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by Tae-Seok Jang and Stephen Sacht

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Keywords: Animal Spirits, Bounded Rationality, Discrete Choice Theory, Euro Area, New-Keynesian Model, Simulated Method of Moments.

JEL classification: C53, D83, E12, E32.

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Tae-Seok Jang* and Stephen Sacht†

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1 Introduction

Rational expectations are a flexible and natural way of modeling market behavior in dynamic stochastic general equilibrium (DSGE) models, which are widely used by macroeconomists. Since the DSGE approach disposes a convenient analytical tractability under the assumption of rational expectations, this modeling framework serves as an efficient toolbox for analyzing monetary and fiscal policy measures. As Selten (2001) states, however, "modern mainstream economic theory is largely based on an unrealistic picture of human decision theory" since evidence from experimental studies supports information processing with limited cognitive ability of agents rather than perfect information (see Hommes (2011) among others). Indeed, a plethora of studies have been done on alternative forms of information processing mechanisms in macroeconomics; see e.g. the literature on learning (Evans and Honkaphohja (2001)), rational inattention (Sims (2003)), sticky information (Mankiw and Reis (2002)) or bounded rationality in general (Sargent (1994) and Kahneman (2003)). Camerer (1998) also offers an informative overview of the discussion on this topic in economics.

For the most part of the behavioral research, we can treat the realization of economic decisions as being a complex and interactive process between different types of agents. Keynes (1936) already attributed significant irrationality to human nature and discussed the impacts of waves of optimism and pessimism - so called *animal spirits* - on economic outcome. According to Akerlof and Shiller (2009), the emotional states are reflected in economic behavior - see also Franke (forthcoming) for his extensive discussion about market behavior and how expectation formation should be treated in macroeconomic models.

In this paper we attempt to empirically examine the hypothesis that the behavioral heterogeneity will have a macroscopic impact on the economy. The point of view taken here is that a behavioral model can provide a conceptual framework for a cognitive ability as well as a substantial degree of inertia in the DSGE models. According to De Grauwe (2011), if agents are known to be either optimists or pessimists, their ability (or better: limitation) to form their expectations affects economic conditions, i.e. movements in employment, the output gap and inflation, more appropriately than standard rational expectation models. Indeed, it is shown in the expectation formation process under bounded rationality that we can explicitly model animal spirits by applying discrete choice theory on group behavior. Then the behavior of optimists and pessimists is considered to be a by-product of the switching mechanisms based on the performance measure from agents' expectations (see also e.g. Westerhoff (2008) as well as Lengnick and Wohltmann (forthcoming) among others).

To the best of our knowledge, however, an empirical evaluation of a bounded rationality model of this type discussed above is missing in the literature so far. We fill the existing gap between the use of the models and their empirical evaluations in the literature by measuring the effects of psychological behavior on the economy under consideration of animal spirits. We show that the moment-based estimation (Franke et al. (2011)) can be easily used to estimate a small-scale DSGE model. Mainly, similarities and dissimilarities between two polar cases of expectation formation processes will be examined: while the underlying model structure is identical to a standard three-equations New-Keynesian model (NKM), we also allow both for rational expectations and for endogenously-formed expectations using the behavioral specification by De Grauwe (2011). In particular, we study his behavioral economic framework and provide an empirical investigation of bounded rationality on economic dynamics in the Euro Area from 1975Q1 to 2009Q4. Accordingly, an important aspect of this paper is to test the bounded rationality hypothesis in order to offer reliable parameter values that can be used for calibration in more realistic-grounded future work, e.g. studying monetary and fiscal policy analysis in a DSGE model without the assumption of rational expectations.

In our empirical application, we show that the NKM with rational expectations or bounded rationality can generate auto- and cross-covariances of the output gap, the inflation gap and the interest gap, which can mimic real data well. A quadratic objective function is used in the estimation to measure the distance between the model-generated and empirical moments. As the usual procedure of the method of moments, the global minimum of the objective function provides consistent parameter estimates of the model. Then we evaluate the goodness-of-fit of the model to the data from the value of the quadratic object function; i.e. the lower this value, the better the fit of the model-generated moments to their empirical counterparts. The empirical application using the method of moment approach stays in line with the work of Franke et al. (2011), who estimate a similar version of the NKM presented here for two sub-samples, i.e. the Great Inflation and Great Moderation period in the US. They come to the conclusion that inflation dynamics are primarily driven by intrinsic rather than extrinsic persistence - which is the total opposite of the results when applying Bayesian estimation. This is reflected by a high degree of price indexation and a low degree of persistence in the assumed AR(1) cost-push shock. In general, this kind of estimation technique is closely related to the approaches of indirect inference with the difference that in our case the structural form of a DSGE model is used instead of an auxiliary model like a SVAR (cf. Smith (1993) and Christiano et al. (2005) among others).

Main findings can be summarized as follows. First, over the whole time interval the agents had expected moderate deviations of the future output gap from its steady state value with low uncertainty. Second, we find strong evidence for an autoregressive expectation formation process regarding the inflation gap, which is in line with the scientific consensus among experimental economists (Roos and Schmidt (2012)).

The remainder of the paper is structured as follows. Section 2 introduces a small-scale NKM and discusses two model specifications, i.e. one with rational expectations and the other under consideration of the animal spirits. The estimation methodology is presented in section 3. Section 4 then estimates two versions of the model by the moment-based estimation and discusses their empirical results. Afterwards, the properties of the moment-based procedure for estimation are examined through a Monte Carlo study and a sensitivity analysis in section 5. Finally, section 6 concludes. The appendix collects all relevant technical details.

2 The Model: Rational Expectations vs. Bounded Rationality

The New-Keynesian three-equations model reads as follows:

$$y_t = \frac{1}{1+\chi}\tilde{E}_t^j y_{t+1} + \frac{\chi}{1+\chi} y_{t-1} - \tau(\hat{r}_t - \tilde{E}_t^j \hat{\pi}_{t+1}) + \varepsilon_{y,t} \tag{1}$$

$$\hat{\pi}_t = \frac{\nu}{1+\alpha\nu}\tilde{E}_t^j \hat{\pi}_{t+1} + \frac{\alpha}{1+\alpha\nu}\hat{\pi}_{t-1} + \kappa y_t + \varepsilon_{\hat{\pi},t} \tag{2}$$

$$\hat{r}_t = \phi_{\hat{r}}(\phi_{\hat{\pi}}\hat{\pi}_t + \phi_y y_t) + (1-\phi_{\hat{r}})\hat{r}_{t-1} + \varepsilon_{\hat{r},t} \tag{3}$$

where the superscript $j = \{\text{RE, BR}\}$ refers to the rational expectation (RE) and the bounded rationality (BR) model, which we describe below. The corresponding expectations operator is $\tilde{E}_t^j$, which has to be specified for both models. It goes without saying that all variables are given in quarterly magnitudes. Equation (1) describes a hybrid dynamic IS curve and results from the standard utility maximization approach of a representative household. Here the current output gap depends negatively on the real interest rate, i.e. it is stemming from intertemporal optimization of consumption and saving resulting in consumption smoothing. The parameter $\tau \geq 0$ denotes the inverse intertemporal elasticity of substitution. Equation (2) is known as the hybrid New-Keynesian Phillips Curve (NKPC) where the output gap (y_t) is the driving force of inflation due to monopolistic competition and the Calvo price-setting scheme. The slope of the Phillips Curve is given by the parameter $\kappa \geq 0$. The parameter ν denotes the discount factor ($0 < \nu < 1$). According to the Taylor rule with interest rate smoothing (3), the nominal interest gap is a predetermined variable while the monetary authority reacts directly to movements in the output ($\phi_y \geq 0$) and inflation ($\phi_{\hat{\pi}} \geq 0$) gap. We account for intrinsic persistence in this stylized version of the well-known Smets and Wouters (2003, 2005 and 2007) model due to the assumption of backward-looking behavior indicated by the parameters for habit formation χ, price indexation α and interest rate smoothing $\phi_{\hat{r}}$, respectively ($0 \leq \chi \leq 1$, $0 \leq \alpha \leq 1$, $0 \leq \phi_{\hat{r}} \leq 1$). We assume that the exogenous driving forces in the model variables follow idiosyncratic shocks $\varepsilon_{z,t}$, which are drawn from multivariate normal distributions around mean zero and variance σ_z^2 with variables $z = \{y, \hat{\pi}, \hat{r}\}$.

Note here that we consider the gaps instead of the levels and therefore account explicitly for a time-varying trend in inflation and the natural rate of interest. The corresponding gaps are simply given by taking the difference of the actual value for output, inflation and the interest rate from their trends (i.e. time-varying steady state values) respectively where the latter is computed by applying the Hodrick-Prescott filter with a standard value of the corresponding smoothing parameter of 1600. Accordingly, the set of equations is used to describe the dynamics in the output gap y_t, the inflation gap $\hat{\pi}_t$ and the nominal interest rate gap $\hat{r}_t$, where $\hat{x}_t$ with $x = \{\pi, r\}$ denotes the deviations in both variables from the time-varying trend explicitly.

The results of many studies show that assuming a constant trend, like a zero-inflation steady state, leads to misleading results. For example, Ascari and Ropele (2009) observe that the dynamic properties (i.e. mainly the stability of the system) depend on the variation in trend inflation. Cogley and Sbordone (2008) also provide evidence for the explanation of inflation persistence by considering a time-varying trend in inflation. In the same vein, we can abandon the assumption of a constant natural rate of interest as being empirically unrealistic. In this paper, we follow the empirical approaches proposed by Cogley et al. (2010), Castelnuovo (2010), Franke et al. (2011) among others, who also consider gap specifications for inflation (and the nominal interest rate). Furthermore, inflation and money growth are likely to be non-stationary in the Euro Area data. If that is the case, the estimation methodology such as the method of moments approach presented here (or the generalized method of moments in general) will lead to biased estimates.[1] Taken this into account, in the current study we consider the gaps rather than the levels in order to ensure the stationary of the times series.

