup>16</sup> we obtain an equation for  $e^j$  with  $j = U$ .

However, as already mentioned, we are interested in the proportion of the unskilled employees and unemployed, who enter training ( $\tau^N$  and  $\tau^U$ , respectively). Therefore, we have to illustrate the relationship between  $e^j$  and  $\tau^j$ . The value of  $e^j$  represents the costs of the marginal worker, i.e. the worker who is indifferent between acquiring human capital and remaining unskilled. Ordering the workers in terms of their individual costs, from the lowest to the highest, we let the cumulative distribution of the costs be approximated by a continuum given by the function  $e^j(\tau^j)$ ,  $(\partial e^j / \partial \tau^j) > 0$ .<sup>17</sup> As we are interested in  $\tau^j$ , we use the inverse function in the remainder:  $\tau^j(e^j)$ , with  $(\partial \tau^j / \partial e^j) > 0$ . For simplicity we assume:  $\tau^j = x e^j$  with  $x > 0$ . Using the expression for  $e^j$  with  $j = U$ , calculated as described above, we obtain the following equation for the proportion of unskilled unemployed who enter training:<sup>18</sup>

$$\tau^U = x * [(\Phi_{N1} [-b_u + w_u^c] h_u \delta + \Phi_{N2}) / \Phi_D] \quad (11)$$

where:

$$\begin{aligned} \Phi_{N1} &= [1 - \delta(2 - f_s - h_s - \lambda) + \delta^2((1 - f_s)(1 - \lambda) - h_s)] \\ \Phi_{N2} &= \delta^p [b_u(1 - \delta)[1 - (1 - f_s - h_s)\delta][1 - (1 - f_u - h_u)\delta] \\ &\quad - b_s(1 - \delta)[1 - h_s - (1 - f_s - h_s)\delta][1 - (1 - f_u - h_u)\delta] \\ &\quad + b_u \delta [h_u \delta (1 - \delta(1 - f_s)) + h_s(1 - \delta)(1 - (1 - f_u)\delta)] \lambda \\ &\quad - h_s(1 - \delta)[1 - (1 - f_u - h_u)\delta](1 + \delta \lambda) w_s^c - h_u \delta^2 [1 - h_s - (1 - f_s - h_s)\delta] \lambda w_u^c \\ \Phi_D &= [(-1 + \delta)[1 - (1 - f_u - h_u)\delta](1 - \delta(2 - h_s - f_s - \lambda) + \delta^2((1 - f_s)(1 - \lambda) - h_s))] \end{aligned}$$

<sup>15</sup>This expression is similar to eq. (8) with one exception. In contrast to an unskilled unemployed, a skilled unemployed also faces a certain probability,  $\lambda$ , of losing its human capital and becoming an unskilled unemployed.

<sup>16</sup>See Appendix A2.

<sup>17</sup>Assume, for example, that the expected value of remaining unskilled,  $Y_u^j$ , increases due to the introduction of LWSs. Given a constant value of being skilled, the cost of education  $e^j$  of the marginal worker has to be smaller in order to balance the expected payoff of being skilled and the payoff of remaining unskilled: only workers with relatively low education costs still have an incentive to acquire human capital. Therefore, the proportion of the unskilled employees and unemployed, who enters training ( $\tau^N$  and  $\tau^U$ , respectively), decreases.

<sup>18</sup>With  $b_s = \phi w_s^c$ ,  $b_u = \phi w_u^c$ ,  $w_s^c = w_s(1 - t_s)$  and  $w_u^c = w_u(1 - t_u + \sigma)$ .



After having modeled the decision making of an unskilled unemployed, we model the decision making of an unskilled employee. Based on an analog reasoning, we obtain a similar equation for the decision making of the marginal unskilled employee:

$$Y_t^{T,N} = Y_{u,t}^N$$

Analogously, we obtain the following equation for the proportion of unskilled employees who enter training:

$$\tau^N = x * [-w_u^c + ( \Phi_{N1} [-b_u(1 - \delta + h_u\delta) + \delta((1 - f_u)(1 - \delta) + h_u\delta)w_u^c] + \Phi_{N2} ) / \Phi_D] \quad (12)$$

## 2.3 Productivities and Wages

In order to calculate the transition probabilities and thereby to be able to assess the impact on LWSs on employment and unemployment, we now model two major components of the transition probabilities: the productivities and the wages.

### 2.3.1 Productivities

In the remainder, we assume diminishing returns to labor. This is implemented by using the production function:  $Y_i = \beta_i N_i^\alpha$  with  $0 < \alpha < 1$ , taking into account that each firm uses only one input (skilled labor,  $N_s$ , or unskilled labor,  $N_u$ ). The productivity is calculated as:

$$a_i = \beta_i N_i^{\alpha-1} \quad (13)$$

### 2.3.2 Wages

It is assumed, that the producer wage is the outcome of a Nash bargain. The wage is renegotiated in each period between each employee and the firm. Under bargaining agreement, the employee receives the consumer wage  $w_i^c$ , which is described below, and the firm receives the expected profit  $(a_i - w_i)$  in each period. Under disagreement, the employee's fallback income is  $b_i$ , assumed equal to the unemployment benefit and the firm's fallback position is  $-\varsigma_i$ , i.e. during disagreement the employee imposes the maximal cost on the firm (e.g. through strike, work-to-rule) short of dismissal, this cost is assumed to be equal to the firing costs. Assuming that disagreement in the current period does not affect future returns, the employee's surplus is  $w_i^c - b_i$  and the firm's surplus is  $a_i - w_i + \varsigma_i$ . The bargaining strength of the employee relative to the firm is represented by  $\mu_i$ . In the baseline model we assume progressive taxation. This is introduced by using two different tax rates:  $t_s$  and  $t_u$ , where  $t_s > t_u$ . The following Nash bargaining problem has to be solved in order to calculate the producer wage  $w_i$ .<sup>19</sup>

$$\text{Maximize } \Omega = [w_i^c - b_i]^{\mu_i} [a_i - w_i + \varsigma_i]^{1-\mu_i}$$

With respect to the skilled employee, we set  $w_s^c = w_s(1 - t_s)$ . Setting the first derivative,  $\frac{\partial \Omega}{\partial w_s}$ , equal to zero and then<sup>20</sup> taking into account that  $b_s = \phi w_s(1 - t_s)$  and  $\varsigma_s = c_f w_s$ , we obtain the

<sup>19</sup>For a detailed description of the following calculations see Appendix A3.

<sup>20</sup>In the bargaining process, the unemployment benefits and the firing costs are considered as constants, which cannot be influenced by bargaining. However, in the steady state, the unemployment benefits and the firing costs are calculated in relation to the wage.

following expression

$$w_s = \frac{\mu_s a_s}{1 - c_f \mu_s - \phi + \phi \mu_s} \quad (14)$$

With respect to the unskilled employee, we set  $w_u^c = w_u(1 - t_u) + \kappa$ . Setting the first derivative,  $\frac{\partial \Omega}{\partial w_u}$ , equal to zero and then taking into account that  $b_u = \phi (w_u(1 - t_u) + \kappa)$ ,  $\varsigma_u = c_f w_u$  and  $\kappa = \sigma w_u$ ,<sup>21</sup> we obtain the following expression

$$w_u = \frac{\mu_u a_u (1 - t_u)}{(1 - c_f \mu_u - \phi + \phi \mu_u)(1 - t_u) + \sigma(1 - \mu_u)(1 - \phi)} \quad (15)$$

## 2.4 Government Budget Constraint

Our model of the labor market is closed through a government budget constraint, (i.e. that the government's spending on labor market policy instruments is equal to its tax receipts). The government budget constraint is expressed as follows:

$$\begin{aligned} t_s w_s N_s + t_u w_u N_u &= \phi [w_s(1 - t_s)] U_s + \phi [w_u(1 - t_u) + \sigma w_u] U_u \\ &+ \phi [w_u(1 - t_u) + \sigma w_u] (T_t + T_{t-1} + T_{t-2} + \dots T_{t+1-p}) \\ &+ \sigma w_u N_u \end{aligned} \quad (16)$$

where the left-hand side stands for the tax receipts, to be paid by the skilled and unskilled employees. The term in the first row on the right-hand side represents the unemployment benefits, which are paid to the skilled and unskilled unemployed. Moreover, as already mentioned, it is assumed, that the people being in training receive an income which is equal to the unemployment benefits of the unskilled (second row). Finally, the LWSs have to be financed.