To make the description of the expectation formation processes more explicit, first we examine two polar cases in the theoretical model framework of the NKM. First, under rational expectations, the forward-looking terms, which are the expectations of the output gap and inflation gap at time $t+1$ in equations (1) and (2), are just given by

$$\tilde{E}_t^{RE} y_{t+1} = E_t y_{t+1} \quad (4)$$
$$\tilde{E}_t^{RE} \hat{\pi}_{t+1} = E_t \hat{\pi}_{t+1} \quad (5)$$

where E_t denotes the expectations operator conditional on information given at time t. Second, as regards the other specification, we depart from rational expectations by considering a behaviorial model of De Grauwe (2011). It is generally assumed that agents will be either *optimists* or *pessimists* (in the following indicated by the superscripts O and P, respectively) who form expectations based on their beliefs regarding movements in the future output gap:

$$E_t^O y_{t+1} = d_t \quad (6)$$
$$E_t^P y_{t+1} = -d_t \quad (7)$$

where

$$d_t = \frac{1}{2} \cdot [\beta + \delta \sigma(y_t)] \quad (8)$$

"can be interpreted as the divergence in beliefs among agents about the output gap" (De Grauwe (2011, p. 427)). In contrast to the RE model, both types of agents are uncertain about the future dynamics of the output gap and therefore predict a fixed value of y_{t+1} denoted by $\beta \geq 0$. We can interpret the latter as the *predicted subjective mean value* of y_t. However, this kind of subjective forecast is generally biased and therefore depends on the volatility in the output gap; i.e. given by the unconditional standard deviation $\sigma(y_t) \geq 0$. In this respect, the parameter $\delta \geq 0$ measures the *degree of divergence* in the movement of economic

[1] See also Russel and Banerjee (2008) as well as Aussenmacher-Wesche and Gerlach (2008) among others for methodological issues related to non-stationary inflation in the US and the Euro Area.

activity. Note that due to the symmetry in the divergence in beliefs, optimists expect that the output gap will differ positively from the steady state value (which for consistency is set to zero) while pessimists will expect a negative deviation by the same amount. The value of δ remains the same across both types of agents.

The expression for the market forecast regarding the output gap in the bounded rationality model is given by

$$\tilde{E}_t^{BR} y_{t+1} = \alpha_{y,t}^O \cdot E_t^O y_{t+1} \; + \; \alpha_{y,t}^P \cdot E_t^P y_{t+1} = (\alpha_{y,t}^O \; - \; \alpha_{y,t}^P) \cdot d_t \tag{9}$$

where $\alpha_y^O + \alpha_y^P = 1$. A specific forecasting rule chosen by agents, i.e. (6) or (7), is indicated by the probability of $\alpha_{y,t}^O$ and $\alpha_{y,t}^P$, respectively. In particular, α_y^O (or α_y^P) can also be interpreted as the probability being an optimist (or pessimist). In the following, we show explicitly how these probabilities are computed. Indeed, the selection of the forecasting rules (6) or (7) depends on the forecast performances of optimists and pessimists U_t^k given by the mean squared forecasting error, which can be simply updated in every period as

$$U_t^k \;\; = \;\; \rho U_{t-1}^k \; - \; (1-\rho)(E_{t-1}^k y_t - y_t)^2 \tag{10}$$

where $k = O, \; P$ and the parameter ρ denotes the measure of the memory of agents $(0 \leq \rho \leq 1)$. Here $\rho = 0$ means that agents have no memory of past observations while $\rho = 1$ means that they have infinite memory instead. By applying discrete choice theory under consideration of the forecast performances, agents revise their expectations in which different performance measures will be utilized for $\alpha_{y,t}^O$ and $\alpha_{y,t}^P$:[2]

$$\alpha_{y,t}^O \;\; = \;\; \frac{\exp(\gamma U_t^O)}{\exp(\gamma U_t^O) + \exp(\gamma U_t^P)} \tag{11}$$

$$\alpha_{y,t}^P \;\; = \;\; \frac{\exp(\gamma U_t^P)}{\exp(\gamma U_t^O) + \exp(\gamma U_t^P)} = 1 - \alpha_{y,t}^O \tag{12}$$

where the parameter $\gamma \geq 0$ denotes the intensity of choice: if $\gamma = 0$, the self-selecting mechanism is purely stochastic ($\alpha_{y,t}^O = \alpha_{y,t}^P = 1/2$), whereas if $\gamma = \infty$, it is fully deterministic ($\alpha_{y,t}^O = 0, \; \alpha_{y,t}^P = 1$ or vice versa; see De Grauwe (2011), p. 429). For clarification, if $\gamma = 0$ agents are indifferent in being optimist or pessimist while if $\gamma = \infty$ their expectation formation process is independent of their emotional state, i.e. they react quite sensitively to infinitesimal changes in their forecast performances.

We explain this revision process as follows. Given the past value of the forecast performance (U_{t-1}^k), the lower the difference between the expected value of the output gap (taken from the previous period, i.e. $E_{t-1}^k y_t = |d_{t-1}|$) and its realization in period t, the higher the corresponding forecast performance U_t^k will be. In other words, if e.g. the optimists predict future movements in

[2] See also Westerhoff (2008, p. 199) and Lengnick and Wohltmann (forthcoming) among others for an application of discrete choice theory to models in finance and macroeconomics.

y_t more accurately compared to the pessimists, then this results in $U_t^O > U_t^P$. Hence, the pessimists revise their expectations by switching to the forecasting rule used by the optimists, which we can express as $E_t^O y_{t+1} = d_t$. Finally, this forecasting rule becomes dominant and the share of pessimistic group in the market decreases. Based on the equations (10) to (12), we can rationalize equation (9) by using simple substitution. This results in a higher degree of volatility in the expectation formation process regarding the output gap when compared to the outcome in the RE model (we refer to section 4.2 for a clarification).

The same logic can be applied for the inflation gap expectations. Following the behavioral heterogeneity approach proposed by De Grauwe (2011, pp. 436), we assume that agents will be either so called *inflation targeters* (tar) or *extrapolators* (ext).[3] In the former case, the central bank anchors expectations by announcing a target for the inflation gap $\bar{\hat{\pi}}$. From the view of the inflation targeters, we consider this pre-commitment strategy to be fully credible. Hence the corresponding forecasting rule becomes

$$E_t^{tar} \hat{\pi}_{t+1} = \bar{\hat{\pi}} \tag{13}$$

where we assume $\bar{\hat{\pi}} = 0$.[4] The extrapolators form their expectations in a static way and will expect that the future value of the inflation gap equals simply its past value, i.e.

$$E_t^{ext} \hat{\pi}_{t+1} = \hat{\pi}_{t-1}. \tag{14}$$

This results in the market forecast for the inflation gap similar to (9):

$$\tilde{E}_t^{BR} \hat{\pi}_{t+1} = \alpha_{\hat{\pi},t}^{tar} E_t^{tar} \hat{\pi}_{t+1} + \alpha_{\hat{\pi},t}^{ext} E_t^{ext} \hat{\pi}_{t+1} = \alpha_{\hat{\pi},t}^{tar} \bar{\hat{\pi}} + \alpha_{\hat{\pi},t}^{ext} \hat{\pi}_{t-1}. \tag{15}$$

The forecast performances of inflation targeters and extrapolators are given by the mean squared forecasting error written as

$$U_t^s = \rho U_{t-1}^s \; - \; (1-\rho)(E_{t-1}^s \hat{\pi}_t - \hat{\pi}_t)^2 \tag{16}$$

where $s = (tar, ext)$, and finally we may write:

$$\alpha_{\hat{\pi},t}^{tar} \;\; = \;\; \frac{\exp(\gamma U_t^{tar})}{\exp(\gamma U_t^{tar}) + \exp(\gamma U_t^{ext})} \tag{17}$$

$$\alpha_{\hat{\pi},t}^{ext} \;\; = \;\; \frac{\exp(\gamma U_t^{ext})}{\exp(\gamma U_t^{tar}) + \exp(\gamma U_t^{ext})} = 1 - \alpha_{\hat{\pi},t}^{tar}. \tag{18}$$

Here $\alpha_{\hat{\pi},t}^{tar}$ denotes the probability to be an inflation targeter, which is the case if the forecast performance using the announced inflation gap target is superior to the extrapolation of the inflation gap expectations and vice versa. Note here that the memory (ρ) as well as the intensive of choice parameter (γ) do

[3] This concept of behavioral heterogeneity has been widely used in financial market models, see e.g. Chiarella and He (2002) as well as Hommes (2006) among others.

[4] In this respect (based on a optimal monetary policy strategy), an inflation *gap* target of zero percent implies that the European Central Bank seeks to minimize the deviation of its (realized) target *rate* of inflation from the corresponding time-varying steady state value, where in the optimum this deviation should be zero.

not differ across the expectation formation processes in terms of the output and inflation gap. In the end, the bounded rationality model turns out to be purely backward-looking (cf. equations (10) and (16)) while the forward- and backward-looking behavior is contained in the rational expectation model. The solution to both systems can be computed by backward-induction and the method of undetermined coefficients respectively, which are shown in appendix A.

Finally, one may argue that the presented model is not suitable for e.g. policy analysis since it is not based completely on micro-foundations. In particular, the expectation mechanisms are imposed ex post on a system of structural equations which themselves have been derived from maximizing behavior under the assumption of rational expectations. However, evidence from experimental economics can help us to motivate the assumption of the divergence in beliefs (reflects guessing) and the existence of the extrapolators (which might be seen as pattern-based time-series forecasting) done by De Grauwe (2011) and adopted in our study. Roos and Schmidt (2012) find evidence for a backward-looking behavior in forming expectations by non-professionals in economic theory and policy. In their experimental study, they show that the projections of the future realizations in the output gap and inflation are based either on historical patterns of the time series or - in the case of no available information - on simple guessing.

From a theoretical point of view, Branch and McGough (2009) introduce heterogeneous expectations into a New Keynesian framework where the forward looking expressions in the IS curve and NKPC are convex combinations of backward- and forward-looking behavior. The authors show that a micro-founded NKM under bounded rationality can be derived if specific axioms are considered within the optimizing behavior of households and firms. These axioms ensure the ability of agents to forecast future realization of the output gap and inflation on the micro level as well as the aggregation of this behavior on the macro level. In comparison, De Grauwe (2010) allows for a switching mechanism based on discrete choice theory. It is an open question if the latter fulfills the axioms imposed by Branch and McGough (2009) which may help to overcome the (neglected) problem of mis-specification. To sum up, there is no doubt that an extensive elaboration on the mircofoundation of expectations formation is needed, even though up to now it is a fact that among neuroscientists the evidence on information processing in the human brain is ambiguous.

3 The Estimation Methodology

Over the last decade the Bayesian estimation became the most popular method for the estimation of DSGE models while pushing classical estimation methods aside such as the generalized method of moments and the pure maximum likelihood approach. Indeed, the Bayesian approach certainly has the advantage over the others: on the one hand, the distributions of the parameters in a system of equations framework can be easily computed from user friendly software like e.g. Dynare. On the other hand, however, there are two major disadvantages when we apply Bayesian techniques to our empirical study.

First, the Bayesian approach to the DSGE model requires the choice of appropriate prior distributions associated with the underlying economic interpretation of the structural parameters. It is still an open question what criteria are suited best in order to identify the most accurate prior information. For instance, Lombardi and Nicoletti (2011) discuss the sensitivity of posterior estimation results to the choice of different expressions of the prior knowledge; Del Negro and Schorfheide (2008) also provide an explicit method for constructing prior distributions based on the beliefs regarding macroeconomic indicators. However, so far the existing knowledge by neuroscientists does not allow for pinning down a general micro-founded model on information processing (De Grauwe (2011)). In addition, the Bayesian estimation must be designed to cope with the shape of the prior distribution, which is often unspecified, i.e. 'uninformative' priors; as a result, the estimated posterior becomes quite similar to the prior distribution. In this respect, the Bayesian analysis is not a panacea for the BR model, since prior information is not available at least for the behavioral parameters β, δ and ρ. Second, due to the fact that a logistic function is applied on the parameters of the BR model (as a result of applying the discrete choice theory), a researcher must use a Bayesian full-information analysis such as a particle filter. Especially, as long as this filter method is applied for evaluating the likelihood function, the estimation can be subjected to e.g. an increase in approximation errors of the non-linear model (DeJong and Dave (2007), Chap. 11).

To avoid these disadvantages of the Bayesian approach, in this paper we seek to match the model-generated autocovariances of the interest gap, the output gap and inflation gap with their empirical counterparts. We minimize the distance between these model-generated and empirical moments under consideration of a quadratic function, which summarizes the characteristics of empirical data. This method is called simply moment matching (cf. Franke et al. (2011)). Main advantage of this econometric method is that we can check transparently the goodness-of-fit of the model to data, since the empirical comparison (graphically) between the match of the estimated and simulated autocovariances is direct.

The method of moment approach comprises distributional properties of empirical data X_t, $t = 1, \cdots, T$. The sample covariance matrix at lag k is defined by

$$m_t(k) = \frac{1}{T} \sum_{t=1}^{T-k} (X_t - \bar{X})(X_{t+k} - \bar{X})' \tag{19}$$

where $\bar{X} = (1/T) \sum_{t=1}^{T} X_t$ is the vector of the sample mean. The sample average of discrepancy between the model-generated and the empirical moments is denoted as

$$g(\theta; X_t) \equiv \frac{1}{T} \sum_{t=1}^{T} (m_t^* - m_t) \tag{20}$$

where m_t^* is the empirical moment function and m_t the model-generated moment function (cf. equation (19)). θ is a $l \times 1$ vector of unknown structural parameters with a parameter space Θ. Given that the length of the business cycles lies between (roughly) one and eight years in the Euro Area. A reasonable compromise is a length of two years. Therefore we will use auto- and cross-covariances of the interest rate gap, the output gap and the inflation gap at a lag k, where $k = 0, \cdots, 8$. We have a p-dimensional vector of moment conditions ($p = 78$) by avoiding double counting at the zero lags in the cross relationships.[5]

We obtain the parameter estimates from the following quadratic objective function (or loss function) as a result of the minimization process:

$$Q(\theta) = \arg \min_{\theta \in \Theta} \; g(\theta; X_t)' \; \widehat{W} \; g(\theta; X_t) \tag{21}$$

with the weight matrix $\widehat{W}$ estimated consistently in several ways (see Andrews (1991)). Here we use the heteroscedasticity and autocorrelation consistent (HAC) covariance matrix estimator suggested by Newey and West (1987). The kernel estimator has the following general form with the covariance matrix of the appropriately standardized moment conditions:

$$\widehat{\Gamma}_T(j) = \frac{1}{T} \sum_{t=j+1}^{T} (m_t - \bar{m})(m_t - \bar{m})' \tag{22}$$

where $\bar{m}$ once again denotes the sample mean. Following an automatic selection for the lag length, we use a popular choice of $j \sim T^{1/3}$ leading to $j = 5$ when estimating the covariance matrix (Newey and West (1994)):

$$\widehat{\Omega}_{NW} = \widehat{\Gamma}_T(0) + \sum_{j=1}^{5} \Big(\widehat{\Gamma}_T(j) + \widehat{\Gamma}_T(j)' \Big). \tag{23}$$

The weight matrix $\widehat{W}$ is computed from the inverse of the estimated covariance matrix. However, a high correlation between the moment conditions that we consider makes the estimated covariance matrix nearly singular. In addition, the moment conditions and the elements of the weight matrix are highly correlated when the small sample size is used (Altonji and Segal (1996)). Therefore, we use the diagonal matrix entries as the weighting scheme, i.e. we ignore the off-diagonal components of the matrix $\widehat{W} = \widehat{\Omega}_{ii}^{-1}$. The estimated confidence bands, then, become wider since the sandwich elements in the covariance of parameter estimates cannot cancel out with this weighting scheme (see also Anatolyev and Gospodinov (2011)).

[5] The Delta method is used to compute the confidence bands in the auto- and cross-covariance moment estimation (see appendix B for details).

Under certain regularity conditions, one can derive the following asymptotic distribution of the method of moments estimation of the parameters:

$$\sqrt{T}(\widehat{\theta}_T - \theta_0) \sim N(0, \Lambda) \tag{24}$$

where $\Lambda = [(DWD')^{-1}]D'W\Omega WD[(DWD')^{-1}]'$, and D is the gradient vector of moment functions evaluated around the point estimates:

$$\widehat{D} = \left.\frac{\partial m(\theta; X_T)}{\partial \theta}\right|_{\theta=\widehat{\theta}_T}. \tag{25}$$

Under RE, we can obtain the simple analytic moment conditions of the model. However, for the BR model, the analytic expressions for the moment conditions are not readily available due to the non-linear discrete choice framework. To circumvent this problem, we use the simulated method of moments to estimate the behavioral parameters in the BR model. The simulated method of moments is particularly suited to a situation where the model is easily simulated by replacing theoretical moments. Then the model-generated moments in Equation (21) are replaced by their simulated counterparts:

$$m_t = \frac{1}{S \cdot T} \sum_{t=1}^{S \cdot T} \widetilde{m}_t. \tag{26}$$

We can simulate the data from the model and compute the moment conditions ($\widetilde{m}_t$) in order to approximate the theoretical moments (m_t). The simulation size is denoted by S. The asymptotic normality of the simulated method of moments holds under certain regularity conditions (Duffie and Singleton (1993), Lee and Ingram (1991)):

$$\sqrt{T}(\widehat{\theta}_{SMM} - \theta_0) \ \sim \ N(0, \Lambda_{SMM}) \tag{27}$$

where $\Lambda_{SMM} = (B'WB)^{-1}B'W\ (1 + 1/S)\ \Omega\ WB(B'WB)^{-1'}$, i.e. a covariance matrix of the SMM estimates. A gradient vector of the moment function is defined as $B \equiv E\left[\left.\frac{\partial m_t}{\partial \theta}\right|_{\theta=\widehat{\theta}}\right]$. Since the covariance matrix becomes less accurate than the estimation where the analytic moments are used, the model estimation is now subjected to simulation errors. To reduce the simulation error, we set the simulation size to a reasonably large value 100.

Finally, we use the J test to evaluate compatibilities of the moment conditions:

$$J \ \equiv \ T \cdot Q(\widehat{\theta}) \ \xrightarrow{\text{d}} \ \chi^2_{p-l} \tag{28}$$

where the J-statistic is asymptotically χ^2 distributed with $(p - l)$ degrees of freedom (over-identification).[6] A striking feature of the method of moments approach is its transparency. In particular, it is easy to check the goodness-of-fit of the model from the moment conditions of interest, i.e. the dynamic properties of the model can be tested by evaluating graphically the match of the estimated and model-generated moments.

4 Empirical Application to the Euro Area

In this section, we first present the data for our empirical application. Then we discuss our empirical results of the structural and behavioral parameters. Finally, we examine the finite sample properties of the moment-based estimator via a Monte Carlo study and investigate three-dimensional parameter space of the BR model.

4.1 Data

The data source for the New Keynesian model is the 10th update of the Area-wide Model quarterly database described in Fagan et al. (2001). The output gap and interest rate gap are computed from real GDP and nominal short-term interest rate respectively using the Hodrick-Prescott filter with a standard smoothing parameter of 1600. The inflation measure is the quarterly log-difference of the Harmonized Index of Consumer Prices (HICP) instead of the GDP deflator. The inflation gap is also computed using the Hodrick-Prescott filter.[7] The sample for this data set is available from 1970:Q1 onwards. As we use the data over five years in a rolling window analysis to estimate the perceived volatility of the output gap $\sigma(y_t)$, the data applied in this study cover the period from 1975:Q1 to 2009:Q4.