## 2.5 Labor Market Equilibrium and the Role of LWSs

In the remainder, we assume that total population is normalized to unity. The equations (1)-(4), (6)-(7) and (11)-(16) describe the complete labor market equilibrium. In order to calculate employment and unemployment through the equations (1)-(4), we need to know the transition probabilities. They are calculated by using the equations (6) and (7) for  $i = s, u$ , which determine the hiring and firing rates and the equations (11) and (12), which determine the training rates. This calculation requires the values of the wages as well as the values of the productivities which are given by the equations (13)-(15). And finally, given the rate of the LWSs,  $\sigma$ , and assuming that the tax rate which is relevant for skilled workers is 25 percent higher than the tax rate which is relevant for unskilled workers ( $t_s = 1.25 * t_u$ ),<sup>22</sup> equation (16) yields the tax rate,  $t_u$ , that balances the budget.<sup>23</sup> In this context, LWSs affect employment through three channels (see figure 2):<sup>24</sup>

(i) Channel A illustrates the *direct employment effect*. LWSs directly reduce the producer wage for unskilled employment,  $w_u$ , and thereby increase the hiring rate,  $h_u$ , and decrease the firing rate,  $f_u$ . Thus, the demand for unskilled labor rises and unskilled employment,  $N_u$ , increases.

<sup>21</sup>In the bargaining process, also the low-wage subsidy is considered as a constant, which cannot be influenced by bargaining. This is a better mapping of the reality than expressing it in relation to the wage. However, in the steady state, the subsidies are calculated in relation to the wage.

<sup>22</sup>See section 3 for an illustration of the derivation of this value.

<sup>23</sup>The equations describing the steady state are given in Appendix A4.

<sup>24</sup>Naturally however, the channels are interdependent. In figure 2 the three channels are illustrated through black arrows. Interdependencies are denoted by gray arrows.

(ii) Channel B illustrates the *skill-acquisition effect*. LWSs increase the consumer wages for unskilled employees,  $w_u^c$ . Thus, they reduce the incentive to acquire skills. The proportion of unskilled employees and unemployed who enters training ( $\tau^N$  and  $\tau^U$ , respectively) decreases. Finally, the unskilled labor force increases. Everything else equal, unskilled employment increases and skilled employment decreases.

(iii) Channel C illustrates the *government budget effect*. LWSs have to be financed via taxes on wages of the skilled workers, thus the negative effect on the incentive to acquire skills via channel (B) is amplified.

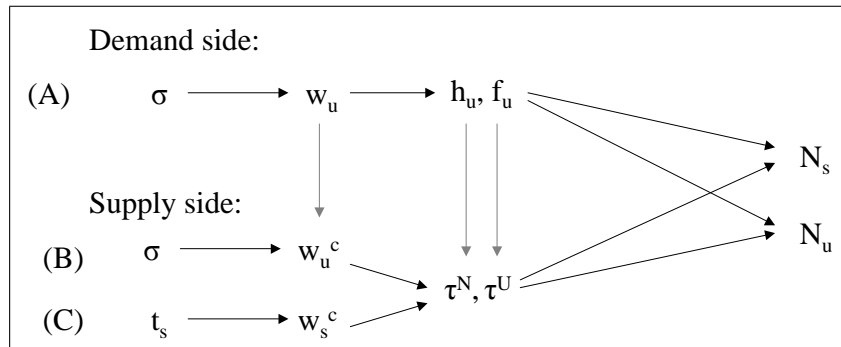


Figure 2: The transmission channels

### 3 Calibration

The steady state solutions cannot be studied analytically but only numerically. Thus, we first calibrate the model for the initial steady state (economy without LWSs,  $\sigma = 0$ ). The model is calibrated in order to match the characteristics of the German labor market for the period 1997-2003.<sup>25</sup> The period of analysis is one year. The calibration is done in several steps. In a first step, the exogenously given parameters are described. The interest rate,  $i$ , is set at 2.5 %, <sup>26</sup> and we set the discount rate  $\delta = \frac{1}{1+i}$ . We define the skilled labor force ( $\theta = N_s + U_s$ ) by an educational attainment level at least equal to upper-secondary education.<sup>27</sup> Using available OECD data for educational attainment (OECD 1999, 2000, 2001, 2002, 2003, 2004b, 2005), we obtain the relative values for  $N_s^r$ ,  $U_s^r$ ,  $N_u^r$  and  $U_u^r$  as fraction of the labor force.<sup>28</sup> The number of periods,  $p$ , necessary to acquire human capital to become a skilled worker, is set equal to 4.<sup>29</sup> The proportion  $\lambda$  of skilled unemployed, which loses its complete human capital in one period, is set to 0.04.<sup>30</sup> The aggregate

<sup>25</sup>Due to missing data for educational attainment, the period 1991-1996 is not considered.

<sup>26</sup>This is the average real interest rate over the whole period, calculated as the yearly money market interest rate minus the inflation rate. All variables are measured in real terms.

<sup>27</sup>This definition corresponds to the definition in Moreno-Galbis and Sneessens (2004:17) as well as OECD (2004a:122).

<sup>28</sup>The underlying labor force contains people between 15 and 64 years.

<sup>29</sup>This roughly corresponds to the additional average time of education of people with at least upper secondary education in comparison to the people with less than upper secondary education.

<sup>30</sup>It is assumed, that this variable corresponds to the depreciation rate of human capital, 0.04 is the intermediate value reported by Jones et al. (2000:19).

producer wage,  $w_a$ , is calculated as average gross wage per employee plus social security payments. This value as well as the value for the aggregate productivity,  $a_a$ , is calculated as average over the period 1997 - 2003 using the data from the German national accounts.<sup>31</sup> In order to get the wages for the unskilled and skilled workers, OECD indices for the relative earnings of the population with income from employment for different skill groups are used (OECD 1999, 2000, 2001, 2002, 2003, 2004b, 2005), they yield the following ratio:  $\frac{w_s}{w_u} = 1.41$ . The hiring and the firing costs are set in relation to the labor costs. According to Chen and Funke (2005), we set the hiring costs to 10 % of labor costs and the firing costs are set to 60 % of labor costs, thus the corresponding parameters are  $c_h = 0.1$  and  $c_f = 0.6$ . Moreover, in order to introduce a progressive tax system, we have to quantify the ratio of the tax rates  $t_s$  and  $t_u$ . This is done by using the income tax scale of the year 2002 described in Boss and Elendner (2003: 379).<sup>32</sup> We obtain the following ratio:  $\frac{t_s}{t_u} = 1.25$ . The ratio of the firing rates of skilled and unskilled workers is set at  $f_u/f_s = 0.82$ .<sup>33</sup> The firing rate of the skilled worker is set at  $f_s = 0.08$ .<sup>34</sup> We set  $\alpha = 0.7$  and the parameter of the distribution function  $\Gamma = 0.000000001$ .<sup>35</sup> Table 2 summarizes the exogenously given parameter values for the initial steady state. The values of the variables in table 2 as well as in the tables below, which are denoted with a star remain constant, the value of all other variables will change in the presence of LWSs ( $\sigma > 0$ ).

In a second step, we can derive more parameter values, given the parameter values so far. We calculate  $T^r$  by using equation (A4.5) as  $T^r = \lambda U_s^r$ . Then, we can calculate the size of the total population  $P^r = N_s^r + U_s^r + N_u^r + U_u^r + pT^r$ . In the remainder, we normalize total population (and not, as yet, the labor force) to 1. Thus,  $N_s^r$ ,  $U_s^r$ ,  $N_u^r$ ,  $U_u^r$  and  $T^r$  are divided by  $P^r$ . So we get the initial values for  $N_s$ ,  $U_s$ ,  $N_u$  and  $U_u$  and  $T$ . In a next step, we calculate the employment rates of the skilled and unskilled labor force  $\varepsilon_i = \frac{N_i}{N_i + U_i}$  (with  $i = s, u$ ). Using the wage ratio ( $\frac{w_s}{w_u}$ ) as well as the fact, that the given aggregate wage,  $w_a$ , is the average of the skilled and the unskilled wage, weighted with the corresponding employment, we can calculate,  $w_s$  and  $w_u$ . The derived values are summarized in table 3.

In a third step, we are able to calculate some further missing values for the initial steady state by using the following system of equations. First, we use the equations describing the ratio of the tax rates ( $t_s = 1.25 t_u$ ), the ratio of the firing rates ( $f_u = 0.82 f_s$ ), besides we use the equation describing the relation of the training rates:  $\tau^N = 7.030(0.003 - 0.023\tau^U)$ .<sup>36</sup> We assume

<sup>31</sup>Statistisches Bundesamt (2006). Nominal values are transformed to real values by using the consumption deflator.

<sup>32</sup>Given the wages of each skill groups, it is possible to calculate the tax levels and thereby the tax rates of each skill group - in this context, we ignore that there is a difference between the labor cost of the employer (gross wage plus social security payments) and the labor income of the employee which is subject to taxation. In the remainder, we do not use the tax rates being the result of the calculation because the rates refer to a budget which contains more expenditure than unemployment benefits. In the context of this paper, only the ratio is important in order to map the tax progression in a realistic way.

<sup>33</sup>The ratio of the firing rates is calculated as the inverse of the ratio of the corresponding average employment durations (AED):  $\frac{f_u}{f_s} = \frac{AED_s}{AED_u}$ . According to Delacroix (2003), the ratio of the average employment duration of a skilled employee and the average employment duration of an unskilled employee is  $\frac{AED_s}{AED_u} = 0.82$ . Delacroix considers only people with post-secondary education as being skilled. However, our definition also contains the nearest higher level (tertiary education) and the nearest lower level (upper secondary education). Thus, it is assumed, that the value reported by Delacroix can also be used for our definition.

<sup>34</sup>Thus, the firing rate of the total workforce is 0.077. Wilke (2004) reports a value of around 0.08 for West-Germany. Assuming, that the firing rate for East-Germany does not have a big impact on the aggregate firing rate for Germany as a whole, our value is in accordance with the result of Wilke.

<sup>35</sup>This combination implies plausible values for the elasticity of unskilled labor demand. According to Riphahn et al. (1999:27), the wage elasticity of the demand for unskilled labor is in the range between -0.3 and -0.9.

<sup>36</sup>This equation is based on eq. (5):  $T = \tau^N N_u + \tau^U U_u$ . The values of  $T$ ,  $N_u$  and  $U_u$  are known.

parameter	variable	value
interest rate	$i$	0.025*
skilled employment (relative value)	$N_s^r$	0.767
skilled unemployment (relative value)	$U_s^r$	0.066
unskilled employment (relative value)	$N_u^r$	0.144
unskilled unemployment (relative value)	$U_u^r$	0.024
proportion of skilled unemployed losing their human capital	$\lambda$	0.04*
periods of training	$p$	4*
replacement rate	$\phi$	0.06*
firing costs per worker in relation to the wage	$c_f$	0.6*
hiring costs per worker in relation to the wage	$c_h$	0.1*
average producer wage per employee plus social security payments	$w_a$	31,020
average productivity (aggregate)	$a_a$	52,575
ratio of wages	$w_s / w_u$	1.41
ratio of tax rates (progression parameter)	$t_s / t_u$	1.25
ratio of firing rates	$f_u / f_s$	0.82
firing rate (skilled)	$f_s$	0.08
production function parameter	$\alpha$	0.70*
distribution parameter	$\Gamma'$	0.000000001*

Table 2: Exogenous parameter values in the initial steady state

parameter	variable	value
discount rate	$\delta$	0.976
inflow into / outflow from training	$T$	0.003
skilled employment	$N_s$	0.759
skilled unemployment	$U_s$	0.065
unskilled employment	$N_u$	0.142
unskilled unemployment	$U_u$	0.023
employment rate (skilled)	$\varepsilon_s$	0.921
employment rate (unskilled)	$\varepsilon_u$	0.859
average producer wage (skilled)	$w_s$	32,512
average producer wage (unskilled)	$w_u$	23,058

Table 3: Derived parameter values in the initial steady state (1)

that the bargaining power is independent of the skill level ( $\mu_s = \mu_u = \mu$ ). Secondly, we use the following three equations in order to calculate the employment rates  $\varepsilon_s$  and  $\varepsilon_u$ <sup>37</sup> and the aggregate productivity  $a_a$ :

$$\varepsilon_s = \frac{h_s(1 + \lambda)}{f_s + h_s(1 + \lambda)} \quad (17)$$

$$\varepsilon_u = \frac{h_u}{f_u + h_u + \tau^N} \quad (18)$$

$$a_a = \frac{a_s N_s + a_u N_u}{(N_s + N_u)} \quad (19)$$

Thirdly, we use the equations already mentioned: equations (14) and (15) describing the producer wage for the skilled and unskilled employees, respectively, the budget constraint (16)<sup>38</sup> and the equations describing the training rates (11) and (12). Now, we can calculate the missing parameter values. One of these parameter is  $x$  which describes the ratio between the education costs  $e^j$  and the training rate  $\tau^j$ . Given the values of  $x$  and  $\tau^j$ , it is possible to calculate  $e^j$ .<sup>39</sup>

In a final step, we can now calculate the parameter values of the productivity function. Given the values for  $a_i$  and  $N_i$  and given the value of  $\alpha$ , (for simplicity independent of the skill level),  $\beta_i$  can be calculated by using the productivity equation (13). The so calculated variables are listed in table 4.

parameter	variable	value	variable	value
bargaining power	$\mu$	0.24*		
average productivity	$a_s$	55,104	$a_u$	39,081
tax rate	$t_s$	0.06	$t_u$	0.05*
hiring rate	$h_s$	0.90	$h_u$	0.50
firing rate			$f_u$	0.07
proportion of unskilled entering training	$\tau^N$	0.0156	$\tau^U$	0.0165
education costs of the marginal worker	$e^N$	252,627	$e^U$	268,009
production function parameter	$\beta_s$	50729.7	$\beta_u$	21770.7

Table 4: Derived parameter values in the initial steady state (2)

## 4 Results

Given the underlying model and the calibrated values, we now illustrate in detail the impact of LWSs on skills and employment. In order to calculate the effects of the LWSs, we use the parameter values calculated so far and being valid of  $\sigma > 0$  as well as the system of equations described in section 2.5 but with the following modifications: concerning the labor markets states  $N_s$ ,  $U_s$ ,  $N_u$ ,  $U_u$  and  $T$ , and the government budget constraint, we use the equations describing the steady

<sup>37</sup>The employment rate is calculated as:  $\varepsilon_i = N_i / (N_i + U_i)$  where  $N_i$  and  $U_i$  are substituted by their corresponding steady state expressions (see Appendix A4).

<sup>38</sup>In the initial steady state, unemployment benefits and the transfers to people being in training are the sole expenditures (i.e.  $\sigma = 0$ ).