4.2 Basic results

We first estimate the RE and BR model parameters using the moment-based estimation presented in the previous section. Afterwards we make a comparison between the two models and examine the effects of divergence in beliefs on the inflation and output gap dynamics. As it is common in a persuasive amount of

[6] However, if the off-diagonal components in the estimated Newey and West matrix are discarded, the the distribution in the J-statistic is likely to have a larger dispersion than the χ^2-distribution with degrees of freedom of $p - l$. Indeed, when the weight matrix is not optimal or some moment conditions are not valid, the J-statistic is no longer χ^2 distributed. We check the validity of the weight matrix with our chosen moment conditions via a Monte Carlo study.

[7] We resort to the HICP instead of the conceptually more appropriate implicit GDP-deflator which is common in the literature, since the former is more in line with micro data evidence. For instance, Forsells and Kenny (2004) show that inflation expectations can be approximated by micro-level data like consumer surveys (i.e. in the European Commission survey indicators). Also see Ahrens and Sacht (2011, pp. 10–11) for a more detailed discussion on using the HICP instead of the GDP-deflator in macroeconomic studies.

empirical studies, the discount parameter ν is calibrated to 0.99. We also fix γ to unity, which is in line with De Grauwe (2011, p. 439) and accounts for a moderate degree in the intensity of choice.[8] By fixing those parameters in the final estimation, we can reduce problems in high-dimensional parameter space and cope with the uncertainty of the estimates. Given these assumptions, we can separately obtain the estimates for remaining parameters from the rational and bounded rationality model via the moment-based estimation. They are presented in Table 1.

Several observations are worth mentioning. The parameter estimate of the degree of price indexation α is much higher in the RE (0.765) than the BR (0.203) model. It follows that the expressions, which are in front of the forward- and backward-looking terms in the Phillips Curve, indicate a higher weight on future inflation $\tilde{E}_t^j \hat{\pi}_{t+1}$ (i.e. $\frac{\nu}{1+\alpha\nu} > \frac{\alpha}{1+\alpha\nu}$); the result is more pronounced for the BR ($0.82 > 0.18$) compared to the RE model ($0.56 > 0.43$). For the latter, this indicates that there is strong evidence for a hybrid structure of the NKPC. The empirical applications of the BR model show that the dynamics of the inflation gap are primarily driven by the expectations (i.e. the evaluation of the forecast performance) for the inflation gap if cognitive limitation of agents is assumed. This is not necessarily true under rational expectations. In other words, we find strong evidence for an autoregressive expectation formation process, since the estimated value for α is high; one group assumes a central bank inflation target of zero percent (equation (13)), while the other group of the agents form their expectations in a purely static way (equation (14)). Regarding the dynamic IS equation, the output gap is influenced by the forward- and backward-looking terms at the same proportion, since the empirical estimates show that $\chi = 1$ and $\chi = 0.950$ hold for the RE and the BR models, respectively. In particular, this degree of habit persistence suggests that past observations strongly matter for the dynamics of the output gap. Finally, the parameter estimate for the degree of interest rate smoothing shows that there is a moderate degree of persistence ($\phi_{\hat{r},t}$) in the nominal interest rate gap for both models.

Furthermore, while the empirical estimates for κ and τ in the RE model indicate a small degree of inherited persistence due to changes in the real interest rate gap and the output gap respectively, this does not hold for the BR model. Here the changes in the output gap have a strong impact ($\kappa = 0.219$) on movements in the inflation gap relative to the RE case ($\kappa = 0.035$). For the output gap, inherited persistence plays a fundamental role in shaping the dynamics of this economic indicator, which can be seen through the high values of inverse intertemporal elasticity of substitution. For the BR model, this value ($\tau = 0.387$) is much larger than the one for the RE model ($\tau = 0.079$). This implies that the tendency towards risk aversion in the BR is stronger than the RE model. To sum up, our results show that in the BR model cross-movements

[8] Goldbaum and Mizrach (2008) estimated the intensity of choice parameter in the dynamic model for mutual fund allocation decision. In our application, the system with many parameters is likely to have a likelihood with multiple peaks, some of which are located in uninteresting or implausible regions of the parameter space. By fixing the intensity of choice parameter, it makes it easier to concentrate on our objective of empirical application, i.e. the interpretation of the role of bounded rationality in the NKM.

Table 1: Estimates of the RE and BR model

Label	RE	BR
α	0.765	0.203
	(0.481 - 1.000)	(0.000 - 0.912)
χ	1.000	0.950
	-	(0.000 - 1.000)
τ	0.079	0.387
	(0.000 - 0.222)	(0.000 - 0.927)
κ	0.035	0.219
	(0.011 - 0.058)	(0.075 - 0.362)
ϕ_y	0.497	0.673
	(0.058 - 0.936)	(0.404 - 0.942)
$\phi_{\hat{\pi}}$	1.288	1.073
	(1.000 - 1.944)	(1.000 - 1.775)
$\phi_{\hat{r}}$	0.604	0.673
	(0.411 - 0.797)	(0.523 - 0.824)
σ_y	0.561	0.827
	(0.354 - 0.768)	(0.463 - 1.190)
$\sigma_{\hat{\pi}}$	0.275	0.743
	(0.097 - 0.453)	(0.449 - 1.046)
$\sigma_{\hat{r}}$	0.421	0.244
	(0.140 - 0.701)	(0.000 - 0.624)
β	-	2.221
		(0.000 - 9.747)
δ	-	0.665
		(0.000 - 7.877)
ρ	-	0.003
		(0.000 - 1.000)
J	56.30	40.30
p-value	0.8436	0.9931
5% crit. of χ^2 dist.	88.25	84.82

Note: The data cover the period spanning 1975:Q1 - 2009:Q4 (T=140 observations). The parameters ν and γ are set to 0.99 and unity, respectively. We use the rolling window of 5 years (20 observations) to compute the perceived volatility of the output gap, i.e. the unconditional standard deviation of y_t is denoted by $\sigma(y_t)$. The 95% asymptotic confidence intervals are given in brackets.

in the output and inflation gap account for persistence in both variables (under consideration of perfect habit formation $\chi = 1$) rather than price indexation alone. This can be seen through the high values of κ and τ compared to α. For the RE model, the opposite holds.

The output and inflation gap shocks, whose magnitudes are estimated to be $\sigma_y = 0.827$ and $\sigma_{\hat{\pi}} = 0.743$ respectively, are larger for the BR than those of the RE model. The results reveal that the volatilities of the output and inflation gap are strengthened by the effects of behavioral heterogeneity on the consumption and pricesetting rules. For instance, the waves of optimism and pessimism act as a persistent force in the output gap fluctuations with peaks and troughs. Figure 1 illustrates that the peak of the fluctuation in the simulated output gap (middle-left panel) corresponds to the market optimism (lower-left panel) and vice versa. The qualitative interpretation remains almost the same for the inflation gap dynamics (middle- and lower-right panel respectively) - but the dynamics of extrapolators are highly volatile reflecting the large second moment of the empirical inflation gap (upper-right panel). The goodness-of-fit of the models could not be directly compared by illustrating the simulated time series (middle-panels), but we can see that the series resemble qualitatively their empirical counterparts (upper-panels). Finally, the nominal interest rate shocks $\sigma_{\hat{r}}$ in the RE model are estimated to be roughly twice as large as in the BR model.

The remaining parameter estimates confirm the known results from the literature where the monetary policy coefficient on the output gap is low while the opposite holds for the coefficient on the inflation gap. The latter indicates that the Taylor principle holds over the whole sample period. Nevertheless, the results for the BR model indicate a stronger concern in the output gap movements relative to the dynamics in the inflation gap. Again, the opposite is true for the RE model. It is worth mentioning that the estimation results indicate a monetary policy coefficient on the output gap ϕ_y of 0.673, which is in line with the observations of De Grauwe (2011, pp. 443-445). His simulations show that flexible inflation targeting can reduce both output gap and inflation (gap) variability at a minimum level if ϕ_y lies in the range of 0.6 to 0.8.

The interpretation of this observation is two-fold. First, consider the case of strict inflation targeting, where the central bank does not account for the volatility in the output gap. As a result, the forecast performance of the optimists and pessimists are not affected since the (real) interest rate gap in the dynamic IS curve does not response directly to monetary policy. However, there is still an indirect effect (even highly volatile movements in y_t are not dampened by the policy makers) indicated by κ in the NKPC. Hence, due to the high degree of inherited persistence the strict inflation targeting can fail to control strong fluctuations in the output and inflation gap. Second, in the case of strong output gap stabilization (relative to the inflation gap) the central bank dampens its pre-commitment to an inflation target. The amplification effects of this kind of policy on the forecast performances of the inflation extrapolators will then result in higher inflation variability. We conclude that our empirical findings account for neither the first nor the second extreme case, but for a optimal flexible inflation targeting in the Euro Area over the observed time