<sup>39</sup>As already mentioned, we assume:  $\xi^j = x e^j$ .

state.<sup>40</sup> Concerning the hiring and firing rates, we use the linearized versions of the corresponding equations.<sup>41</sup>

## 4.1 Effect on Employment

Figure 3 shows the effect of different rates of the LWSs ( $\sigma$ ). Skilled employment decreases and unskilled employment increases in the presence of LWSs. The higher  $\sigma$ , the higher is the effect. The effect on total employment is marginally negative.<sup>42</sup> On the supply side, the introduction of subsidies causes a decrease of the differential of the consumer wages as  $w_s^c$  decreases and  $w_u^c$  increases. Thus, the incentive to enter training and thereby the skilled labor force,  $\theta$ , decreases. On the demand side, the employment rate of the unskilled labor force,  $\varepsilon_u$ , increases because of a sufficient decrease of the producer wage.<sup>43</sup> As the unskilled labor force also increases, unskilled employment increases. But the positive impact on unskilled employment is marginally overcompensated by the negative impact on skilled employment.

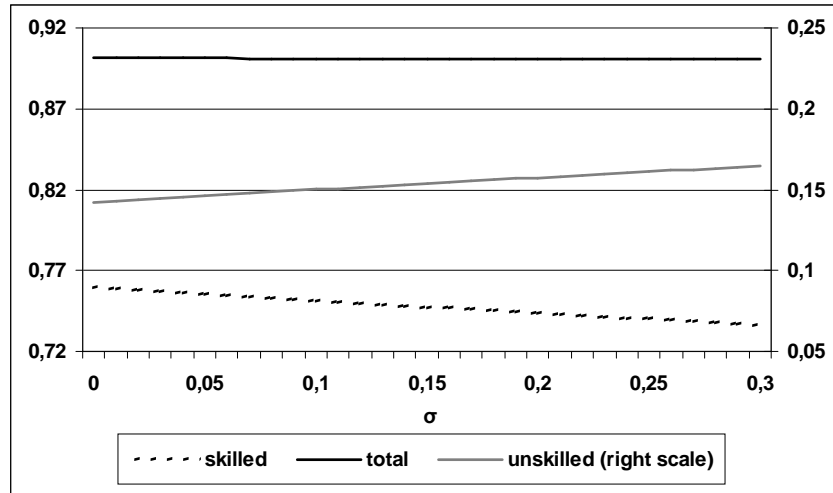


Figure 3: Employment as a function of  $\sigma$

## 4.2 Effect on Output

Given the marginally negative effect on employment, it is interesting to analyze the effect on output. In this context not only the net impact of subsidies on total employment but also the shift from skilled to unskilled employment becomes relevant as the productivity of an unskilled is lower than the productivity of a skilled employee. Output is approximated by the following very simple production

<sup>40</sup>See Appendix A4.

<sup>41</sup>See Appendix A1.

<sup>42</sup>See also second column of table 8 describing the effect of a 30%-low-wage subsidy for the initial model.

<sup>43</sup>Thus, the hiring (firing) rate of the unskilled labor force increases (decreases) which has a positive impact on the corresponding employment rate.

function:  $Y_{total} = Y_s + Y_u = \beta_s N_s^\alpha + \beta_u N_u^\alpha$ . Given the values for  $N_s$ ,  $N_u$ ,  $a_s$  and  $a_u$  in the initial steady-state and for different values of  $\sigma$ , it is possible to calculate the effects of  $\sigma$  on output  $Y$  and – for the sake of comparison – on employment  $N$ . Table 5 surveys the corresponding growth rates,  $\hat{Y}$  and  $\hat{N}$ .<sup>44</sup>

$\sigma$	$\hat{N}$	$\hat{Y}$
10	-0.02	-0.21
20	-0.04	-0.41
30	-0.06	-0.61

Table 5: Effect of LWSs on employment and output

For a given level of  $\sigma$ , the decrease of output is higher than the decrease of employment. Thus, when analyzing LWSs not only the net impact on employment should be taken into account. It is also necessary to pay attention to the shift from skilled employment to unskilled employment which has a negative impact on aggregate productivity.

### 4.3 Robustness

The results calculated so far are based on certain values concerning the firing and hiring cost parameters,  $c_f$  and  $c_h$ , respectively, as well as the deskilling parameter  $\lambda$ , the replacement rate  $\phi$  and the firing rate  $f_s$ . In the tables 6 and 7,<sup>45</sup> the effects of nine alternatives on the growth rates of employment are shown for  $\sigma = 30\%$ . The initial calibration serves as a benchmark.<sup>46</sup>

	<i>initial</i>	$c_f = 0.5$	$c_f = 0.7$	$c_h = 0.2$	$\lambda = 0.02$	$\lambda = 0.06$
$\hat{N}$	-0.06115	-0.07213	-0.05074	-0.06115	-0.11083	-0.01669
$\hat{N}_s$	-3.01934	-3.54597	-2.52063	-3.01934	-3.00294	-3.03533
$\hat{N}_u$	15.72442	18.46513	13.12914	15.72444	15.32213	16.09151

Table 6: The impact of different parameter values on the growth rates of employment (1)

	<i>initial</i>	$\phi = 0.5$	$\phi = 0.7$	$f_s = 0.075$	$f_s = 0.085$
$\hat{N}$	-0.06115	-0.06109	-0.06122	-0.05686	-0.06502
$\hat{N}_s$	-3.01934	-3.01679	-3.02208	-3.02400	-3.01522
$\hat{N}_u$	15.72442	15.71122	15.73864	15.77653	15.67799

Table 7: The impact of different parameter values on the growth rates of employment (2)

The results show, that in a plausible parameter range, in most case the change in the growth rate of total employment is small with respect to a change of the parameter values.

<sup>44</sup>The values for  $\sigma$  are in % of the wage. The values of  $\hat{N}$  and  $\hat{Y}$  are in %.

<sup>45</sup>The values are in %.

<sup>46</sup> $c_f = 0.6$ ,  $c_h = 0.1$ ,  $\lambda = 0.04$ ,  $\phi = 0.6$ ,  $f_s = 0.08$ .



The qualitative conclusions remain unchanged when the progressive taxation is replaced by a flat tax.<sup>47</sup>

#### 4.4 Channels of Employment Effects

In the following we want to shed light on the relative strengths of the different channels through which LWSs affect total employment (see figure 2). This is done by doing the same calculation as in section 4.1, but now, in each calculation, one effect is suppressed.

Figure 4 shows aggregate employment as a function of LWSs for four different types of modeling. The black line at the bottom represents the total employment as a function of  $\sigma$  in the initial model, it corresponds to the black line in figure 3 and serves as a benchmark.

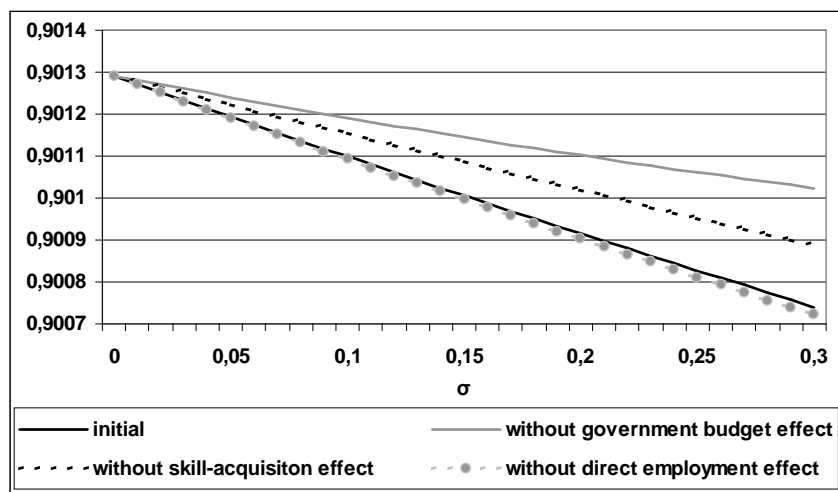


Figure 4: Employment as a function of  $\sigma$  in the absence of different effects

In addition, three modifications are considered. In order to explain the results, it is helpful to regard table 8 which shows the results for  $\sigma = 30\%$ .

First, we suppress the *direct employment effect* (channel A).<sup>48</sup> The supply side is hardly affected, the change in the skilled labor force,  $\theta$ , is roughly the same as in the initial model. The demand side (illustrated by the employment rates,  $\varepsilon_s$  and  $\varepsilon_u$ ) is marginally affected. The employment rate of the unskilled increases to a smaller effect than in the initial model. Thus, the increase of unskilled employment is smaller than in the initial model. The reduction in skilled employment is roughly the same. In total, the decrease of total employment is marginally stronger than in the initial model. Thus, this channel, if considered separately, has a marginally positive effect on employment. The effect of LWSs on total employment in the absence of the direct employment effect is illustrated by the dashed and dotted gray line in figure 4.

<sup>47</sup>See Appendix A5 for details.

<sup>48</sup>In the equations (6) and (7) determining the firing and the hiring rate, respectively, the producer wage for the unskilled worker,  $w_u$ , is assumed to remain on its initial level and is therefore independent of  $\sigma$ .

parameter	initial model	without channel A (without direct employment effect)	without channel B (without skill-acquisition effect)	without channel C (without government budget effect)
$w_s^c$	-3.05573	-3.05665	-3.16816	0.45779
$w_u^c$	1.45013	1.45374	0.00000	3.61071
$\theta$	-3.01955	-3.01957	-2.23931	-1.51109
$w_s$	0.92400	0.92400	0.68169	0.45779
$w_u$	-22.85770	-22.85495	-22.02652	-21.21480
$\varepsilon_s$	0.00022	0.00022	0.00163	0.00010
$\varepsilon_u$	0.44128	0.42927	0.34061	0.24136
$N_s$	-3.01934	-3.01936	-2.23915	-1.51098
$N_u$	15.72442	15.71068	11.66330	7.87436
$N_t$	-0.06115	-0.06334	-0.04503	-0.02976

Table 8: Strengths of different effects

In a second case, we suppress the *skill-acquisition effect* (channel B).<sup>49</sup> Given a constant and not increasing consumer wage,  $w_u^c$ , the opportunity costs of becoming skilled do not increase from this side. Thus, the incentive to acquire human capital and thus the skilled labor force,  $\theta$ , decreases to a smaller extent than in the initial model. On the demand side, the employment rate of the unskilled,  $\varepsilon_u$ , increases to a lower extent than in the initial model due to the smaller reduction in the corresponding producer wage. Thus, the increase of unskilled employment is lower than in the initial case. However, the reduction of skilled employment is also lower. All in all, the negative impact on total employment is lower than in the initial model. In other words, the *skill-acquisition effect*, if considered separately, has a negative impact on aggregate employment. The effect of LWSs on total employment in the absence of the skill-acquisition effect is illustrated by the black dashed line in figure 4.

In a third case, the *government budget effect* (channel C) is suppressed.<sup>50</sup> As the tax rate,  $t_s$ , does not increase, the consumer wage,  $w_s^c$  does not fall.<sup>51</sup> Thus, from this side there is no negative impact on the incentive to enter training and thereby on the training rate  $\tau^j$ .<sup>52</sup> In total, the negative impact on  $\tau^j$  is smaller than in the initial model and thus the decrease of  $\theta$  (-1.51%) is also smaller than in the initial model (-3.02%). On the demand side, the employment rate of the unskilled,  $\varepsilon_u$ , increases to a lower extent due to the smaller reduction in the corresponding producer wage. Finally, when comparing this case with the initial case, skilled employment decreases to a lower extent and unskilled employment increases, but also to a lower extent. The negative impact on total employment is smaller than in the initial model. In other words, the government budget effect, if considered separately, has a negative impact on aggregate employment. The effect of LWSs

<sup>49</sup>The consumer wage  $w_u^c$  is assumed to remain on its initial level in the equations (11) and (12) and is therefore independent of  $\sigma$ .

<sup>50</sup>The tax rate  $t_s$  is assumed to remain on its initial level, and therefore does not increase with  $\sigma$ .

<sup>51</sup>Here,  $w_s^c$  even increases. This is caused by a feedback effect. As the reduction of skilled employment,  $N_s$ , is lower than in the initial model, the productivity (see equation (13)) and thereby the wage increases.

<sup>52</sup>Whereas the increase of  $w_s^c$  has a positive impact on  $\theta$ , there is a negative impact due to the increase of  $w_u^c$  (over channel B). Here,  $w_u^c$  increases even stronger than in the initial model. This can be explained via the feedback effects. As the increase in  $N_u$  is smaller than in the initial model, the decrease in the productivity (see equation (13)) and thereby the decrease in the producer wage is smaller. Thus, the increase in the consumer wage is higher than in the initial model.

on total employment in the absence of the government budget effect is illustrated by the gray line in the figure 4.

## 5 Conclusion

This paper has examined three channels whereby low-wage subsidies affect employment: (i) the direct employment effect, (ii) the skill-acquisition effect and (iii) the government budget effect. Our calibration results indicate that although LWSs raise unskilled employment, they reduce the incentives to acquire human capital and also need to be financed through taxes. These latter two effects imply that LWSs lead to a less skilled labor force. This implication is potentially important since skilled workers have a much higher employment probability than unskilled workers. Our numerical analysis shows that LWSs may stimulate unskilled employment by less than they reduce skilled employment, so that total employment falls. Furthermore, the shift from skilled to unskilled employment reduces aggregate productivity and thus LWSs adversely affect output and the standard of living.

## References

- Bassanini, A., J.H. Rasmussen and S. Scarpetta, 1999. The Economic Effects of Employment-Conditional Income Support Schemes for the Low-Paid: An Illustration from a CGE Model applied to four OECD Countries. Economics Department Working Paper 224. OECD, Paris.
- Boss, A. and T. Elendner, 2003. Steuerreform und Lohnsteueraufkommen in Deutschland. Die Weltwirtschaft 4, 368-387.
- Boss, A., 2006. Brauchen wir einen Kombi-Lohn? Kiel Working Papers (1279). Kiel Institute for the World Economy, Kiel.
- Buslei, H. and V. Steiner, 1999. Beschäftigungseffekte von Lohnsubventionen im Niedriglohnbereich. Nomos Verlagsgesellschaft, Baden-Baden.
- Cardullo, G. and B. Van der Linden, 2006. Employment Subsidies and Substitutable Skills: An Equilibrium Matching Approach. IZA Discussion Paper 2073. IZA, Bonn.
- Chen, Y.-F., and M. Funke, 2005. Non-Wage Labour Costs, Policy Uncertainty and Labour Demand - A Theoretical Assessment. The Scottish Journal of Political Economy, 52 (1), 687-709.
- Delacroix, A., 2003. Transitions into unemployment and the nature of firing costs. Review of Economic Dynamics, 6 (3), 651-671.
- Dietz, M, S. Koch and U. Walwei, 2006. Kombilohn. Ein Ansatz mit Haken und Ösen. IAB Kurzbericht 3. Institut für Arbeitsmarkt- und Berufsforschung der Bundesagentur für Arbeit, Nürnberg.