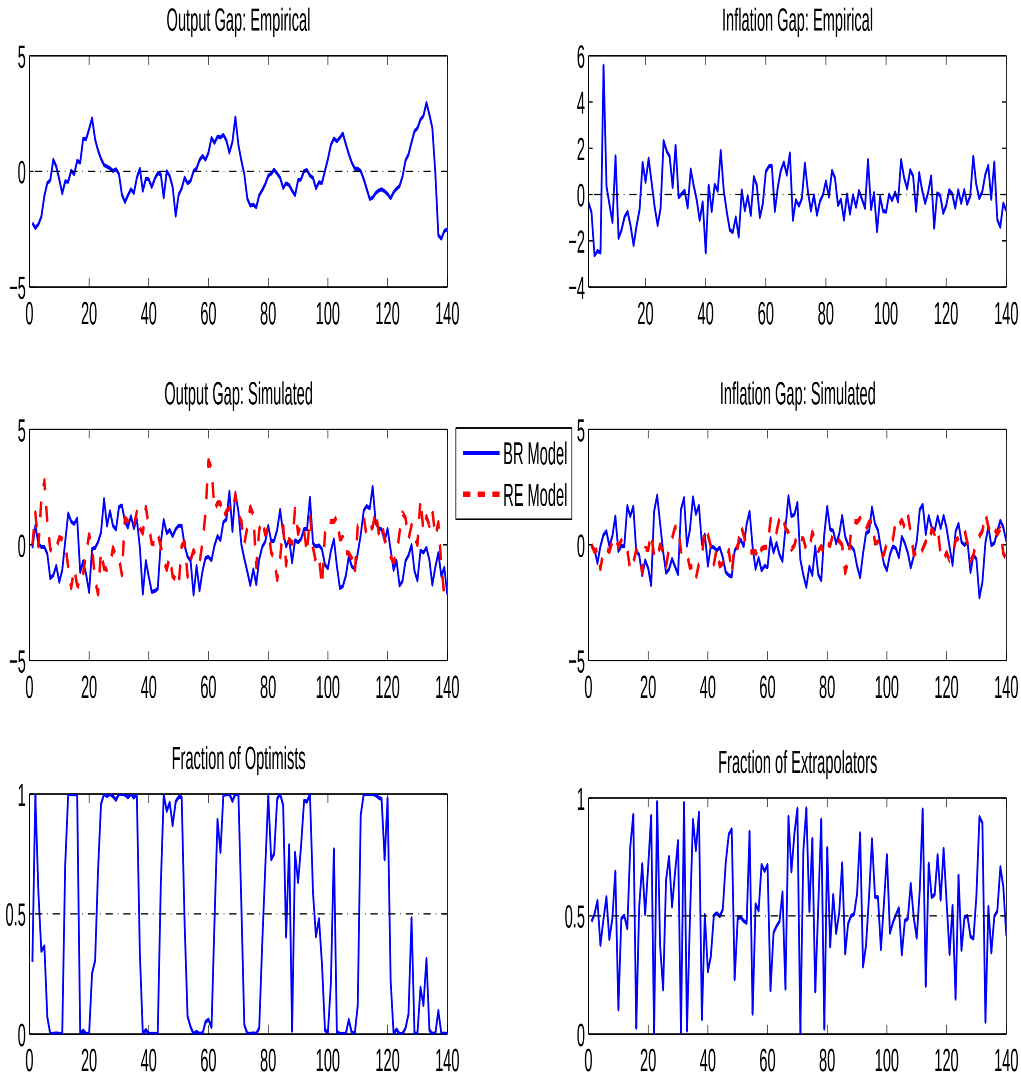

Figure 1: Dynamics in the output gap and the inflation gap.

Upper and middle panels plot empirical and simulated values for the output gap (left) and the inflation gap (right), while lower panels plot the corresponding fraction of market optimists (left) and extrapolators (right). The simulated time series are computed using the parameter estimates for both models given in Table 1.

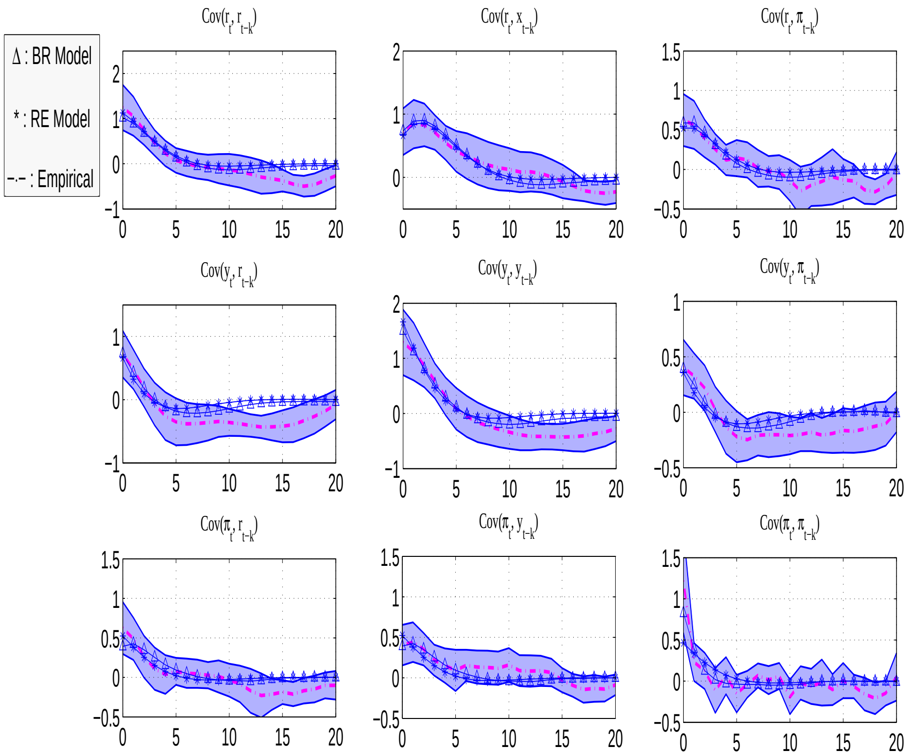

Figure 2: Model covariance (Cov) profiles in Euro area.

The dashed line results from the empirical covariance estimates. The shaded area is the 95% confidence bands around the empirical moments. The triangle (BR) and star (RE) lines indicate the model generated ones. The confidence bands are computed via the Delta method (see Appendix B).

interval instead.

As already noted, the present study focuses on the estimation of the bounded rationality parameters. First, we come to the conclusion that over the whole sample period, the optimistic agents have expected a fixed divergence of belief of $\beta = 2.221$. Roughly speaking, the optimists have been really optimistic that the future output gap will differ *positively* by slightly above one percent on average from its steady state value.[9] Due to the symmetric structure of the divergence in beliefs, over the same sample period pessimistic agents were moderately pessimistic instead, since from their point of view the future output gap was expected to be around one percent on average *below* its steady state value. Furthermore, both types of agents felt safely about their expectations due to the fact that the estimate for the variable component in the divergence of pessimistic beliefs is very low ($\delta = 0.665$) - this implies that there is a low degree of uncertainty connected to the expected future value of y_t. In line with the results for (and assumptions of) the parameters, which indicate endogenous and inherited persistence (α, χ, κ and τ), the highly subjective expected mean value of the output gap β - in conjunction with the dynamics induced by the self-selecting mechanisms (see the corresponding fractions in the lower-panels in Figure 1) - explains the high volatility of the output gap. Based on discrete choice theory, this strengthens the optimistic agents' belief about the future output gap to diverge in the data, since they can over(or under)-react to underlying shocks that occur across the Euro Area. The same observation holds for the inflation gap dynamics. The proportion of the extrapolators in the economy corresponds to the inflation gap movements (cf. lower right vs. upper-right panels in Figure 1): the higher the fraction of extrapolators is, the more volatile the inflation gap dynamics will be. Finally, ρ is estimated to be zero, i.e. past errors are not taken into account (cf. equations (10) and (16)). This leads to the conclusion that strict forgetfulness or cognitive limitation holds, which is a requirement for observing animal spirits (cf. De Grauwe (2011, p. 440)).

Indeed, visual inspection shows a fairly remarkable goodness-of-fit of the models to data (see Figure 2). The match both models achieve looks clearly good over the first few lags and still fairly good over the higher lags until the lag 8. In any case, all of the moments are now inside the confidence intervals of the empirical moments. This even holds true for some covariances up to lag 20. This is also confirmed by the values of the loss function J for the RE (56.30) and BR (40.30) model given in the last row of Table 1. The asymptotic χ^2 distributions for the J-test have the degrees of freedom of 68 and 65 for the RE and BR model, respectively. Since the critical values at 5% level are 85.25 and 84.82 respectively, and the estimated loss function values are smaller than these criteria, we do not reject the null hypothesis that these models are valid. Moreover, the picture shows a remarkable fit of the BR model, which leads to some confidence in the estimation procedure. We conclude that a bounded rationality model with cognitive limitation provides good fits for auto- and cross-covariances of the data.

[9]Note that expected future value of the output gap is given by $E^i_t y_{t+1} = |d_t| = \frac{1}{2}\beta$ *on average* with $i = \{O, P\}$.

Note here that the significant differences between two models have to be tested by a formal model comparison method, since the models do not have any difficulties to fit the empirical moments at the 5% significant interval (see also Jang (2012) among others). In other words, the J-test only evaluate the validity of the model along the lines of the chosen moment conditions. Therefore we cannot provide a direct comparison between the fits of the two models. More rigorous test will be a priority for future research.

Finally, our empirical results indicate that the empirical test of bounded rationality (viz. the assumption of the divergence in beliefs) has to be treated carefully, because all parameters (especially the behavioral ones) within the non-linear modeling approach are generally poorly determined, i.e. wide confidence bands occur. We delve into this problem by examining the finite size properties of the moment-based procedure through a Monte Carlo study and a sensitivity analysis presented in the next section. Our results from these exercises will achieve confidence in the parameter estimates given in Table 1.

4.3 Comparison with other studies

There exists a plethora of studies on the estimation of (small, medium or large) NKM with rational expectations using Euro Area data. However, to the best of our knowledge these studies are different to our contribution in several dimensions. While we apply a moment-based estimation on the Euro Area data over a specific time interval up to the end of 2009, most of the investigations are based on the generalized method of moments and Bayesian estimations using data just to the beginning of the 21st century instead. Furthermore, we consider gap specifications of $\hat{\pi}_t$ and $\hat{r}_t$ explicitly while in the literature the majority of time series are not detrended. Hence, a comparison of our results with those from the literature has to be done with some caution.

More generally, one of the representative studies in this field is the empirical application of Smets and Wouters (2003). Here the sample period captures the period from 1980:Q2 to 1999:Q4. In their paper, they apply Bayesian estimation on a medium scale model for the Euro Area. Compared to the cases of the RE and BR presented here, they found different values for the parameters τ and $\phi_{\hat{\pi}_t}$, which are estimated to be higher (0.739 and 1.684). In contrast, the estimated values for κ and ϕ_y are relatively small (0.01 and 0.10). Finally, $\phi_{\hat{r}} = 0.673$ is slightly lower than in Smets and Wouters (2003, $\phi_r = 0.956$).

Moons et al. (2007) give a good overview on the results stemming from different studies using different techniques except for the Bayesian one. Most of the parameter estimates are in line with those reported in column 1 of our Table 1, i.e. in case of the RE model. According to Table 1 in Moons et al. (2007, p. 888) τ and κ vary in a range of (0.03, 0.08) and (0.02, 0.17), while we find $\tau = 0.079$ and $\kappa = 0.035$. The results for the policy parameters $\phi_{\hat{y}} = 0.604$, $\phi_{\hat{\pi}} = 1.288$ and $\phi_{\hat{r}} = 0.497$ are slightly below the estimates reported in Moons et al. (2007) where $\phi_y = (0.77, 0.90)$, $\phi_\pi = (0.87, 2.02)$ and $\phi_r = (1, 3.2)$. For the latter, note once again that the level and not the gap of the corresponding time series is considered. The composite parameter, which indicate backward-looking behavior in the dynamic IS curve and the NKPC, can be denoted by

$\psi_1 = \frac{\chi}{1+\chi}$ and $\psi_2 = \frac{\alpha}{1+\alpha\nu}$. It can be stated that our results for the RE model, $\psi_1 = 0.5$ and $\psi_2 = 0.43$, mimic roughly those found in the literature, i.e. $\psi_1 = (0.22, 0.97)$ and $\psi_2 = (0.13, 0.54)$.

Comparing the results discussed in the previous paragraph with those presented in column 2 of Table 1, it can be seen that in the case of the BR model these results differ substantially from the those reported in the literature. Not surprisingly, this stems from the fact that the behavioral model of De Grauwe exhibits a different kind of expectation channel which can substitute the absence of rational expectations for the model dynamics. Nevertheless, Moons et al. (2007) estimate a small scale NKM of an open-economy under consideration of a fiscal policy rule (in the spirit of the European Stability and Growth Pact) with Bayesian techniques and found the parameter estimates, which are similar with our results. In particular, τ is estimated to be high (0.24) which is in line with the BR model (0.387). The authors also find that a high value of the monetary policy coefficient concerning the output gap is estimated to be $\phi_y = 0.75$, while we find a value of 0.673.

5 Robustness Checks

In this section, we report the variation of the parameter estimates under both the RE and BR model. First, we study the finite size properties of the moment-based estimation using the Monte Carlo study. The result shows that we can reduce the estimation uncertainty presented here with a large sample size. Compared to the RE model, however, the parameter estimates of the BR model have wide confidence intervals, because the non-linearity of the model gives rise to additional parameter uncertainty during the estimation. This affects the corresponding values of the bounded rationality parameters β, δ and the memory parameter ρ in the forecasting heuristics (11) and (12) as well as (17) and (18). Second, we investigate the sensitivity of these behavioral parameters in the objective function by investigating three-dimensional parameter space. We vary these parameters in a reasonable range to find the lowest value of the loss function (21).

5.1 Monte Carlo study

To analyze the finite sample properties in the macro data, we use three sampling periods in the data generating process (T=100, 200, 500). The experimental true parameters are drawn from the parameter estimates in the previous section. After 550 observations are simulated, we discard the first 50 observations to trim a transient period. In the RE model, we compute the empirical moment conditions and its Newey-West weight matrix of each artificial time series, and estimate the parameters using the method of moment estimator over 500 replications. The same procedure is carried out to estimate the parameters of the BR model. However, this makes the computation expensive for the simulated method of moment estimator. We reduce the computational cost by setting the simulation size to $S = 10$.[10]

Table 2: Monte Carlo study for the RE model

		T=100		T=200		T=500	
Label	True (θ^0)	Mean	RMSE	Mean	RMSE	Mean	RMSE
α	0.750	0.802	0.174	0.778	0.125	0.763	0.079
		S.E: 0.155		S.E: 0.112		S.E: 0.073	
χ	1.000	0.943	0.128	0.939	0.127	0.946	0.103
		S.E: 0.365		S.E: 0.293		S.E: 0.202	
τ	0.085	0.100	0.062	0.088	0.043	0.083	0.029
		S.E: 0.079		S.E: 0.061		S.E: 0.041	
κ	0.035	0.047	0.026	0.042	0.016	0.039	0.009
		S.E: 0.016		S.E: 0.011		S.E: 0.071	
ϕ_y	0.500	0.518	0.267	0.487	0.167	0.487	0.107
		S.E: 0.236		S.E: 0.162		S.E: 0.104	
$\phi_{\hat{\pi}}$	1.250	1.350	0.309	1.322	0.217	1.296	0.146
		S.E: 0.343		S.E: 0.222		S.E: 0.144	
$\phi_{\hat{r}}$	0.600	0.623	0.111	0.615	0.076	0.611	0.046
		S.E: 0.094		S.E: 0.069		S.E: 0.045	
σ_y	0.600	0.632	0.127	0.627	0.090	0.623	0.059
		S.E: 0.125		S.E: 0.095		S.E: 0.062	
$\sigma_{\hat{\pi}}$	0.275	0.249	0.075	0.263	0.049	0.270	0.030
		S.E: 0.062		S.E: 0.046		S.E: 0.031	
$\sigma_{\hat{r}}$	0.400	0.234	0.240	0.289	0.181	0.345	0.105
		S.E: 7.487		S.E: 1.456		S.E: 1.026	
J		30.58		24.12		20.10	
# of rejections		4		6		0	

Note: ν is set to the value of 0.99. The reported statistics are based on 500 replications. RMSE is the root mean square error. S.E denotes the mean of standard error.

Table 2 summarizes the results from the MC experiment for the RE model. We report the mean, the root mean square error (RMSE) and the standard error (S.E). The true values of the parameters are stated in the second column. The results show that the method of moment estimation of the RE model has good finite sample properties; see the RMSE sensitivity to variations in sample size. The large sample size remarkably improves the asymptotic efficiency of the method of moments estimator, since the mean of standard error for the estimates becomes the smallest. However, the estimated value for the policy

[10] The implementation of the MC study on the model with a large simulation size (i.e. S=100) does not have a drastic change in parameter estimates; see appendix C. The approximation error rates of analytic moments are 10% and 1% for the simulation sizes $S = 10$ and 100, respectively. The computation becomes expensive when the large simulation size is used.

shock parameter σ_r often hit the boundary (i.e. $\sigma_r = 0.0$) and makes the numerical derivative of the moment conditions unstable. This leads to the large asymptotic error for this parameter.[11]

The J-statistic is used to evaluate the validity of the two models when fitting the artificial data. On average, the null hypothesis that the model is the true one is not rejected according to the over-identification test for both the RE and BR model; e.g. for the sample size of T=100, the J test rejects the validity of the RE and BR model for 4 and 16 times, respectively. The number of rejection is very small, since the simulated replications are 500. And we do not find any rejection of both models when a large sample size is used (T=500). In addition, it can be seen from the J test that the BR model fits the data slightly better than the RE model on average. Nevertheless, the direct diagnostic comparison between two models must be made with caution, because the BR model has more parameters than the RE model, i.e. their χ^2-distributions are different.

In comparison with the results of the RE model, we found that the simulated method of moments regarding the BR model has more or less poor finite sample properties when inspecting the parameters α, τ, β, and δ (see Table 3). However, the large uncertainty for the parameter estimates can be mitigated by more observations in the data. On the other side, note here that we can consistently recover the true values for the other parameter estimates. Put differently, the parameter estimates almost converge to the true ones as the sample size increases (i.e. T=500). In this case the RMSE gets smaller. The large sample allows us to make more accurate inference about the group behavior in the market expectation formation processes. Indeed, as market behavior is unobservable in most cases, we need a large sample size to consistently estimate the behavioral parameters. Nevertheless, the estimated results for the behavioral parameters can be seen as confident starting values used for calibration exercises like for e.g. (optimal) monetary and fiscal policy analysis.

5.2 Sensitivity of the behavioral parameters

In this sensitivity analysis we investigate the region of the objective function with respect to different values of β, δ and ρ. The findings from the MC study indicate that the RMSE values for these behavioral parameters in the discrete choice theory are higher than those for the other structural parameters even for a large sample size. We discuss the poor finite sample properties of these crucial parameters in the BR model by evaluating the loss function under consideration of different pairs for β, δ and ρ. The remaining parameters are fixed on their estimated values taken from the second column of Table 1. It is our aim to pin down those values from the parameter space, which are associated with the lowest value of the loss function.

[11] Note here that we use the optimization tool (Matlab version R2010a) with the *fmincon* solver. Especially the interior-point algorithm has a number of advantages over other algorithms (i.e., active-set, trust-region-reflective, and sqp). For example, the implementation of the interior-point algorithm for large-scale linear programming is considerably simpler than for the other algorithms. Furthermore, it can handle nonlinear non-convex optimization problems of the BR model.