- Dilnot, A. and J. McCrae, 2000. The Family Credit System and the Working Families Tax Credit in the United Kingdom. OECD Economic Studies 31 (2). OECD, Paris.
- Eissa, N. and H. Hoynes, 2005. Behavioral Responses to Taxes: Lessons from the EITC and Labor Supply. NBER Working Paper Series (11729).
- Eissa, N. and J. B. Liebman, 1996. Labor Supply Responses to the Earned Income Tax Credit. Quarterly Journal of Economics, 111 (2), 605-637.
- Hamermesh, D.S., 1978. Subsidies for Jobs in the Private Sector. In: Palmer, J. (Ed.), Creating Jobs. Brookings Institution, Washington, D.C., pp. 87-122.
- Haveman, R. H. and J. L. Palmer, 1982. Jobs for Disadvantaged Workers: The Economics of Unemployment Subsidies. Brookings Institution, Washington, D.C.
- Heckman, J.J., L. Lochner and R. Cossa, 2003. Learning-by-doing versus on-the-job training: using variation by the EITC to distinguish between models of skill formation. In: Phelps, E.S. (Ed.), Designing Inclusion - Tools to Raise Low-end Pay and Employment in Private Enterprise. Cambridge University Press, Cambridge, MA, pp. 74-130.
- Hoon, H.T. and E.S. Phelps, 2003. Low-wage employment subsidies in a labor-turnover model of the "natural rate". In: Phelps, E.S. (Ed.), Designing Inclusion - Tools to Raise Low-end Pay and Employment in Private Enterprise. Cambridge University Press, Cambridge, MA, pp. 16-43.
- Hotz, V.J. and J.K. Scholz, 2003. The Earned Income Tax Credit. In: Moffit, R.A. (Ed.), Means-Tested Transfer Programs in the United States. The University of Chicago Press, Chicago and London, pp. 141-197.
- Jones, L. E., R. E. Manuelli and H. E. Sui, 2000. Growth and Business Cycles. Research Department Staff Report 271. Federal Reserve Bank of Minneapolis. Minneapolis, MN.
- Kaldor, N., 1936. Wage Subsidies as a Remedy for Unemployment. Journal of Political Economy, 44 (6), 721-742.
- Layard, R. and S. Nickell, 1980. The case of subsidising extra jobs. The Economic Journal, 90 (March), 51-73.
- Layard, R., S. Nickell and R. Jackman, 1991. Unemployment: Macroeconomic Performance and the Labour Market. Oxford University Press, Oxford.
- Liebman, J.B., 2001. The Optimal Design of the Earned Income Tax Credit. In: Meyer, B.D., Holtz-Eakin, D. (Eds.), Making Work Pay. Russell Sage Foundation, New York, NY, pp. 196-233.
- Meyer, B.D., 2002. Labor Supply at the Extensive and Intensive Margins: The EITC, Welfare, and Hours Worked. American Economic Review, 92 (2), Papers and Proceedings, 373-379.
- Moreno-Galbis, E. and H. R. Sneessens, 2004. Low-skilled Unemployment, Capital-Skill Complementarity and Embodied Technical Progress. mimeo.

- Mortensen, D. and C. Pissarides, 2003. Taxes, subsidies and equilibrium labor market outcomes. In: Phelps, E.S. (Ed.): *Designing Inclusion - Tools to Raise Low-end Pay and Employment in Private Enterprise*. Cambridge University Press, Cambridge, MA, pp. 44-73.
- Michalopoulos, C., P. K. Robins and D. Card, 2005. When financial work incentives pay for themselves: evidence from a randomized social experiment for welfare recipients. *Journal of Public Economics*, 89 (1), 5-29.
- OECD, 1999. *Education at a Glance*. Paris.
- OECD, 2000. *Education at a Glance*. Paris.
- OECD, 2001. *Education at a Glance*. Paris.
- OECD, 2002. *Education at a Glance*. Paris.
- OECD, 2003. *Education at a Glance*. Paris.
- OECD, 2004a. *Economic Outlook*. Paris.
- OECD, 2004b. *Education at a Glance*. Paris.
- OECD, 2005. *Education at a Glance*. Paris.
- Orszag, J.M. and D.J. Snower, 2003. Unemployment vouchers versus low-wage subsidies. In: Phelps, E.S. (Ed.): *Designing Inclusion - Tools to Raise Low-end Pay and Employment in Private Enterprise*. Cambridge University Press, Cambridge, MA, pp. 131-160.
- Phelps, E.S., 1994a. Low-Wage Employment Subsidies versus the Welfare State. *American Economic Review*, 84 (2), Papers and Proceedings, 54-58.
- Phelps, E.S., 1994b. *Structural Slumps: The Modern Equilibrium Theory of Unemployment, Interest, and Assets*. Harvard University Press, Cambridge, MA.
- Phelps, E.S., 1997a. *Rewarding Work: How to Restore Participation and Self-Support to Free Enterprise*. Harvard University Press, Cambridge, MA.
- Phelps, E.S., 1997b. Wage Subsidy Programmes: Alternative Designs. In: Snower, D.J., de la Dehesa, G. (Eds.), *Unemployment Policy: Government Options for the Labour Market*. Cambridge University Press, Cambridge, MA, pp. 206-244.
- Pigou, A.C., 1932. *The Economics of Welfare*. Macmillan, London.
- Riphahn, R., A. Thalmeier and K. F. Zimmermann, 1999. *Schaffung von Arbeitsplätzen für Geringqualifizierte*. IZA Research Report 2. IZA, Bonn.
- Sinn, H.-W., C. Holzner, W. Meister, W. Ochel and M. Werding, 2002. *Aktivierende Sozialhilfe 2006. Ein Weg zu mehr Beschäftigung und Wachstum*. ifo Schnelldienst, 55 (9), 3-52.
- Sinn, H.-W., C. Holzner, W. Meister, W. Ochel and M. Werding, 2006. *Aktivierende Sozialhilfe 2006: Das Kombi-Lohn-Modell des ifo Instituts*. ifo Schnelldienst, 59 (2), 6-27.

- Snower, D.J., 1994. Converting Unemployment Benefits into Employment Subsidies. In: American Economic Review, 84 (2), Papers and Proceedings, 65-70.
- Spermann, A., 2003. Ergebnisse und Lehren aus Modellversuchen mit Kontrollgruppen: Einstiegsgehalt in Baden-Württemberg und Hessischer Kombilohn. In: Institut für Arbeitsmarkt- und Berufsforschung der Bundesagentur für Arbeit (Ed.), Beschäftigungsförderung im Niedriglohnsektor. Nürnberg. pp. 91-99.
- Spermann, A. and H. Strotmann, 2005. The Targeted Negative Income Tax (TNIT) in Germany: Evidence from a Quasi Experiment. ZEW Discussion Paper 05-68. ZEW, Mannheim.
- Statistisches Bundesamt, 2006. Fachserie 18: Volkswirtschaftliche Gesamtrechnungen. Reihe 1.2.: Vierteljahresergebnisse der Inlandsproduktberechnung. February. Wiesbaden.
- Ventry, Jr., D.J., 2001. The Collision of Tax and Welfare Politics: The Political History of the Earned Income Tax Credit. In: Meyer, B.D., Holtz-Eakin, D. (Eds.), Making Work Pay. Russell Sage Foundation, New York, NY, pp. 15-66.
- Wilke, R., 2004. New Estimates of the Duration and Risk of Unemployment for West Germany. ZEW Discussion Paper 04-26, ZEW, Mannheim.

## A Appendix

### A1 Hiring and Firing Rates

The expected present value of the firm's profit is calculated as follows, with  $i = s, u$ :

$$V_{i,t} = (a_i - w_i) - \epsilon + \sum_{t=1}^{\infty} \delta^t (1 - f_i)^t (a_i - w_i) - \sum_{t=0}^{\infty} \delta^{t+1} (1 - f_i)^t f_i \varsigma_i \quad (\text{A1.1})$$

This can be rewritten as:

$$V_{i,t} = (a_i - w_i) - \epsilon - (a_i - w_i) + \sum_{t=0}^{\infty} \delta^t (1 - f_i)^t (a_i - w_i) - \delta f_i \varsigma_i \sum_{t=0}^{\infty} \delta^t (1 - f_i)^t \quad (\text{A1.1a})$$

The term on the right hand side, can be simplified so that the equation becomes:

$$V_{i,t} = -\epsilon + \frac{a_i - w_i}{1 - \delta(1 - f_i)} - \frac{\delta f_i \varsigma_i}{1 - \delta(1 - f_i)} \quad (\text{A1.1b})$$