Table 3: Monte Carlo study for the BR model

Label	True (θ^0)	T=100		T=200		T=500	
		Mean	RMSE	Mean	RMSE	Mean	RMSE
α	0.200	0.326	0.286	0.383	0.278	0.285	0.187
		S.E: 0.312		S.E: 0.232		S.E: 0.142	
χ	1.000	0.666	0.724	0.798	0.679	0.850	0.655
		S.E: 1.802		S.E: 1.614		S.E: 1.470	
τ	0.385	1.075	0.837	0.620	0.370	0.550	0.292
		S.E: 0.810		S.E: 0.341		S.E: 0.216	
κ	0.215	0.246	0.153	0.223	0.142	0.225	0.139
		S.E: 0.076		S.E: 0.051		S.E: 0.035	
ϕ_y	0.675	0.757	0.458	0.697	0.435	0.694	0.430
		S.E: 0.248		S.E: 0.116		S.E: 0.065	
$\phi_{\hat{\pi}}$	1.100	1.090	0.703	1.069	0.699	1.089	0.699
		S.E: 0.326		S.E: 0.174		S.E: 0.106	
$\phi_{\hat{r}}$	0.670	0.681	0.427	0.675	0.425	0.681	0.424
		S.E: 0.073		S.E: 0.046		S.E: 0.028	
σ_y	0.825	0.872	0.549	0.888	0.533	0.874	0.527
		S.E: 0.290		S.E: 0.182		S.E: 0.133	
$\sigma_{\hat{\pi}}$	0.740	0.606	0.496	0.647	0.477	0.699	0.470
		S.E: 0.090		S.E: 0.053		S.E: 0.034	
$\sigma_{\hat{r}}$	0.240	0.165	0.182	0.180	0.176	0.165	0.180
		S.E: 0.169		S.E: 0.140		S.E: 0.113	
β	2.250	2.831	1.867	2.440	1.608	2.330	1.543
		S.E: 5.638		S.E: 4.149		S.E: 3.670	
δ	0.650	1.293	1.021	0.925	0.750	0.862	0.663
		S.E: 4.000		S.E: 3.596		S.E: 3.223	
ρ	0.000	0.213	0.218	0.104	0.134	0.093	0.122
		S.E: 0.422		S.E: 0.386		S.E: 0.335	
J		29.34		22.30		20.74	
# of rejections		16		1		0	

Note: ν is set to the value of 0.99. The reported statistics are based on 500 replications. RMSE is the root mean square error. S.E denotes the mean of standard error.

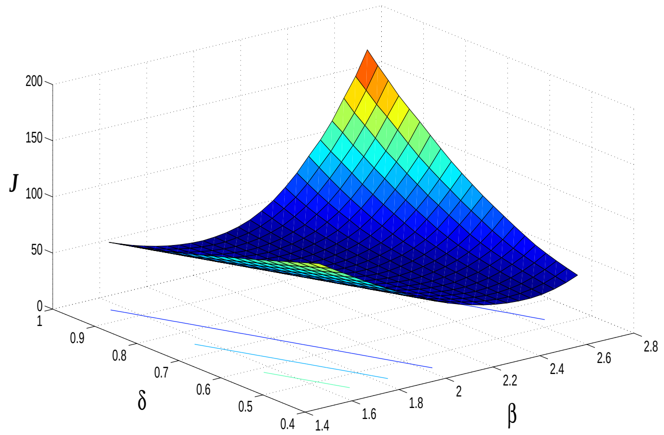

Figure 3: 3-D contour plot of the parameter space with β and δ

Note: The value of the quadratic objective function J is given on the vertical axis.

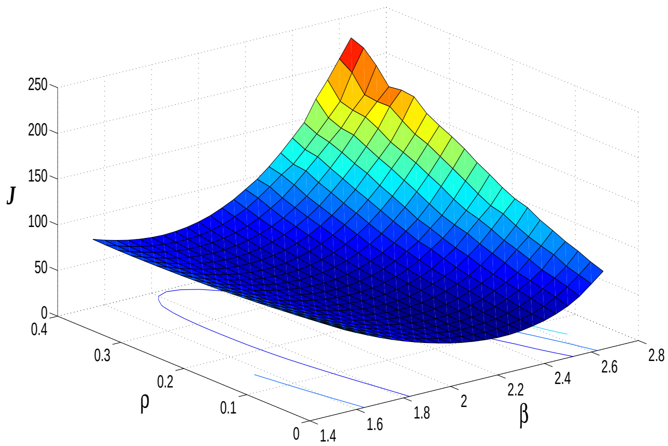

Figure 4: 3-D contour plot of the parameter space with β and ρ

Note: The value of the quadratic objective function J is given on the vertical axis.

Figures 3 to 5 illustrate three contour plots, from which we can examine the region of the loss function J under consideration of the pairwise variation in all three parameters over a reasonable range. We see from Figure 3 that the minimum value of the loss function is centered around $(\delta, \beta) = (0.6, 2.2)$. This observation is in line with our results given in Table 1, and indicates that applying the method of moment approach leads to consistent parameter estimates. However, our result emphasizes that the shape of the contour plot is moderately flat for specific combinations of δ and β, i.e. which still indicates the existence of wide confidence bands. Note that the value of the loss function increases dramatically if δ and β deviate strongly from their estimated values. In this case a trade-off arises: a highly predicted subjective mean value β requires a low degree of divergence δ in order to ensure a minimum value of J.

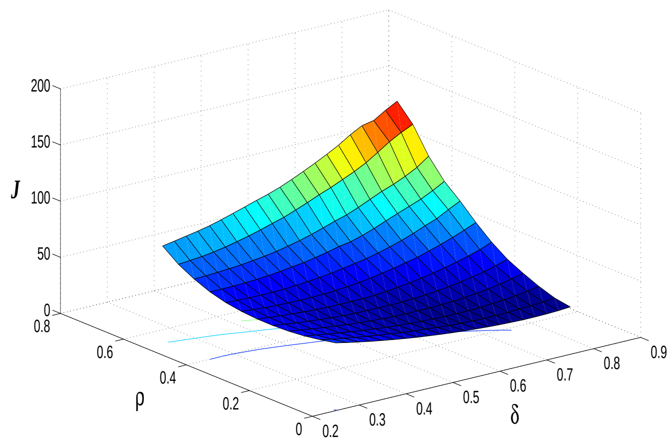

Figure 5: 3-D contour plot of the parameter space with δ and ρ

Note: The value of the quadratic objective function J is given on the vertical axis.

Figure 4 and 5 show that the minimum of the loss function is given by a value of the memory parameter ρ equal to zero in conjuncture with the estimated values of β and δ around 2.2 and 0.6, respectively. This result confirms the estimate of ρ given in Table 1 and strengthens our argumentation in section 4 since strict forgetfulness holds as a requirement for observing animal spirits.

In sum, this simulation results show that for a small sample size, the results from a MC study and a sensitive analysis confirm the absence of statistical accuracy of these behavioral parameters (i.e. the case of wide confidence bands) when applying the method of moment approach.

6 Conclusion

In this paper, we attempt to provide empirical evidence for the behavioral assumption in the model of De Grauwe (2011). The validity of the model assumption on the cognitive limitation (e.g. because of different individual emotional states) is empirically tested using historical Euro Area data. We attempt to identify the so-called behavioral parameters, which account for animal spirits in the Euro Area; i.e. we hypothesize that historical movements of macro dynamics are influenced by waves of optimism and pessimism.

To examine the effects of the group behavior on the output and inflation gap, we follow the behavioral approach of De Grauwe (2011), who assumes divergence in beliefs about the future value of both variables. The corresponding decision rules for market optimism and pessimism are given by the forecast performance of the agents from the discrete choice theory. To see this, we contrast a standard hybrid version of the three-equations New-Keynesian model of rational expectations with a version of the same model where we assume bounded rationality in expectation formation processes using the moment-based estimation.

Our main empirical findings show that a bounded rationality model with cognitive limitation provides a reasonable fit to auto- and cross-covariances of the Euro Area data. Therefore our empirical results of the BR model offer some new insights into expectation formation processes for the Euro Area. First, over the whole time interval the agents had expected moderate deviations of the output gap from its steady state value with low uncertainty. Second, in the absence of rational behavior we find strong evidence for an autoregressive expectation formation process regarding the inflation gap. Both observations explain a high degree of persistence in the output gap and the inflation gap. Based on the discrete choice theory and the self-selection process of the agents, we found that animal spirits strengthen the optimistic's belief about the future output gap to diverge in the historical Euro Area data.

To the best of our knowledge, such kind of empirical studies have not been extensively investigated before in the literature. However, the empirical test of bounded rationality (viz. the assumption of the divergence in beliefs) has to be treated carefully, because the parameters (especially the behavioral ones) within the non-linear modeling approach are poorly determined, i.e. wide confidence bands occur. We delve into this problem by examining the finite size properties of the moment-based procedure through a Monte Carlo study and a sensitivity analysis. In the end, we provide empirical evidence in support of De Grauwe (2011, fn. 4) for understanding the group's over- and under-reaction to the economy. In order to identify the effects of individual expectation formation processes on the economy, in further research, the decision rules i.e. the transition rules from one state of the economy to another can be calculated based on survey data (for example see Lux (2009)). Thus these probabilities are then treated as exogenous and (in contrast to the De Grauwe model) are computed under consideration of the underlying time series using the discrete choice theory. Finally and only if the estimation of small-scale models is considered to be satisfactory, one can further continue the model estimation with much richer models like e.g. the medium-scale version developed by the Smets and Wouters (2005, 2007). We leave these issues to future research.

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Appendix

A: Solution of the NKM

In general, all model specifications are described by the following system in canonical form:

$$AX_t + BX_{t-1} + CX_{t+1} + \varepsilon_t = 0 \tag{29}$$

where

$$X_t = \begin{pmatrix} y_t \\ \hat{\pi}_t \\ \hat{r}_t \end{pmatrix}, \; X_{t-1} = \begin{pmatrix} y_{t-1} \\ \hat{\pi}_{t-1} \\ \hat{r}_{t-1} \end{pmatrix}, \; X_{t+1} \;\; = \;\; \begin{pmatrix} \tilde{E}_t^j y_{t+1} \\ \tilde{E}_t^j \hat{\pi}_{t+1} \\ \tilde{E}_t^j \hat{r}_{t+1} \end{pmatrix}, \; \varepsilon_t = \begin{pmatrix} \varepsilon_{y,t} \\ \varepsilon_{\hat{\pi},t} \\ \varepsilon_{\hat{r},t} \end{pmatrix}.$$

The corresponding system matrices are given by:

$$A = \begin{pmatrix} 1 & 0 & \tau \\ -\lambda & 1 & 0 \\ -\phi_{\hat{r}}\phi_y & -\phi_{\hat{r}}\phi_\pi & 1 \end{pmatrix}, \; B = \begin{pmatrix} -\frac{\chi}{1+\chi} & 0 & 0 \\ 0 & -\frac{\alpha}{1+\alpha\nu} & 0 \\ 0 & 0 & -(1-\phi_{\hat{r}}) \end{pmatrix} \tag{30}$$

and

$$C = \begin{pmatrix} -\frac{1}{1+\chi} & -\tau & 0 \\ 0 & -\frac{\nu}{1+\alpha\nu} & 0 \\ 0 & 0 & 0 \end{pmatrix}. \tag{31}$$

Recall that for the rational expectations model we assume

$$\begin{aligned} \tilde{E}_t^{RE} y_{t+1} &= E_t y_{t+1} \\ \tilde{E}_t^{RE} \hat{\pi}_{t+1} &= E_t \hat{\pi}_{t+1} \end{aligned}$$

and for the bounded rationality model we assume

$$\begin{aligned} \tilde{E}_t^{BR} y_{t+1} &= (\alpha_{y,t}^O - \alpha_{y,t}^P) d_t \\ \tilde{E}_t^{BR} \hat{\pi}_{t+1} &= \alpha_{\hat{\pi},t}^{tar} \bar{\hat{\pi}} + \alpha_{\hat{\pi},t}^{ext} \hat{\pi}_{t-1} \end{aligned}$$

where we also consider equations (10) to (18) with $\bar{\hat{\pi}} = 0$. In the following, we solve for the dynamics of the system (29). In case of the BR model, the solution is given by

$$X_t = -A^{-1}[BX_{t-1} + CX_{t+1} + \varepsilon_t] \tag{32}$$

where the matrix A is of full rank, i.e. its determinant is not equal to zero, given the parameter estimates in section 4. Under consideration of the heuristics for the forecasts regarding the output and inflation gap expectations, the forward looking term X_{t+1} is substituted by the equivalent expressions for the discrete choice mechanism given in section 2. It follows that the model becomes purely backward-looking and thus (32) can be solved by backward-induction.

In contrast, the RE model is both backward- and forward-looking. Therefore we apply the method of undetermined coefficients in order to solve the model. The law of motion which describes the analytical solution is given by

$$X_t = \Omega X_{t-1} + \Phi \varepsilon_t \tag{33}$$

where $\Omega \in \mathbb{R}^{3\times 3}$ and $\Phi \in \mathbb{R}^{3\times 3}$ are the solution matrices. The former is a stable matrix as long as (similar to the matrix A in the BR case) its determinant is not equal to zero, which ensures the invertibility of Ω. Again, this is confirmed given the estimation results in section 4. We substitute (33) into (29) which yields

$$A(\Omega X_{t-1} + \Phi\varepsilon_t) + BX_{t-1} + C(\Omega X_t + \Phi E_t\varepsilon_{t+1}) + \varepsilon_t = 0.$$

This is equivalent to

$$A(\Omega X_{t-1} + \Phi\varepsilon_t) + BX_{t-1} + C(\Omega^2 X_{t-1} + \Omega\Phi\varepsilon_t + \Phi E_t\varepsilon_{t+1}) + \varepsilon_t = 0.$$

Hence, the reduced form can be rewritten as

$$(C\Omega^2 + A\Omega + B)X_{t-1} + (A\Phi + C\Omega\Phi + I)\varepsilon_t = 0 \tag{34}$$

with I being the identity matrix. Note that $\varepsilon_t \sim N(0, \sigma_z^2)$ with $z = \{y, \hat{\pi}, \hat{r}\}$ and thus $E_t\varepsilon_{t+1} = 0$. In order to solve equation (34), all the terms in brackets must be zero.[12] Thus the solution matrices can be uniquely determined. We may write that as

$$C\Omega^2 + A\Omega + B = 0 \Rightarrow \Omega = -(C\Omega + A)^{-1}B. \tag{35}$$

In order to solve the quadratic matrix equation (35) numerically, we employ the brute force iteration procedure mentioned in Binder and Pesaran (1995, p. 155, fn 26). Hence an equivalent recursive relation of (35) is given by

$$\Omega_n = -(C\Omega_{n-1} + A)^{-1}B \tag{36}$$

with an arbitrary number of iteration steps N where $n = \{1, 2, ..., N\}$. We define $\Omega_0 = \xi I$ with $0 \leq \xi \leq 1$. The iteration process (36) proceeds until $||\Omega_n - \Omega_{n-1}|| < \xi$ where ξ is an arbitrarily small number. Given the solution of Ω, the computation of Φ is straightforward:

$$A\Phi + C\Omega_n\Phi + I = 0 \Rightarrow \Phi = -(A + C\Omega_n)^{-1}. \tag{37}$$

B: Delta Method and Confidence Interval for Auto- and Cross-covariances

The Delta method is a common technique for providing the first-order approximations to the variance of a transformed parameter; see chapter 5 of Davidson and Mackinnon (2004) among others. In the study, we use the Delta

[12] Obviously the trivial solution $X_{t-1} = \Gamma_t = \varepsilon_t = 0$ is discarded.

method when computing the standard errors of the estimated auto- and cross-covariances of the data. The covariance is defined as follows:

$$\gamma_{ij}(h) = E[(X_{i,t} - \mu_i)(X_{j,t+h} - \mu_j)'], \quad t = 1, \cdots, T \tag{38}$$

where γ_{ij} is the auto-covariance function when $i = j$. Otherwise γ_{ij} denotes the cross-covariance between $X_{i,t}$ and $X_{j,t+h}$. h denotes the lag in data and μ_i(or μ_j) is the sample mean of the variable X_i(or X_j). The covariance function in Equation (38) proceeds with a simple multiplication:

$$\gamma_{ij}(h) = E[X_{i,t} \cdot X'_{j,t+h}] - \mu_i \cdot E[X'_{j,t+h}] = \mu_{ij} - \mu_i \cdot \mu_j$$

where μ_{ij} denotes $E[X_{i,t} \cdot X'_{j,t+h}]$. Now we see that $\gamma_{ij}(h)$ is a transformed function of the population moments μ_i, μ_j and μ_{ij}. Denote the vector μ as the collection of the moments: $\mu = [\mu_i \quad \mu_j \quad \mu_{ij}]$. We differentiate the covariance function with respect to the vector μ:

$$D = \frac{\partial \gamma_{ij}(h)}{\partial \mu} = \begin{bmatrix} \frac{\partial \gamma_{ij}(h)}{\partial \mu_i} \\ \frac{\partial \gamma_{ij}(h)}{\partial \mu_j} \\ \frac{\partial \gamma_{ij}(h)}{\partial \mu_{ij}} \end{bmatrix} = \begin{bmatrix} -\mu_j \\ -\mu_i \\ 1 \end{bmatrix} \tag{39}$$

Therefore the Delta method provides the asymptotic distribution of the estimate $\widehat{\gamma}_{ij}$ by matching the sample moments of the data.

$$\sqrt{T}(\gamma_{ij} - \widehat{\gamma}_{ij}) \;\sim\; N(0, D'SD). \tag{40}$$

For some suitable lag length q, we use a common HAC estimator of Newey and West (1994) when estimating the covariance matrix of sample moments. Specifically, we follow the advice in Davidson and MacKinnon (2004, p.364) and scale q with $T^{1/3}$. Accordingly we may set $q = 5$ for the Euro area data.

$$\widehat{\Sigma}_\mu = \widehat{C}(0) + \sum_{k=1}^{q} \Big(1 - \frac{k}{q+1}\Big)[\widehat{C}(k) + \widehat{C}(k)'] \tag{41}$$

$$\widehat{C}(k) = \frac{1}{T} \sum_{t=k+1}^{T} [f(z_t) - \widehat{\mu}][f(z_{t-h}) - \widehat{\mu}]'$$

where $f(z_t) = [X_i, \; X_j, \; X_i \cdot X_j]$. We use the optimal weight matrix $S = \widehat{\Sigma}_\mu^{-1}$ in estimating the covariance matrix of moments. Let s_γ be $\sqrt{D'SD}$. Then the 95% asymptotic confidence intervals for auto- and cross-covariance estimates become:

$$[\gamma_{ij} - 1.96 \cdot s_\gamma, \quad \gamma_{ij} + 1.96 \cdot s_\gamma]. \tag{42}$$

C: Large-scale Simulation Study for the BR Model

We report the results of a simulation study for the BR model when a large simulation size is used; S=100. At present, we see that the model estimates using a large simulation size have slightly smaller values for the RMSEs than ones from a small simulation size in the section 4.3.

Table 4: Monte Carlo Study for the BR Model

		T=100	
Label	True (θ^0)	Mean	RMSE
α	0.200	0.249	0.262
		S.E: 0.314	
χ	1.000	0.693	0.716
		S.E: 2.653	
τ	0.385	0.884	0.818
		S.E: 0.699	
κ	0.215	0.236	0.154
		S.E: 0.0800	
ϕ_y	0.675	0.728	0.454
		S.E: 0.181	
$\phi_{\hat{\pi}}$	1.100	1.105	0.701
		S.E: 0.701	
$\phi_{\hat{r}}$	0.670	0.677	0.427
		S.E: 0.066	
σ_y	0.825	0.913	0.561
		S.E: 0.302	
$\sigma_{\hat{\pi}}$	0.740	0.689	0.479
		S.E: 0.081	
$\sigma_{\hat{r}}$	0.240	0.165	0.198
		S.E: 0.246	
β	2.250	2.585	1.812
		S.E: 9.275	
δ	0.650	1.139	0.992
		S.E: 7.656	
ρ	0.000	0.224	0.239
		S.E: 0.547	
J		28.53	
# of rejections		9	

Note: ν is set to the value of 0.99. The reported statistics are based on 500 replications. RMSE is the root mean square error. S.E denotes the mean of standard error. Since the simulation studies become computationally expensive with a large sample size, we only report the case of T = 100.