For a given hiring cost per worker,  $\gamma_i$ , an unemployed is hired, whenever  $V_{i,t} > \gamma_i$ . Substituting  $V_{i,t}$  according to equation (A1.1b), and solving for the random component  $\epsilon$ , the following equation is obtained:

$$\epsilon < \frac{a_i - w_i - \delta f_i \varsigma_i}{1 - \delta(1 - f_i)} - \gamma_i \quad (\text{A1.2})$$

Taking into account that  $\gamma_i = c_h w_i$  and  $\varsigma_i = c_f w_i$ , we get:

$$\epsilon < \frac{a_i - w_i - \delta f_i c_f w_i}{1 - \delta(1 - f_i)} - c_h w_i \quad (\text{A1.3})$$

Thus, the probability of being hired is:

$$h_i = \Gamma\left(\frac{a_i - w_i - \delta f_i c_f w_i}{1 - \delta(1 - f_i)} - c_h w_i\right) \quad (\text{A1.4})$$

We linearize the hiring rate with respect to the anchor (which is the average of the period 1997-2003). All other firing rates in the model are calculated with a first order Taylor series expansion with respect to this point.

$$\begin{aligned} h_{i,t} = h_{i,0} &+ \left(\frac{-1 - \delta f_{i,0} c_f}{1 - \delta(1 - f_{i,0})} - c_h\right) * \Gamma'[(w_{i,t} - w_{i,0})] \\ &+ \left(\frac{-a_{i,0} + w_{i,0}(1 + c_f(-1 + \delta))}{[1 - \delta(1 - f_{i,0})]^2} \delta\right) * \Gamma'[(f_{i,t} - f_{i,0})] \\ &+ \left(\frac{1}{1 - \delta(1 - f_{i,0})}\right) * \Gamma'[a_{i,t} - a_{i,0}] \end{aligned} \quad (\text{A1.4a})$$

For a given firing cost per worker,  $\varsigma_i$ , the employee is fired, when  $V_{i,t} < -\varsigma_i$ . Substituting  $V_{i,t}$  according to equation (A1.1b), and solving for the random component  $\epsilon$ , the following expression is obtained:

$$\epsilon > \varsigma_i + \frac{a_i - w_i - \delta f_i \varsigma_i}{1 - \delta(1 - f_i)} \quad (\text{A1.5})$$

Taking into account that  $\varsigma_i = c_f w_i$ , we get:

$$\epsilon > \frac{a_i - w_i + c_f w_i(1 - \delta)}{1 - \delta(1 - f_i)} \quad (\text{A1.6})$$

Thus, the probability of being fired is:

$$f_i = 1 - \Gamma\left(\frac{a_i - w_i + c_f w_i(1 - \delta)}{1 - \delta(1 - f_i)}\right) \quad (\text{A1.7})$$

We linearize the firing rate with respect to the anchor (which is the average of the period 1997-2003). All other firing rates in the model are calculated with a first order Taylor series expansion with respect to this point.

$$\begin{aligned} f_{i,t} = f_{i,0} &- \left(\frac{-1 + c_f(1 - \delta)}{1 - \delta(1 - f_{i,0})}\right) * \Gamma'[w_{i,t} - w_{i,0}] \\ &- \left(\frac{-a_{i,0} + w_{i,0}(1 + c_f(-1 + \delta))}{[1 - \delta(1 - f_{i,0})]^2} \delta\right) * \Gamma'[f_{i,t} - f_{i,0}] \\ &- \left(\frac{1}{1 - \delta(1 - f_{i,0})}\right) * \Gamma'[a_{i,t} - a_{i,0}] \end{aligned} \quad (\text{A1.7a})$$

## A2 Expected Lifetime Income

Leaving out the time subscript in the equations describing the expected lifetime incomes being associated with different labor market states, the following steady state expressions are obtained:

$$Y_s^N = \frac{1}{1 - \delta(1 - f_s)} [w_s^c + \delta f_s Y_s^U] \quad (\text{A2.1})$$

$$Y_s^U = \frac{1}{1 - \delta(1 - \lambda - h_s)} [b_s + \delta h_s Y_s^N + \delta \lambda Y_u^U] \quad (\text{A2.2})$$

$$Y_u^N = \frac{1}{1 - \delta(1 - f_u)} [w_u^c + \delta f_u Y_u^U] \quad (\text{A2.3})$$

$$Y_u^U = \frac{1}{1 - \delta(1 - h_u)} [b_u + \delta h_u Y_u^N] \quad (\text{A2.4})$$

Given these four equations, we can calculate the steady state solutions for  $Y_s^N, Y_s^U, Y_u^N$  and  $Y_u^U$ :

$$Y_s^N = \frac{1}{1 - (1 - f_s)\delta} [w_u^c + (f_s \delta [b_s(-1 + \delta)(1 - (1 - f_s)\delta(1 - (1 - f_u - h_u)\delta) + \delta(-b_u(1 - (1 - f_s)\delta)(1 - (1 - f_u)\delta)\lambda - h_s(1 - \delta)(1 - (1 - f_u - h_u)\delta)w_s^c - h_u\delta(1 - \delta + f_s\delta)\lambda w_u^c)])] / [(-1 + \delta)(1 - (1 - f_u - h_u)\delta)(1 - \delta(2 - f_s - h_s - \lambda) + \delta^2(1 - f_s - h_s - \lambda + f_s\lambda))] \quad (\text{A2.1a})$$

$$Y_s^U = -[b_s(1 - \delta)(1 - (1 - f_s)\delta)(1 - (1 - f_u - h_u)\delta) + \delta[b_u(1 - (1 - f_s)\delta)(1 - (1 - f_u)\delta)\lambda + h_s(1 - \delta)(1 - (1 - f_u - h_u)\delta)w_s^c + h_u\delta(1 - (1 - f_s)\delta)\lambda w_u^c]] / [(-1 + \delta)(1 - (1 - f_u - h_u)\delta)(1 - \delta(2 - f_s - h_s - \lambda) + \delta^2(1 - f_s - h_s - \lambda + f_s\lambda))] \quad (\text{A2.2a})$$

$$Y_u^N = \frac{b_u f_u \delta + (1 - (1 - h_u)\delta)w_u^c}{(1 - \delta)(1 - (1 - f_u - h_u)\delta)} \quad (\text{A2.3a})$$

$$Y_u^U = \frac{b_u(1 - (1 - f_u)\delta) + h_u w_u^c \delta}{(1 - \delta)(1 - (1 - f_u - h_u)\delta)} \quad (\text{A2.4a})$$

## A3 Wage Bargaining

In order to calculate the wage, first, the bargaining surplus of the firm is calculated. Under bargaining agreement the firm receives the profit  $(a_{i,t} - w_{i,t} - \varepsilon)$  in the first period. The intertemporal profit can be calculated according to equation (A1.1).

$$V_{i,t} = (a_{i,t} - w_{i,t}) - \varepsilon + \sum_{t=1}^{\infty} \delta^t (1 - f_i)^t (a_{i,t} - w_{i,t}) - \sum_{t=0}^{\infty} \delta^{t+1} (1 - f_i)^t f_i c_{i,t} \quad \text{with } i = s, u \quad (\text{A3.1})$$



Under disagreement, the firm's fallback position in the first period is  $-\varsigma_{i,t} - \varepsilon$ . Assuming that disagreement in the first period does not affect future returns, the present value of the firm's intertemporal profit under disagreement is:

$$\tilde{V}_{i,t} = -\varsigma_{i,t} - \varepsilon + \sum_{t=1}^{\infty} \delta^t (1 - f_i)^t (a_{i,t} - w_{i,t}) - \sum_{t=0}^{\infty} \delta^{t+1} (1 - f_i)^t f_i \varsigma_{i,t} \quad \text{with } i = s, u \quad (\text{A3.2})$$

Thus, the surplus of the firm,  $S_{i,t}^F$ , can be expressed as follows:  $S_{i,t}^F = V_{i,t} - \tilde{V}_{i,t} = a_{i,t} - w_{i,t} + \varsigma_{i,t}$ .

Now, the bargaining surplus of the employee has to be calculated. Under bargaining agreement, the employees receives the net wage income  $w_{i,t}^c$  in each period. Thus, the intertemporal wage income of the employee is:

$$Y_{i,t}^N = w_{i,t}^c + \delta[(1 - f_i)Y_{i,t+1}^N + f_i Y_{i,t+1}^U] \quad \text{with } i = s, u \quad (\text{A3.3})$$

Under disagreement, the employee receive the unemployment benefit  $b_{i,t}$ . Given the assumption that future returns are not affected, the intertemporal wage income is:

$$\tilde{Y}_{i,t}^N = b_{i,t} + \delta[(1 - f_i)Y_{i,t+1}^N + f_i Y_{i,t+1}^U] \quad \text{with } i = s, u \quad (\text{A3.4})$$

Thus, the surplus of the employee can be expressed as follows:  $S_{i,t}^E = Y_{i,t}^N - \tilde{Y}_{i,t}^N = w_{i,t}^c - b_{i,t}$ . In order to determine the wage, the following Nash bargaining problem has to solved:

$$\text{Maximize } \Omega = [S_{i,t}^E]^{\mu_i} [S_{i,t}^F]^{1-\mu_s} \quad (\text{A3.5})$$

For sake of simplicity, the time subscript is ignored in the remainder. The system of a progressive taxation is modeled by introducing two tax rates,  $t_s$  and  $t_u$  for the skilled and unskilled employee, respectively, where  $t_s > t_u$ . The surplus of the skilled employee is  $S_s^E = w_s^c - b_s$  with  $w_s^c = w_s(1 - t_s)$ . The Nash bargaining problem to be solved is:

$$\text{Maximize } \Omega = [w_s(1 - t_s) - b_s]^{\mu_s} [a_s - w_s + \varsigma_s]^{1-\mu_s} \quad (\text{A3.6})$$

The first derivative with respect to the wage,  $\frac{\partial \Omega}{\partial w_s}$ , has to be zero:

$$\begin{aligned} & \mu_s [w_s(1 - t_s) - b_s]^{\mu_s - 1} (1 - t_s) [a_s - w_s + \varsigma_s]^{1-\mu_s} \\ & + [w_s(1 - t_s) - b_s]^{\mu_s} (1 - \mu_s) [a_s - w_s + \varsigma_s]^{-\mu_s} (-1) \\ & = 0 \end{aligned} \quad (\text{A3.7})$$

This can be written as:

$$\mu_s (1 - t_s) [a_s - w_s + \varsigma_s] = (1 - \mu_s) [w_s(1 - t_s) - b_s] \quad (\text{A3.8})$$

With  $b_s = \phi w_s(1 - t_s)$  and  $\varsigma_s = c_f w_s$ , we obtain:

$$\mu_s (1 - t_s) [a_s - w_s(1 - c_f)] = (1 - \mu_s) w_s (1 - t_s) (1 - \phi) \quad (\text{A3.9})$$

Solving for  $w_s$ , we get equation (14).

Leaving out the time subscript, the surplus of the unskilled employee is  $S_u^E = w_u^c - b_u$  with  $w_u^c = w_u(1 - t_u) + \kappa$ . The Nash bargaining problem to be solved is:

$$\text{Maximize } \Omega = [w_u(1 - t_u) + \kappa - b_u]^{\mu_u} [a_u - w_u + \varsigma_u]^{1-\mu_u} \quad (\text{A3.10})$$

The first derivative with respect to the wage,  $\frac{\partial \Omega}{\partial w_u}$ , has to be zero:

$$\begin{aligned} & \mu_u [w_u(1-t_u) + \kappa - b_u]^{\mu_u-1} (1-t_s) [a_u - w_u + \varsigma_u]^{1-\mu_u} \\ & + [w_u(1-t_u) + \kappa - b_u]^{\mu_u} (1-\mu_u) [a_u - w_u + \varsigma_u]^{-\mu_u} (-1) \\ & = 0 \end{aligned} \tag{A3.11}$$

This can be written as:

$$\mu_u (1-t_u) [a_u - w_u + \varsigma_u] = (1-\mu_u) [w_u(1-t_u) + \kappa - b_u] \tag{A3.12}$$

With  $b_u = \phi (w_u(1-t_u) + \kappa)$ ,  $\varsigma_u = c_f w_u$  and  $\kappa = \sigma w_u$ , we obtain:

$$\mu_u (1-t_u) [a_u - w_u(1-c_f)] = (1-\mu_u) [(w_u(1-t_u) + \sigma)(1-\phi)] \tag{A3.13}$$

Solving for  $w_u$ , we get equation (15).

## A4 Labor Market Equilibrium

Given that total population is normalized to unity ( $N_s + U_s + N_u + U_u + pT = 1$ ), the equations (1) to (4) describing the laws of motion imply the following steady state levels of employment and unemployment:

$$N_s = \frac{h_s(h_u \tau^N + (f_u + \tau^N)\tau^U) (1+\lambda)}{D} \tag{A4.1}$$

$$U_s = \frac{f_s (h_u \tau^N + (f_u + \tau^N)\tau^U)}{D} \tag{A4.2}$$

$$N_u = \frac{f_s h_u \lambda}{D} \tag{A4.3}$$

$$U_u = \frac{f_s (f_u + \tau^N) \lambda}{D} \tag{A4.4}$$

with  $D = h_s(h_u \tau^N + (f_u + \tau^N)\tau^U) (1+\lambda) + f_s [h_u(\tau^N + \lambda + p\tau^N \lambda) + (f_u + \tau^N)(\tau^U + \lambda + p\tau^U \lambda)]$ .

Moreover, in the steady state, the inflow into training is equal to the outflow from training:

$$T = \lambda U_s \tag{A4.5}$$

As the inflow into training is calculated as:  $T = \tau^N N_u + \tau^U U_u$ , we obtain:

$$\lambda U_s = \tau^N N_u + \tau^U U_u \tag{A4.6}$$

The steady state expression of the budget constraint is given by the following equation:

$$\begin{aligned} t_s w_s N_s + t_u w_u N_u &= \phi [w_s(1-t_s)] U_s \\ &+ \phi [w_u(1-t_u) + \sigma w_u] U_u \\ &+ \phi [w_u(1-t_u) + \sigma w_u] pT \\ &+ \sigma w_u N_u \end{aligned} \tag{A4.7}$$

Now, the inflow into training is the same in every period. Thus, the stock of people being in the training is calculated as:  $pT$ .

## A5 Flat Tax

As alternative to the baseline model, a system with a flat tax rate ( $t_s = t_u = t$ ) is analyzed. In the case of a progressive tax system it was assumed, that  $t_u$  remains constant whereas  $t_s$  is adjusted so that the government budget constraint is satisfied (i.e. the total fiscal burden generated by the LWSs was carried only by the skilled workers). Now, due to the presence of a common tax rate  $t$ , also the unskilled workers carry a fraction of the burden. In order to analyze the effects of LWSs in this tax system, we start by calculating the values of the initial steady state. We use, the same parameter values and the same system of equations described as before but with one exception, now, there is  $t_s = t_u = t$ . Then, we can analyze the effect of LWSs given a flat tax. In this case, the shifting from skilled to unskilled employment is marginally lower than in the case of a progressive tax system. In the latter, LWSs are only financed via higher taxes for the skilled worker, whereas in the presence of a flat tax, also the unskilled have to pay higher taxes. Thus, the wage differential and thereby the incentive to become skilled is higher in the presence of a flat tax system than in the presence of a progressive tax system. This is reflected in the number of skilled and unskilled employment; given a LWS of  $\sigma = 30\%$ , skilled employment decreases by 1.8 % (-3.0% in initial model) and unskilled employment increases by 9.5% (15.7% in initial model). Total employment decreases by 0.04 % (-0.06 % in initial model).